# Why do we care about conductivities?

Ionosphere is a plasma with an embedded magnetic field.

$$\nabla \cdot [\sigma \cdot (\mathbf{E}(\mathbf{r},t) + \mathbf{U}(\mathbf{r},t) \times \mathbf{B})] = 0$$

**"The resulting electric field is as rich and complex as the driving wind field and the conductivity pattern that produce it", Kelley, Ch. 3**

# Equations of Motion

Parallel equation of motion

$$qE = m_i \nu_{in} u_i \qquad -eE = m_e \nu_{en} u_e$$

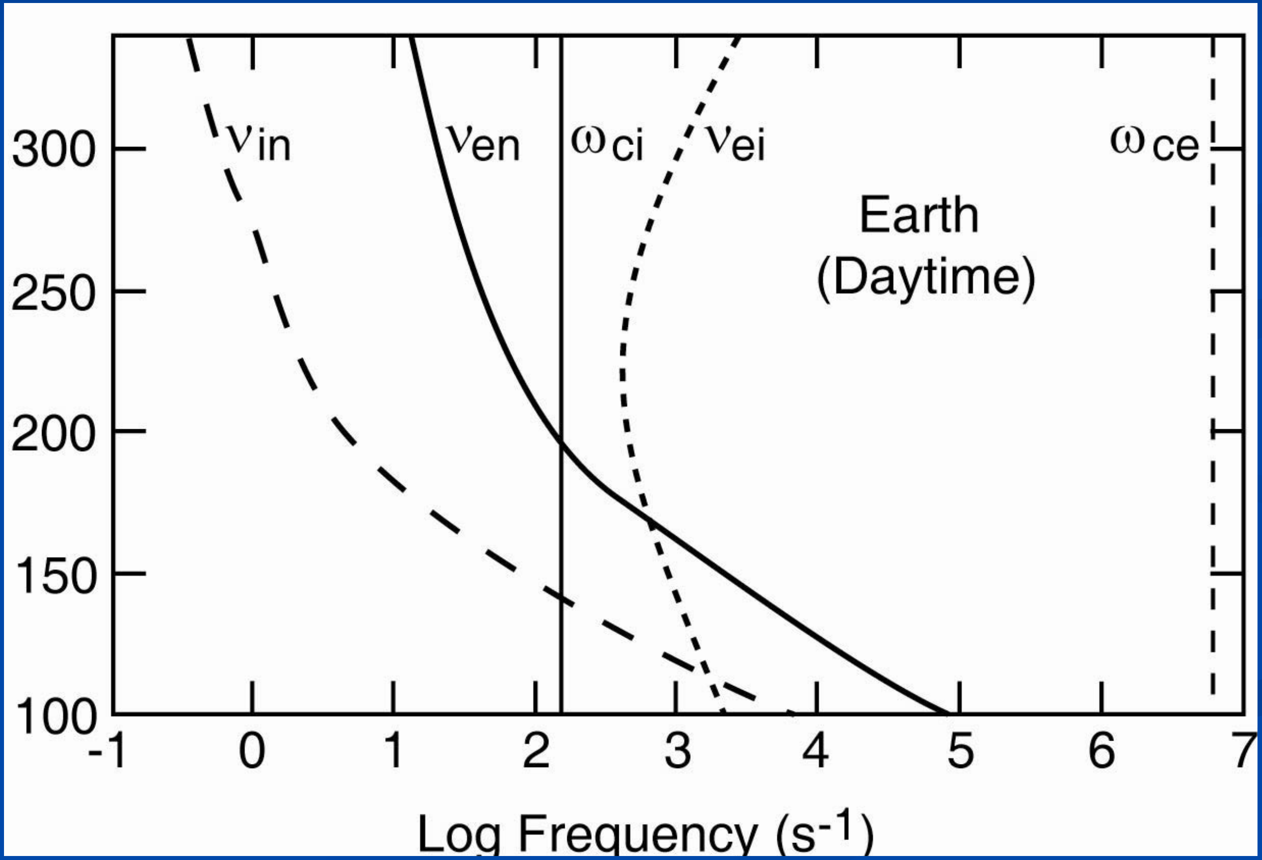
Perpendicular equation of motion

$$q(\mathbf{E}_\perp + \mathbf{u}_i \times \mathbf{B}) = m_i \nu_{in} \mathbf{u}_{\perp i}$$

$$-e(\mathbf{E}_\perp + \mathbf{u}_e \times \mathbf{B}) = m_e \nu_{en} \mathbf{u}_{\perp e}$$

# Collision Frequencies

Ion and electrons collide with neutrals as they gyrate. How they move in response to electric fields depends very much on the collision frequency relative to the gyro-frequency.



# Conductivity

$$\sigma_1 = [\frac{1}{m_e \nu_{en}}(\frac{\nu_{en}^{\;2}}{\nu_{en}^2 + \Omega_e^2}) + \frac{1}{m_i \nu_{in}}(\frac{\nu_{in}^{\;2}}{\nu_{in}^2 + \Omega_i^2})]n_e e^2$$

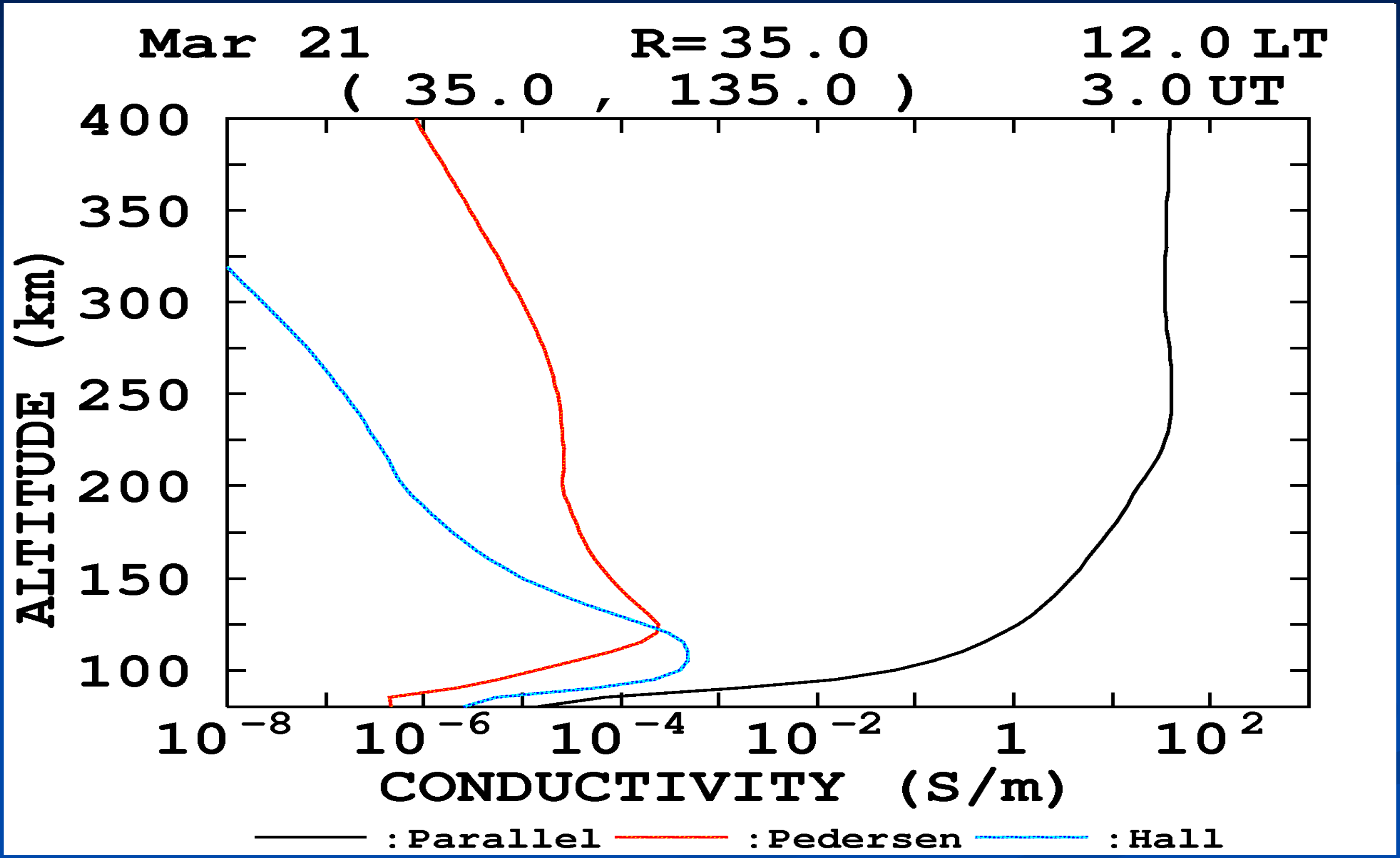
$$\sigma_2 = [\frac{1}{m_e \nu_{en}}(\frac{\Omega_e \nu_{en}}{\nu_{en}^2 + \Omega_e^2}) - \frac{1}{m_i \nu_{in}}(\frac{\Omega_i \nu_{in}}{\nu_{in}^2 + \Omega_i^2})]n_e e^2$$

$$\sigma_0 = [\frac{1}{m_e \nu_{en}} + \frac{1}{m_i \nu_{in}}]n_e e^2$$

$$j = \begin{pmatrix} \sigma_1 & \sigma_2 & 0 \\ -\sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

- **Pedersen conductivity (along $E_\perp$) perpendicular B, parallel E; horizontal**
- **Hall conductivity (along E x B)**
- **Parallel conductivity**
- Conductivity tensor

http://wdc.kugi.kyoto-u.ac.jp/ionocond/exp/icexp.html

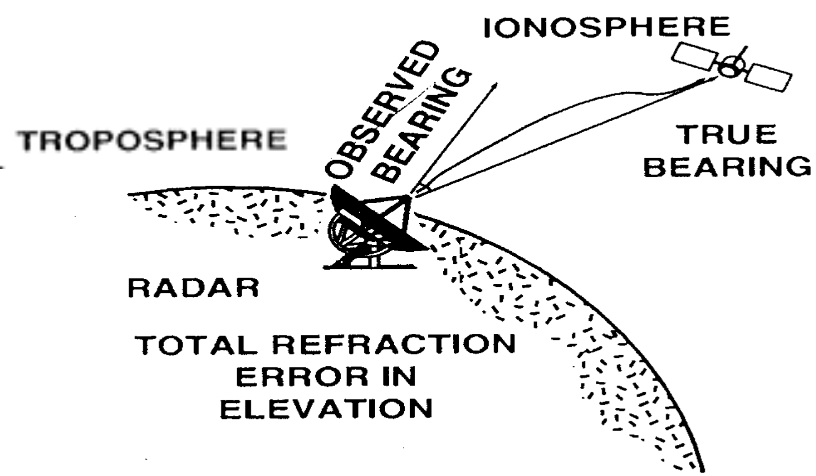


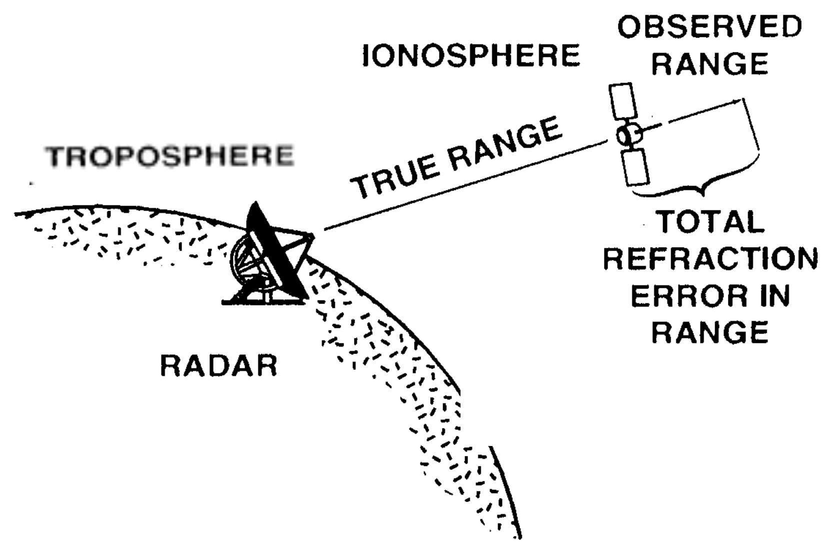
# Introduction to the IONOSPHERE

3. Index of refraction; definition of plasma frequency and gyrofrequency (introduction of magnetic field).

# Radio Waves Refract just like light

# Illustration of Atmospheric Effects



# INDEX OF REFRACTION

$$n^2 = 1 - \frac{X(1-X)}{(1-X) - \frac{1}{2}Y_T^2 \pm \left(\frac{1}{4}Y_T^4 + (1-X)^2 Y_L^2\right)^{1/2}}$$

$$X = \frac{\omega_N^2}{\omega^2} \qquad Y = \frac{\omega_H}{\omega} \qquad \omega_N = \left(\frac{Ne^2}{\varepsilon_0 m_e}\right)^{1/2} \qquad \omega_H = \frac{e|B|}{m_e}$$

$\omega$ = the angular frequency of the radar wave,

$Y_L$ = $Y\cos\theta$, $Y_T$ = $Y\sin\theta$,

$\theta$ = angle between the wave vector $\bar{k}$ and $\bar{B}$,

$\bar{k}$ = wave vector of propagating radiation,

$\bar{B}$ = geomagnetic field, N = electron density

e = electronic charge, $m_e$ = electron mass,

and $\varepsilon_0$ = permittivity constant.

4. Debye length/Debye sphere why important

# 5. Debye length/Debye sphere

The Debye length is a measure of the plasma’s ability to shield out electric potentials that are applied to it.

The Debye length marks the division between different regimes of plasma’s behavior; i.e. collective plasma motion versus that of individual particle motion.

Plasma phenomenon that take place over distances greater than the Debye length must be described in terms of collective behavior of the plasma.

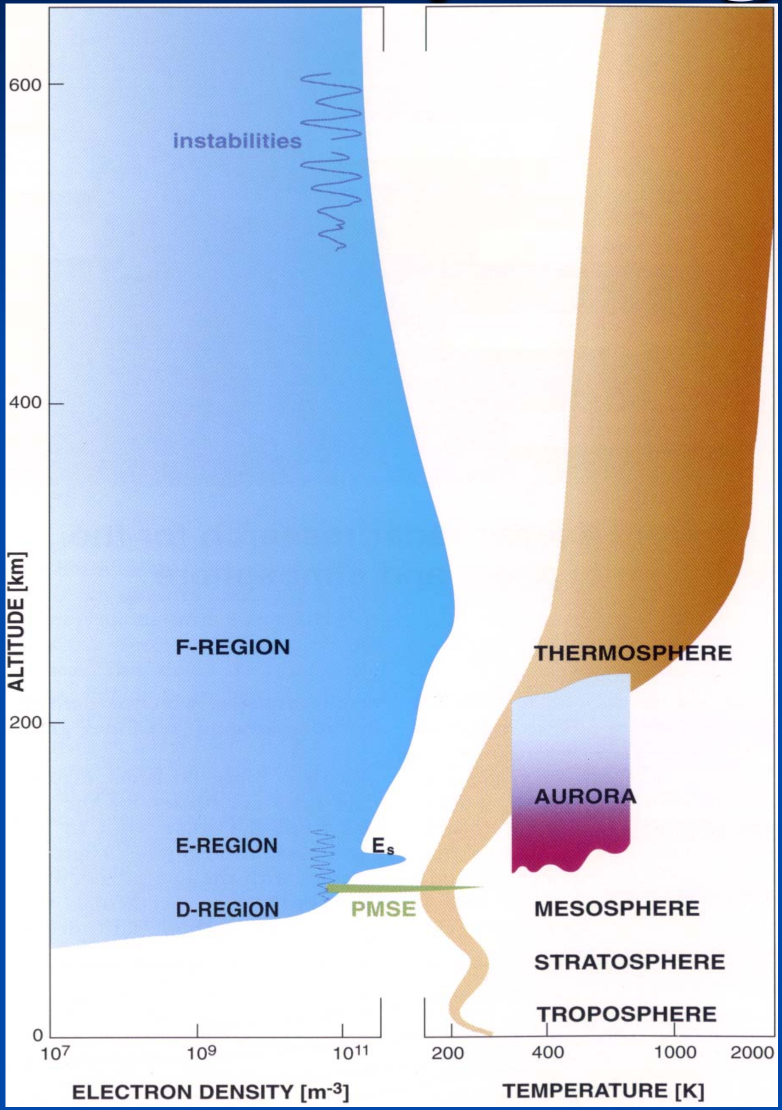
# Debye Length

1. Plasma will not support large potential variations (i.e. will seek to maintain charge neutrality) over distances larger than the Debye length.

2. Potential gradients that do exist have a characteristic length parameter equal to a Debye length

3. These potential gradients are characterized by a natural oscillation frequency known as the plasma frequency.

# Debye length dependence



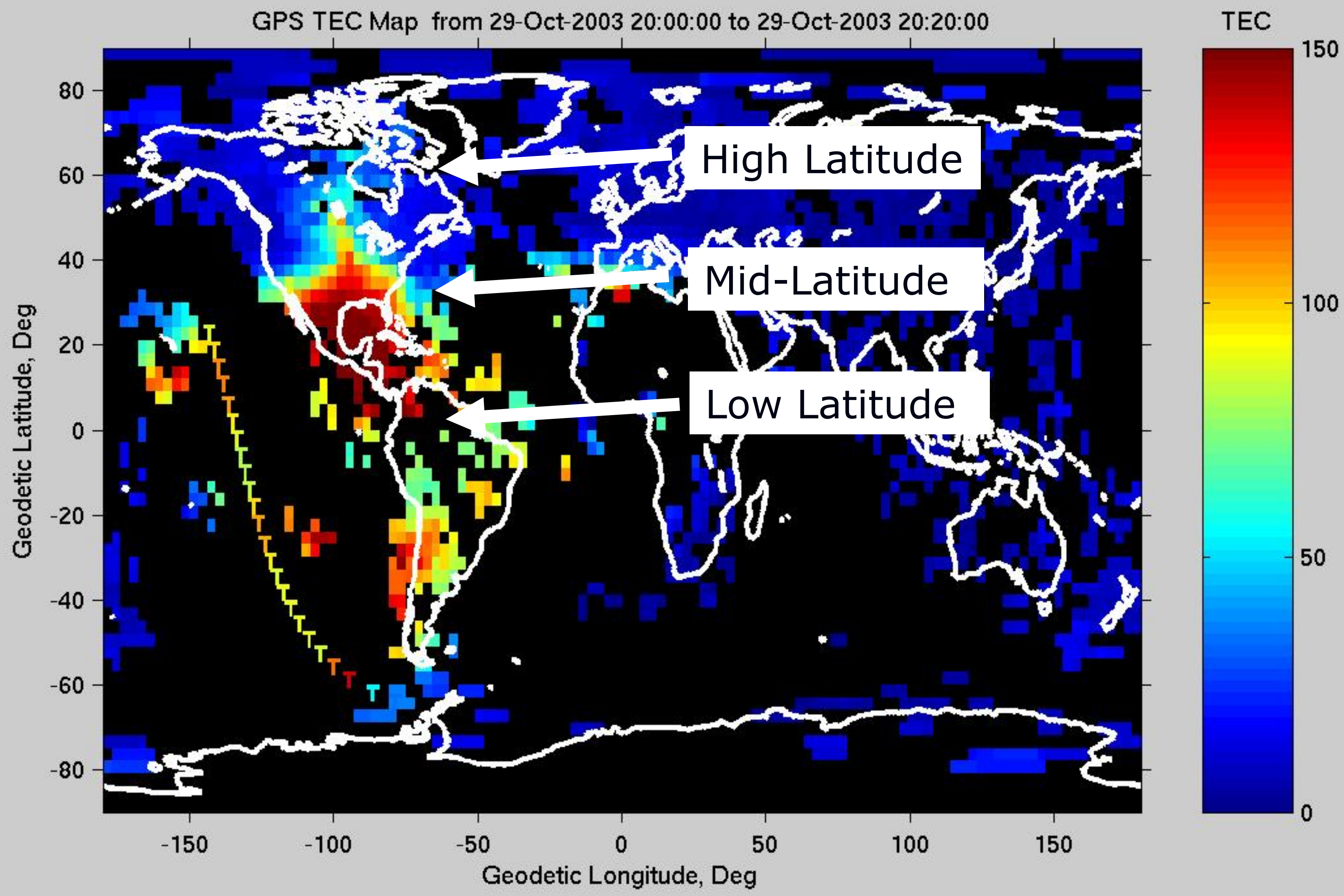

$$\lambda_D \simeq 69\sqrt{T_e/n_e}$$

The Debye length is increasing with altitude - from a few millimeter in the D-region up to meters in the magnetosphere

Debye length in E and F region is 0.1 to 1 cm.

Day302 PM

MIT Haystack Observatory



# World Incoherent Scatter Radars