Introduction to ISR Signal Processing

Joshua Semeter, Boston University

Why study ISR?

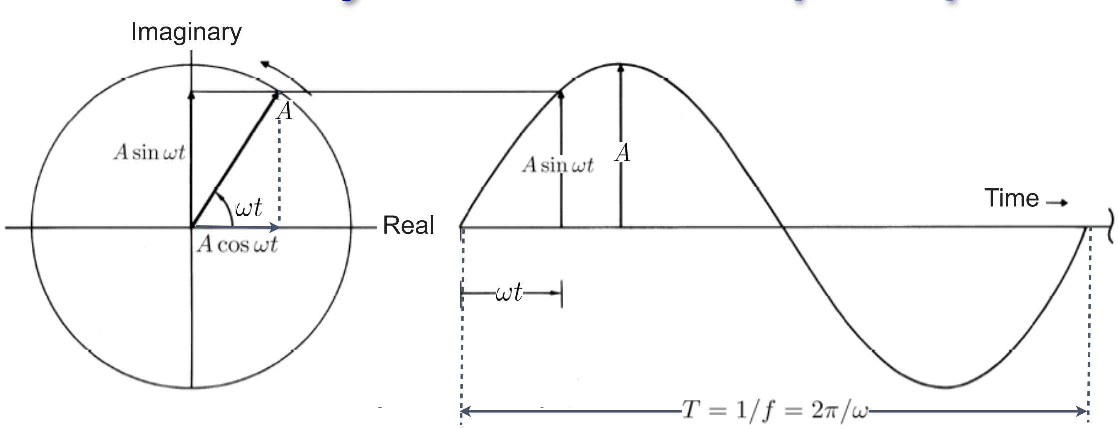
Requires that you learn about a great many useful and fascinating subjects in substantial depth.

- Plasma physics
- Radar
- Coding (information theory)
- Electronics (Power, RF, DSP)
- Signal Processing
- Inverse theory

Outline

- Mathematical toolbox
- Review of basic radar concepts
- Ionospheric Doppler spectrum
- Range resolution and matched filtering
- I/Q demodulation
- Autocorrelation function (ACF) and Power Spectral Density (PSD)

Euler identity and the complex plane



ω is the "angular velocity" (radians/s) of the spinning arrow

f is the number of complete rotations (2π radians) in one second (1/s or Hz)

We need a signal that tells us **how fast** and in **which direction** the arrow is spinning. This signal is the complex exponential. Invoking the Euler identity,

$$s(t) = Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t = I + jQ$$

I = in-phase component
Q = in-quadrature component

Essential mathematical operations

<u>Fourier Transform:</u> Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

<u>Convolution:</u> Expresses the action of a linear, time-invariant system on a function**.**

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(\tau - t) d\tau \qquad f(t) * g(t) \Longleftrightarrow F(f)G(f)$$

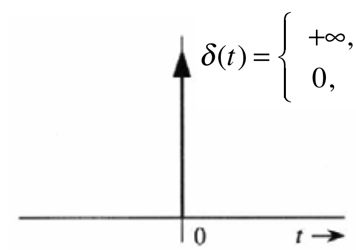
<u>Correlation:</u> A measure of the degree to which two functions look alike at a given offset.

$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau) g(t + \tau) d\tau \qquad f(t) \circ g(t) \Longleftrightarrow F^*(f)G(f)$$

<u>Autocorrelation, Convolution, Power Spectral Density, Wiener-Khinchin Theorem</u>

$$R_{uu} = u(t) \circ u(t) = u(t) * u^*(-t) \qquad R_{uu} \Longleftrightarrow |U(f)|^2$$

Dirac Delta Function



$\delta(t)$ is defined by the property that for all continuous functions

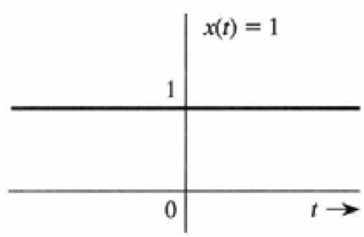
$$f(0) = \int_{-\infty}^{+\infty} \delta(t) f(t) dt$$

$$f(t - T) = f(t) * \delta(t - T)$$

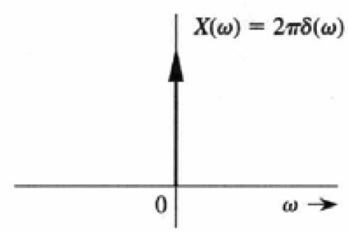
The Fourier Transform of a train of delta functions is a train of delta functions.

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

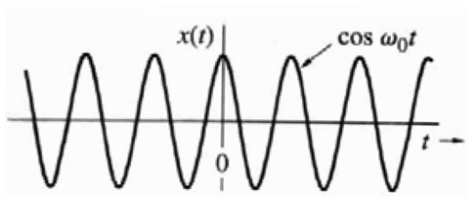
Harmonic Functions



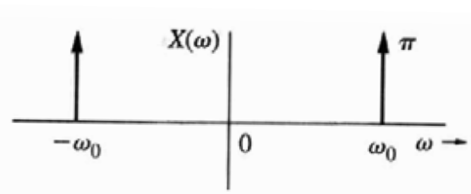
$\Longleftrightarrow$



$$\cos\,\omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$



$\Longleftrightarrow$

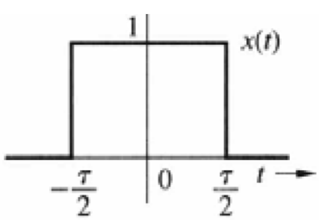


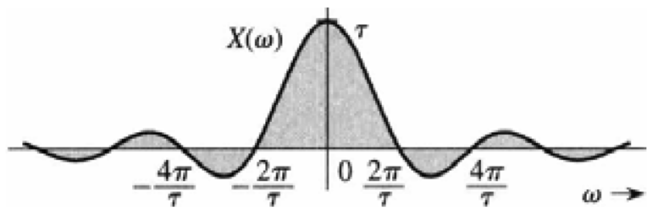
$$\sin\,\omega_0 t \quad \Longleftrightarrow \quad j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$e^{j\omega_0 t} \quad \Longleftrightarrow \quad 2\pi\delta(\omega - \omega_0)$$

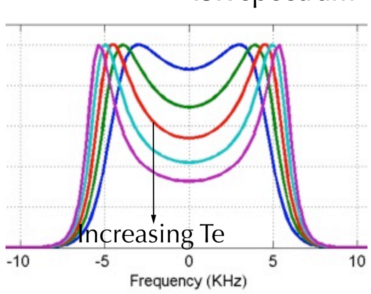
Gate function

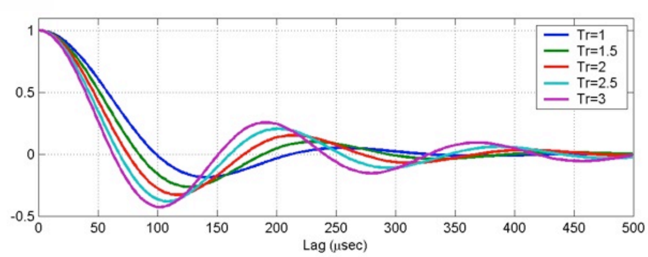
$$\mathrm{rect}(t/\tau) \quad = \begin{cases} 1 & \text{for } -\tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases} \Longleftrightarrow \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\Longleftrightarrow$ Autocorrelation function (ACF)

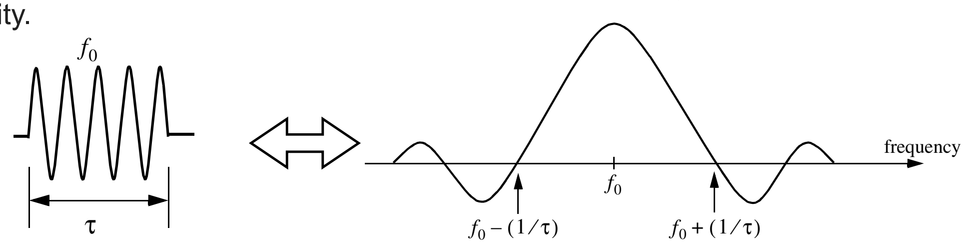




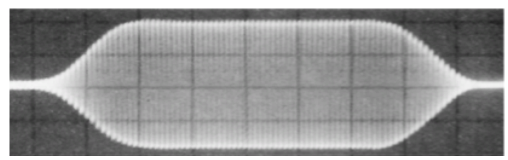
Not surprisingly, the ISR ACF looks like a sinc function…

How it all hangs together.

- Duality:
  - Gate function in the time domain represents amplitude modulation
  - Gate function in the frequency domain represents filtering
- Limiting cases:
  - Gate function approaches delta function as width goes to 0 with constant area
  - A constant function in time domain is a special case of harmonic function where frequency = 0.
  - A constant function in time domain is a special case of a gate function where width = infinity.

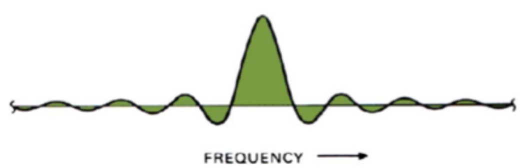


How many cycles are in a typical ISR pulse?
PFISR frequency: 449 MHz
Typical long-pulse length: 480 µs ⇨ 215,520 cycles!

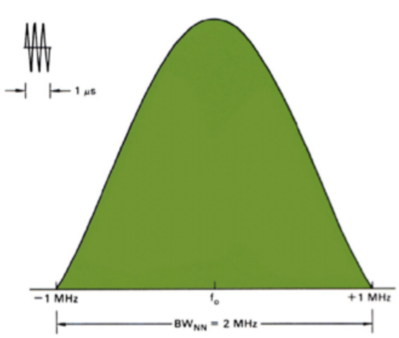


Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe



A 1 microsecond pulse has a null-to-null bandwidth of the central lobe = 2 MHz



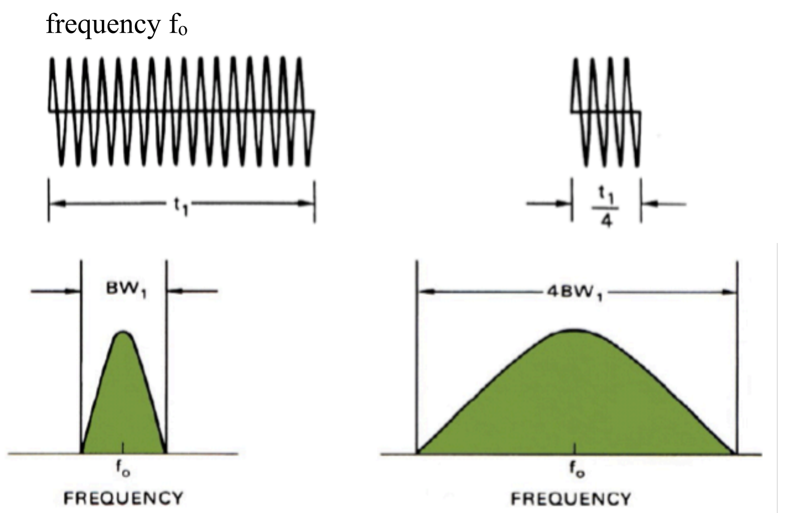
Two possible bandwidth measures:

“null to null” bandwidth $B_{nn} = \frac{2}{\tau}$

“3dB” bandwidth $B_{3dB} = \frac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

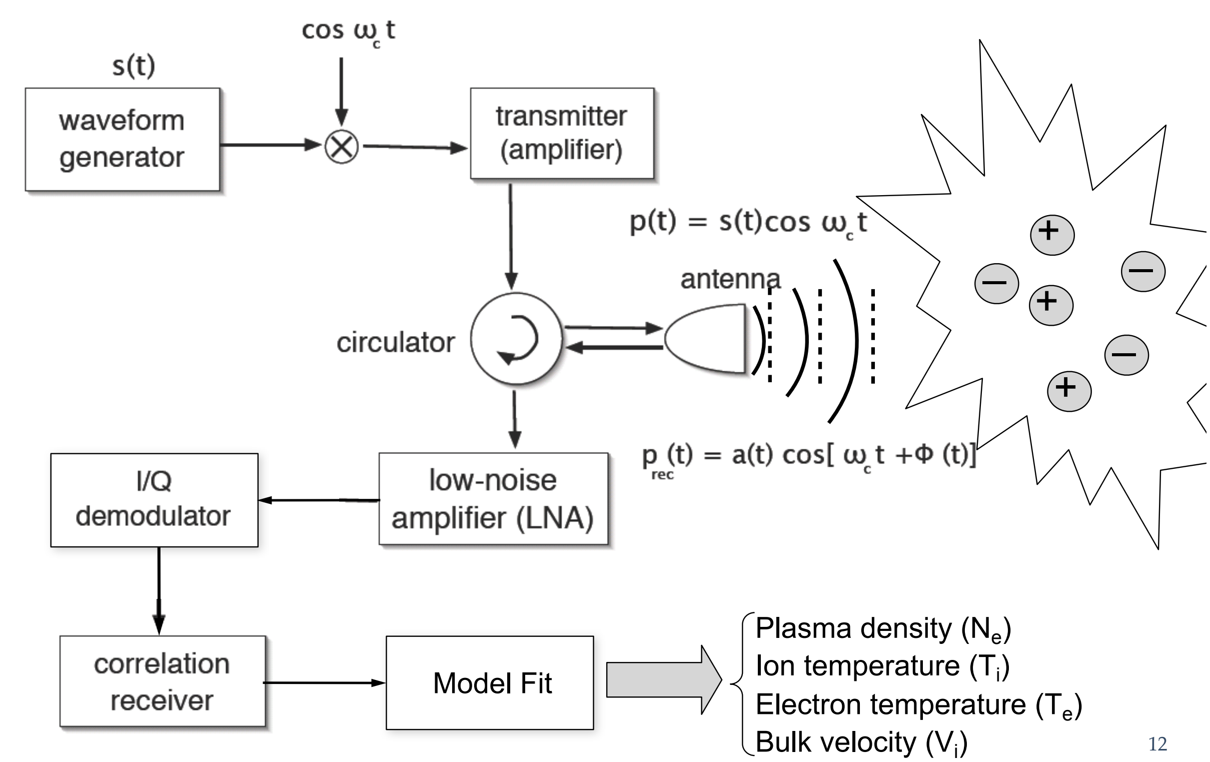
Pulse-Bandwidth Connection



Shorter pulse ⟺ Larger bandwidth

Faster sampling rate ⟺ Larger bandwidth

Components of a Pulsed Doppler Radar



The deciBel (dB)

The relative value of two quantities expressed on a logarithmic scale

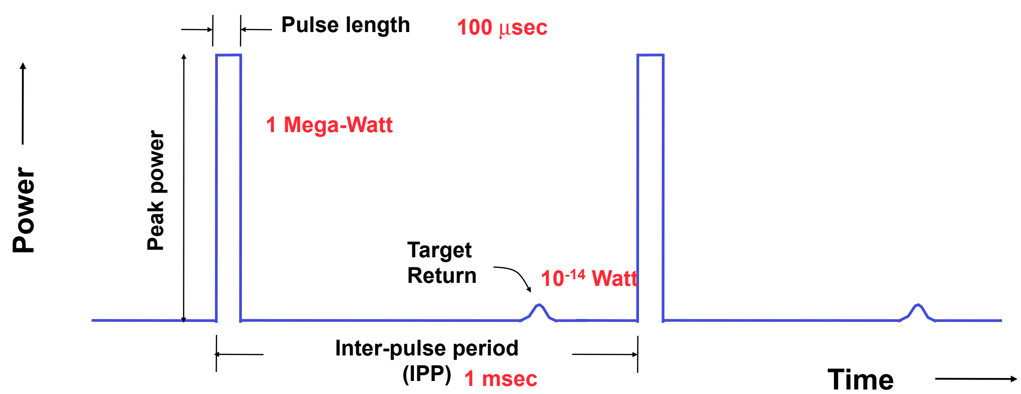
$$SNR = 10 \log_{10} \frac{P_1}{P_2} = 20 \log_{10} \frac{V_1}{V_2} \qquad (Power \propto Voltage^2)$$

Factor of:	Scientific Notation	dB
0.1	10^{-1}	-10
0.5	$10^{0.3}$	-3
1	10^{0}	0
2	$10^{0.3}$	3
10	10^{1}	10
100	10^{2}	20
1000	10^{3}	30
1,000,000	10^{6}	60

Other forms used in radar:

dBW	dB relative to 1 Watt
dBm	dB relative to 1 mW
dBsm	dB relative to 1 m^2 of radar cross section
dBi	dB relative to isotropic radiation

Pulsed Radar



$$\text{Duty cycle} = \frac{\text{Pulse length}}{\text{Pulse repetition interval}} \qquad 10\%$$

Average power = Peak power * Duty cycle — 100 kWatt

Pulse repetition frequency (PRF) = 1/(IPP) — 1 kHz

Continuous wave (CW) radar: Duty cycle = 100% (always on)

Doppler Frequency Shift

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t + \frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + vt$$

Substituting:

$$\cos\left[2\pi\left(f_o + \underbrace{f_o\frac{2v}{c}}_{-f_D}\right)t + \underbrace{\frac{2_o R_o}{c}}_{\text{constant}}\right]$$

$$f_D = -2f_o\left(\frac{v}{c}\right) = -2\left(\frac{v}{\lambda_o}\right) \propto \frac{\text{line-of-sight velocity}}{\text{radar wavelength}}$$

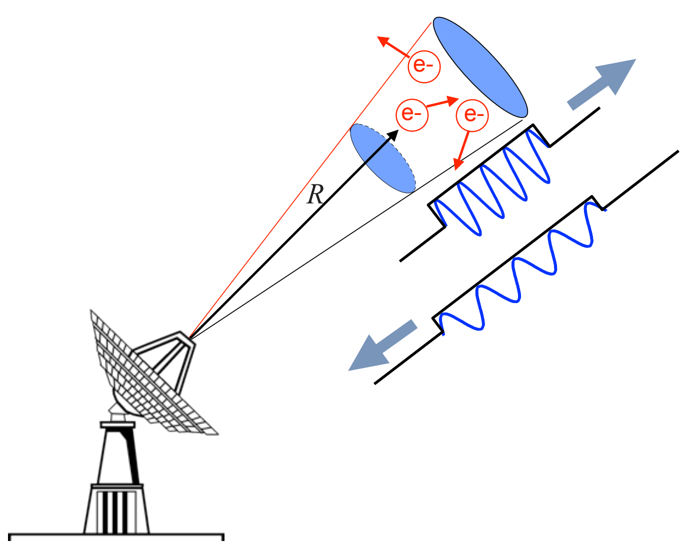
By convention, positive Doppler shift ⟺ Target and radar are "closing"

Two key concepts

Two key concepts:

Distant ⟺ Time
$R = c\Delta t/2$

Velocity ⟺ Frequency
$v = -f_D\lambda_0/2$



A Doppler radar measures backscattered power as a function range and velocity.
Velocity is manifested as a Doppler frequency shift in the received signal.

Two key concepts

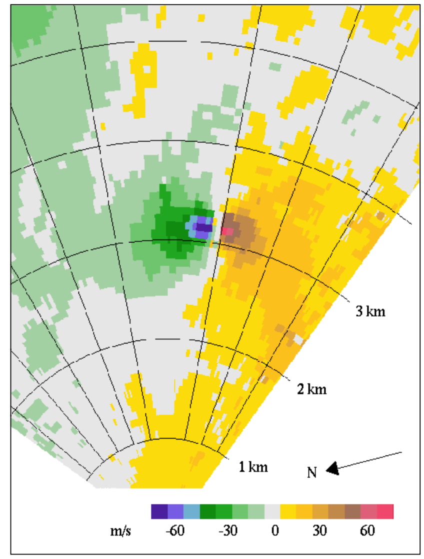
Two key concepts:

Distant ⟺ Time

$R = c\Delta t/2$

Velocity ⟺ Frequency

$v = -f_D\lambda_0/2$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Concept of a "Doppler Spectrum"

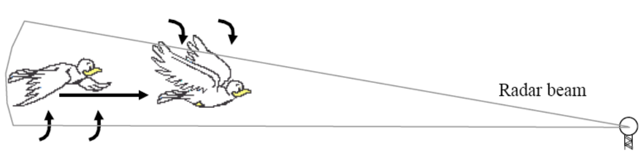
Two key concepts:

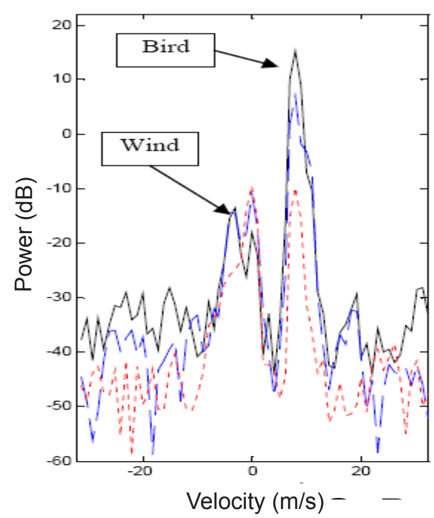
Distant ⟺ Time

$R = c\Delta t/2$

Velocity ⟺ Frequency

$v = -f_D\lambda_0/2$





If there is a distribution of targets moving at different velocities (e.g., electrons in the ionosphere) then there is no single Doppler shift but, rather, a Doppler spectrum.

What is the Doppler spectrum of the ionosphere at UHF (λ of 10 to 30 cm)?

Longitudinal Modes in a Thermal Plasma

Ion-acoustic

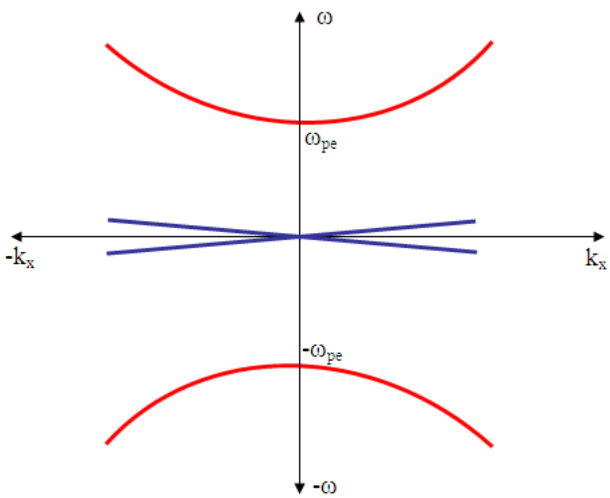
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}^3}{k^3} \frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Simulated ISR Doppler Spectrum

Particle-in-cell (PIC):

$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

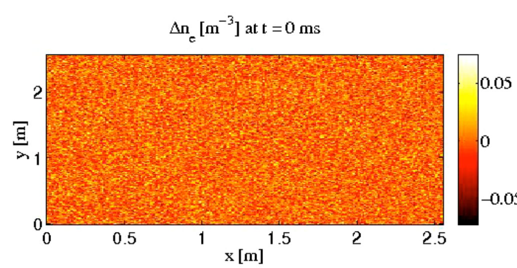
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

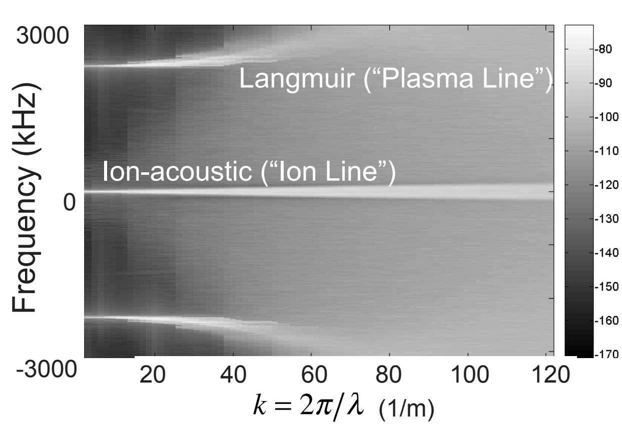
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

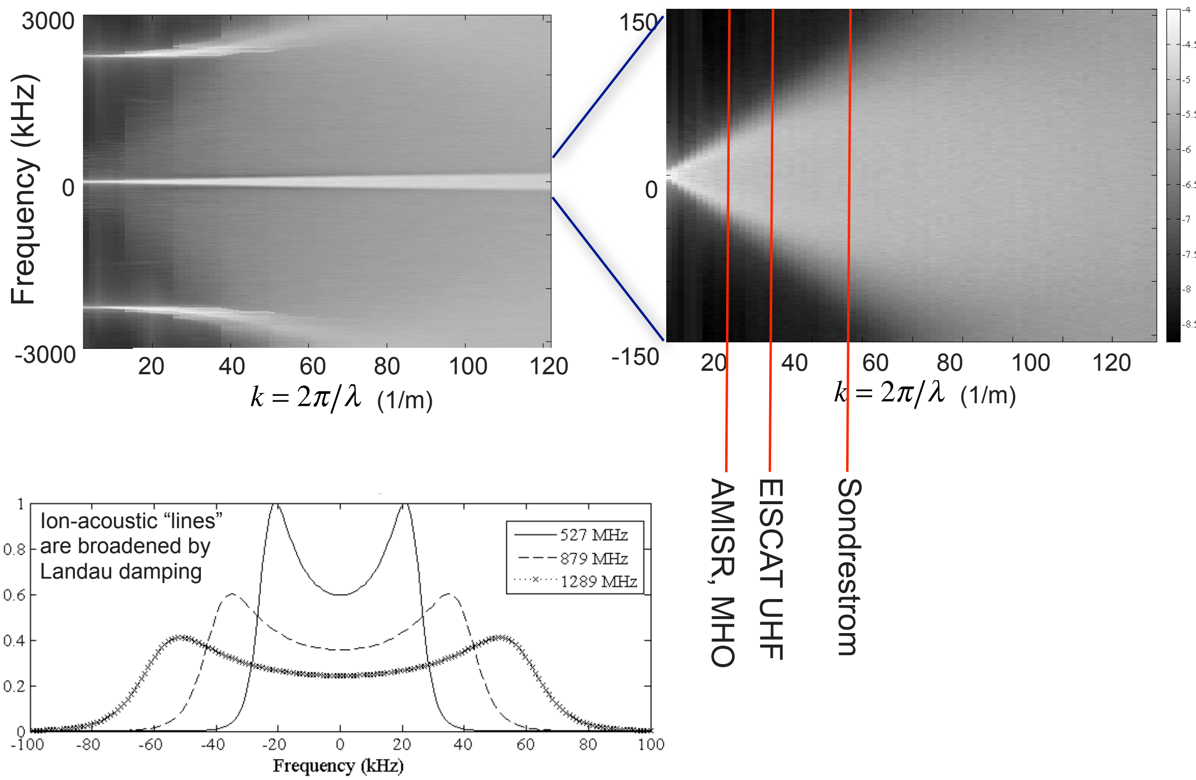
$$\nabla \cdot \mathbf{B} = 0$$

Simple rules yield complex behavior





ISR Measures a Cut Through This Surface



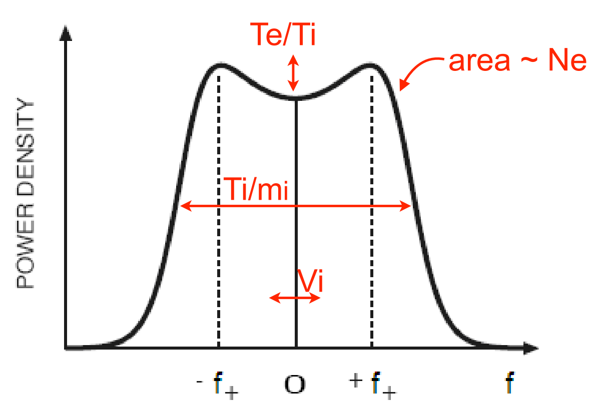
The ISR model

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \left|N_i^0(\omega)\right|^2}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \cdot F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

where:

$$F_e(\omega) = 1 - \omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)d\tau$$

$$F_i(\omega) = 1 - \omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)d\tau$$



From Evans, IEEE Transactions, 1969

ISR in a nutshell

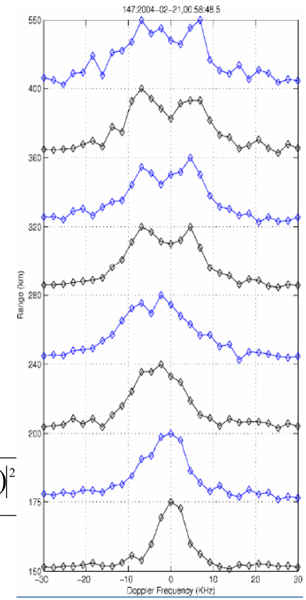
Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

The "radar equation"

$P_r =$ Received power
$P_n =$ Received noise power
$P_t =$ Transmitted power
$\sigma =$ Radar cross section
$G =$ Antenna gain

$A =$ Antenna area
$k_B =$ Boltzman's constant
$T =$ Temperature
$B =$ Bandwidth
$N_{sys} =$ System noise temperature



Here's the theory:

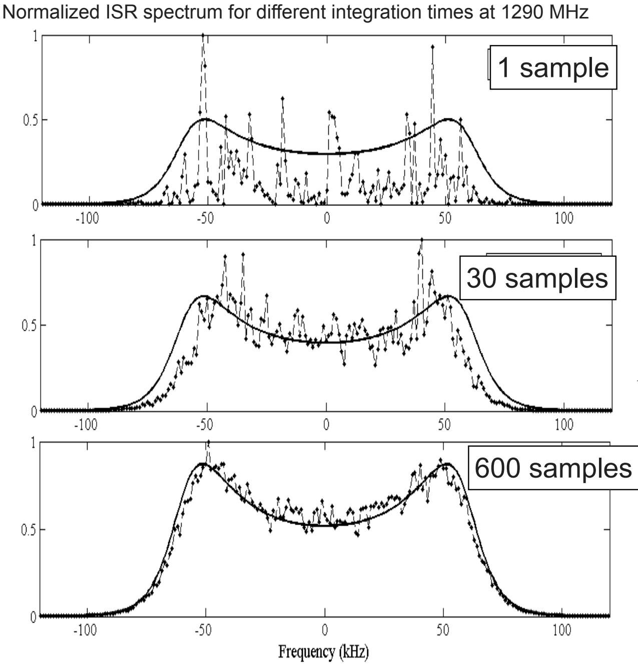
$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{|N_e^0(\omega)|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 |F_e(\omega)|^2 \sum_i \overline{|N_i^0(\omega)|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega\int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

Incoherent Averaging

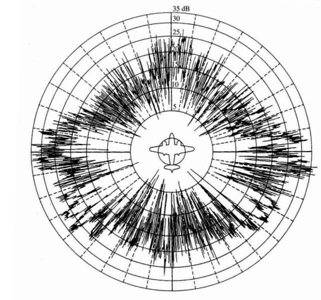
Normalized ISR spectrum for different integration times at 1290 MHz

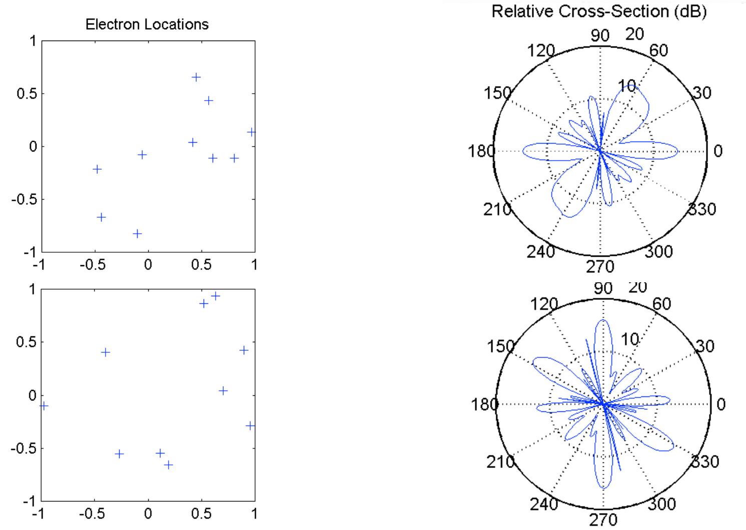


We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

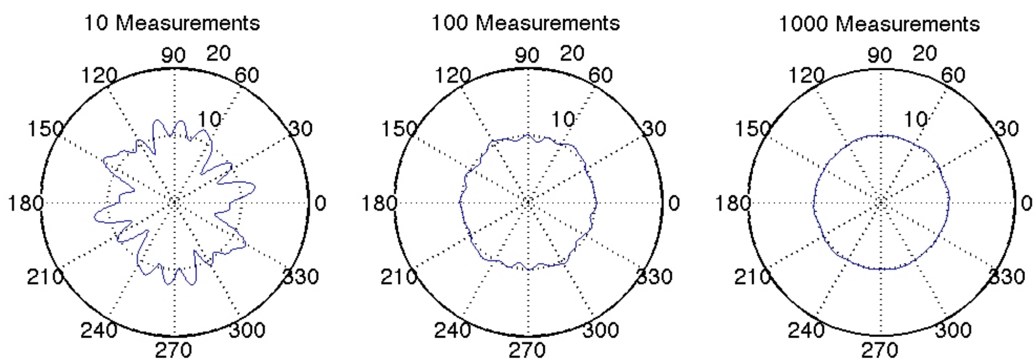
$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

RCS: hard target versus electrons



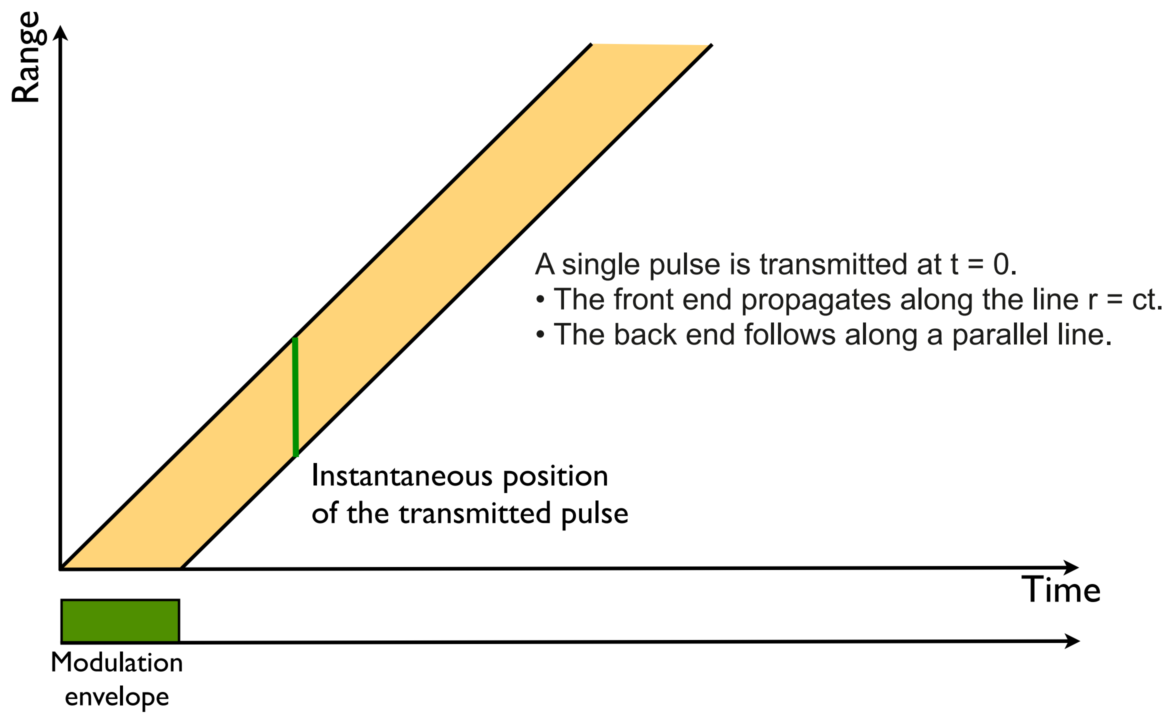


Incoherent integration

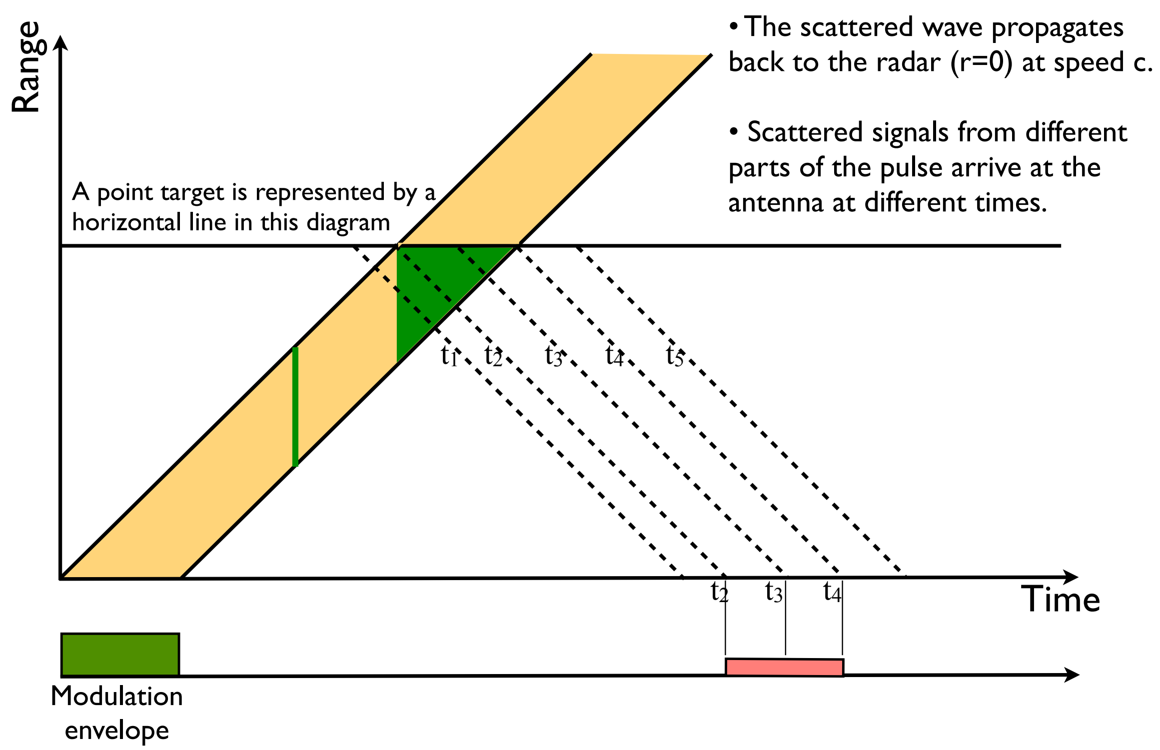


$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

Range-time analysis

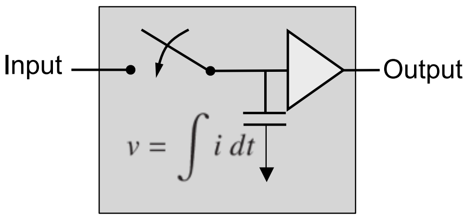


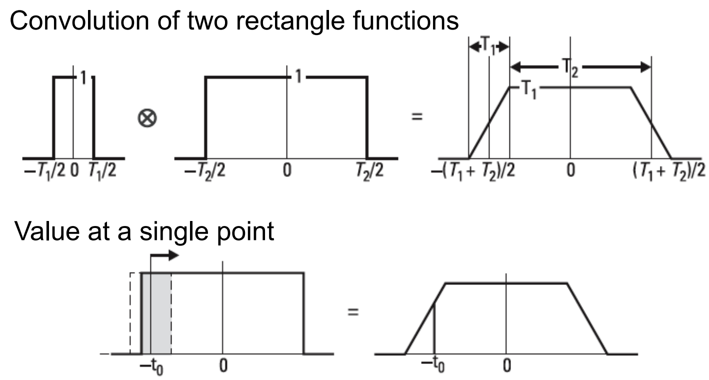
Range-time analysis



Sampling a signal require time-integration

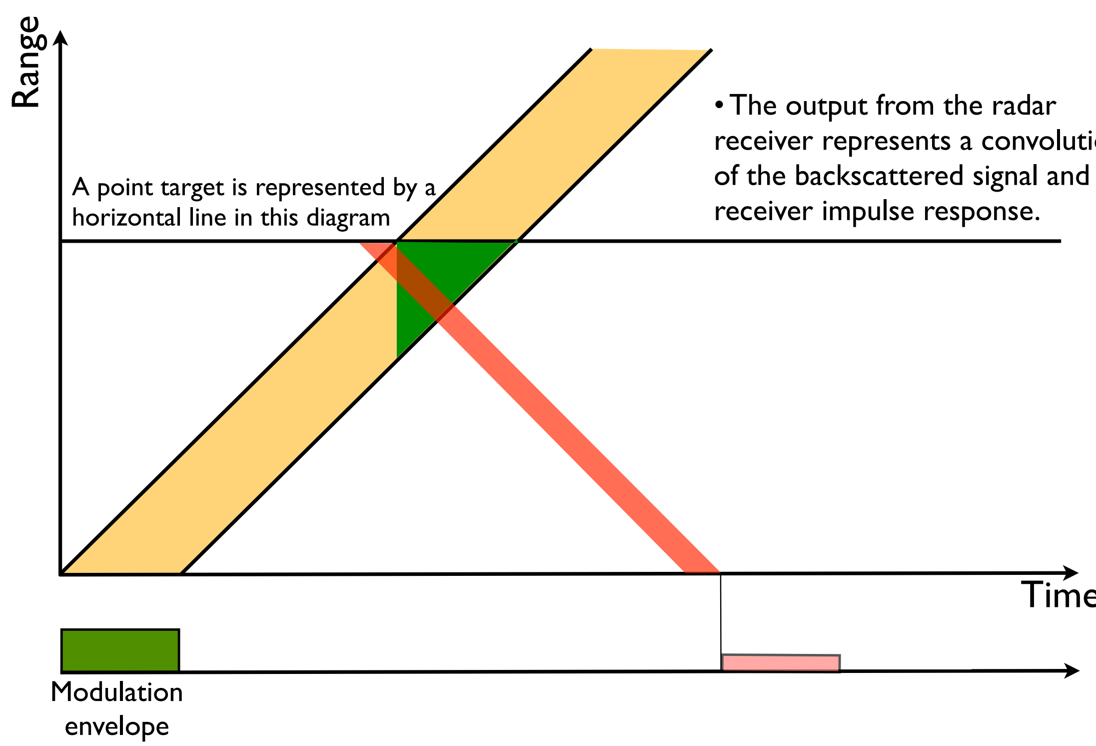
We send a pulse of duration τ. How should we listen for the echo?



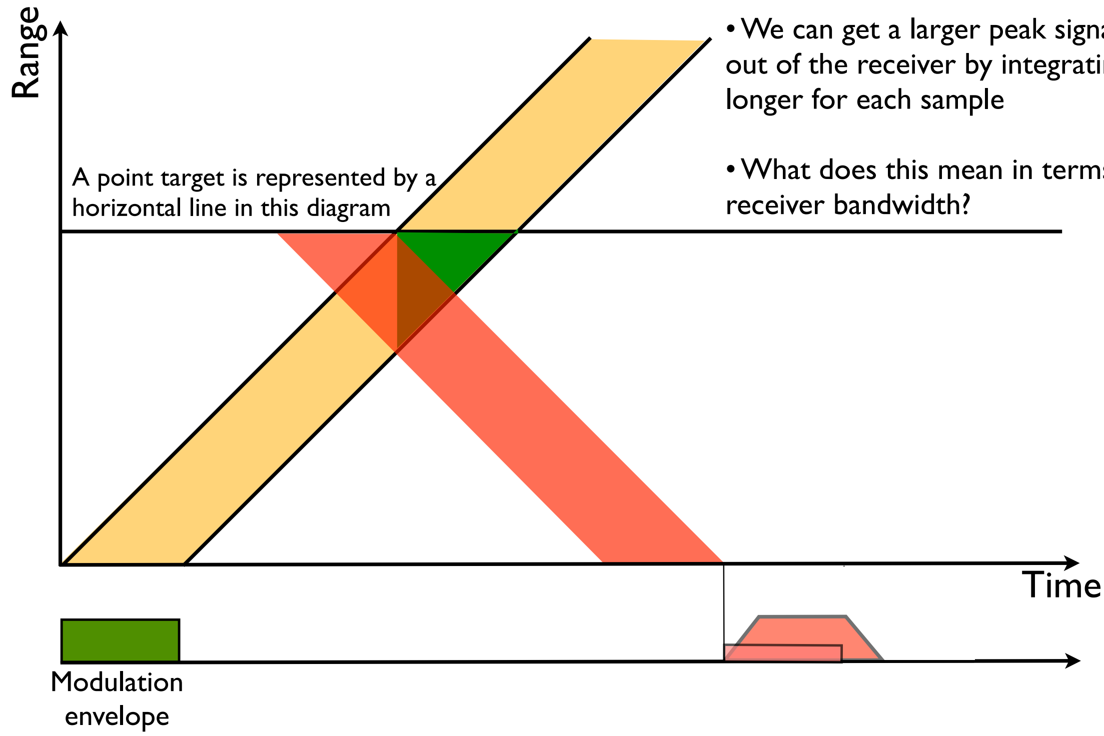


- To determine range to our target, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

Sampling the received signal



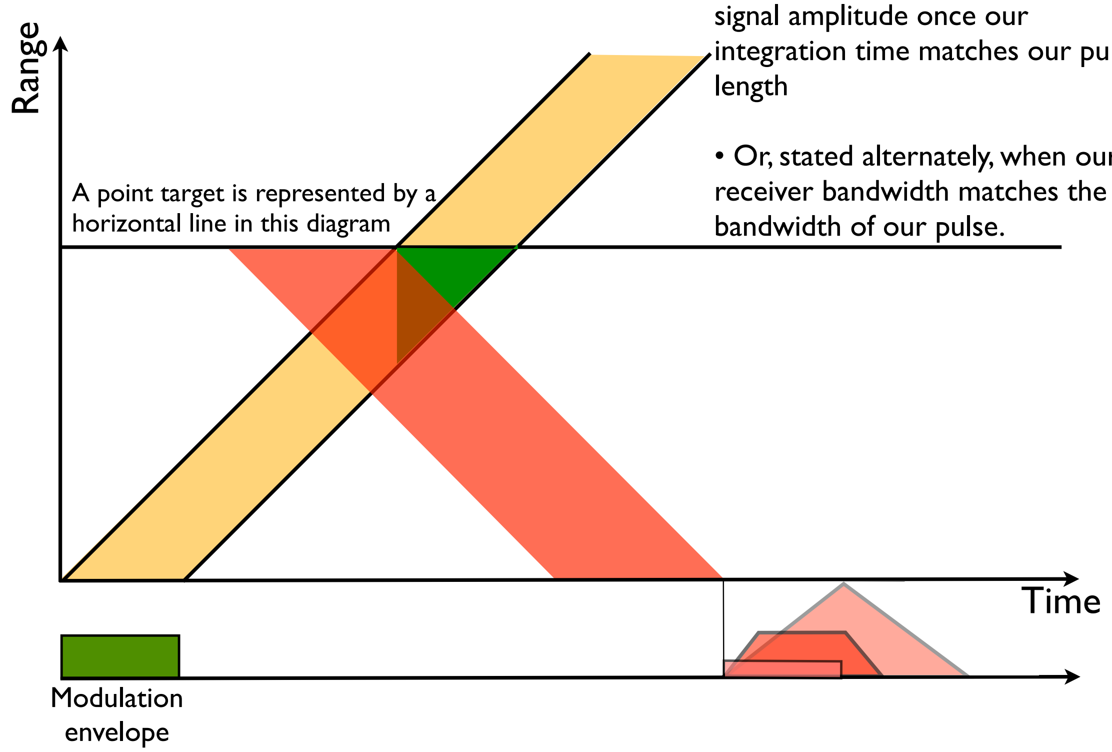
Computing the ACF



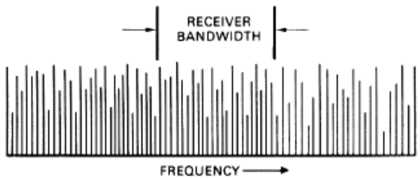
- We can get a larger peak signal out of the receiver by integrating longer for each sample
- What does this mean in terms of receiver bandwidth?

Computing the ACF

- We don't get any more gain in signal amplitude once our integration time matches our pulse length
- Or, stated alternately, when our receiver bandwidth matches the bandwidth of our pulse.

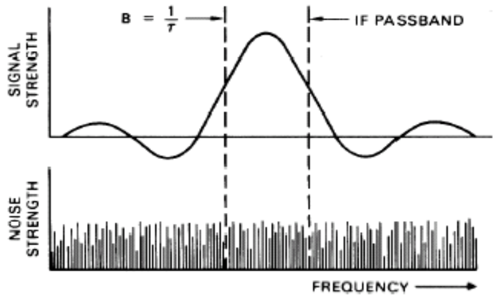


The bandwidth-noise connection



6. Noise in receiver output is proportional to bandwidth of receiver.

The matched filter is a filter whose impulse response, or transfer function, is determined by a given signal, in a way that will result in the maximum attainable signal-to-noise ratio at the filter output when both the signal and white noise are passed through it.

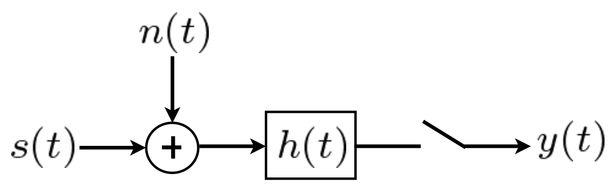


1. Signal-to-noise ratio may be maximized by narrowing the passband of the IF amplifier to the point where only the bulk of the signal energy is passed.

The optimum bandwidth of the filter, B, turns out to be very nearly equal to the inverse of the transmitted pulse width.

To improve range resolution, we can reduce τ (pulse width), but that means increasing the bandwidth of transmitted signal = More noise…

Matched Filter



$$y(t) = \int [s(\tau) + n(\tau)]\, h(t-\tau) d\tau$$

$$= \int H(f) S(f) e^{j2\pi f T} df + \int H(f) N(f) e^{j2\pi f T} df$$

How should we choose *h(t)*⇔*H(f)* such that the output SNR is maximal?

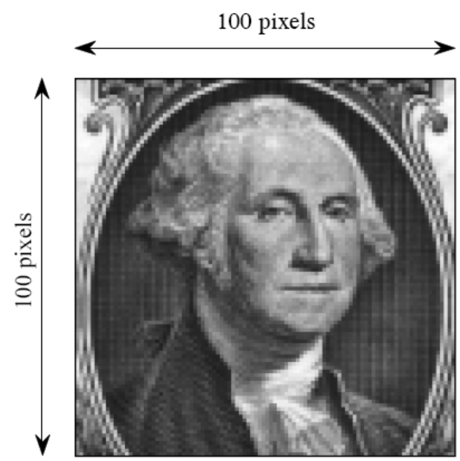
$$SNR = \frac{\left| \int H(f) S(f) e^{j2\pi f T} df \right|^2}{E\left\{ \left| \int H(f) N(f) df \right|^2 \right\}}$$

Assuming white Gaussian noise, it can be shown that max SNR is when

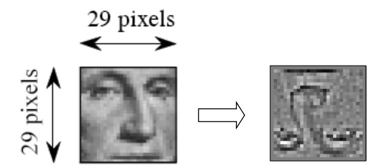
$$\boxed{H(f) = S^*(f) \Longleftrightarrow h(t) = s^*(-t)}$$

Pulse compression and matched filtering

"If you know what you're looking for, it's easier to find."

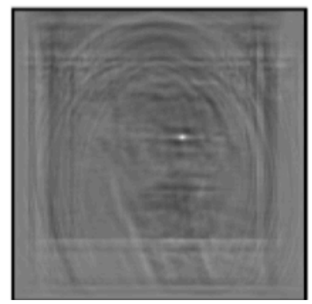


a. Image to be searched



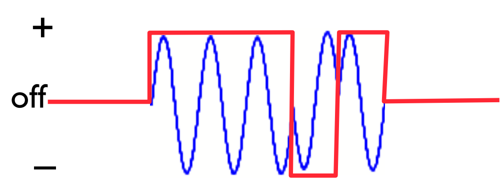
b. Target

c. Kernel



Problem: Find the precise location of the target in the image.
Solution: Correlation

Barker codes



				+	+	+	-	+	correlator output
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

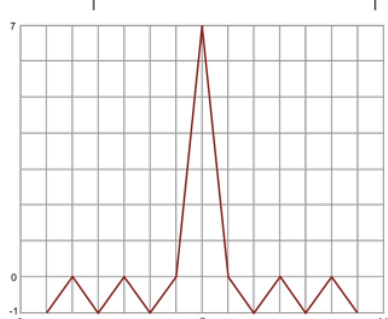
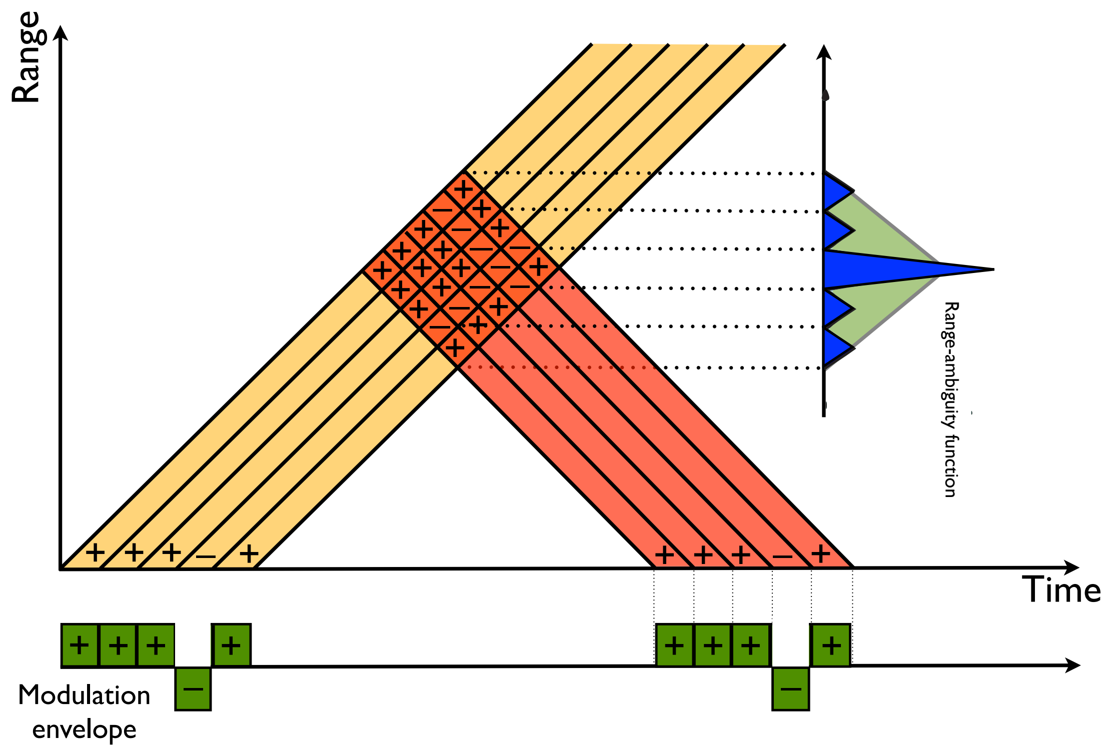


TABLE 6.2 All Known Binary Barker Codes

Code Length	Code
2	11 or 10
3	110
4	1110 or 1101
5	11101
7	1110010
11	11100010010
13	1111100110101

Volume target (e.g., the ionosphere)



Doppler Revisited

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o\left(t + \frac{2R}{c}\right)\right]$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + v_o t$$

Substituting:

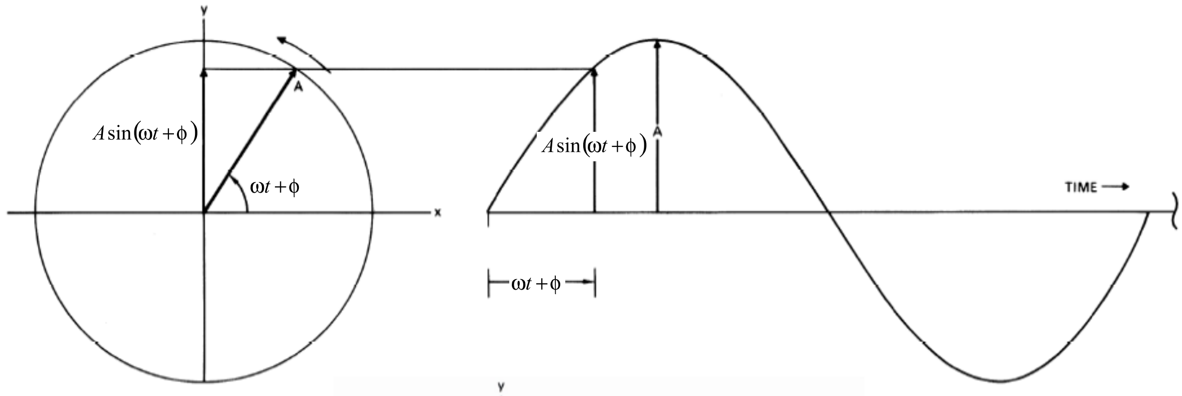
$$\cos\left[2\pi\left(f_o + \underbrace{f_o \frac{2v_o}{c}}_{-f_D}\right)t + \underbrace{\frac{2\pi f_o R_o}{c}}_{\text{constant}}\right]$$

$$\boxed{f_D = \frac{-2 f_o v_o}{c} = \frac{-2v_o}{\lambda_o} = \frac{d\phi}{dt}}$$

By convention, positive Doppler frequency shift $\Longleftrightarrow$ Target and radar closing

Doppler Detection: Intuitive Approach

Phasor diagram is a graphical representation of a sine wave



I & Q components*

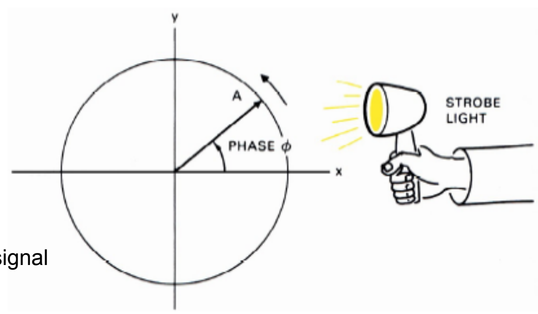
I => in-phase component

Acos(ϕ)

Q => in-quadrature component

Asin(ϕ)

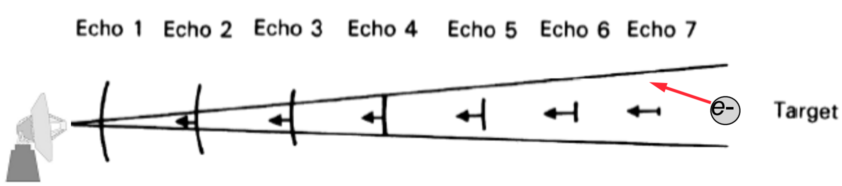
*relative to reference signal



Consider strobe light as cosine reference wave at same frequency but with initial phase = 0

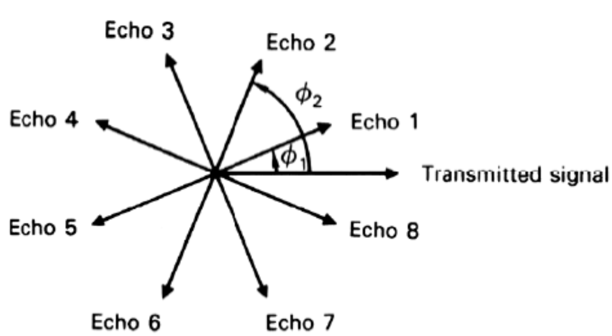
Doppler Detection: Intuitive Approach

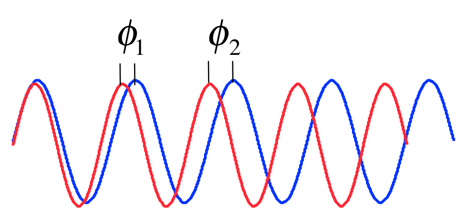
Closing on target – positive Doppler shift



Transmitted

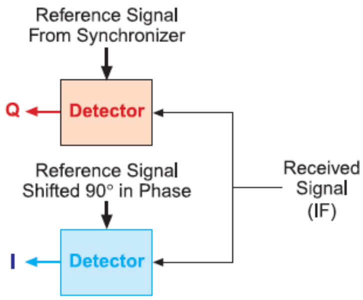
Received

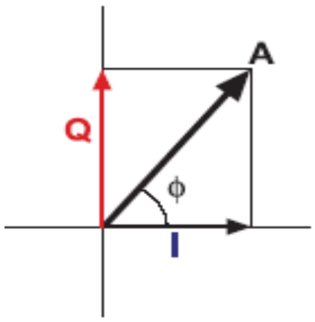




Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

I and Q Demodulation





in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\cos(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t)+2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t)+\omega_c t)\sin(\omega_c t)$$

$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t)+2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

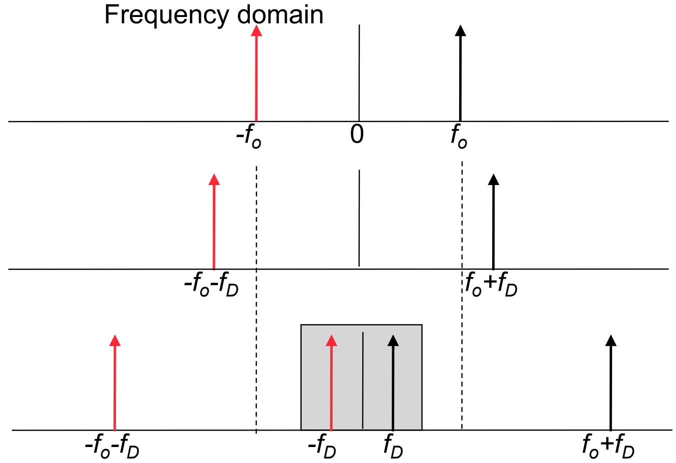
The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

I and Q Demodulation in Frequency Domain

Transmitted signal

$$\cos(2\pi f_o t)$$

Frequency domain



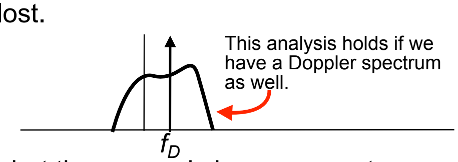
Doppler shifted

$$\cos(2\pi(f_o + f_D)t)$$

Mixed (multiplied) with carrier $\cos(2\pi f_o t)$

Cosine is even function, so sign of f_D (and, hence, velocity) is lost.
What we need instead is:

$$\exp(j2\pi f_D t) = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

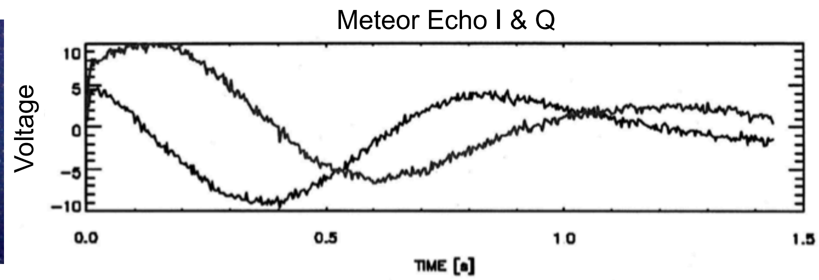


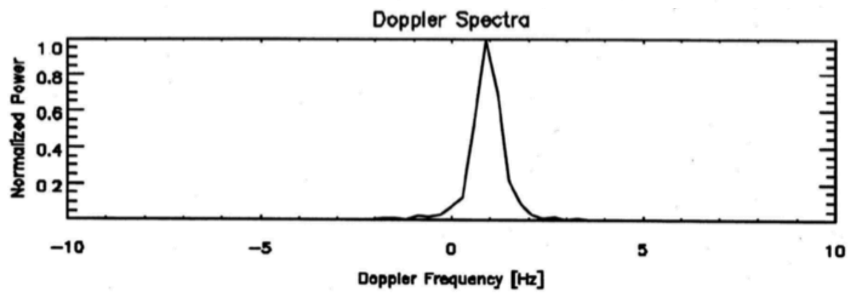
The analytic signal $\exp(j2\pi f_D t)$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called "in phase" (or *I*) and "in quadrature" (or *Q*):

$$A\exp(j2\pi f_D t) = I(t) + jQ(t) \quad \overset{\text{FFT}}{\Longleftrightarrow} \quad A\delta(f_D)$$

Example: Doppler Shift of a Meteor Trail

- **Collect N samples of $I(t_k)$ and $Q(t_k)$ from a target**
- **Compute the complex FFT of $I(t_k)+jQ(t_k)$, and find the maximum in the frequency domain**
- **Or compute "phase slope" in time domain.**



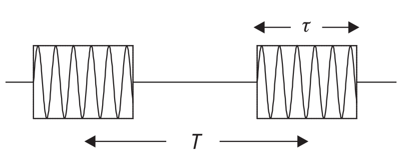


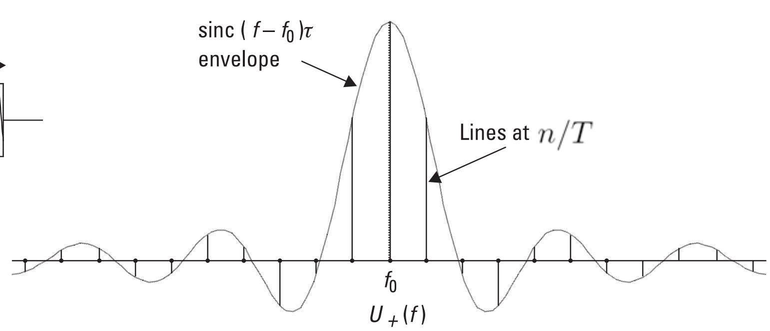
Does this strategy work for ISR?

Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Required PRF to probe ionosphere (500km range): 300 Hz

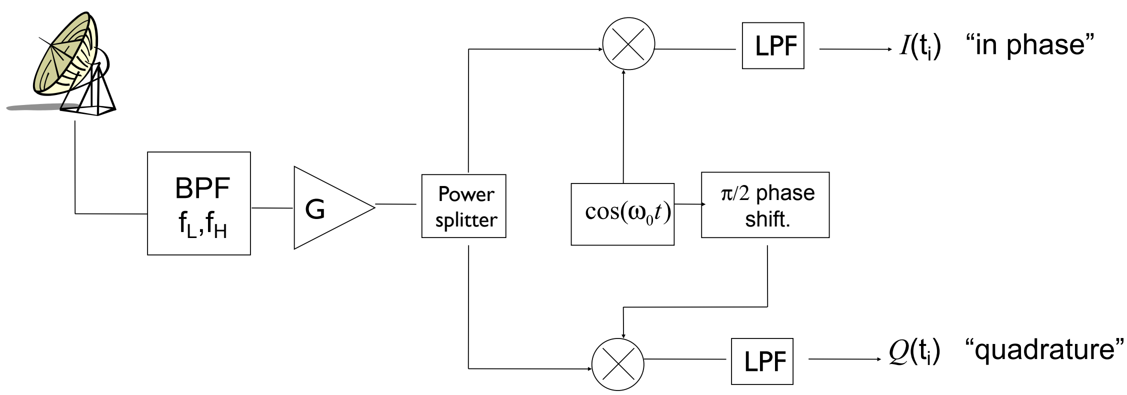
Plasma has completely decorrelated by the time we send the next pulse.

Alternately, the Doppler shift is well beyond the max unambiguous Doppler defined by the Inter-Pulse Period *T*.





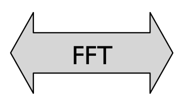
ISR Receiver: I and Q plus correlation



We have time series of *V(t) =I(t) + jQ(t),* how do I compute the Doppler spectrum?

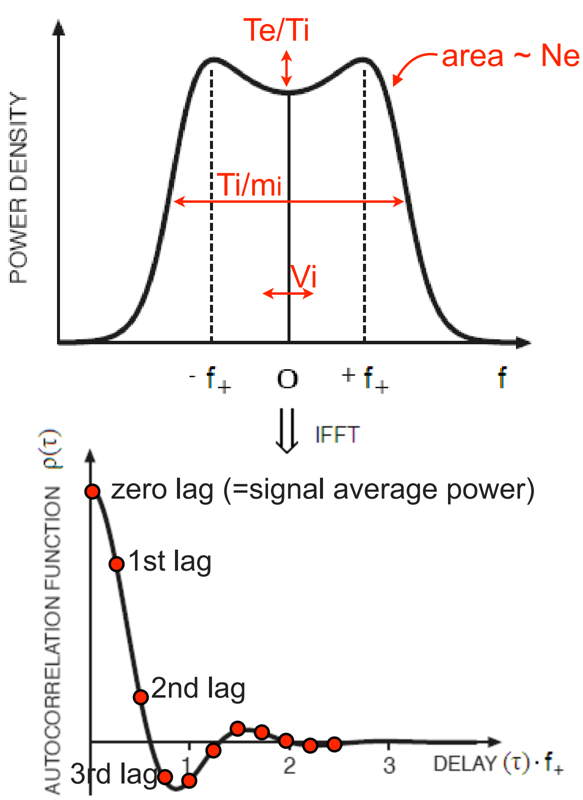
Estimate the autocorrelation function (ACF) by computing products of complex voltages ("lag products")

$$R_{vv}(\tau) = \frac{\langle V(t)V^*(t+\tau)\rangle}{S}$$



Power spectrum is Fourier Transform of the ACF

Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

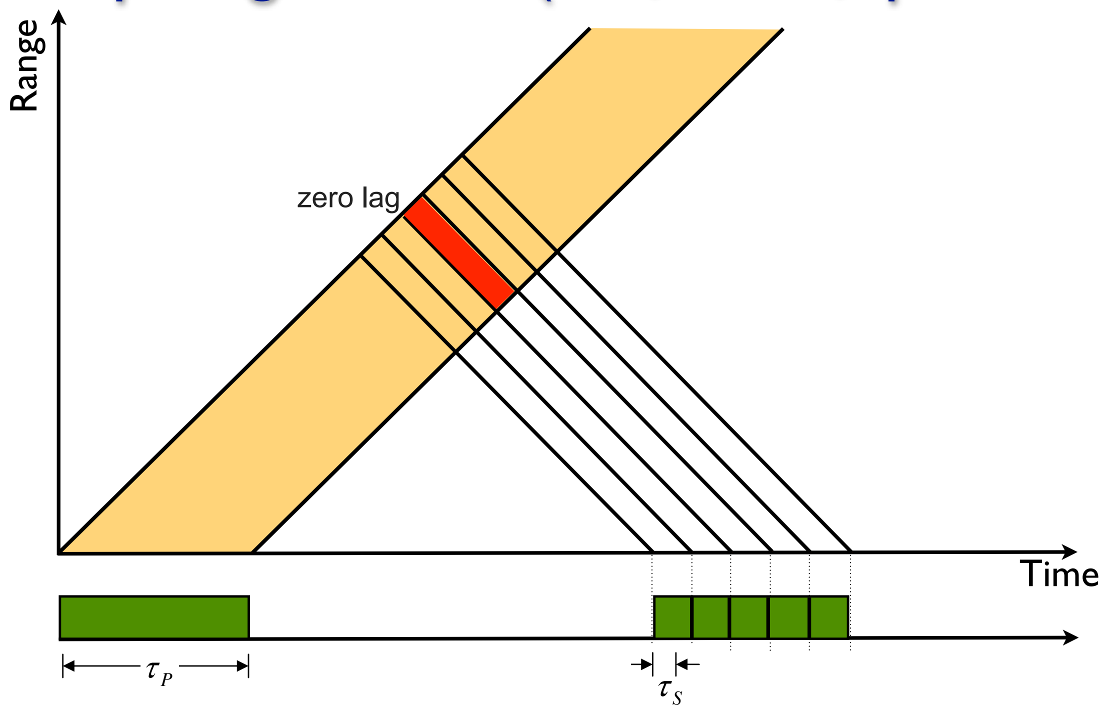
Electron to ion temperature ratio (Te/Ti) from "peak-to-valley" ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to compute lags

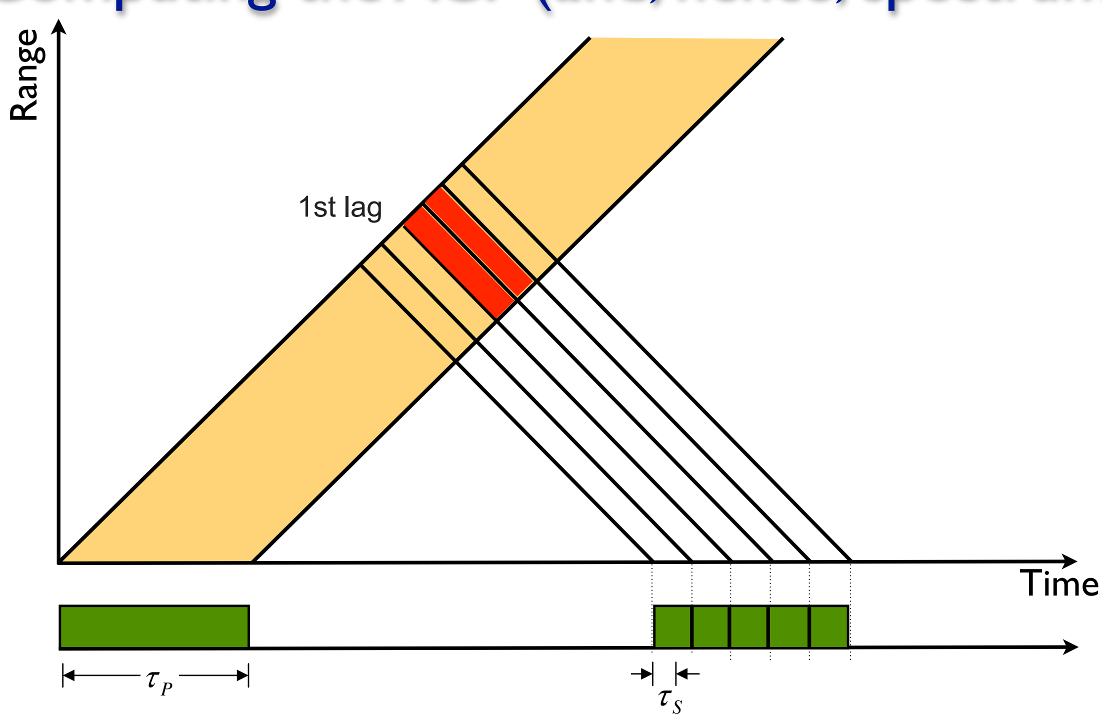
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

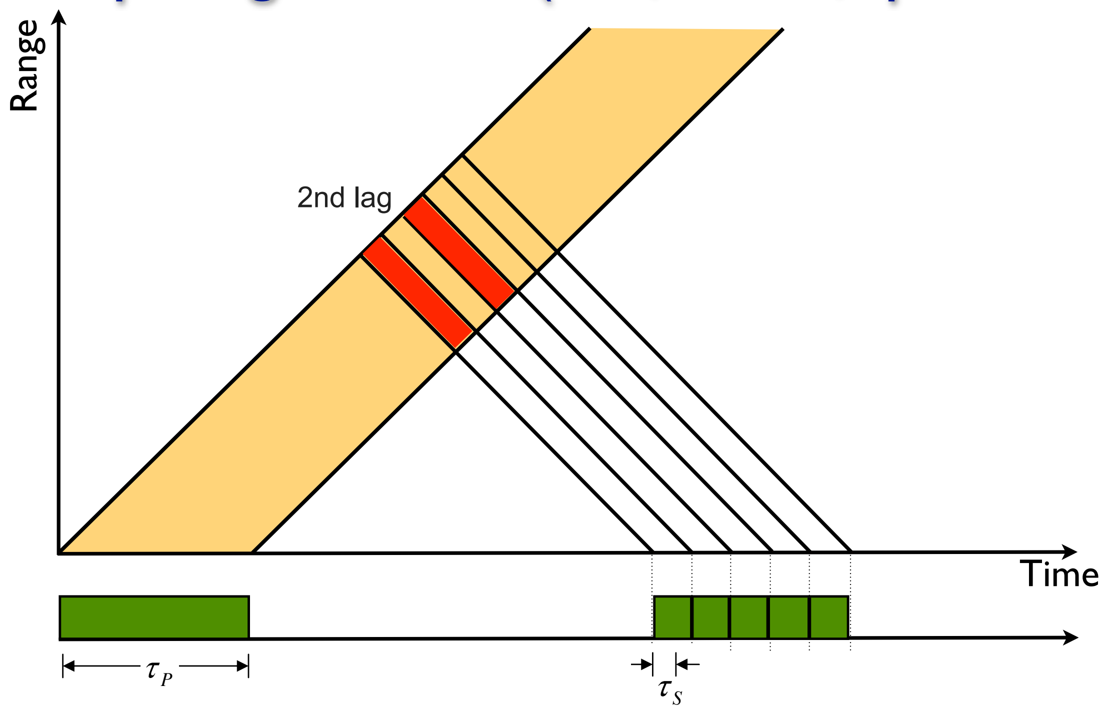
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

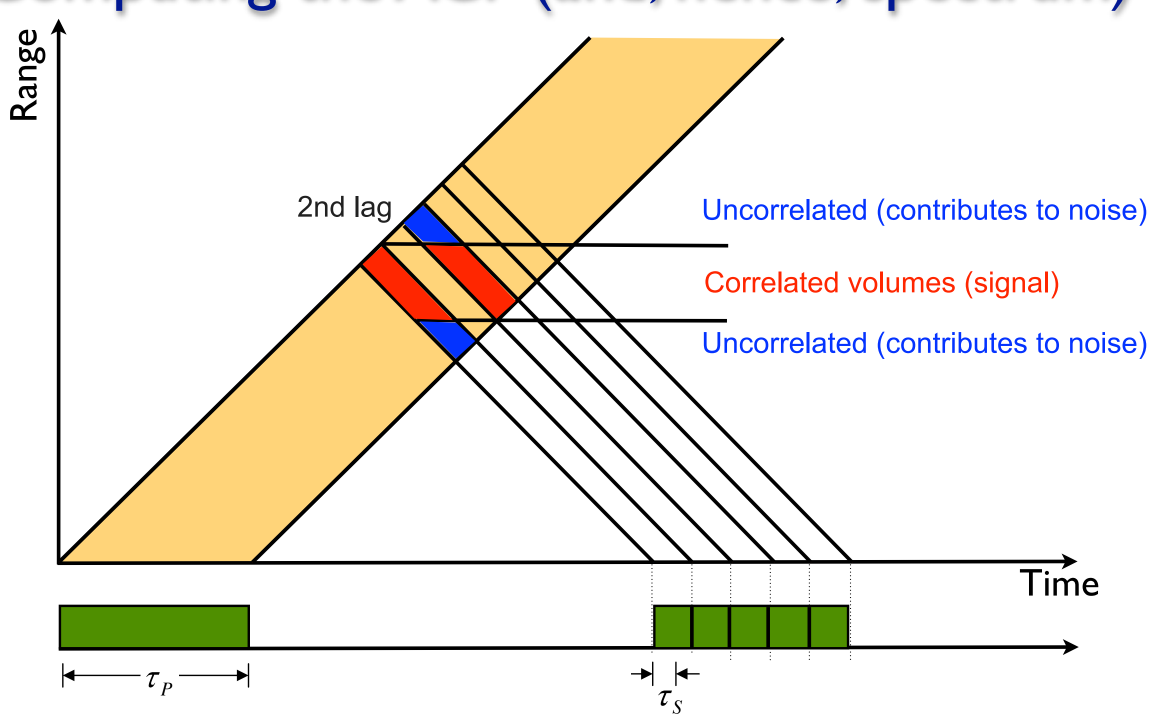
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

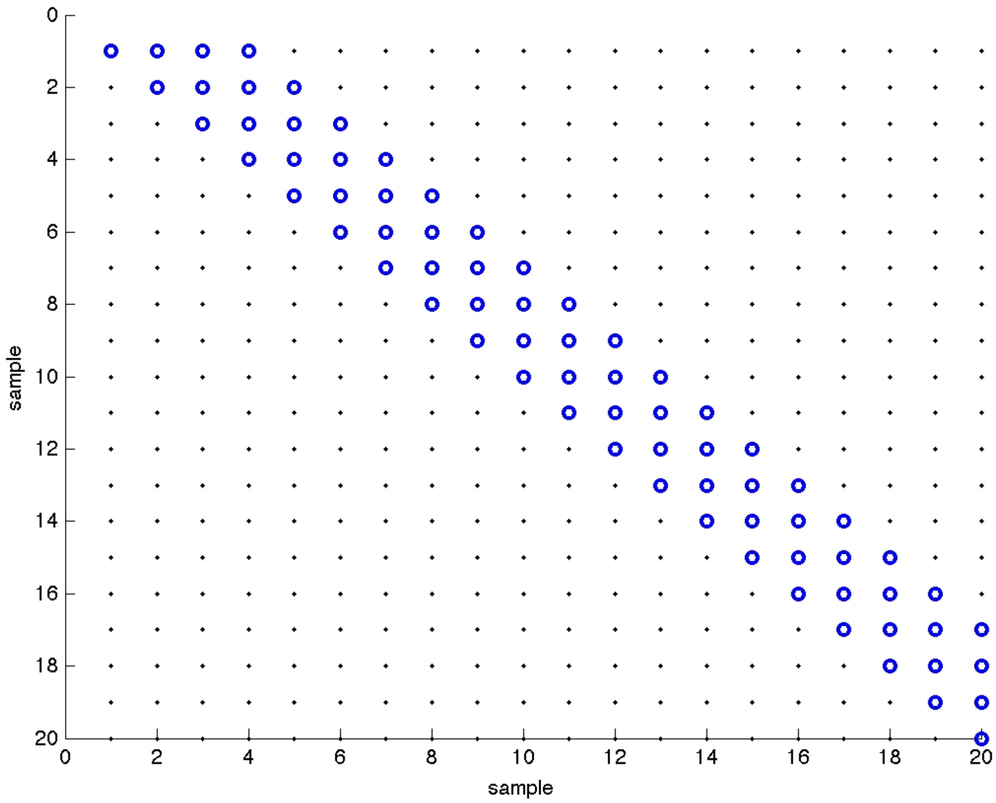
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Lag-product matrix



Bibliography

ISR tutorial material:

- *http://www.eiscat.se/groups/Documentation/CourseMaterials/*

Radar signal processing

- Mahafza, *Radar Systems Analysis and Design Using MATLAB*
- Skolnik, *Introduction to Radar Systems*
- Peebles, *Radar Principles*
- Levanon, *Radar Principles*
- Blahut, *Theory of Remote Image Formation*
- Curlander, *Synthetic Aperture Radar: Systems and Signal Analysis*

Background (Electromagnetics, Signal Processing:

- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
- Oppenheim, Willsky, and Nawab, *Signals and Systems*
- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:

http://mathforum.org/mbower/johnandbetty/frame.htm