Bias in Chisq Estimation

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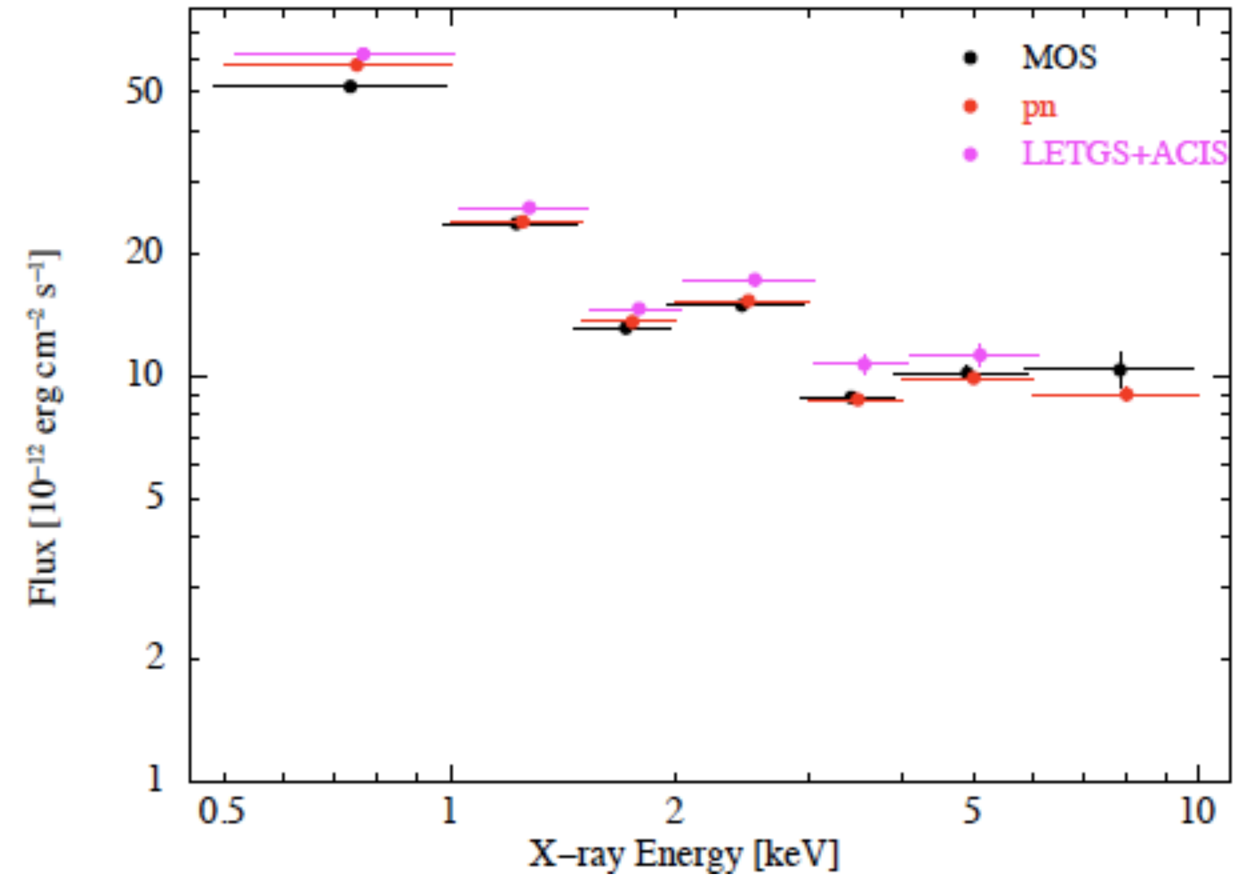
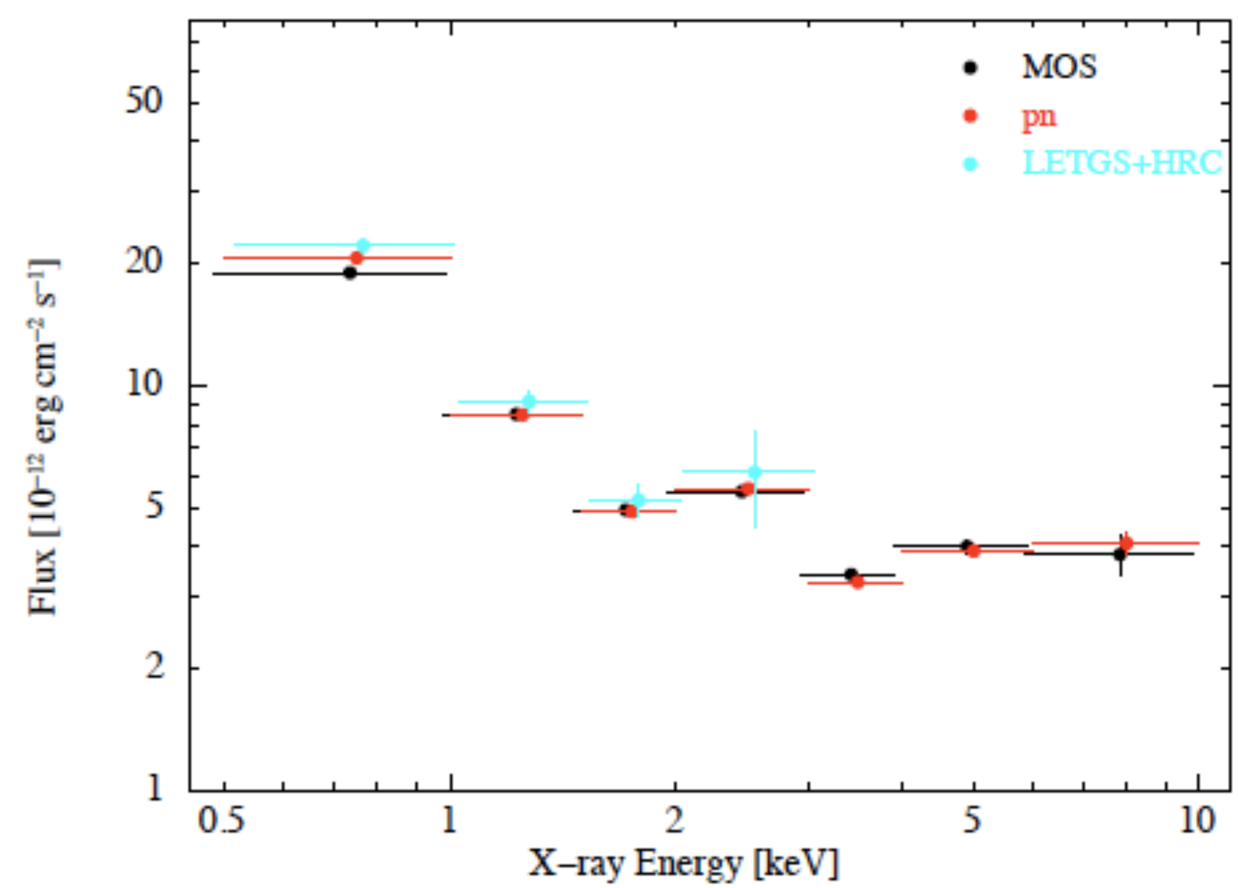
Fitting Power Laws in Narrow Energy Ranges

- Objective: Coarse characterization of systematic errors
- Method:
  - Define narrow energy bands
  - Fit power law to spectrum in each band
  - Compute flux in each band using model
  - Compute confidence interval for each flux
  - Compare fluxes for different instruments
- Claim: flux is robust to error in model
- Shortcut for grating spectra: straight sums

Cross-calibration with PKS 2155

- Ishida et al (2011)
- Direct result of IACHEC
- Joint Suzaku, XMM, & Chandra
- Each combination examined
- Overall fits to power law
- Fluxes in bands (by PL fits)
- No conclusion yet....





Application to HETGS

- Cross-check results with direct method
- Data = $\{C_i, E_i\}$, measured in time t
- Effective area = A_i
- Estimator:

$$F(E_{\rm min}, E_{\rm max}) = \sum_{E_{\rm min}}^{E_{\rm max}} \frac{C_i E_i}{t A_i}$$

- Is this the best estimator?
- Is it biased?

Estimation Methods

- Consider simple situation (Case 1)
  - Source has invariant photon flux n
  - Observe twice with effective area A
  - Exposure times are t_1, t_2, counts C_1, C_2
- One estimate of n (χ^2):

$$n = \frac{n_1/\sigma_1^2 + n_2/\sigma_2^2}{1/\sigma_1^2 + 1/\sigma_2^2}, n_1 = \frac{C_1}{At_1}, n_2 = \frac{C_2}{At_2}, \sigma_1 = \frac{\sqrt{C_1}}{At_1}, \sigma_2 = \frac{\sqrt{C_2}}{At_2}$$

- Maximum Likelihood (Poisson) estimate of n:

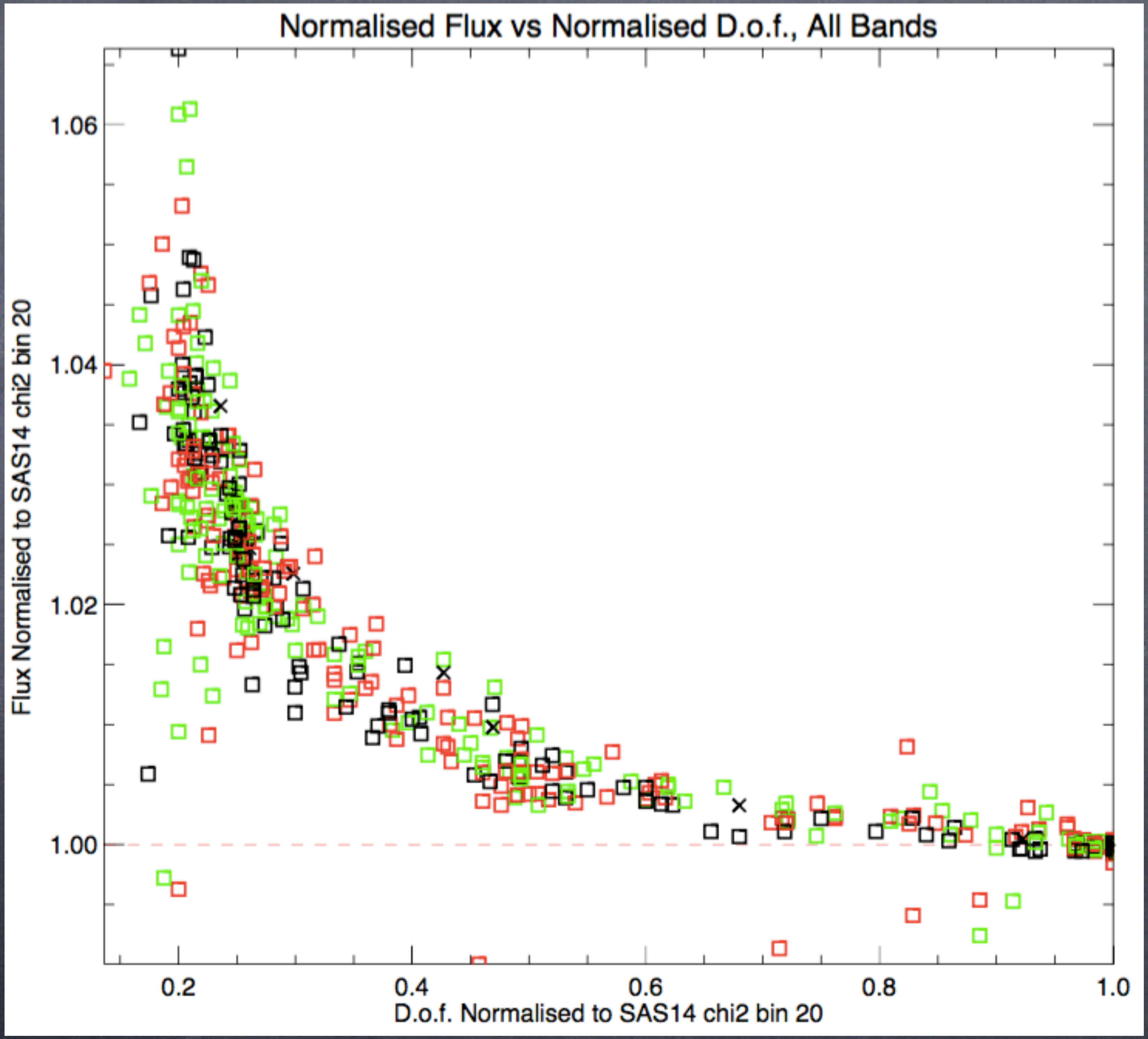
$$n = \frac{C_1+C_2}{A(t_1+t_2)}$$

Chisq v. ML

- Bevington (p. 248)
  - model: $y = \alpha e^{-\beta x^2} + \gamma$; data: $(y_i, \sigma_i), y \sim P(y[x_i])$
  - Fit using χ^2 stat giving (α', β', γ')
  - Define $A = \Sigma y_i, A' = \Sigma y[x_i; \alpha', \beta', \gamma']$
  - Then using χ^2 stat gives $A' = A - \chi^2_{\min}$
  - If $\sigma_i = \sigma$, then A′ = A (but unexplained)
- ML treatment gives A = A′
  - Simple case: y ~ P(α), M equal bins: α' = N/M
    - Fit using χ^2: 1-A′/A = χ^2_{min}/N ≈ (M-1)/N ≈ 1/(SNR)^2
    - Fit using χ^2, $\sigma_i = \alpha$: A′ = A+χ^2_{min}/2
  - Also true for y ~ P(η_i α), with known η_i

Cross-Cal Case

- Example from fitting XMM spectra in bands



Simple Cases

- Case 2: two observations, different areas and exposures:

$$n = \frac{C_1+C_2}{A_1 t_1 + A_2 t_2}$$

- Case 3: estimate narrow band energy flux (two observations, same band)

$$F = E\frac{C_1 + C_2}{A_1 t_1 + A_2 t_2}$$

Extending Chisq v. ML

- Case 4, analogous to counts in HETGS
  - Model: $y = \omega_i \mu_i F, \Sigma\mu_i = 1$
    - μ_i = unknown fractional flux in bin i (at energy E_i) of M
    - $\omega_i = TA_i/E_i$ = known flux/count scaling, total count is $N = \Sigma C_i$
  - ML: $F' = N/\Sigma\omega_i\mu_i, \mu_i' = C_i/(F'\omega_i)$
    - using $\Sigma\mu_i = 1$, then $F' = \Sigma C_i/\omega_i = \Sigma C_i E_i/(TA_i)$
    - flux is sum of flux estimates in each bin
    - Uncertainty: $\sigma_F = F/\sqrt{N}$
  - χ^2: Same answer!
    - M unknowns (F,μ_i), N_{DoF} = 0 –> $\chi^2_{\min} = 0$

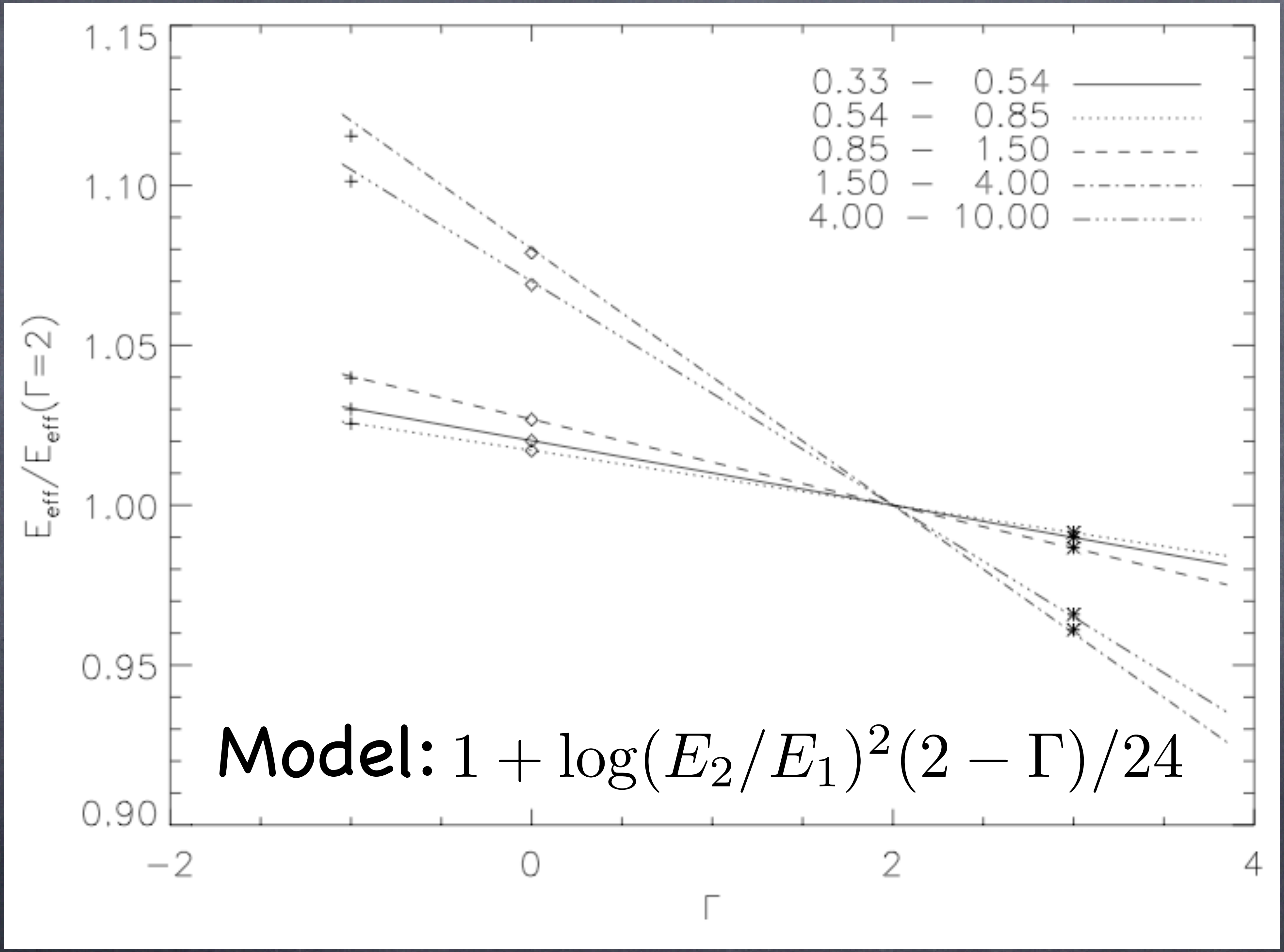
The Case of Interest

- PL spectral model, want broad-band flux
  - known $\Gamma, n(E) = K(E/\hat{E})^{-\Gamma}$
  - data: counts in equal bandpasses of size ΔE

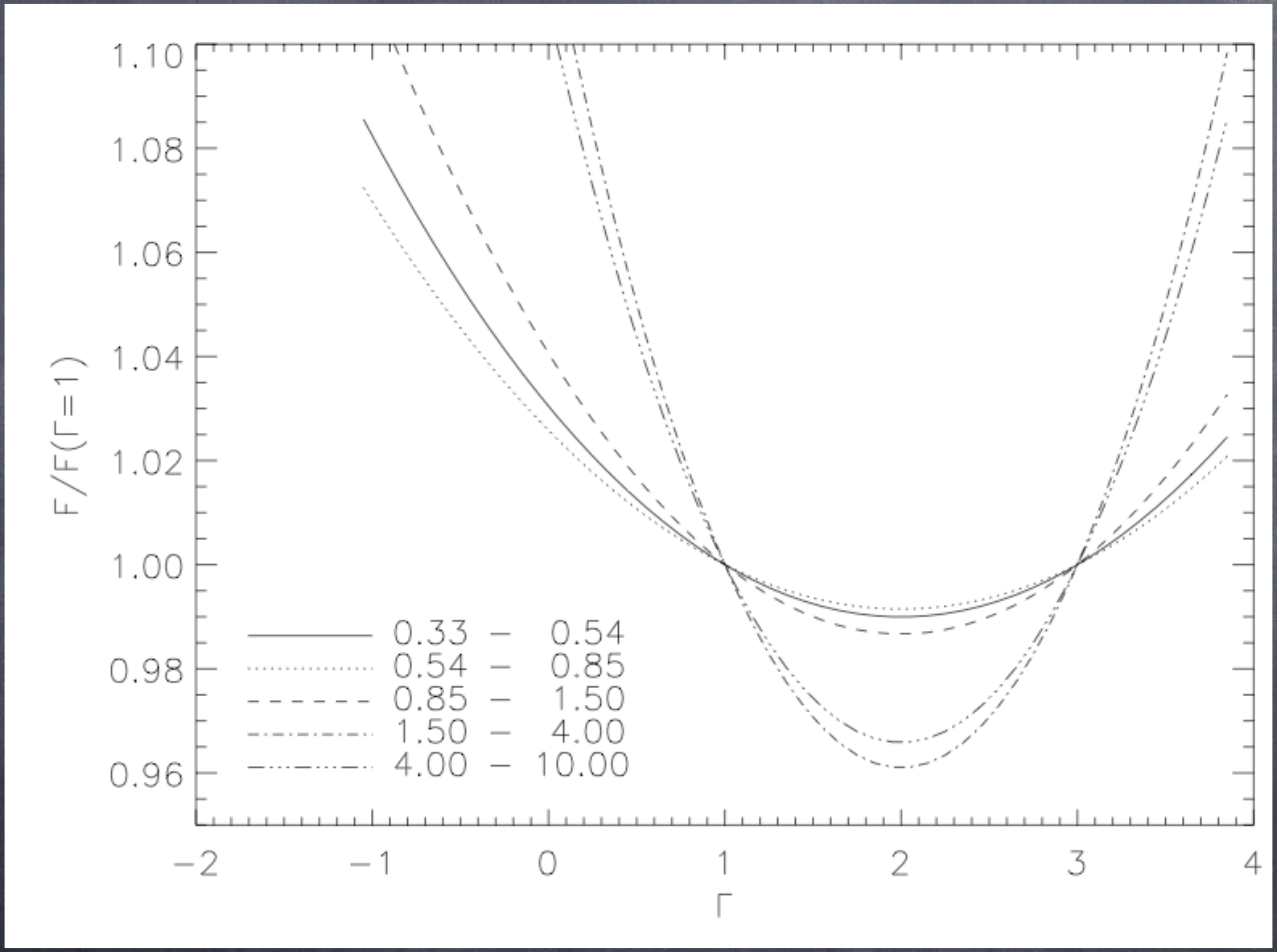
$$K = \frac{\Sigma C_i}{\Delta E\ \Sigma A_i t_i (E_i/\hat{E})^{-\Gamma}}, F = \frac{K\hat{E}^2}{2-\Gamma}[(E_{\max}/\hat{E})^{2-\Gamma} - (E_{\min}/\hat{E})^{2-\Gamma}]$$

- χ^2: fractional error in F = χ^2_{min}/N ≈ (M-1)/N
- Set reference energy to $\hat{E} = \log E_{\max}/E_{\min}$
  - How does $\hat{E}$ depend on assumed Γ (=2)?
- What's the error in F if assumed Γ is wrong?

Central Energy



Flux Sensitivity



Summary

- Chisq fits: systematically low flux estimates
  - Fractional flux bias is ~ 1/(cnt/bin)
  - Applies to fluxes in lines as well
    - emission lines: underestimated
    - optical depths: overestimated
  - Results from approx. model of stat. variations
- Maximum likelihood fluxes are unbiased
- Flux summing method is same for ML and χ^2
  - Not "best" estimator if spectral shape is known
  - Biased if full band is not represented
    - e.g. PL model of 4-10 keV is larger than sum of 4-8 keV
  - "Best" if spectrum is not easily characterized