**LECTURE 3. POWER AMPLIFIER DESIGN FUNDAMENTALS** 

3.1. Main characteristics (two-port networks, gain, delivered power)

3.2. Gain and stability

3.3. Stabilization circuit technique

3.4. Class-A,-B,-C operation modes

3.5. Linearity

3.6. DC biasing

3.7. Push-pull amplifiers

3.8. Practical aspect of RF and microwave power amplifiers

### 3.1. Main characteristics

Generalized single-stage power amplifier circuit



**Two-port active device is characterized by immitance W-parameters which means system of impedance Z-parameters or admittance Y-parameters Matching circuits are necessary to transform source W<sub>s</sub> and load W<sub>L</sub> immitances into definite values between points 1-2 and 3-4, respectively** 

If source of input signal is presented by current source with internal admittance Ys



 $\Rightarrow$  device is characterized by Y-parameters

$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases}$$

 $\begin{cases} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ V_2 = Z_{21}I_1 + Z_{22}I_2 \end{cases}$ 

If source of input signal is presented by voltage source with internal impedance  $Z_s$  $Z_s$  1  $I_1$   $I_2$  3  $\Rightarrow$  device is characterized by Z-parameters



3.1. Main characteristics Power amplifier gain (in terms of Y-parameters)

Operating power gain G<sub>P</sub> = P<sub>L</sub>/P<sub>in</sub> - ratio of power dissipated in active load G<sub>L</sub> to power delivered to input port of active device with admittance Y<sub>in</sub> : this gain is independent of G<sub>S</sub> but is strongly dependent on G<sub>I</sub>

 Available power gain G<sub>A</sub> = P<sub>out</sub>/P<sub>S</sub> - ratio of power available at output port of active device with admittance Y<sub>out</sub> to power available from source G<sub>S</sub> : this gain depends on G<sub>S</sub> but is independent of G<sub>L</sub>

Transducer power gain G<sub>T</sub> = P<sub>L</sub>/P<sub>S</sub> - ratio of power dissipated in active load G<sub>L</sub> to power available from source G<sub>S</sub> : this gain strongly depends on both G<sub>S</sub> and G<sub>L</sub>

 Maximum available gain MAG - theoretical power gain of active device when its reverse transfer function Y<sub>12</sub> is set equal to zero : represents theoretical gain limit that can be achieved with given device under assumption of conjugate input and output impedance matching 3.1. Main characteristics **Operating power gain** 

Two types of power gain are widely used: operating power gain  $G_P$  and transducer power gain  $G_T$ ,

- operating power gain to characterize device amplifying capability and multistage power amplifier
  - transducer power gain to evaluate input matching and stability

Power flowing from input port

Output power dissipated in load

operating power gain

From  $\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases}$ 

input admittance

in view of  $I_2 = -Y_1 V_2$ 

 $P_{\rm in} = 0.5 V_1^2 \,{\rm Re} Y_{\rm in}$ 

$$Y_{in} = \frac{I_1}{V_1} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L}$$
$$P_L = 0.5V_2^2 \operatorname{Re} Y_L$$
$$G_P = \frac{P_L}{P_{in}} = \frac{|Y_{21}|^2 \operatorname{Re} Y_L}{|Y_{22} + Y_L|^2 \operatorname{Re} Y_{in}}$$

4

[*Y*]

2

#### 3.1. Main characteristics

Transducer power gain

Transducer power gain  $G_{\tau}$  includes assumption of conjugate matching both load and source

From  $\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{cases}$ 

**Power flowing from input port**  $P_{\rm S} = \frac{I_{\rm S}^2}{8 {\rm Re} Y_{\rm S}}$ 



in view of  $I_{\rm S} = Y_{\rm S}V_1 + I_1$ 

$$I_{\rm S} = \frac{(Y_{11} + Y_{\rm S})(Y_{22} + Y_{\rm L}) - Y_{12}Y_{21}}{Y_{22} + Y_{\rm L}} V_{1}$$

1 \_\_ 12 \_\_

4 Re  $Y_{11}$  Re  $Y_{22}$ 

5

Output power dissipated in load

source current

$$P_{\rm L} = 0.5 V_2^2 \,{\rm Re} Y_{\rm L}$$

 $MAG = |Y_{21}|^2$ 

$$\Rightarrow \text{ transducer power gain } G_{\rm T} = \frac{P_{\rm L}}{P_{\rm S}} = \frac{4|Y_{21}|^2 \operatorname{Re}Y_{\rm S} \operatorname{Re}Y_{\rm L}}{|(Y_{11} + Y_{\rm S})(Y_{22} + Y_{\rm L}) - Y_{12}Y_{21}|^2}$$

Maximum available gain  $(Y_{12} = 0, Y_S = Y_{11}^*, Y_1 = Y_{22}^*)$ 

### 3.1. Main characteristics

Small-signal FET power amplifier



Equivalent circuit with  $C_{ad} = 0$ 

$$Y_{11} = j\omega C_{gs} / (1 + j\omega R_{gs}C_{gs})$$
$$Y_{12} = 0$$
$$Y_{21} = g_m / (1 + j\omega R_{gs}C_{gs})$$
$$Y_{22} = (1/R_{ds}) + j\omega C_{ds}$$

 $G_{\rm T}(f) = G_{\rm T}(f_{\rm T}) \left(\frac{f_{\rm T}}{f}\right)^2$  - gain estimation at any frequency vs gain at transition frequency

Input and output conjugate matching

$$R_{\rm s} = R_{\rm gs} \qquad R_{\rm L} = R_{\rm ds}$$
$$L_{\rm in} = 1/\omega^2 C_{\rm gs} \qquad L_{\rm out} = 1/\omega^2 C_{\rm ds}$$

$$G_{\rm T} \left( C_{\rm gd} = 0 \right) = MAG = \left( \frac{f_{\rm T}}{f} \right)^2 \frac{R_{\rm ds}}{4R_{\rm gs}}$$

 $f_{\rm T} = g_{\rm m} / 2\pi C_{\rm gs}$  - transition frequency  $f_{\text{max}} = \frac{f_{\text{T}}}{2} \sqrt{\frac{R_{\text{ds}}}{R_{\text{gs}}}}$  - maximum frequency where MAG = 1

#### 3.2. Gain and stability

Principle of power amplifier design - to provide maximum power gain and efficiency for given output power with predictable degree of stability

Main reasons of instability:

• positive feedback from output to input through intrinsic feedback capacitance or inductance of common-grounded terminal

oscillation conditions due to external elements forming positive feedback loop

In terms of immitance approach, circuit will be unconditionally stable if for both hypothetical conditions of open-circuited input and output ports:

 $\begin{cases} \operatorname{Re}\left[W_{\rm s}(\omega) + W_{\rm in}(\omega)\right] > 0 & \left\{\operatorname{Re}\left[W_{\rm L}(\omega) + W_{\rm out}(\omega)\right] > 0 \\ \operatorname{Im}\left[W_{\rm s}(\omega) + W_{\rm in}(\omega)\right] = 0 & \left[\operatorname{Im}\left[W_{\rm L}(\omega) + W_{\rm out}(\omega)\right] = 0 \\ \end{array}\right. \end{cases}$ 

In case of opposite signs, active two-port network can be treated as unstable or potentially unstable (having negative input or output immitance)

When  $\operatorname{Re}[W_{S}(\omega)] > 0$ 

Requirements of power amplifier stability can be simplified to

 $\operatorname{Re}[W_{in}(\omega)] > 0 \quad \operatorname{Re}[W_{out}(\omega)] > 0$ 

7

 $\operatorname{Re}\left[W_{I}\left(\omega\right)\right] > 0$ 

# 3.2. Gain and stability

#### **Device** stability

In common case, value of  $ReW_{out}$  depends on  $W_S \Rightarrow$ within definite values of W<sub>s</sub>, ReW<sub>out</sub> < 0 and two-port network will be potentially unstable

To provide unconditional stability

 $\operatorname{Re}\left[W_{\mathrm{out}}(\omega)\right]\Big|_{\mathrm{min}} > 0$ 

 $\partial \operatorname{Re} W_{out} / \partial \operatorname{Im} W_{S} = 0$ 

Minimum positive value when  $ReW_s = 0$ :

$$ReW_{out} = ReW_{22} - \frac{|W_{12}W_{21}| + Re(W_{12}W_{21})}{2Re(W_{11} + W_{S})}$$
$$ReW_{out} = ReW_{22} - \frac{|W_{12}W_{21}| + Re(W_{12}W_{21})}{2Re(W_{11})}$$

 $W_{\rm out} = W_{22} - \frac{W_{12}W_{21}}{W_{11} + W_{22}}$ 

 $|W_{12}W_{21}| + \operatorname{Re}(W_{12}W_{21})$ 

$$2\operatorname{Re}W_{11}\operatorname{Re}W_{22} - |W_{12}W_{21}| - \operatorname{Re}(W_{12}W_{21}) > 0$$

$$K = \frac{2\operatorname{Re}W_{11}\operatorname{Re}W_{22} - \operatorname{Re}(W_{12}W_{21})}{|W_{12}W_{21}|} \qquad \text{Unconditional s}$$

- device stability factor

Unconditional stability: K > 1 Potential instability: -1 < K < 1

### 3.2. Gain and stability

#### Circuit stability

When active device is potentially unstable, power amplifier stability can be improved with proper choice of source and load immitances, W<sub>s</sub> and W<sub>L</sub>:

$$K_{\rm T} = \frac{2 \operatorname{Re} (W_{11} + W_{\rm S}) \operatorname{Re} (W_{22} + W_{\rm L}) - \operatorname{Re} (W_{12} W_{21})}{|W_{12} W_{21}|} >$$

### Maximum gain with unconditionally stable device

When K > 1, it is necessary to choose load immitance W<sub>L</sub> to maximize finite value of operating power gain  $G_{P}$ :

$$G_{\rm P} = \frac{P_{\rm L}}{P_{\rm in}} = \frac{|W_{21}|^2 \,{\rm Re}W_{\rm L}}{|W_{22} + W_{\rm L}|^2 \,{\rm Re}W_{\rm in}}$$

$$\frac{\partial G_{\rm P}}{\partial \operatorname{Re} W_{\rm L}} = 0$$

$$\frac{\partial G_{\rm P}}{\partial \operatorname{Im} W_{\rm L}} = 0$$

$$\operatorname{Re} W_{\rm L}^{\circ} = \frac{\left| \frac{W_{12} W_{21}}{2 \operatorname{Re} W_{11}} \sqrt{K^2 - 1} \right|}{2 \operatorname{Re} W_{11}}$$

$$\operatorname{Im} W_{\rm L}^{\circ} = \frac{\operatorname{Im}(W_{12} W_{21})}{2 \operatorname{Re} W_{11}} - \operatorname{Im} W_{11}^{\circ}$$

$$G_{\rm Pmax} = \left| \frac{W_{21}}{W_{12}} \right| / \left( K + \sqrt{K^2 - 1} \right)$$

$$- \operatorname{maximum gain (maximum value at K = 1)}$$

 $ImW_{22}$ 

3.3. Stabilization circuit technique Frequency domains of BJT potential instability

Stability factor through Z-parameters:

$$K = \frac{2 R_{11} R_{22} - \operatorname{Re} (Z_{12} Z_{21})}{|Z_{12} Z_{21}|}$$

#### BJT equivalent circuit Z-parameters:

$$Z_{11} = r_{\rm b} + \frac{1}{g_{\rm m}} / \left(1 + j\frac{\omega}{\omega_{\rm T}}\right)$$
$$Z_{12} = \frac{1}{g_{\rm m}} / \left(1 + j\frac{\omega}{\omega_{\rm T}}\right)$$
$$Z_{21} = \left(\frac{1}{g_{\rm m}} - \frac{1}{j\omega C_{\mu}}\right) / \left(1 + j\frac{\omega}{\omega_{\rm T}}\right)$$
$$Z_{22} = \left(\frac{1}{g_{\rm m}} + \frac{1}{\omega_{\rm T}}C_{\mu}\right) / \left(1 + j\frac{\omega}{\omega_{\rm T}}\right)$$



**BJT** stability factor



#### Maximum value at higher frequencies:

$$K = 2r_{\rm b}g_{\rm m} \left(1 + \frac{g_{\rm m}}{\omega_{\rm T}C_{\mu}}\right)$$
 10

#### Frequency domains of BJT potential instability

At low frequencies if to take into account dynamic base-emitter resistance  $r_{\pi}$  and Early collector-emitter resistance  $r_{o} \Rightarrow K > 1$  Only one unstable frequency domain with low  $f_{p1}$  and high  $f_{p2}$  boundary frequencies

For K = 1 
$$f_{p2} = \frac{g_m}{2\pi C_{\mu}} / \sqrt{\left(2r_b g_m\right)^2 \left(1 + \frac{g_m}{\omega_T C_{\mu}}\right)^2 - 1}$$
 or  $f_{p2} \cong \frac{1}{4\pi r_b C_{\pi}}$ 

In common case, at higher frequencies with parasitic emitter lead inductance  $L_e$  :

**Expression for low**  $f_{p3}$  **and high**  $f_{p4}$  **boundary frequencies of second domain of BJT potential instability** 

$$f_{p3,4} = f_{T} \sqrt{\frac{1 - 4\omega_{T} r_{b} C_{\mu}}{8\omega_{T} r_{b} C_{\mu}}} \mu \sqrt{\left(\frac{1 - 4\omega_{T} r_{b} C_{\mu}}{8\omega_{T} r_{b} C_{\mu}}\right)^{2} - \frac{1 + \kappa}{\omega_{T} r_{b} C_{\mu} \kappa^{2}}$$

where  $\kappa = \omega_{\rm T} L_{\rm e} / r_{\rm b}$ 



3.3. Stabilization circuit technique Frequency domains of BJT potential instability

Appearance of second frequency domain of BJT potential instability is result of simultaneous effect of feedback capacitance  $C_{\mu}$  and emitter lead inductance  $L_{e}$ 

• first case for  $L_e = 0$  and reactive load  $X_L$ : one frequency domain of potential instability

 $Z_{\rm in} = r_{\rm b} + \frac{1}{g_{\rm m}} \cdot \frac{1}{1 + j\frac{\omega}{\omega_{\rm T}}} \cdot \frac{1 + \frac{g_{\rm m}}{\omega C_{\mu}}}{1 + \frac{g_{\rm m}}{\omega_{\rm T}} C_{\mu}} (1 - \omega C_{\mu} X_{\rm L}) + jg_{\rm m} X_{\rm L}}$ 

#### Hartley oscillator



#### Boundary condition of first potential instability domain:

$$\frac{L_{\rm L}}{L_{\rm S}} \approx \frac{1}{\omega_{\rm T} r_{\rm b} C_{\mu}}$$

to prevent oscillations ⇒ reduce value of collector choke inductance and increase value of base choke inductance 3.3. Stabilization circuit technique Frequency domains of BJT potential instability

Appearance of second frequency domain of BJT potential instability is result of simultaneous effect of feedback capacitance  $C_{\mu}$  and emitter lead inductance  $L_{e}$ 

• second case for  $L_e \neq 0$  and reactive load  $X_L$ : two frequency domains of potential instability second frequency domain

first frequency domain

- parasitic oscillator with inductive source and load reactances





- parasitic oscillator with capacitive source and inductive load reactances 3.3. Stabilization circuit technique Frequency domains of MOSFET potential instability

Stability factor through Yparameters:

$$K = \frac{2 G_{11} G_{22} - \operatorname{Re} (G_{12} G_{21})}{|Y_{12} Y_{21}|}$$

MOSFET equivalent circuit Y-parameters:





**MOSFET** stability factor:

$$K = \left[1 + \frac{2}{g_{\rm m}R_{\rm ds}} \left(1 + \frac{C_{\rm gs}}{C_{\rm gd}}\right)\right] \frac{\omega R_{\rm gs}C_{\rm gs}}{\sqrt{1 + (\omega R_{\rm gs}C_{\rm gs})^2}}$$

Maximum value at higher frequencies:

$$K = \left[1 + \frac{2}{g_{\rm m}R_{\rm ds}} \left(1 + \frac{C_{\rm gs}}{C_{\rm gd}}\right)\right]$$

14

#### Frequency domains of MOSFET potential instability

At low frequencies if to take into account gate leakage resistance  $\Rightarrow K > 1$  Only one unstable frequency domain with low  $f_{p1}$  and high  $f_{p2}$  boundary frequencies

For K = 1 
$$f_{p2} = \frac{1}{4\pi R_{gs}C_{gs}} \cdot \frac{g_{m}R_{ds}}{\sqrt{1 + \frac{C_{gs}}{C_{gd}}}} \cdot \frac{1}{\sqrt{1 + \frac{C_{gs}}{C_{gd}}} + g_{m}R_{ds}}$$
 or  $f_{p2} \approx \frac{1}{4\pi R_{gs}C_{gs}}$ 

In common case, parasitic emitter lead inductance L<sub>e</sub> creates second frequency domain of potential instability at higher frequencies



Frequency domains of MOSFET potential instability

Appearance of second frequency domain of MOSFET potential instability is result of simultaneous effect of feedback capacitance C<sub>ad</sub> and source lead inductance L<sub>s</sub>

• first case for  $L_s = 0$  and reactive load  $X_L$ : one frequency domain of potential instability

$$Y_{\rm in} = \frac{j\omega C_{\rm gs}}{1 + j\omega R_{\rm gs}C_{\rm gs}} \left[ 1 + g_{\rm m}R_{\rm ds} \frac{1 - j\frac{\omega}{\omega_{\rm T}} \left(1 + j\omega R_{\rm gs}C_{\rm gs}\right)}{1 + j\omega R_{\rm ds}C_{\rm ds} \left(1 + \frac{C_{\rm gd}}{C_{\rm ds}} + \frac{B_{\rm L}}{\omega C_{\rm ds}}\right)} \right]$$

16

Frequency domains of MOSFET potential instability

Appearance of second frequency domain of MOSFET potential instability is result of simultaneous effect of feedback capacitance C<sub>ad</sub> and source lead inductance L<sub>s</sub>

 second case for L<sub>s</sub> ≠ 0 and reactive load X<sub>L</sub>: two frequency domain of potential instability

second frequency domain

first frequency domain

- parasitic oscillator with inductive source and load reactances





- parasitic oscillator with capacitive source and inductive load reactances

General requirements to provide stable operation of power amplifier:

use active device with stability factor K > 1

 if it is impossible to choose active device with K > 1, provide circuit stability factor K<sub>T</sub> > 1 on operating frequency by appropriate choice of real parts of source and load immitances

disrupt equivalent circuit of possible parasitic oscillators

 choose such reactive parameters of matching circuits adjacent to input and output of active device which are necessary to avoid self-oscillation conditions

In common case, it is difficult to propose unified approach to provide stable operation of different power amplifiers especially for multistage power amplifier

Stability analysis must be done in different frequencies ranges:

 at lower frequencies when frequency of parasitic oscillations f<sub>p</sub> is significantly smaller operating frequency f<sub>0</sub> (f<sub>p</sub> << f<sub>0</sub>)





 using stabilizing resistor R<sub>1</sub> with series bypass
 capacitance C<sub>2</sub> in parallel to power supply



 using stabilizing resistor R<sub>1</sub> in parallel to RF choke - using stabilizing resistor R<sub>1</sub> in parallel to additional RF choke to avoid degradation of RF performance

 using additional RF choke if impedance of series R<sub>1</sub>C<sub>2</sub> circuit is too high



#### Stability analysis must be done in different frequencies ranges:

 at higher frequencies when frequency of parasitic oscillations f<sub>p</sub> is significantly higher operating frequency f<sub>0</sub> (f<sub>p</sub> >> f<sub>0</sub>)



#### Stability analysis must be done in different frequencies ranges:

• near operating frequency frequency when frequency of parasitic oscillations  $f_p$  is close to operating frequency  $f_0$  ( $f_p \approx f_0$ )

series connection of stabilizing RLC circuit connected in series between active device and output matching circuit series L<sub>1</sub>C<sub>1</sub> circuit is tuned on operating frequency





parallel connection of stabilizing RLC circuit between active device and output matching circuit







- $I_{a} = -I \cos \theta$  quiescent current as function of half-conduction angle  $\theta$ 
  - when  $\theta > 90^{\circ} \Rightarrow \cos \theta < 0 \Rightarrow I_q > 0$  Class AB operation mode
  - when  $\theta = 90^{\circ} \Rightarrow \cos \theta = 0 \Rightarrow I_q = 0$  Class B operation mode
  - when  $\theta < 90^{\circ} \Rightarrow \cos \theta > 0 \Rightarrow I_q < 0$  Class C operation mode
  - $i = I_0 + I_1 \cos \omega t + I_2 \cos 2\omega t + I_3 \cos 3\omega t + K$  Fourier series
  - where  $I_0 = \frac{1}{2\pi} \int I(\cos \omega t \cos \theta) d(\omega t) = I \gamma_0$  DC component

 $I_{1} = \frac{1}{\pi} \int I(\cos \omega t - \cos \theta) \cos \omega t d(\omega t) = I \gamma_{1} - fundamental component$ 

where  $\gamma_0 = \frac{1}{\pi} (\sin \theta - \theta \cos \theta), \quad \gamma_1 = \frac{1}{\pi} (\theta - \sin \theta \cos \theta)$  - current coefficients  $\eta = \frac{P_1}{P_0} = \frac{1}{2} \frac{I_1}{I_0} \xi = \frac{1}{2} \frac{\gamma_1}{\gamma_0} \xi - \text{collector efficiency}$ When  $\theta = 90^{\circ}$  and  $\xi = 1 \implies \eta = \frac{\pi}{4} \cong 0.785$  - maximum collector efficiency in Class B24



$$i = \left(I_{q} + \frac{V_{cc}}{\gamma_{1}R}\right) - \frac{v}{\gamma_{1}R}$$

$$\tan \beta = \frac{I}{V(1 - \cos \theta)} = \frac{1}{\gamma_1 R}$$

- dynamic characteristic of power amplifier or load line function within  $-\theta \le \omega t < \theta$ 

- slope of load line

25



 load line represents broken line with three sections: KL- saturation region (depression in collector current waveform) KM – active region

MP – pinch-off region



 collector current becomes asymmetrical for complex load impedance

asymmetrical load line

• for inductive load impedance, depression in collector current waveform is shifted to the left (a)

• for capacitive load impedance, depression in collector current waveform is shifted to the right (b)

Reason: different phase conditions for higher-order harmonics

To evaluate nonlinear properties of power amplifier, consider transfer function of active device in common form of i = f(v)where i - output current, v - input voltage

$$f(v) = f(V_0) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^{(n)} f(v)}{\partial v^n} \bigg|_{v=V_0} (v - V_0)^n \qquad \text{where } V_0 - DC \text{ bias voltage}$$

Usual method to determine nonlinear properties is to to apply twotone excitation test signal

For two signals with equal amplitudes  $V_1 = V_2 = V$ :  $v = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t$  $= 2V \cos \omega t \cos \Omega t$ where  $\omega = (\omega_1 + \omega_2)/2$  $\Omega = (\omega_1 - \omega_2)/2$  $\Omega = (\omega_1 - \omega_2)/2$ Peak envelope power PEP corresponds to maximum amplitude of 2V:  $P_{\text{PEP}} = (2V)^2/2R$ Total power due to each tone:  $P_{\text{total}} = P_{\omega 1} + P_{\omega 2} = V^2/R$  $P_{\text{PEP}} = 2P_{\text{out}} = 4P$ where  $P = P_{\omega 1} = P_{\omega 2}$  28

For two-tone excitation test signal  $v = V_0 + V_1 \cos \omega_1 t + V_2 \cos \omega_2 t$ 

Taylor's expansion of output current for first three derivatives results in

$$i = f(v) = f(V_0) + \frac{1}{4} \frac{\partial^2 f(v)}{\partial v^2} \bigg|_{v=V_0} \left( V_1^2 + V_2^2 \right)$$

$$= \left[ f'(V_0) + \frac{1}{4} \frac{\partial^3 f(v)}{\partial v^3} \right|_{v=V_0} \left( \frac{1}{2} V_1^2 + V_2^2 \right) \right] V_1 \cos \omega_1 t + \left[ f'(V_0) + \frac{1}{4} \frac{\partial^3 f(v)}{\partial v^3} \right|_{v=V_0} \left( V_1^2 + \frac{1}{2} V_2^2 \right) \right] V_2 \cos \omega_2 t$$

$$+ \frac{1}{4} \frac{\partial^2 f(v)}{\partial v^2} \bigg|_{v=V_0} \left( V_1^2 \cos 2\omega_1 t + V_2^2 \cos 2\omega_2 t \right) + \frac{1}{24} \frac{\partial^3 f(v)}{\partial v^3} \bigg|_{v=V_0} \left( V_1^3 \cos 3\omega_1 t + V_2^3 \cos 3\omega_2 t \right)$$

$$+ \left. \frac{1}{2} \frac{\partial^2 f(v)}{\partial v^2} \right|_{v=V_0} V_1 V_2 \cos\left(\omega_1 \pm \omega_2\right) t$$

 $+ \left. \frac{1}{8} \frac{\partial^3 f(v)}{\partial v^3} \right|_{v=V_0} \left[ V_1^2 V_2 \cos\left(2\omega_1 \pm \omega_2\right) t + V_1 V_2^2 \cos\left(\omega_1 \pm 2\omega_2\right) t \right] \mathbf{K}$ 

#### Main conclusions:

• variation of DC bias point is directly proportional to second derivative (in common case - even derivatives) of transfer function

- device transfer function will be linear only if third derivative (in common case - odd derivatives) is equal to zero
- even harmonic components are result of even derivatives of transfer function; odd harmonic components are result of odd derivatives of transfer function
- first-order mixing products (total and differential) depend on even derivatives of transfer function
  - mixing products of third and higher order are mainly determined by odd derivatives of transfer function

Distortions which are determined by second derivatives of device transfer function are called second-order intermodulation distortions; distortion which are determined by third-order derivatives are called third-order intermodulation distortions

Output current amplitude of fundamental, second and third harmonic or intermodulation components depends on first, second and third degree of input voltage, respectively



Consequently, output powers of linear, second- or third-order component show straight-line behavior and vary by 1 dB, 2 dB and 3 dB, respectively, with 1-dB variation of input power

These straight lines intersect at some points which are called intercept points IP<sub>n</sub>

$$P_{\text{IM}_{n}} = nP_{\omega_{1}} - (n - 1)IP_{n} \text{ (dBm)}$$

Second harmonic component

$$P_{2\omega_1} = 2P_{\omega_1} - IP_2 \text{ (dBm)}$$

Third-order intermodulation component

$$P_{2\omega_1-\omega_2} = 3P_{\omega_1} - 2IP_3 \,(\mathrm{dBm})$$

 $P_{1\rm dB} = IP_3 - 9 (\rm dBm)$ 

- 1-dB gain compression point

31

For MOSFET device, there is optimum bias point with drain quiescent current  $I_{dg}$  in limits of 0.1...0.15  $I_{dss}$  when  $IM_3$  can be minimized providing high-power and high-efficiency operation because of quadratic transfer function in this region



Output spectrum containing n-order intermodulation components



 $IM_{3} = 10 \log_{10} \left( P_{2\omega_{1}-\omega_{2}} / P \right) = P_{2\omega_{1}-\omega_{2}} - P (dBc) - third-order intermodulation product$  $IM_{5} = 10 \log_{10} \left( P_{3\omega_{1}-2\omega_{2}} / P \right) = P_{3\omega_{1}-2\omega_{2}} - P (dBc) - fifth-order intermodulation product$  $where P = P_{\omega_{1}} = P_{\omega_{2}}$ 32

### 3.6. DC biasing

DC biasing of active device provides required operation condition which should be stable over input power, temperature or technology process variations

For MOSFETs as voltage controlled devices, at normal conditions it is enough to use resistive divider to set gate bias voltage





However, in wide temperature range when device threshold voltage varies with temperature (2 mV/°C), to reduce quiescent current variation, it is possible to use silicon diode in series to variable resistor L1 When device threshold voltage is too high, it is best to connect several silicon diodes in series

 $Q V_{dd}$ 

 $VT_1$ 

 $R_2$ 

 $VD_1$ 

 $VD_2$ 

Such simple bias circuit configurations for MOSFETs become possible in view of extremely small gate DC current equal to its leakage current only

### 3.6. DC biasing

#### **Current mirror bias circuits**

For bipolar transistor as current-controlled device, to stabilize quiescent current it is best to use current-mirror type of bias circuits where reference diode is formed using same diode-connected transistor with substantially smaller area





To minimize quiescent current variations over temperature, ratio of ballast resistors  $R_1/R_0$ must be equal to device area ratio  $Q_0/Q_1$ 

However, to fix current flowing from reference source through resistor  $R_2$ , its value should be much higher than base current of RF device  $Q_0$  To reduce current from reference source and to increase current driving capability for high power RF device, driving transistor  $Q_3$  is used to feed DC base current for RF device  $Q_0$  34

### 3.6. DC biasing



Current mirror bias circuits

Popular configuration of temperature compensated bias circuit contains one reference transistor and one driving device



It is very important to provide ratio of ballast resistors  $R_1/R_0$  equal to ratio of device areas  $Q_0/Q_1$  which minimizes variations over temperature as well as stabilizes DC bias point over input power  $V_{be0}$ . V



1 - required value of ballast resistor R<sub>1</sub>

 $2 - R_1 = 0$ 

To minimize current from reference voltage source, emitter follower configuration can be used where this current is equal to extremely small base current of emitter follower device Q<sub>3</sub>

 $l_{\rm cc}$ 

 $I_{\rm c}$ 

 $I_{\rm c0}$ 

2π

Push-pull operation helps to increase values of input and output impedances and to additionally suppress even harmonics

2π

π



 $i_{\rm cL}$ 

 $I_{\rm c}$ 

 $2\pi$ 

2π

π

π

 $I_{\rm c}$ 

 $i_{c2}$ 

 $I_{\rm c}$ 

For 50% duty cycle of each device (ideal Class B) with driving sinusoidal voltage:

first transistor collector current

$$i_{c1} = \begin{cases} +I_c \sin \theta, & 0 \le \theta < \pi \\ 0, & \pi \le \theta < 2\pi \end{cases}$$

second transistor collector current

$$i_{c2} = \begin{cases} 0, & 0 \le \theta < \pi \\ -I_c \sin \theta, & \pi \le \theta < 2\pi \end{cases}$$

Being transformed through output transformer T<sub>2</sub>, total collector current:

$$i_{\rm L}(\theta) = i_{\rm c1}(\theta) - i_{\rm c2}(\theta) = I_{\rm c} \sin(\theta)$$

Current flowing in center tap of primary winding of transformer T<sub>2</sub>:

 $i_{cc}(\theta) = i_{c1}(\theta) + i_{c2}(\theta) = I_c |\sin(\theta)|$ 36



**Total DC collector current** 

$$I_{\rm co} = \frac{1}{2\pi} \int_{0}^{2\pi} i_{\rm cc}(\theta) d\theta = \frac{2}{\pi} I_{\rm cc}$$

For zero saturation resistance when collector voltage amplitude  $V_c = V_{cc}$  and equal turns of winding when  $V_L = V_c$ , DC and fundamental output powers

$$P_0 = \frac{2}{\pi} I_c V_{cc} \qquad P_{out} = \frac{1}{2} I_c V_{cc}$$

Ideally, even-order harmonics are canceled as they are in-phase and combined in center tap of primary winding of output transformer

To eliminate losses, it is necessary to connect bypass capacitance to this center point

As for 50% duty cycle, third- and higherorder odd harmonics do not exist, ideally sinusoidal signal will appear in load

$$v_{\rm L}(\theta) = I_{\rm c} R_{\rm L} \sin(\theta) = V_{\rm L} \sin(\theta)$$

Maximum theoretical collector efficiency that can be achieved in Class B operation

$$\eta = \frac{P_{\text{out}}}{P_0} = \frac{\pi}{4} \cong 78.5\%$$



Matching conditions for single-ended transistor



In balanced circuit, identical sides carry 180° out-of phase signals of equal amplitude

If perfect balance is maintained, there are midpoints where signal amplitudes are zero

Such a condition is called virtual grounding

Being inside device package with two balanced transistors, virtual ground reduces commonmode inductance and simplify matching circuit

> Simplification for balanced transistors where matching parallel capacitances are combined and DC blocking capacitances are not required



38

For push-pull operation, unbalance-to-balance transformation is required



 as shortened stubs produce inductive impedances, series capacitors C<sub>1</sub> and C<sub>2</sub> are used forming high-pass matching sections • most suitable approach is to use 1:1 coaxial transformer

• for 50-ohm source and load, its characteristic impedance = 50 Ohm and each balanced part sees 25 Ohm

• to minimize transformer size and provide broadband operation with minimum return loss, coaxial transformers are mounted and soldered along shortened microstrip lines and additional shortened stubs are added for symmetry39

# 3.8. Practical aspect of RF and microwave power amplifiers

Typical microwave power amplifier topology



matching circuits in form of L-transformers:
 parallel microstrip open stubs represent capacitive reactances,
 series microstrip lines represent inductive reactances

• bias circuits contain quarterwave loaded and opened microstrip lines for RF signal isolation

#### 3.8. Practical aspect of RF and microwave power amplifiers

Microwave 2.5-2.7 GHz 5 W GaAs MESFET power amplifier topology



• matching circuits are combinations of L-transformers (parallel capacitors and series microstrip lines) and quarterwave lines with different characteristic impedances

• drain bias circuit contains additional RC-circuit to prevent high-frequency oscillations and large capacitance to prevent low-frequency oscillations

#### 3.8. Practical aspect of RF and microwave power amplifiers



 to combine output powers from two or more transistors at microwaves, 90-degree branch-line hybrids are widely used where active devices are isolated from each other

• at equal reflection coefficients from loads connected to output terminals, reflection wave is absent at input terminal and flowing to 50-ohm ballast resistor  $50 \Omega$ 

> branch-line hybrid also can work as impedance transformer with characteristic impedances of its microstrip branches as

 $Z_1 = Z_{\text{in}}$   $Z_3 = Z_{\text{out}}$   $Z_2 =$ 

 $P_{\rm out}$ 

50 Ω

 $Z_1$ 

 $\sqrt{\frac{Z_{\rm in} Z_{\rm out}}{12}}$ 

# 3.8. Practical aspect of RF and microwave power amplifiers Microwave 5.5 GHz 2.5 W GaAs MESFET power amplifier topology



 for monolithic microwave applications, when output resistance of transistor is slightly less or higher than 50ohm, it is convenient to realize parallel connection of active devices (easy to provide circuit symmetry for packaged Pout devices)

50 Ohm

 to combine power from two transistors, it is necessary simply to transform impedance from each device to 100 Ohm and then parallel connection results in required 50-ohm load

• input matching circuits represent quarterwave microstrip line transformer and L-transformer with series microstrip line and parallel capacitance each 43