Introduction to Radar Signal Processing

Joshua Semeter

Boston University

Jls@bu.edu

Contents

- Review of yesterday's material.
- Essential frequency domain concepts.
- Bandwidth and noise.
- Conceptual description of matched filtering.
- Doppler processing.
- Peculiarities in ISR Doppler analysis.
- Introduction to pulse compression.

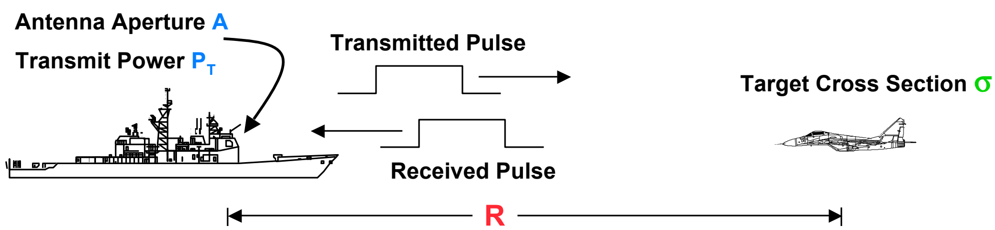
Bibliography

- Mahafza, *Radar Systems Analysis and Design Using MATLAB*
- Skolnik, *Introduction to Radar Systems*
- Peebles, *Radar Principles*
- Levanon, *Radar Principles*
- Blahut, *Theory of Remote Image Formation*
- Curlander, *Synthetic Aperture Radar: Systems and Signal Analysis*

Background material:
- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
- Oppenheim, Willsky, and Nawab, *Signals and Systems*
- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:
- http://mathforum.org/johnandbetty/

Review: Basic operation of a pulsed radar



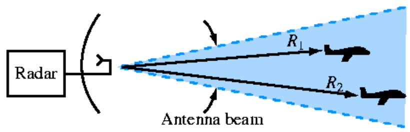
Received Signal Energy (Transmit Power × Transmit Gain × Spread Factor × Losses × Target RCS × Spread Factor × Receive Aperture × Dwell Time):

$$\text{Received Signal Energy} = \left[P_T \right] \left[\frac{4\pi A}{\lambda^2} \right] \left[\frac{1}{4\pi R^2} \right] \left[\frac{1}{L} \right] \left[\sigma \right] \left[\frac{1}{4\pi R^2} \right] \left[A \right] \left[\tau \right]$$

Information about target properties is embodied in the RCS or "Radar Cross Section"

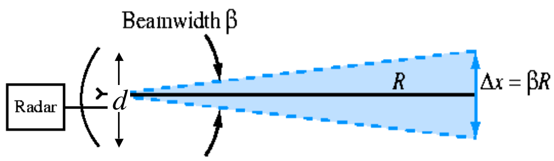
Review: basic radar parameters

Range resolution: **Set by pulse length, given in units of time, τ_p, or length, c τ_p**



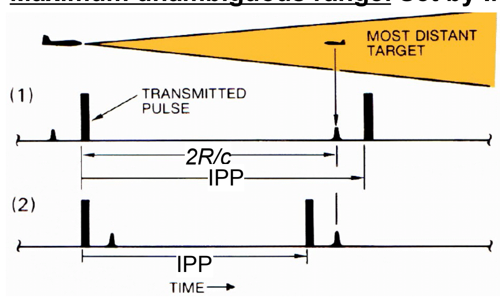
$$! \ R = R_2 \quad R_1 = \frac{c\#_p}{2}$$

Cross-range resolution: **Set by "beam width" (in degrees) and target range**



$$! \quad \frac{\#}{d} \ \text{radians}$$

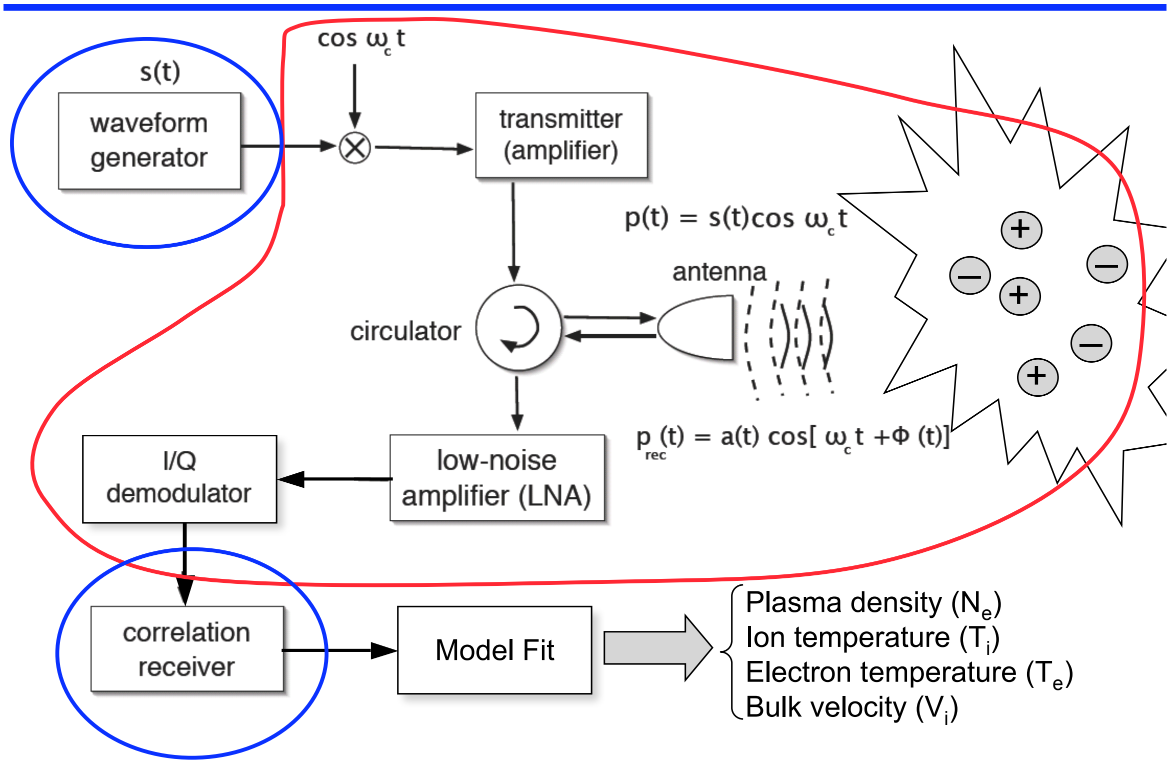
Maximum unambiguous range: **Set by Inter-pulse Period (IPP)**



IPP = Interpulse period (s)
PRF = pulse repetition frequence
= 1/IPP (Hz)

$$R_u = \frac{c \ \text{IPP}}{2}$$

Components of a pulsed Doppler radar

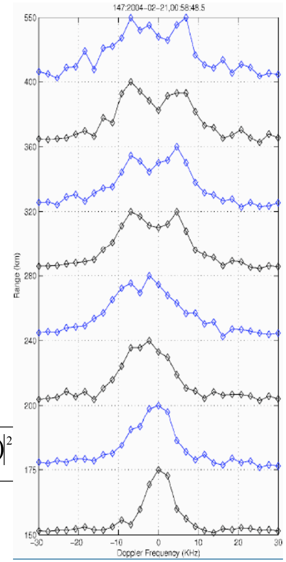


Review: ISR in a nutshell

Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

The "radar equation"



$P_r =$ Received power

$P_n =$ Received noise power

$P_t =$ Transmitted power

$\sigma =$ Radar cross section

$G =$ Antenna gain

$A =$ Antenna area

$k_B =$ Boltzman's constant

$T =$ Temperature

$B =$ Bandwidth

$N_{sys} =$ System noise temperature

Here's the theory:

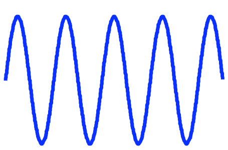
$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

$$F_e(\omega) = 1 - \omega\int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)d\tau$$

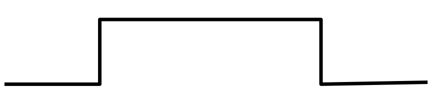
$$F_i(\omega) = 1 - \omega\int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)d\tau$$

Waves versus pulses

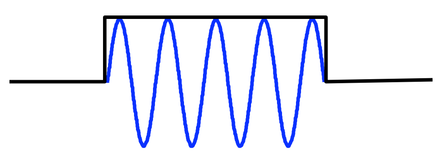
What do radars transmit?



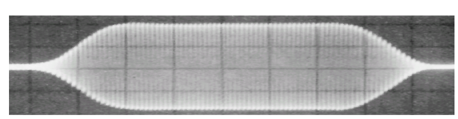
Waves?



or Pulses?



Waves, modulated by "on-off" action of pulse envelope



How many cycles are in a typical pulse?

PFISR frequency: 449 MHz

Typical long-pulse length: 480 μs } **215,520 cycles!**

Essential mathematical operations

Fourier:

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} dt \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Euler:

$$\begin{aligned} Ae^{j\phi t} &= A\cos(\phi) + jA\sin(\phi) \\ &= I + jQ \end{aligned}$$

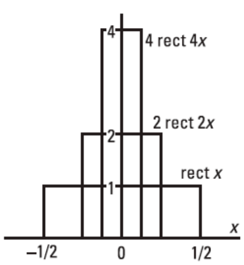
Convolution:

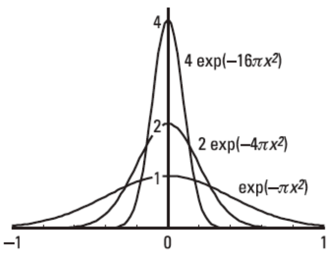
$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) \cdot g(t-\tau) d\tau \qquad f(t) * g(t) \Longleftrightarrow F(\omega) \cdot G(\omega)$$

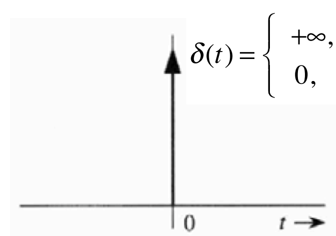
Correlation:

$$f(t) \star g(t) = \int_{-\infty}^{+\infty} f^*(\tau) \cdot g(t+\tau) d\tau \qquad f(t) \star g(t) \Longleftrightarrow F(f)^* \cdot G(f)$$

Dirac delta function





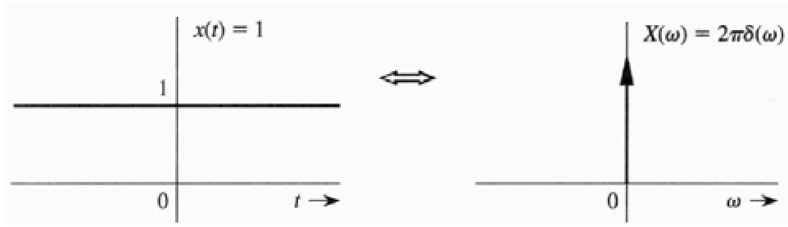


$\delta(t)$ is defined by the property that for all continuous functions

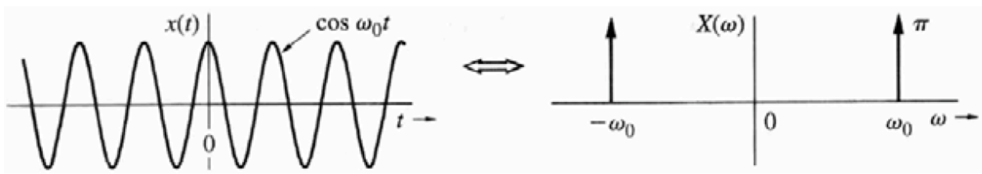
$$f(0) = \int_{-\infty}^{+\infty} \delta(t) f(t) dt$$

$$f(t-T) = f(t) * \delta(t-T)$$

Harmonic functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$



$$\sin \omega_0 t \Longleftrightarrow j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$

$$e^{j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega-\omega_0)$$

Fourier transform of an impulse train

Consider an impulse train $\delta_{T_0}(t)=\sum_{-\infty}^{\infty}\delta(t-nT_0)$

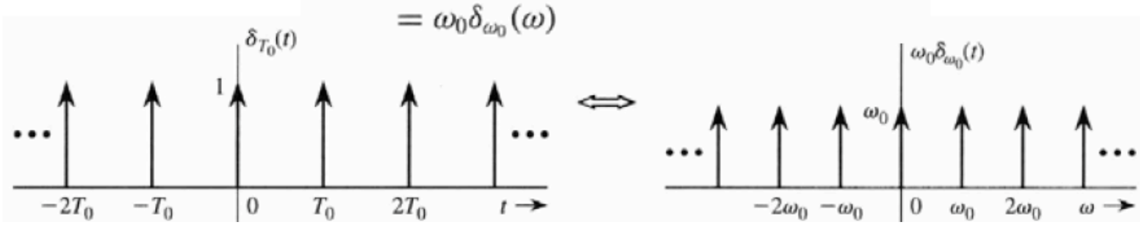
The Fourier series of this impulse train can be shown to be:

$$\delta_{T_0}(t)=\sum_{-\infty}^{\infty}D_n e^{jn\omega_0 t} \quad \text{where} \quad \omega_0=\frac{2\pi}{T_0} \quad \text{and} \quad D_n=\frac{1}{T_0}$$

Therefore using results from the last slide (slide 11), we get:

$$X(\omega)=\frac{2\pi}{T_0}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0) \qquad \omega_0=\frac{2\pi}{T_0}$$

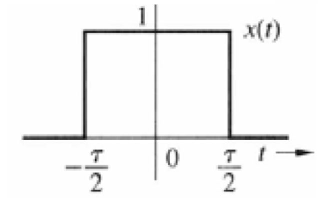
$$=\omega_0\delta_{\omega_0}(\omega)$$

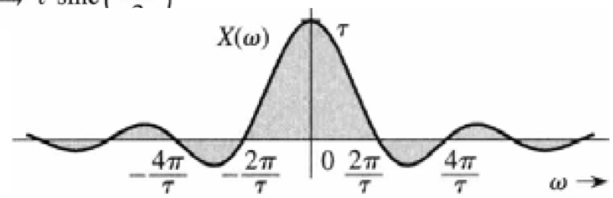


The gate function and its Fourier transform

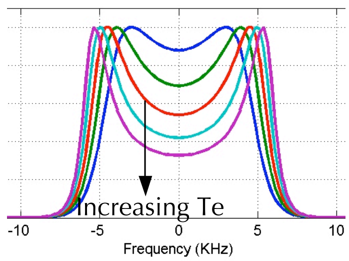
$$\mathrm{rect}(t/\tau) = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

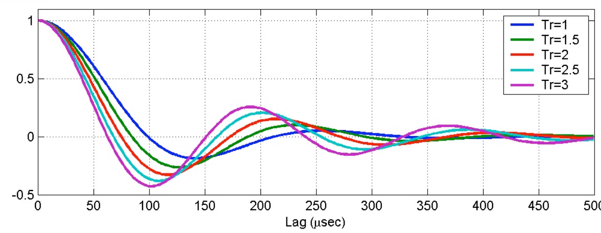
$$\mathrm{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\ \mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\Longleftrightarrow$ Autocorrelation function (ACF)

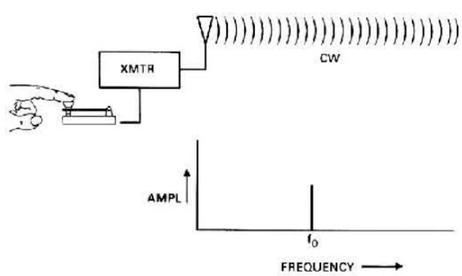




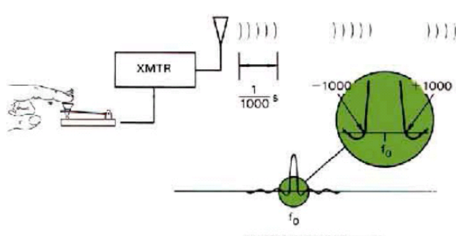
Not surprisingly, the ISR ACF looks like a sinc function…

A pulsed signal has a continuous spectrum

A continuous wave (CW) signal at frequency *f_o* produces an output from the receiver only when it is tuned to *discrete* frequencey *f_o*

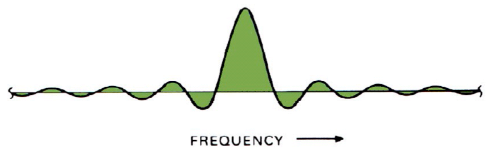


The receiver output for a train of independent pulses with pulse width 10 ms, constant PRF, and random phase, is *continuous* over a band of frequencies 2 kHz wide.

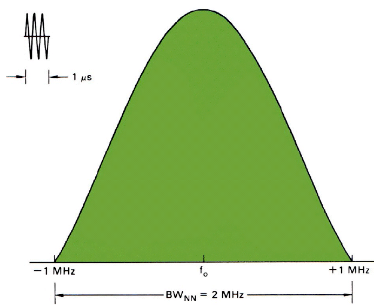


Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe



A 1 microsecond pulse has a null-to-null bandwidth of the central lobe = 2 MHz



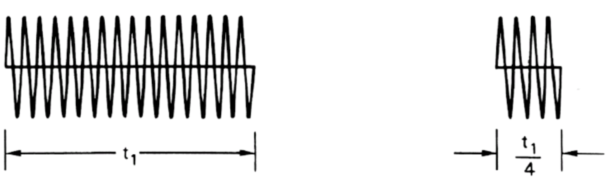
Two possible bandwidth measures:

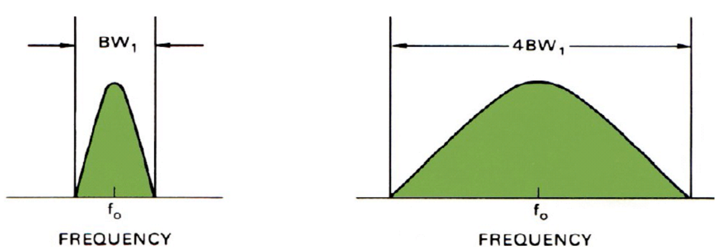
"null to null" bandwidth $B_{\mathrm{nn}} = \dfrac{2}{\tau}$

"3dB" bandwidth $B_{\mathrm{3dB}} = \dfrac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

Bandwidth is inversely proportional to pulse length

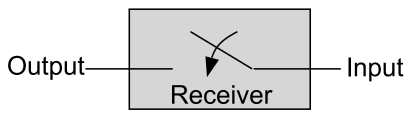




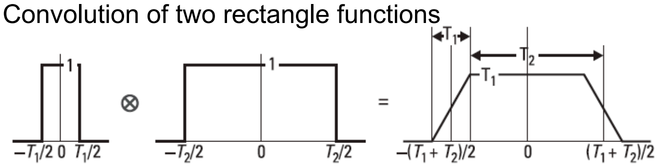
Shorter pulse ⟺ Larger bandwidth

Strategy for radar reception

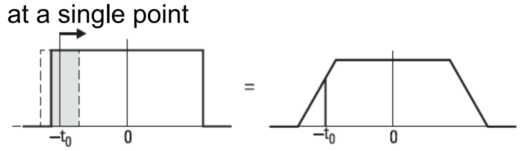
We send a pulse of duration τ. How should we listen for the echo?



Convolution of two rectangle functions

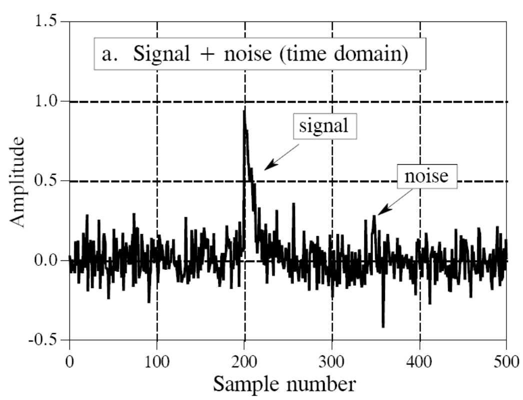


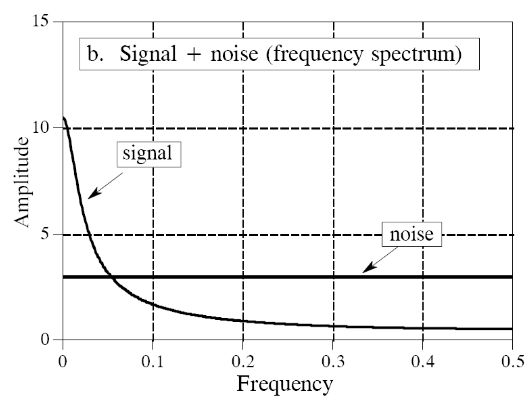
Value at a single point



- To determine range, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

Signal in White Gaussian Noise





Exponential pulse buried in random noise. Sine the signal and noise overlap in both time and frequency domains, the best way to separate them is not obvious.

Most important thing is to match bandwidth of the signal you are looking for

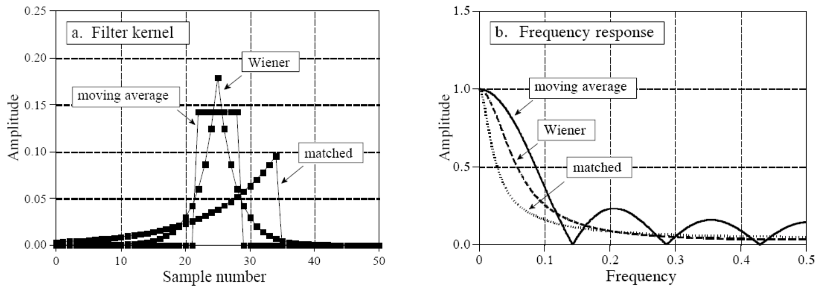
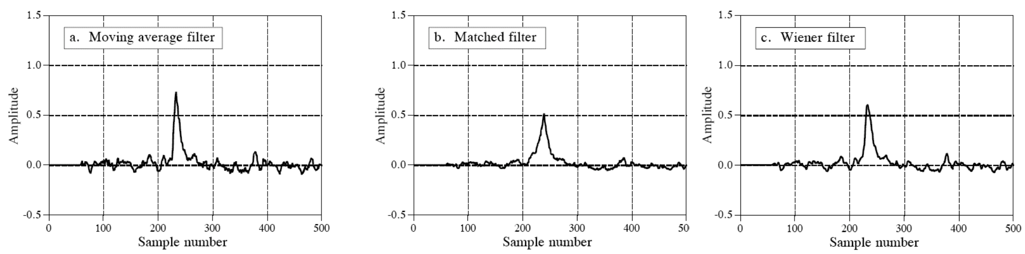
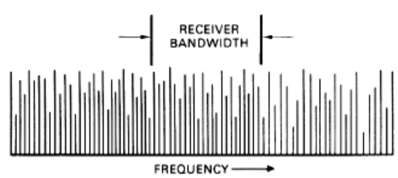


FIGURE 17-8
Example of optimal filters. In (a), three filter kernels are shown, each of which is optimal in some sense. The corresponding frequency responses are shown in (b). The moving average filter is designed to have a rectangular pulse for a filter kernel. In comparison, the filter kernel of the matched filter looks like the signal being detected. The Wiener filter is designed in the frequency domain, based on the relative amounts of signal and noise present at each frequency.

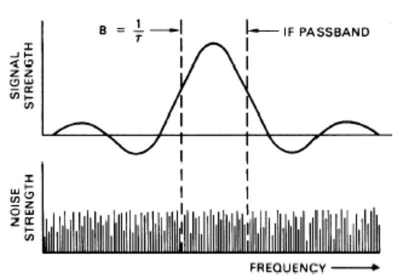


Matched Filter Concept



6. Noise in receiver output is proportional to bandwidth of receiver.

The matched filter is a filter whose impulse response, or transfer function, is determined by a given signal, in a way that will result in the maximum attainable signal-to-noise ratio at the filter output when both the signal and white noise are passed through it.

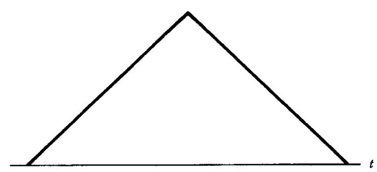


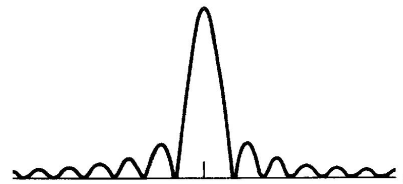
11. Signal-to-noise ratio may be maximized by narrowing the passband of the IF amplifier to the point where only the bulk of the signal energy is passed.

The optimum bandwidth of the filter, B, turns out to be very nearly equal to the inverse of the transmitted pulse width.

To improve range resolution, we can reduce τ (pulse width), but that means increasing the bandwidth of transmitted signal = More noise…

Matched filter for a simple RF pulse





- For an uncoded pulse, matched filter is the baseband filter whose bandwidth matches the bandwidth of the transmitted pulse ($1/\tau$)
- For point target range is determined by locating the time of the maximum in either I or Q (range cut through Ambiguity function)
- For distributed target with 0 Doppler, there is no need to sample more than once for each pulse.
- In this case we must reduce τ (pulse width) to improve range resolution, which means increasing bandwidth of transmitted signal = More noise…

"Range resolution—detectability tradeoff"

Review: Doppler frequency shift

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t+\frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + v_o t$$

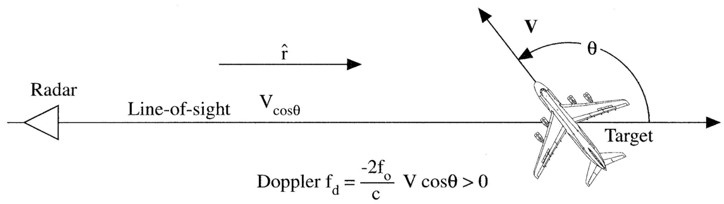
Substituting:

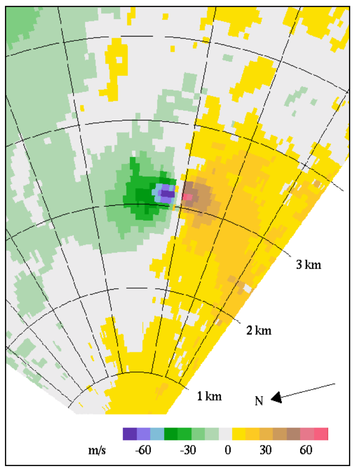
$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R_o}{c}}_{\text{constant}}\right]$$

$$\boxed{f_D = \frac{-2f_o v_o}{c} = \frac{-2v_o}{\lambda_o}}$$

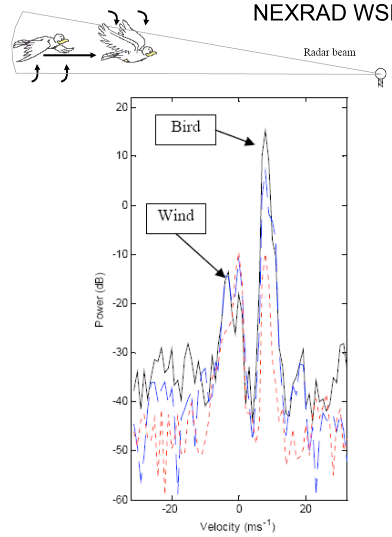
By convention, positive Doppler frequency shift $\Longleftrightarrow$ Target and radar approaching

Doppler frequency gives projection of velocity onto LOS



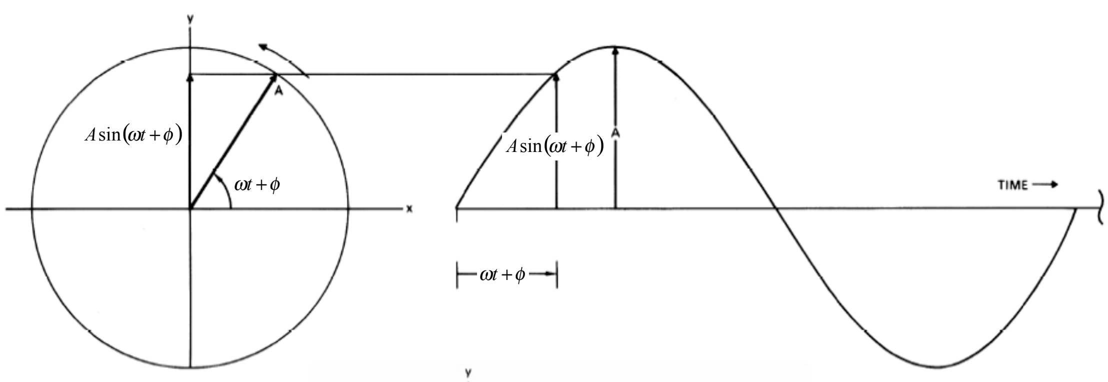


NEXRAD WSR-88D



Doppler analysis in complex plane

Phasor diagram is a graphical representation of a sine wave



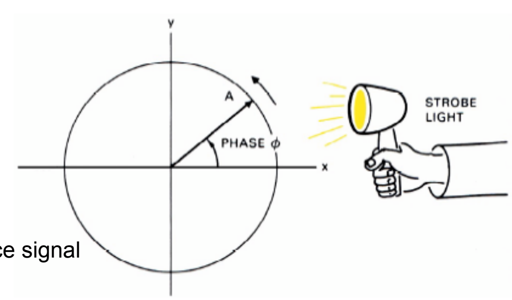
I & Q components*

I => in-phase component

Acos(ϕ)

Q => in-quadrature component

Asin(ϕ)

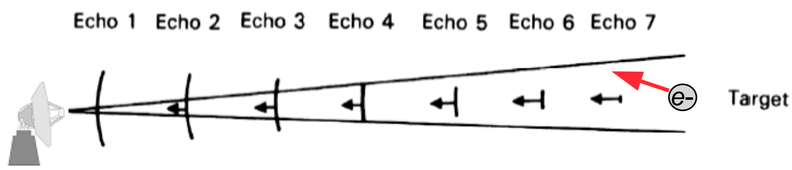


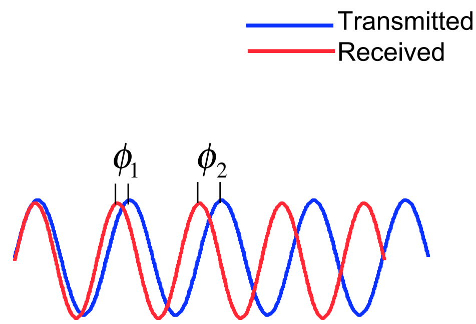
Consider strobe light as cosine reference wave at same frequency but with initial phase = 0

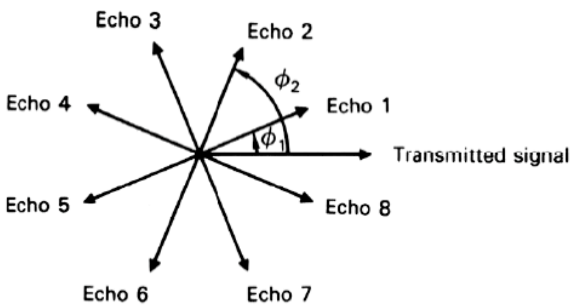
*relative to reference signal

Doppler analysis in complex plane

Closing on target – positive Doppler shift







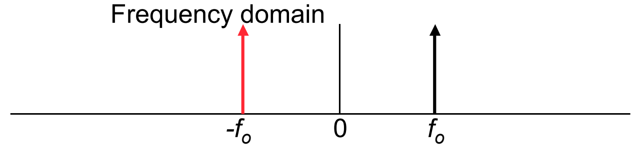
Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

Baseband mixing: Conceptual presentation

Transmitted signal

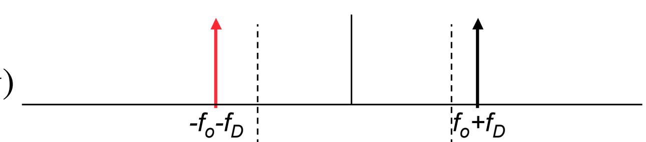
$$\cos(2\pi f_o t)$$

Frequency domain



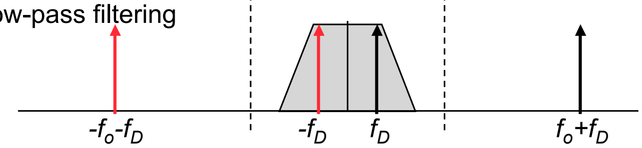
Doppler shifted

$$\cos(2\pi(f_o + f_D)t)$$



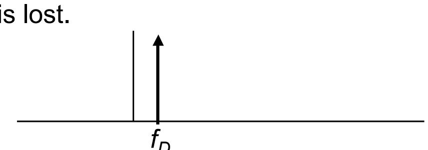
After multiplication by $\cos(2\pi f_o t)$ and low-pass filtering

$$\cos(2\pi f_D t)$$



Cosine is even function, so sign of f_D (and, hence, velocity) is lost.
What we need instead is:

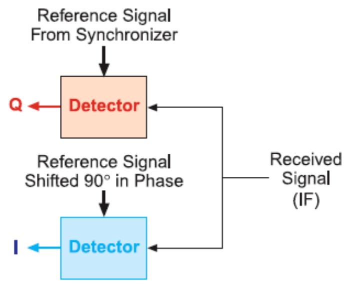
$$\exp(j2\pi f_D t) = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

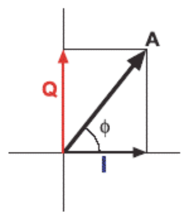


The analytic signal $\exp(j2\pi f_D t)$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called "in phase" (or *I*) and "in quadrature" (or *Q*):

$$A\exp(j2\pi f_D t) = I + jQ$$

I and Q demodulation





in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\cos(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t)+2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\sin(\omega_c t)$$

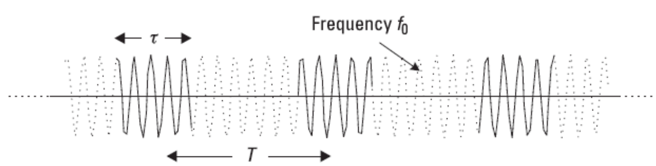
$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t)+2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

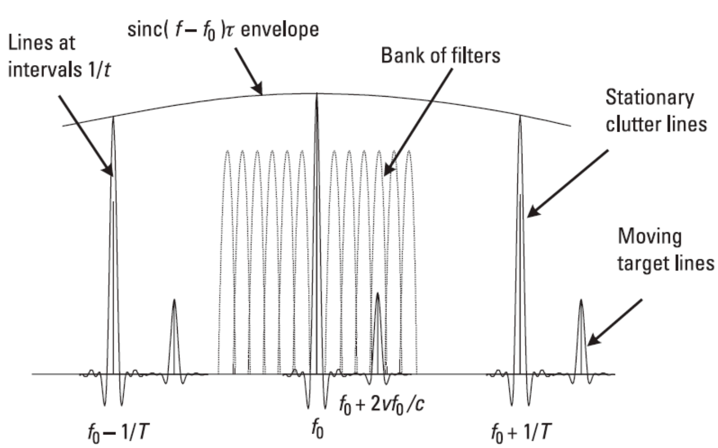
I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

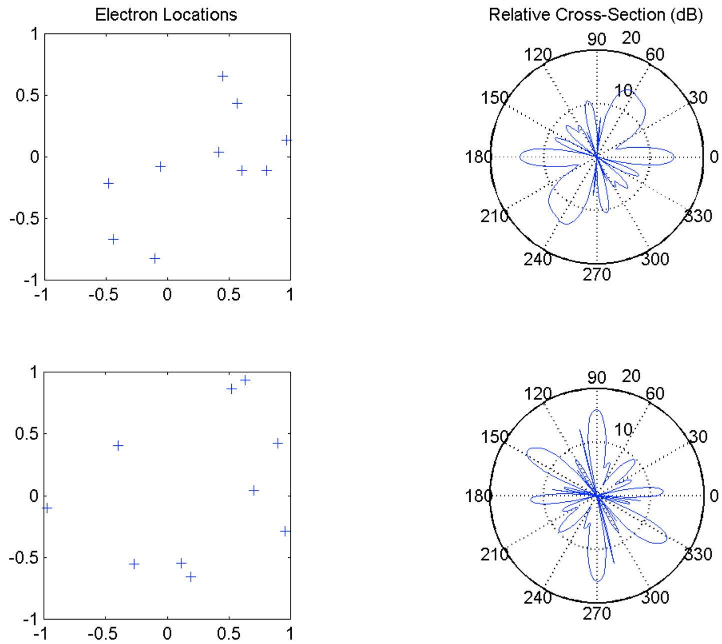
The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

How do pull out the Doppler frequency from the baseband signal?





How is ISR different?



Electrons change their trajectory long before the next pulse is sent. So can't do Doppler as previously described.

Alternately stated:

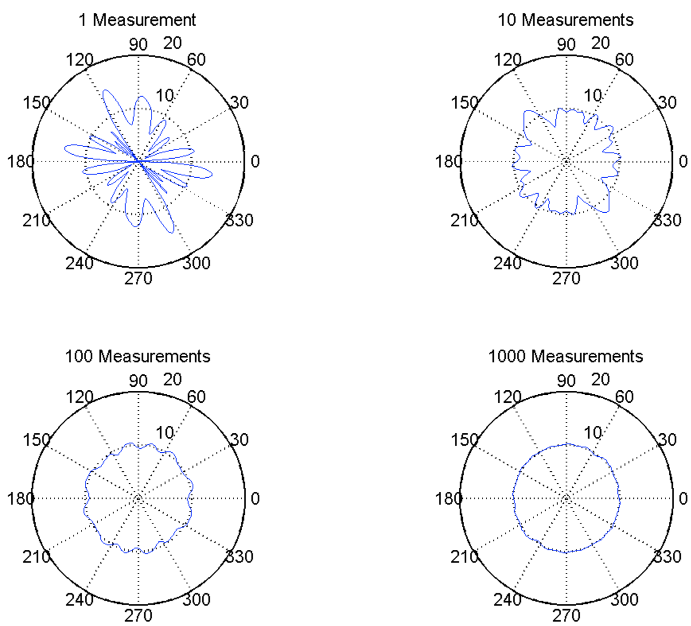
1. Correlation time is short copared to the IPP

2. Dopper frequency shift is large compared to PRF

Target is "Overspread"

How is ISR different?

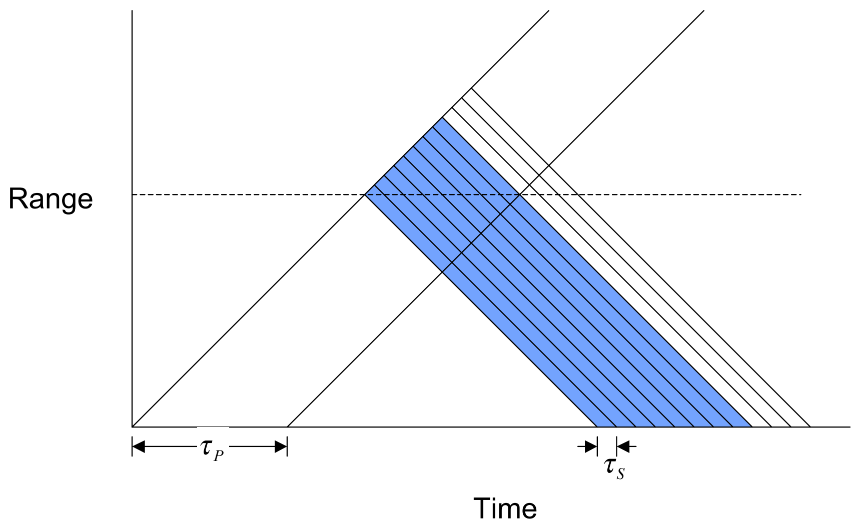
Averaging in time removes the randomness. We're uncover the mean behavior:
The power spectral density of the stochastic process



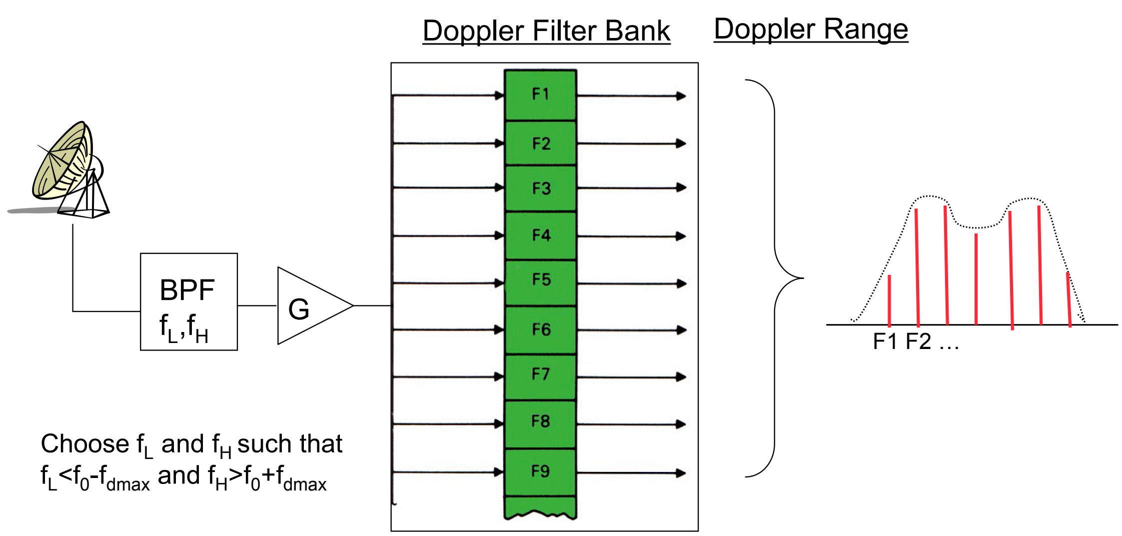
Sampling in ISR

$f_d >> 1/\tau$ (Doppler changes significantly during one pulse)

- Must sample multiple times per pulse
- Result: Doppler can be determined from single pulse.



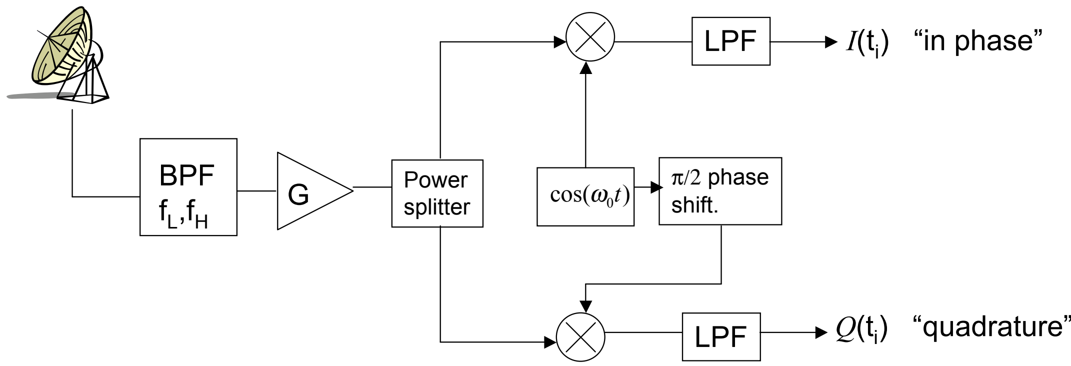
Receiver Approach 1: Doppler Filter Bank



Practical Problem: It is hard to make narrow band (High Q) RF filters:

$$Q = \frac{f_0}{f_H - f_L}$$

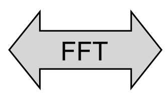
Receiver Approach 2: *I* and *Q* plus correlation



We have time series of *V(t) =I(t) + jQ(t)*, how do I compute the Doppler spectrum?

Estimate the autocorrelation function (ACF) by computing products of complex voltages ("lag products")

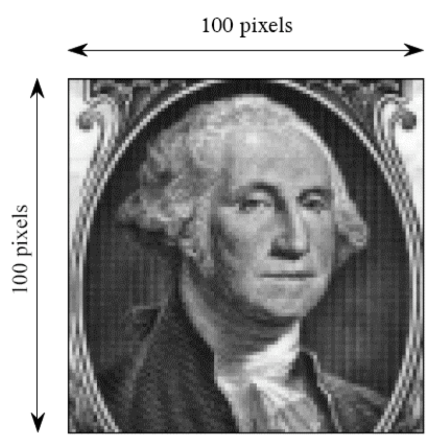
$$\rho(\tau) = \frac{\left\langle V(t)V^*(t+\tau)\right\rangle}{S}$$



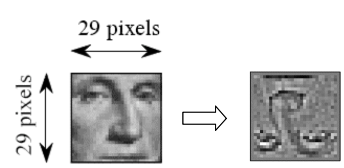
Power spectrum is Fourier Transform of the ACF

Pulse compression and matched filtering

"If you know what you're looking for, it's easier to find."

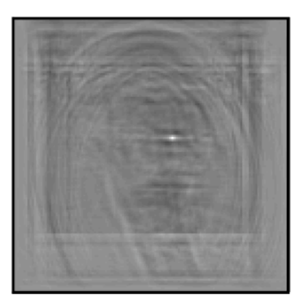


a. Image to be searched



b. Target

c. Kernel



Problem: Find the precise location of the target in the image.
Solution: Correlation

Range detection: revisited

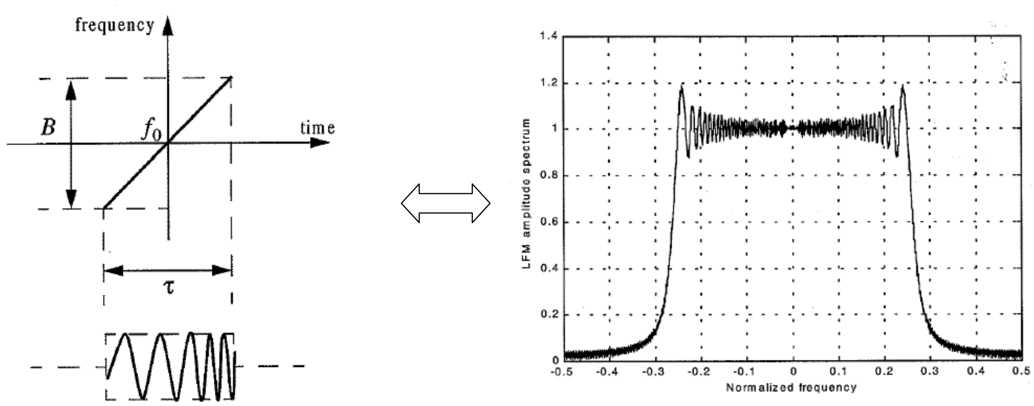
$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B}$$

$\tau =$ Pulse length

$B =$ Bandwidth

- **For high range resolution we want short pulse ⇔ large bandwidth**
- **For high SNR we want long pulse ⇔ small bandwidth**
- **Long pulse also uses a lot of the duty cycle, can't listen as long, affects maximum range**
- **The Goal of pulse compression is to increase the bandwidth (equivalent to increasing the range resolution) while retaining large pulse energy.**

Linear Frequency Modulation (LFM or "Chirp")

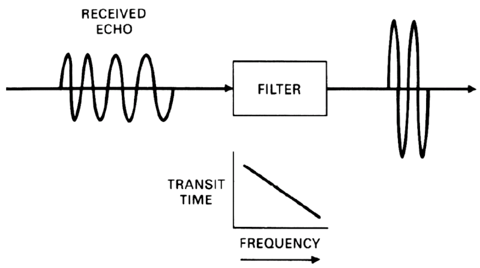


$$s_1(t) = e^{j2\pi f_0 t} s(t)$$

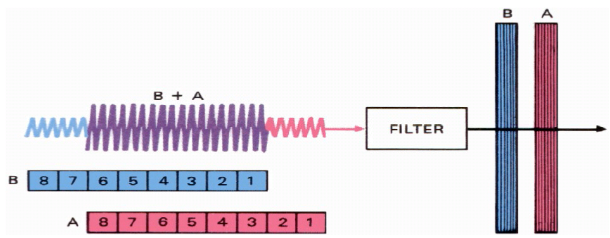
where

$$s(t) = Rect\left(\frac{t}{\tau}\right) e^{j\pi\mu t^2}$$

Matched filter detection of Chirp

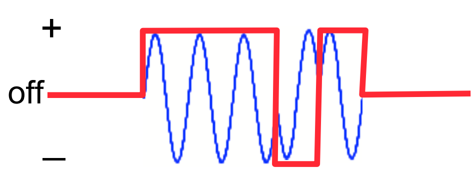


Since trailing portions of echo take less time to pass through filter, successive portions tend to bunch up: Amplitude of pulse is increased and width is decreased.



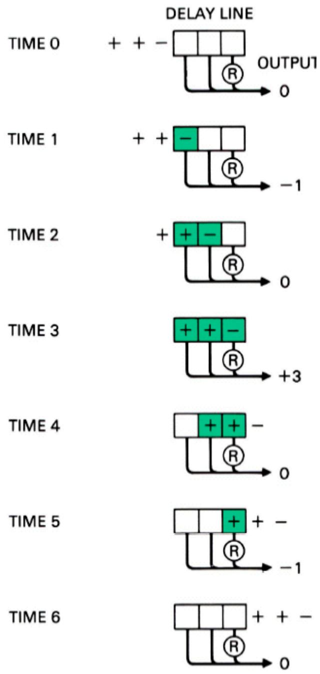
Echoes from closely spaced targets, A and B, are merged but, because of coding, separate in output of filter.

Example: 5-baud Barker coded pulse



				+	+	+	-	+	correlator output
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

Matched filtering with Barker Codes



15. Step-by-step progress of a 3-digit binary phase modulated pulse through a tapped delay line.

3 bit code

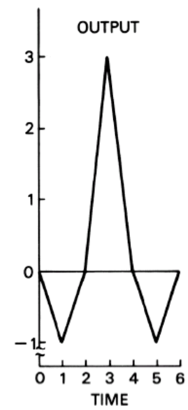


TABLE 6.2 All Known Binary Barker Codes

Code Length	Code
2	11 or 10
3	110
4	1110 or 1101
5	11101
7	1110010
11	11100010010
13	1111100110101