Incoherent Scatter Theory and the effects of Coulomb collisions

Marco Milla[1] and Erhan Kudeki[2]

[1] Radio Observatorio de Jicamarca, Instituto Geofísico del Perú, Lima, Perú.

[2] Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL.

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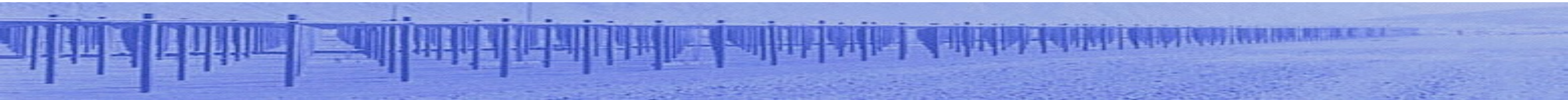
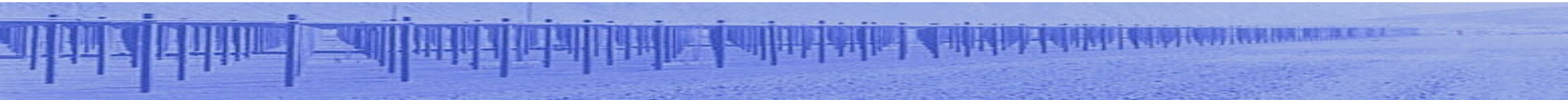


Some introduction



1958 - First ISR observations by Bowles

- Bowles (1958) carries first successful experiment at Long Branch, Illinois.
  - Operating frequency: 40.92 MHz
  - Peak pulse power: 4 to 6 MW
  - Antenna cross section: 116 x 140 m^2
- First observations showed that the electrons do not control the spectrum shape. It is the ions.

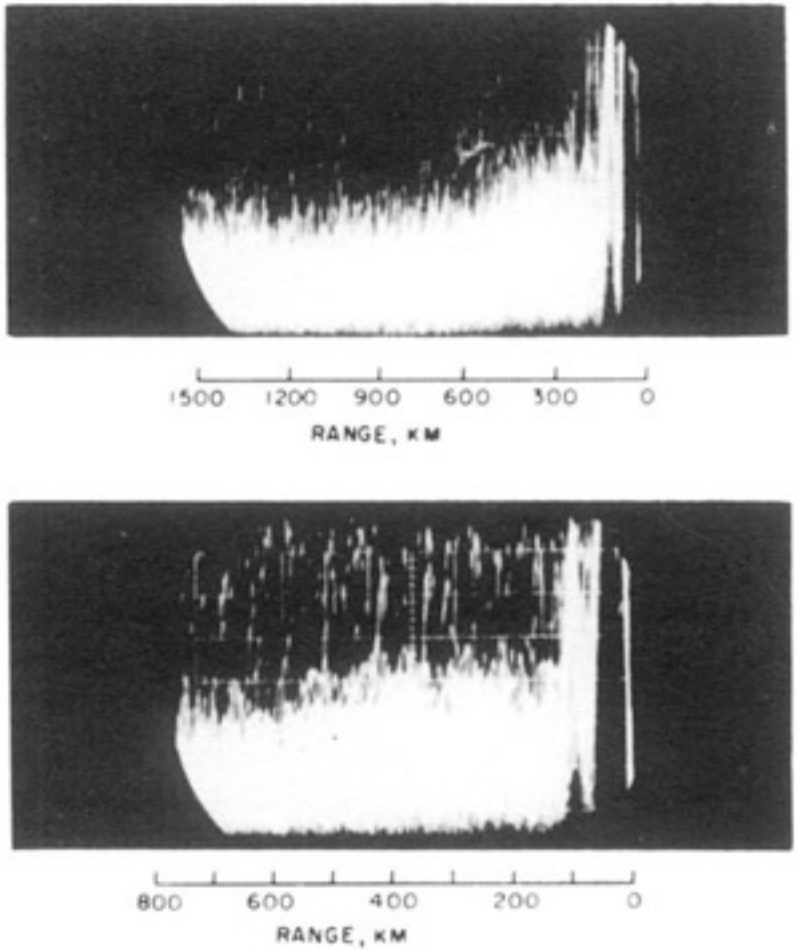
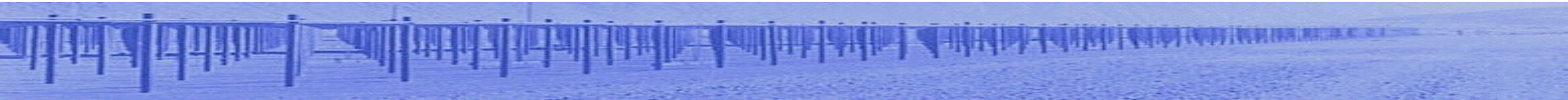
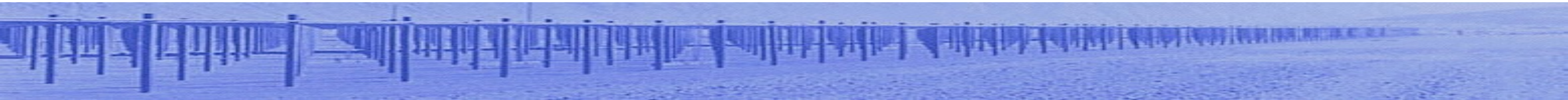


FIG. 1. (a) Pulse width 140 μsec (21 km); bandwidth 10 kc. (b) Pulse width 120 μsec (18 km); bandwidth 15 kc

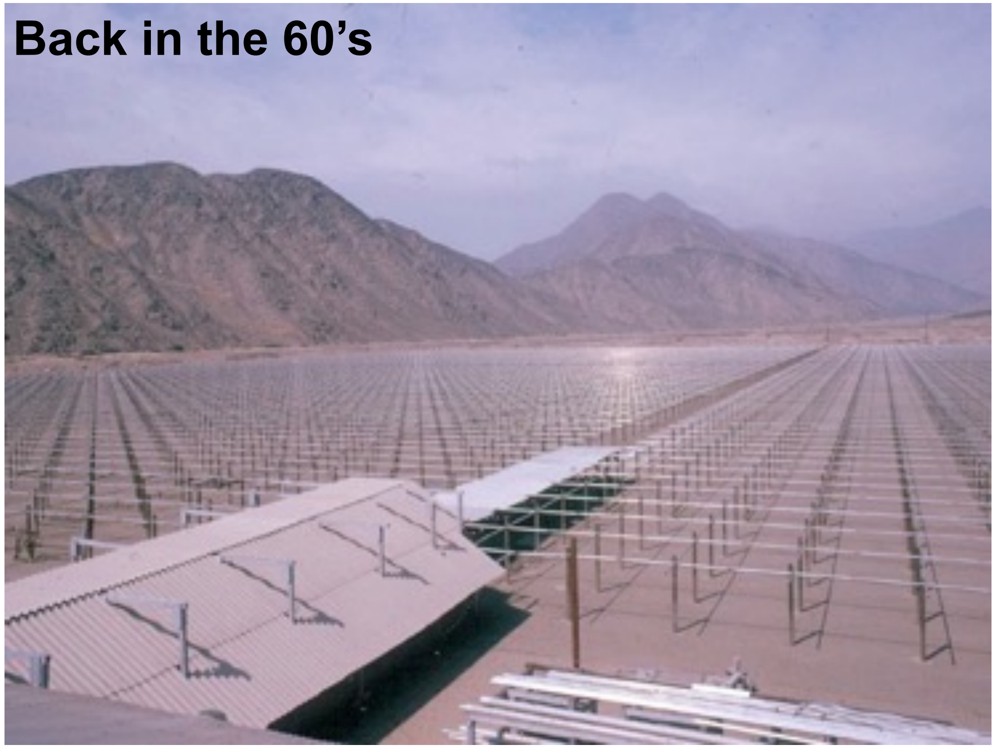


1960s - Development of the ISR theory

- During the 1960s different elements of the theory were developed by multiple authors following different approaches.
  - Collision-free plasmas:
    Fejer (1960, 1961), Dougherty and Farley (1960),
    Farley et al (1961), Salpeter(1960, 1961), Farley (1966), Hagfors (1961),
    Rosenbluth and Rostoker (1962)
  - Collisional plasmas:
    Dougherty and Farley (1963), Woodman (1967), etc.



1960-1962 Construction of JRO



Lat: 11.95° S, Lon: 76.87° W - Lima, Perú.
(at the Magnetic Equator)

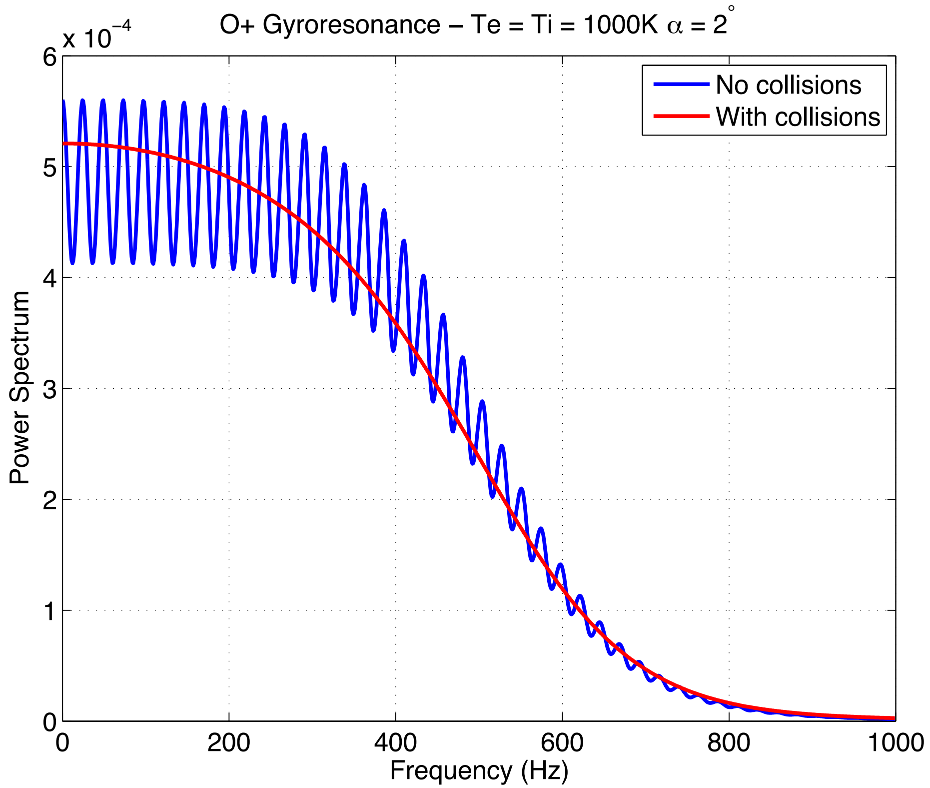
- Frequency: 50 MHz (VHF)
- Antenna area: 300x300 m2
- 18432 dipole elements
- 4 big transmitters - 1.5 MW each.

- Farley et. al (1961) - It should be possible, although difficult, to use an incoherent scatter sounder having a sufficiently large antenna as a mass spectrometer in the ionosphere.

Lack of gyro resonance in JRO measurements

Farley [1964]

Attempts have been made to observe the ion gyroresonances in the ionosphere, but these have so far been unsuccessful (Bowles, 1963, private communication), even though the equipment used could easily have detected the effects predicted by the collision-free theory. Recently a theory of incoherent scattering has been developed [*Dougherty and Farley*, 1963] which takes into account the effect of collisions of electrons and ions with neutral particles (Coulomb collisions are neglected). One of the results



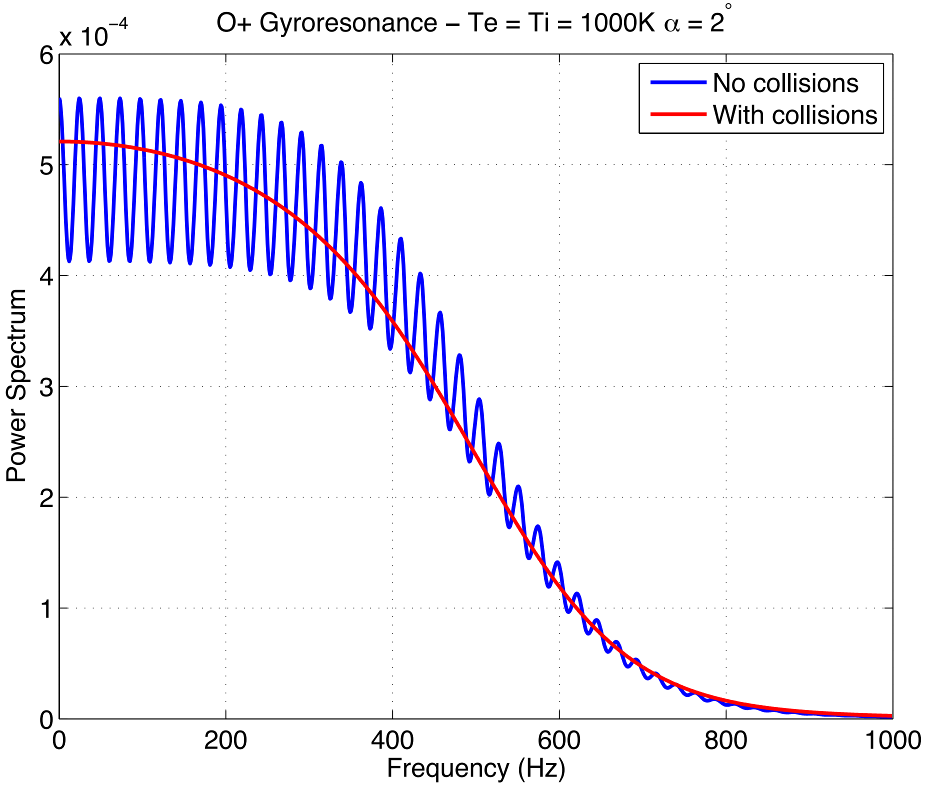
The Effect of Coulomb Collisions on Incoherent Scattering of Radio Waves by a Plasma

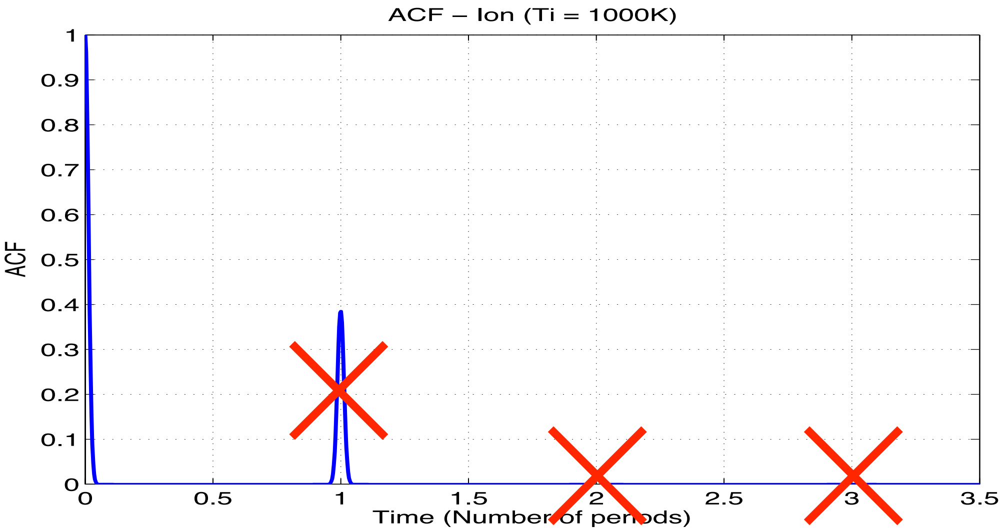
D. T. Farley, Jr.

Jicamarca Radar Observatory, Apartado 3747, Lima, Peru

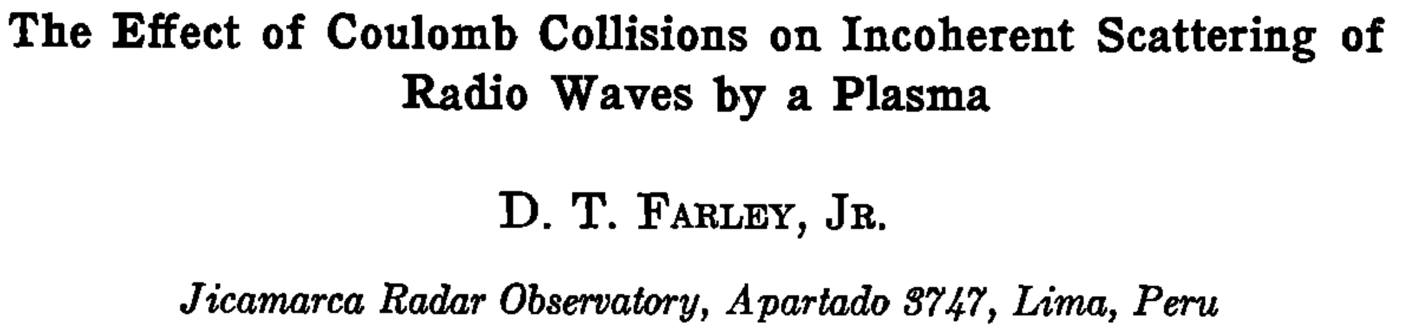
Lack of gyro resonance in JRO measurements

- The non-collisional theory predicted that at small aspect angles (<3 deg), the ISR spectrum should be highly oscillatory, with a peak-to-peak separation approximated equal to the ion gyro-frequency.





However, collisions destroy the periodicity of the O+ single particle ACF, therefore, no O+ gyro-resonance was observed.



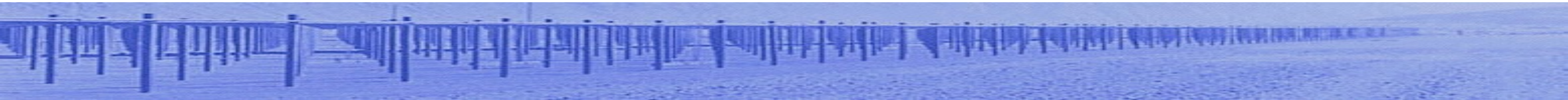
The Effect of Coulomb Collisions on Incoherent Scattering of Radio Waves by a Plasma

D. T. FARLEY, JR.

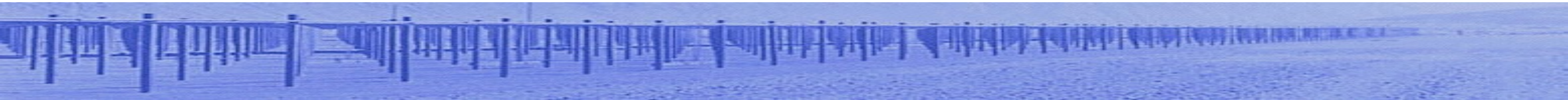
Jicamarca Radar Observatory, Apartado 3747, Lima, Peru

Farley [1964] was the starting point for the different studies that have been made on understanding the Coulomb collision effects on the IS radar observations taken at Jicamarca (with radar beams pointing close to perpendicular to B).

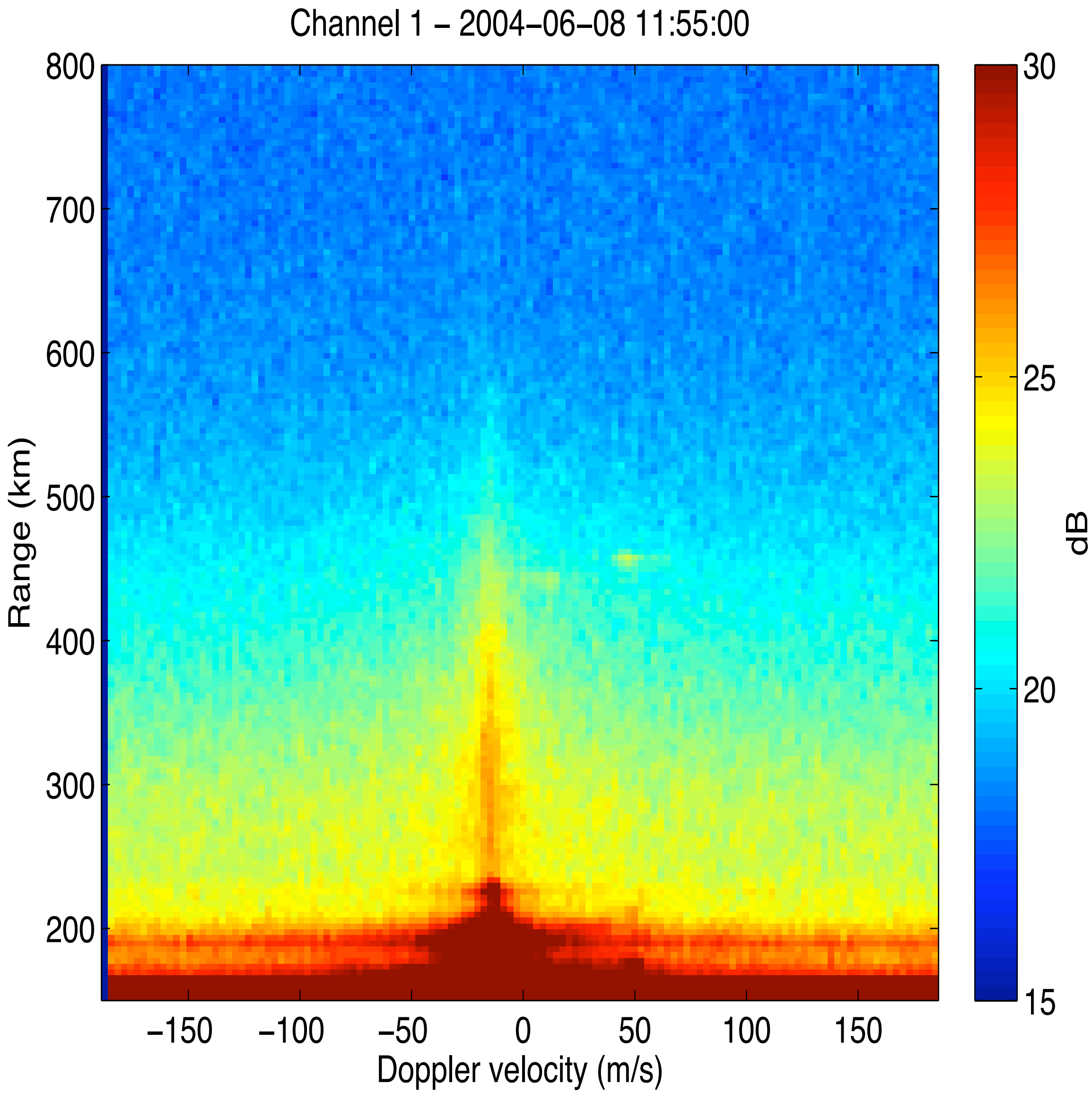
- Woodman [1967]
- Sulzer & Gonzalez [1999]
- Aponte et al [2001]
- Woodman [2004]
- Milla & Kudeki [2011]
- …



Understanding Coulomb collision effects in multi-component plasmas



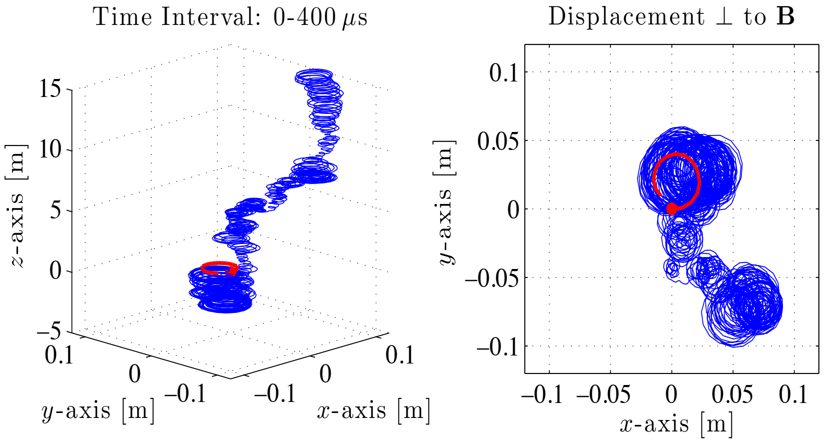
Motivation: ISR spectrum perp. to B

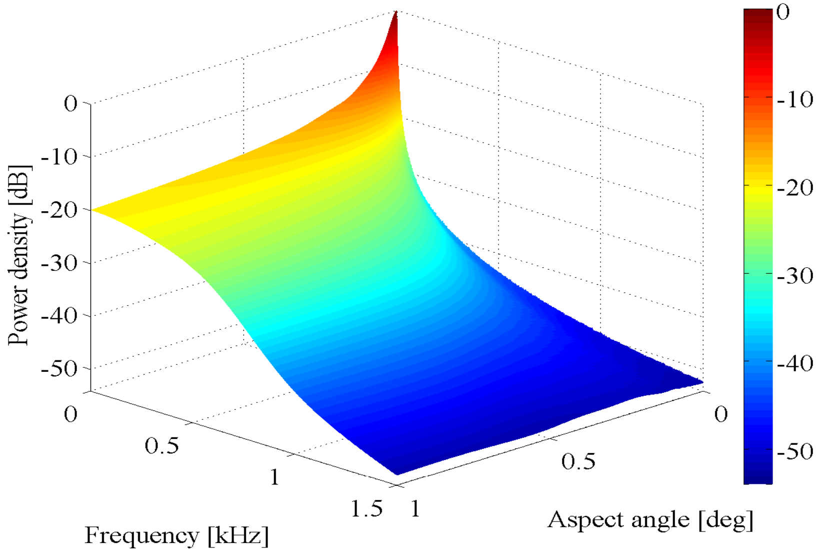


To develop a model for the IS spectra measured with antenna beams pointed perpendicular-to-B at Jicamarca with the goal of estimating ionospheric physical parameters (e.g. densities, temperatures).

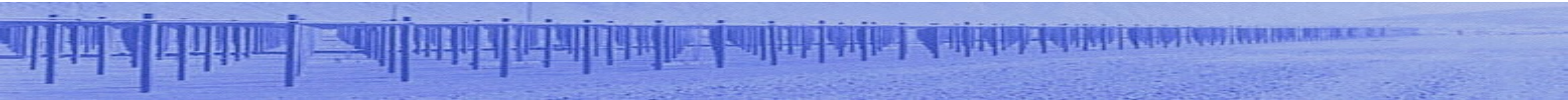
Coulomb collision effects on H^+, He^+, and O^+ plasmas

- Massive simulation of particle trajectories in H^+, He^+ and O^+ plasmas (Langevin equation and Fokker-Planck collision model).
- Statistical analysis of the simulated trajectories and construction of a numerical library of single-particle ACF's.
- Comparison of the collisional model with standard incoherent scatter theories.
- Application of the model to ISR experiments at Jicamarca.





Simulation of particle trajectories based on Langevin equation



IS spectrum and Gordeyev integrals

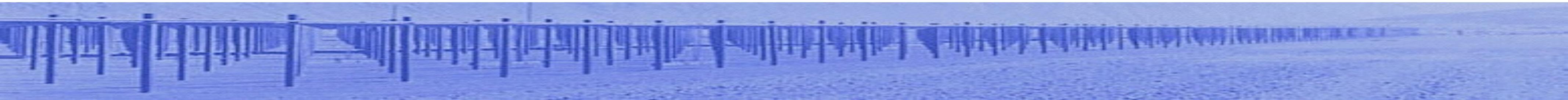
- The power spectrum of the incoherent scatter signals is proportional to the spectrum of electron density fluctuations in a plasma (e.g., Kudeki & Milla, 2011)

$$\langle|n_e(\vec{k},\omega)|^2\rangle = \frac{|j\omega\epsilon_o + \sigma_i(\vec{k},\omega)|^2\langle|n_{te}(\vec{k},\omega)|^2\rangle + |\sigma_e(\vec{k},\omega)|^2\langle|n_{ti}(\vec{k},\omega)|^2\rangle}{|j\omega\epsilon_o + \sigma_e(\vec{k},\omega) + \sigma_i(\vec{k},\omega)|^2}$$

- The fluctuation-dissipation (or Nyquist) theorem - Self-spectra of thermal density fluctuations and species conductivities are link to each other. Moreover, they can be written in the following forms

$$\frac{\langle|n_{ts}(\vec{k},\omega)|^2\rangle}{N_s} = 2\mathrm{Re}\{J_s(\omega)\} \qquad \frac{\sigma_s(\omega,\vec{k})}{j\omega\epsilon_o} = \frac{1 - j\omega J_s(\omega)}{k^2h_s^2}$$

where $J_s(\omega)$ denotes the so-called Gordeyev integral for each particle species.



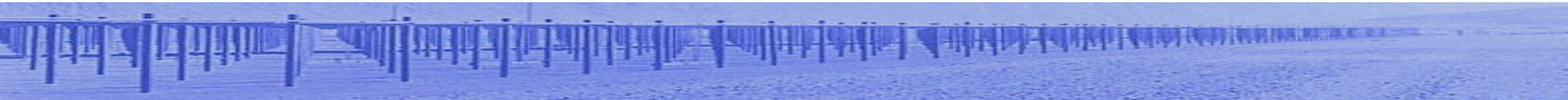
Gordeyev integral, Fokker-Planck collision model and Langevin equation

- The Gordeyev integrals are effectively Fourier transforms of the electron or ion particle ACFs (Hagfors & Brockelman, 1971)

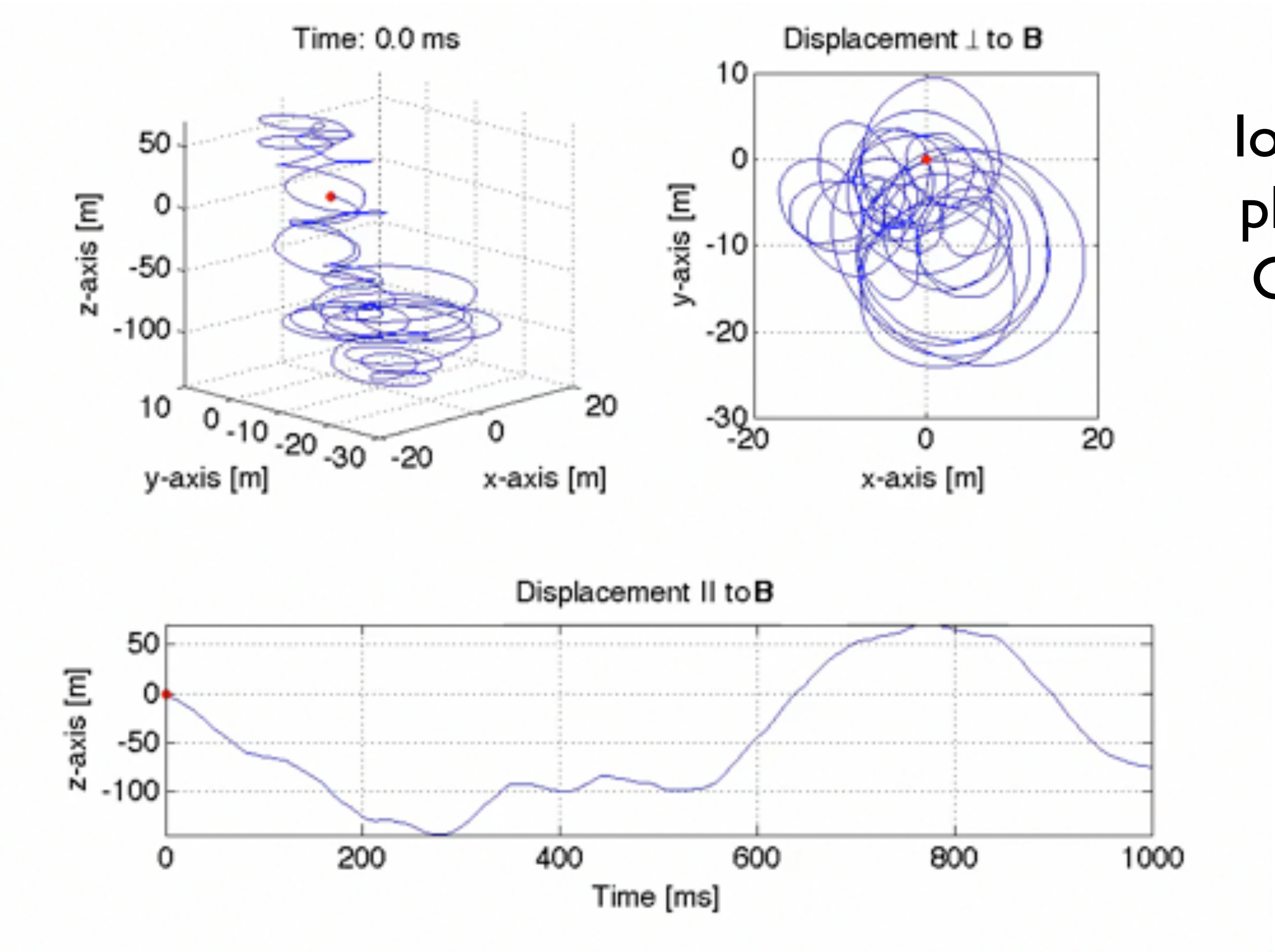
$$J_s(\omega) = \int_0^\infty d\tau e^{-j\omega\tau} \langle e^{j\vec{k}\cdot\Delta\vec{r}_s} \rangle \qquad \langle e^{j\vec{k}\cdot\Delta\vec{r}_s} \rangle = \langle e^{j\vec{k}\cdot(\vec{r}_s(t+\tau)-\vec{r}_s(t))} \rangle$$

- Instead of computing the integrals solving a kinetic equation with the Fokker-Planck collision operator, we decided to compute them from simulated electron and ion trajectories.
- The trajectories are simulated using a Generalized version of the Langevin equation in which Coulomb collisions are modeled by a deterministic friction force and random diffusion forces acting on a test particle.

$$\begin{aligned} \frac{d\vec{v}(t)}{dt} \quad = \quad & \frac{q}{m}\,\vec{v}(t) \times \vec{B} - \beta(v)\,\vec{v}(t) + \sqrt{D_{||}(v)}\,\mathcal{W}_1(t)\,\hat{v}_{||}(t) \\ & \sqrt{\frac{D_\perp(v)}{2}}\,\mathcal{W}_2(t)\,\hat{v}_{\perp 1}(t) + \sqrt{\frac{D_\perp(v)}{2}}\,\mathcal{W}_3(t)\,\hat{v}_{\perp 2}(t) \end{aligned}$$



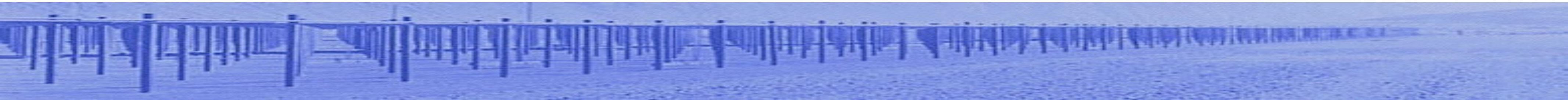
3D particle trajectory sample



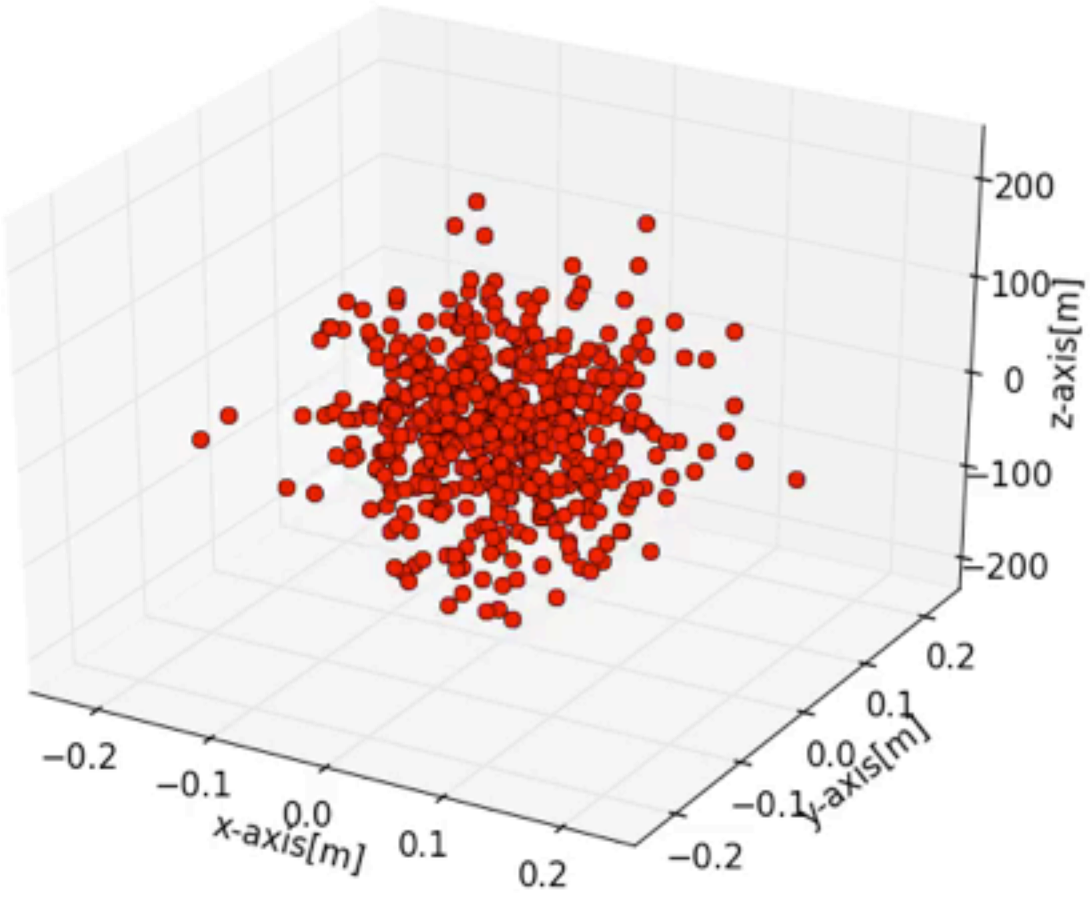
Ion moving in an O+ plasma experiencing Coulomb collisions

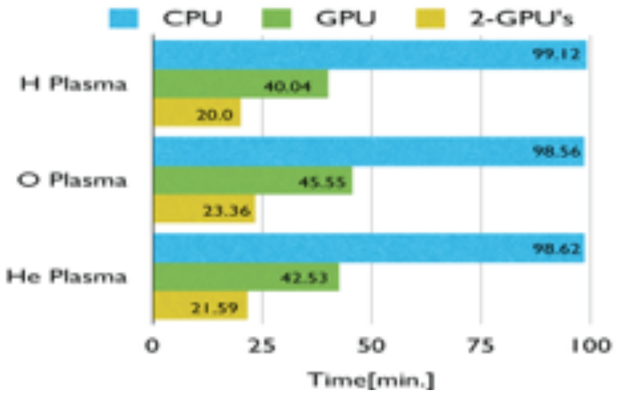
$N_e = 10^{11}\ m^{-3}$
$T_e = 2000$ K
$T_i = 2000$ K
B = 20000 nT

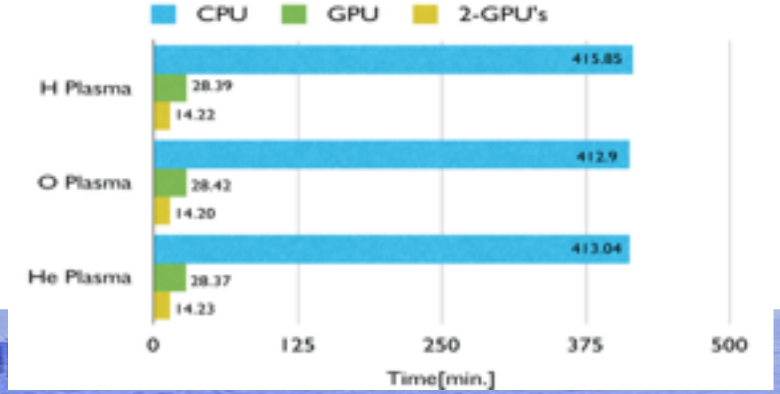
10^4 sequences of 2^{17} samples are generated (~30 GB), however, only the statistics (pdfs and ACFs) are stored (~60 MB).



Simulation of multiple trajectories using CUDA

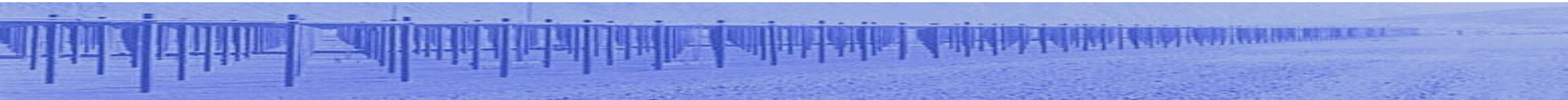




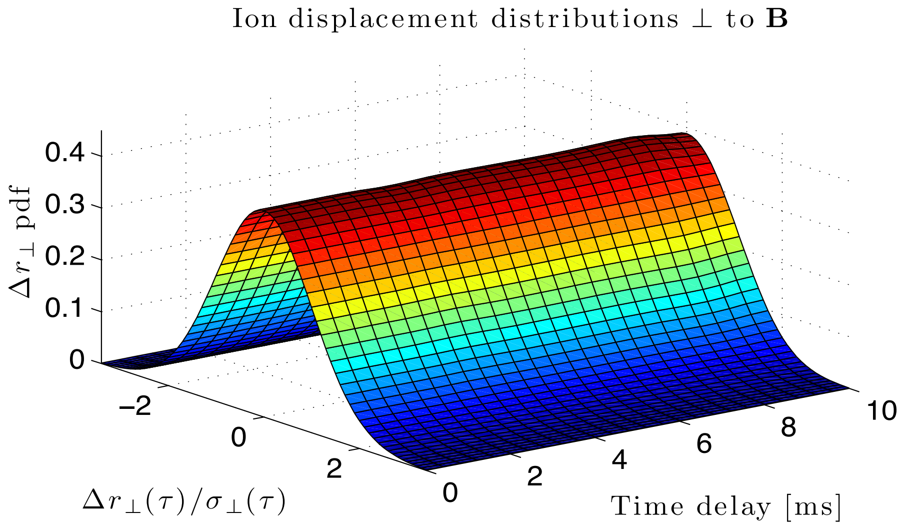


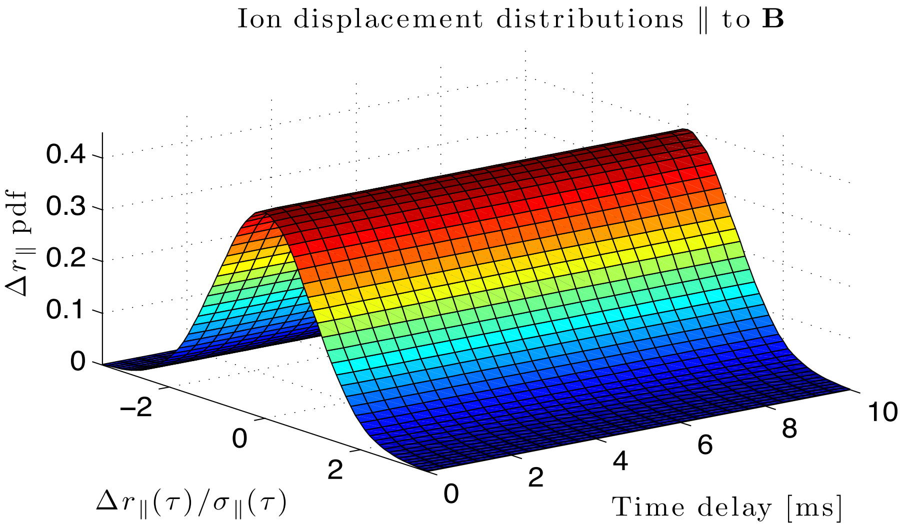
Significant saving in simulation and processing time.

Statistics of test-ion displacements in H^+, He^+, and O^+ plasmas and comparison to the Brownian model



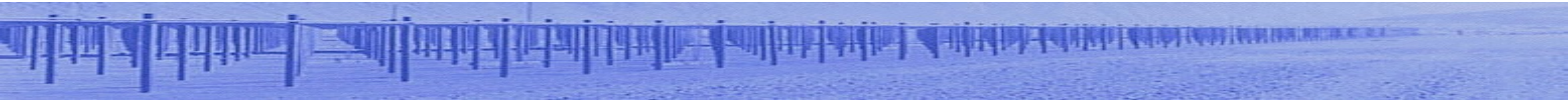
Ion displacement distributions



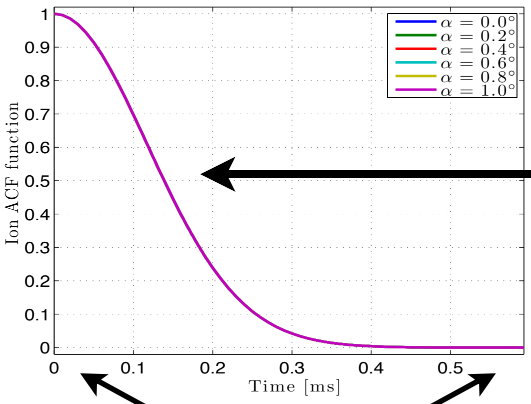


- H^+, He^+, and O^+ ion displacement distributions are Gaussian in the direction perpendicular to B as function of delay τ.
- In the parallel direction, the distributions also look gaussian.
- A Brownian motion model with Gaussian trajectories is a good representation of the ion process (Woodman, 1967).
- The single-ion ACF can be approximated by

$$\left\langle e^{j\vec{k}\cdot\Delta\vec{r}}\right\rangle = e^{-\frac{1}{2}k^2\sin^2\alpha\langle\Delta r_\parallel^2\rangle} \times e^{-\frac{1}{2}k^2\cos^2\alpha\langle\Delta r_\perp^2\rangle}$$

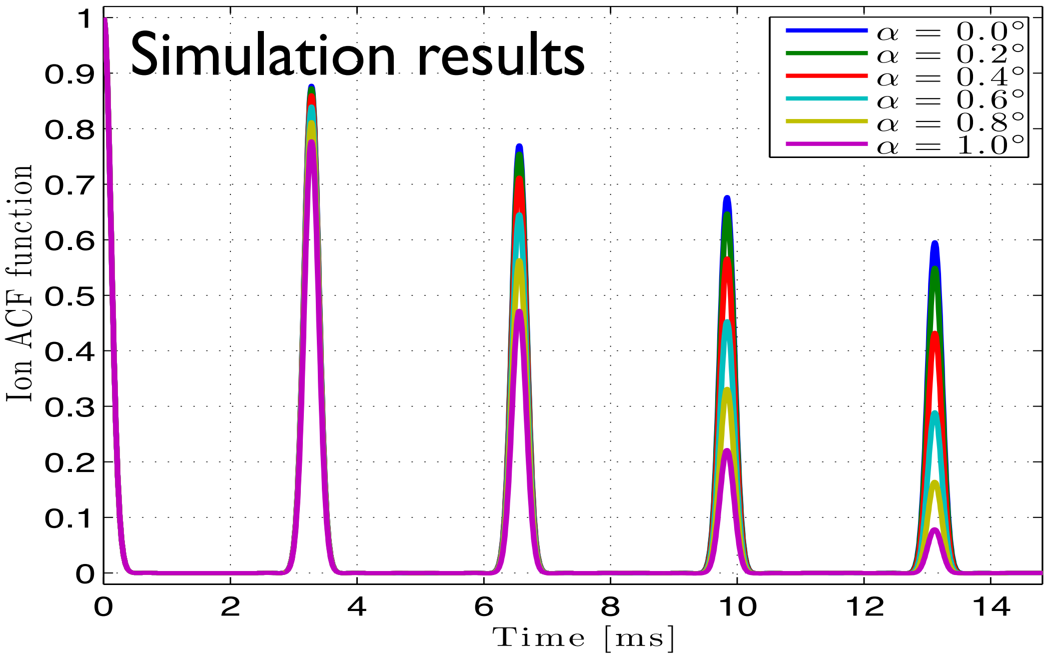


H$^+$ single-ion ACFs



Correlation time proportional to

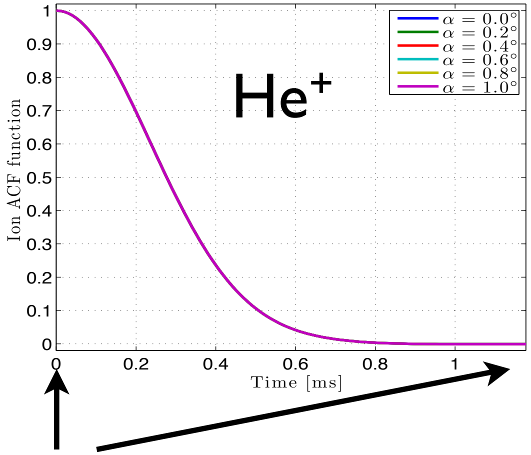
$$\frac{1}{k_B C_i}$$


Simulation results

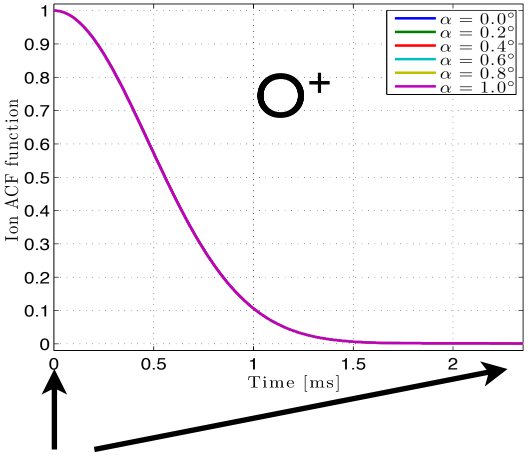
At perpendicular to B, periodicity of the H$^+$ ion ACF is damped by ion-ion collisions. At larger angles is a mixed effect of collisional and non-collisional damping.

Brownian-motion model captures well all the details of the ion ACFs.

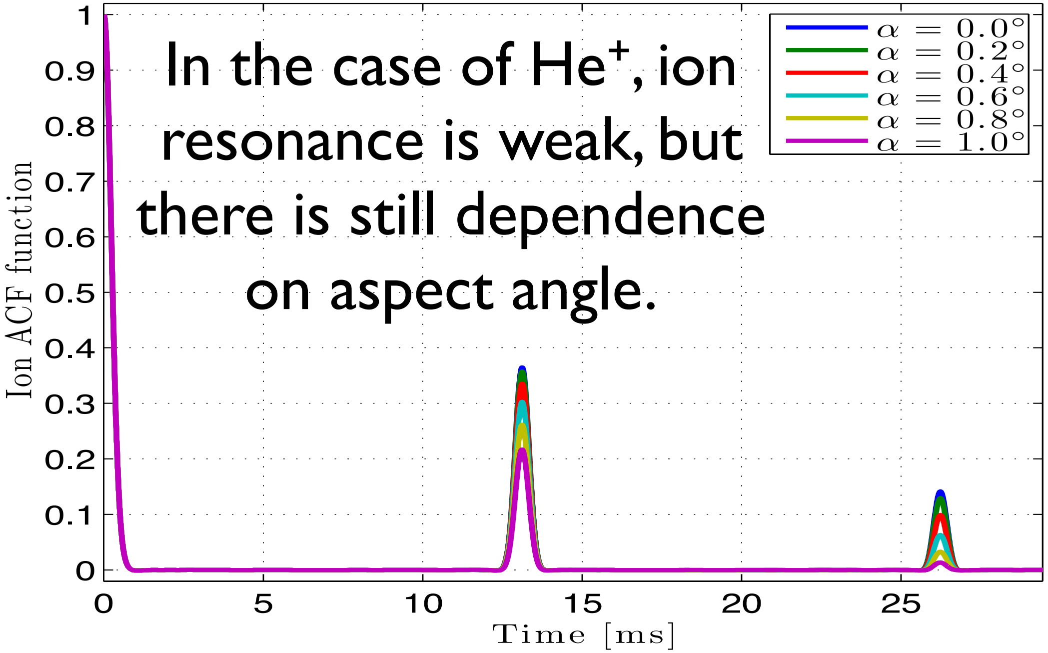
He^+ and O^+ single-ion ACFs



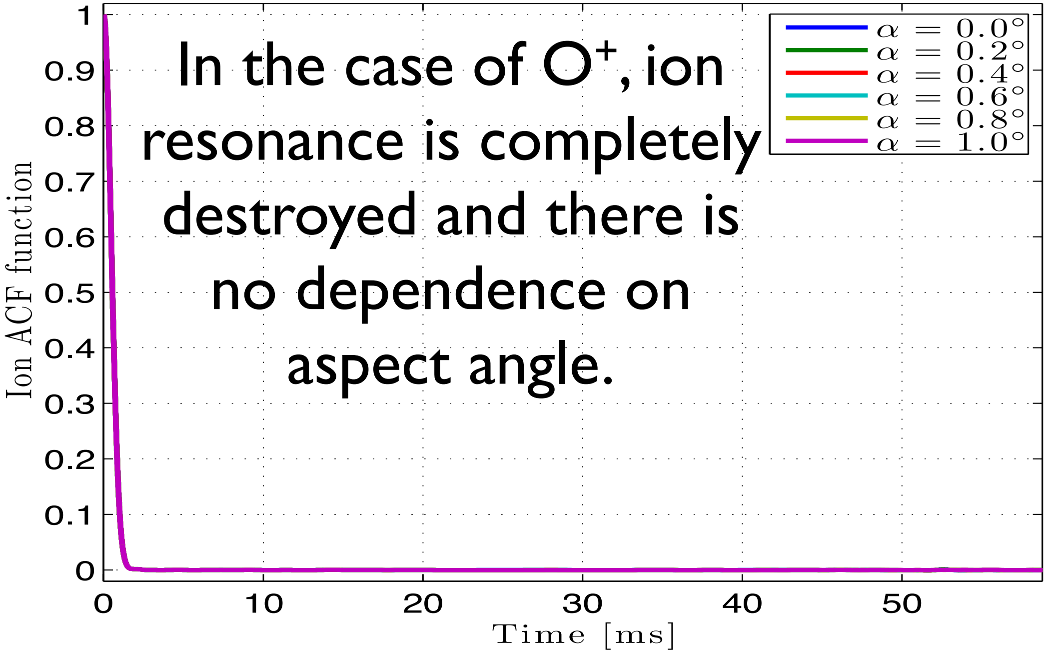
Correlation times are proportional to $\frac{1}{k_B C_i}$



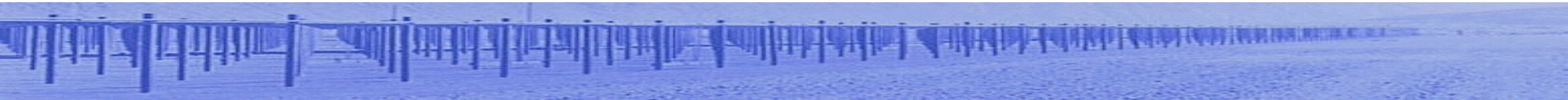
In both cases, the Brownian motion model captures all the details of the ion ACFs.



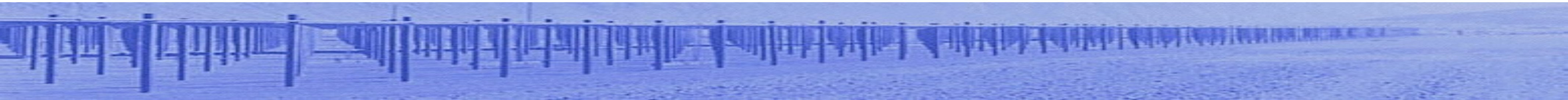
In the case of He^+, ion resonance is weak, but there is still dependence on aspect angle.



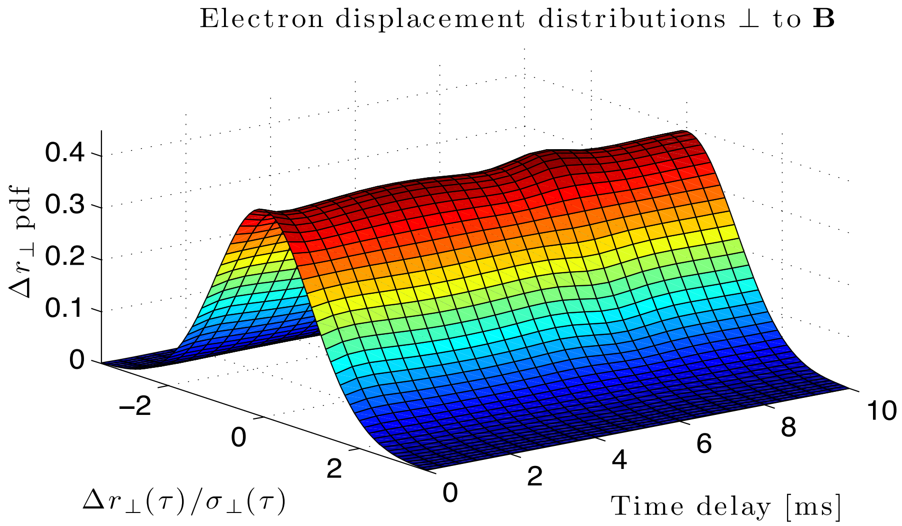
In the case of O^+, ion resonance is completely destroyed and there is no dependence on aspect angle.



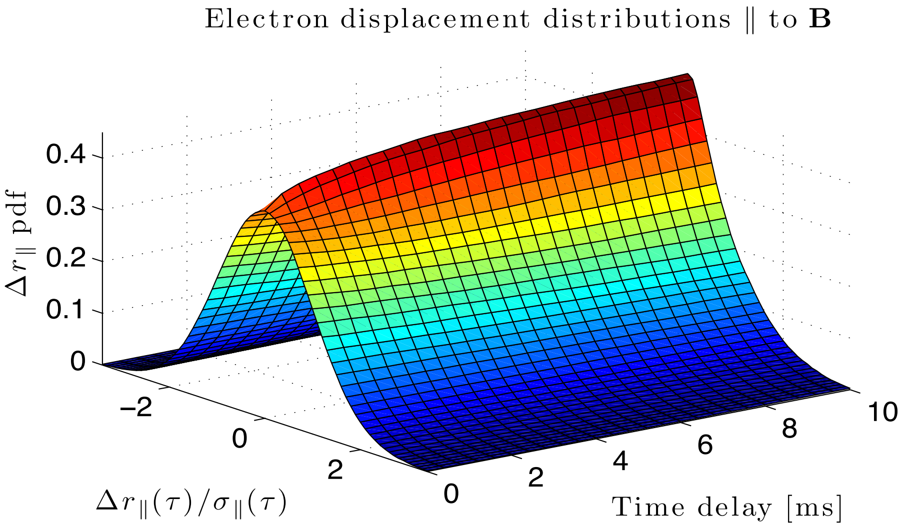
Statistics of test-electron displacements in H^+, He^+, and O^+ plasmas and comparison to the Brownian model



Electron displacement distributions



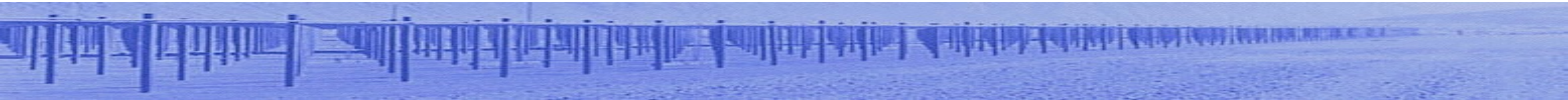
Electron displacement distributions $\perp$ to $\mathbf{B}$



Electron displacement distributions $\parallel$ to $\mathbf{B}$

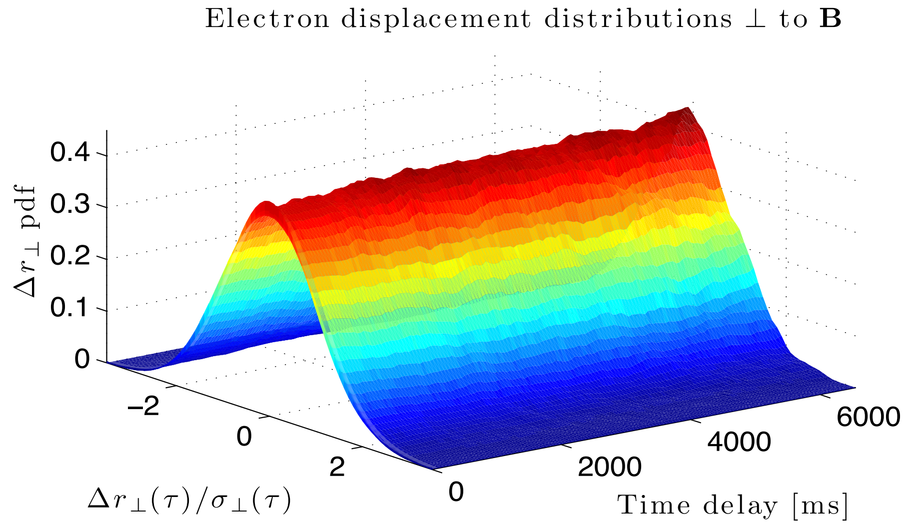
- Electron distributions look the same for H^+, He^+, and O^+ plasmas.
- In the direction perp. to B, the distribution is approximately Gaussian as function of time delay.
- In the parallel direction, the distribution looks Gaussian at short delays, but becomes narrower within a “collision time”.
- Brownian motion (gaussian displacements) is not a good model for the electron motion.

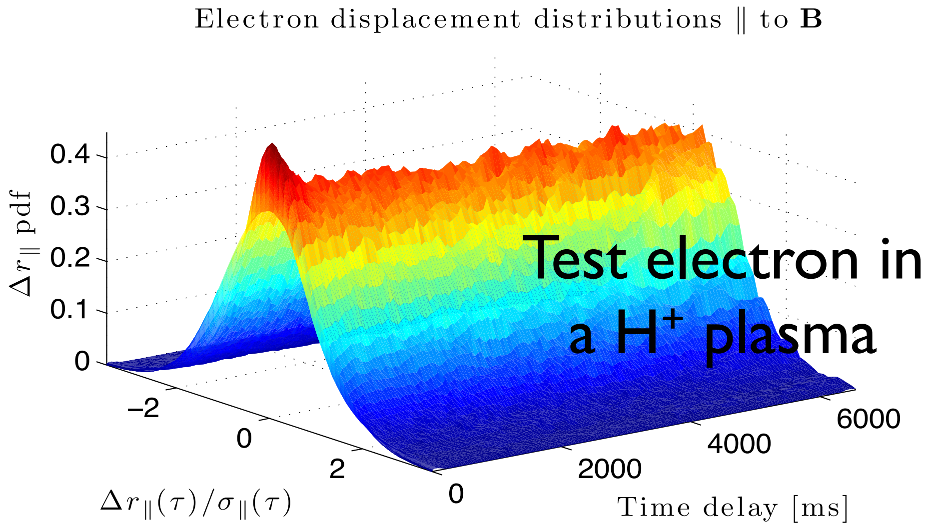
$$\xcancel{\left\langle e^{j\vec{k}\cdot\Delta\vec{r}}\right\rangle = e^{-\frac{1}{2}k^2\sin^2\alpha\langle\Delta r_\parallel^2\rangle}\times e^{-\frac{1}{2}k^2\cos^2\alpha\langle\Delta r_\perp^2\rangle}}$$



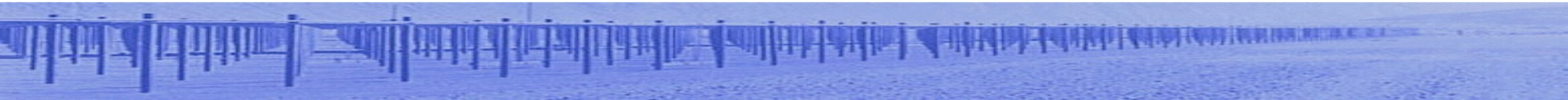
More on electron displacement distributions

- The distribution in the direction perpendicular to B remains Gaussian for long time delays.
- In the parallel direction, the distribution becomes Gaussian again after a few "collision times".
- The electron distribution is independent of the ion type.
- For ISR applications around perpendicular to B, the correlation times are of the order of the electron "collision time", therefore the choice of the collision model does matter to define the shape of the spectrum.

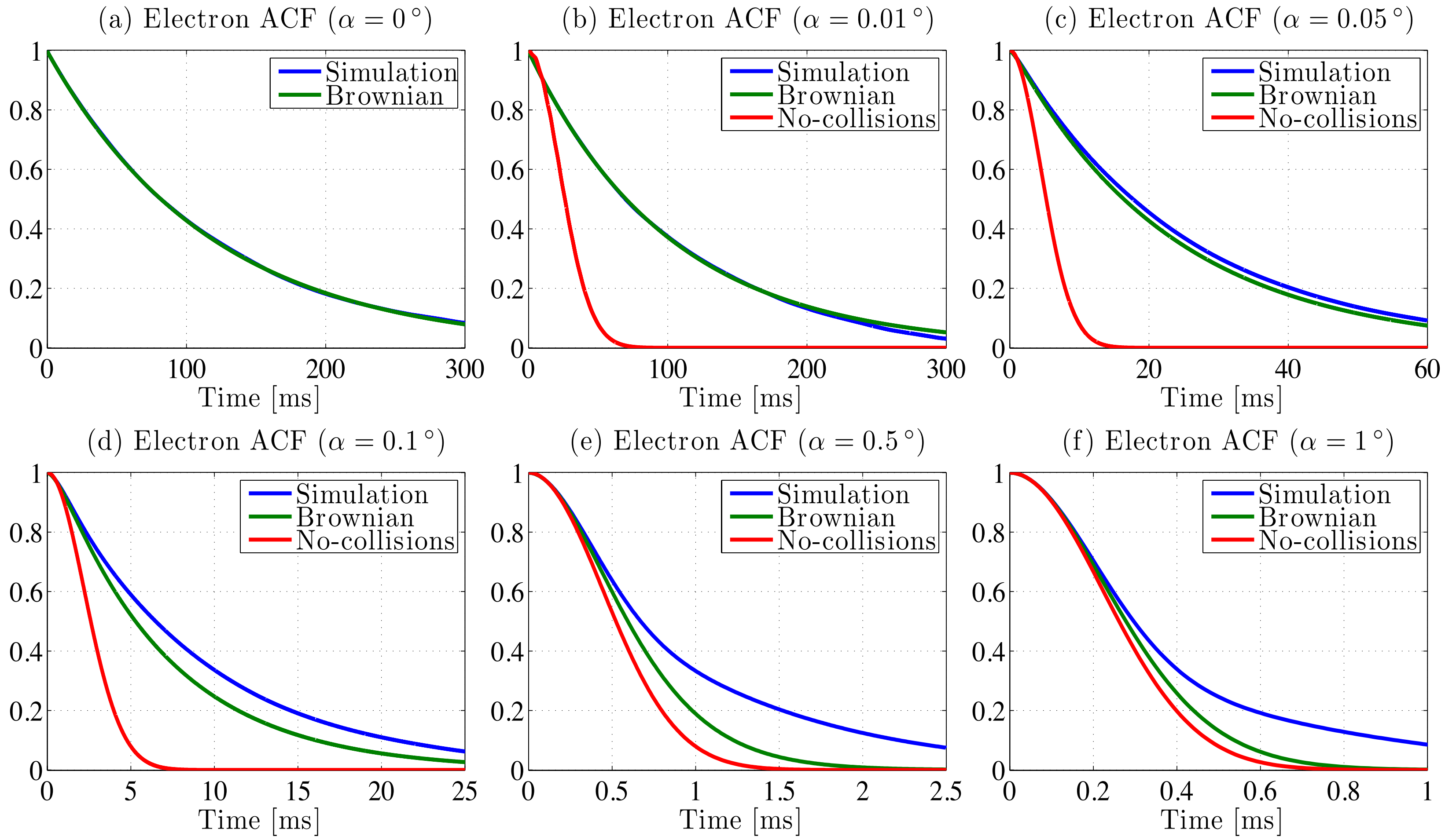




Test electron in a H^+ plasma



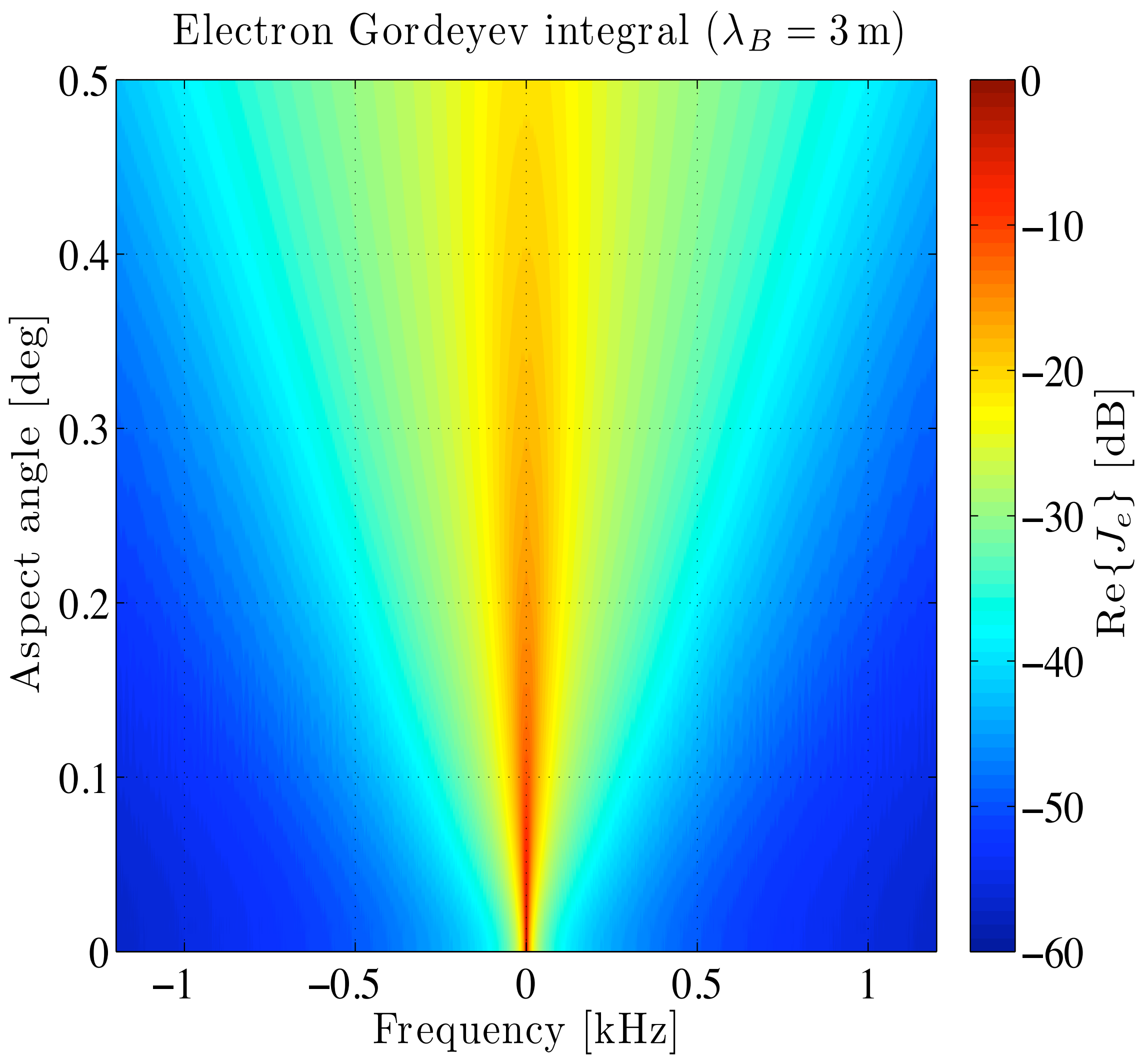
Comparison of single-electron ACFs



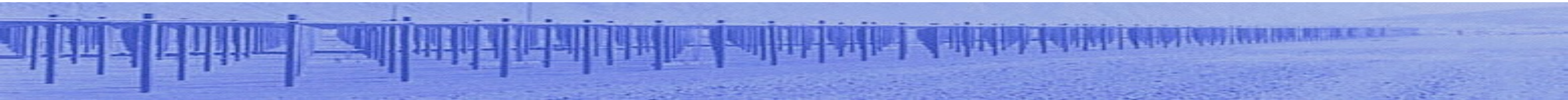
The Brownian motion model does not capture all the details of the electron ACFs.

Simulated electron ACF's for λ_B=3m at different magnetic aspect angles: (a) α=0°, (b) α=0.01°, (c) α=0.05°, (d) α=0.1°, (e) α=0.5°, and (f) α=1°.

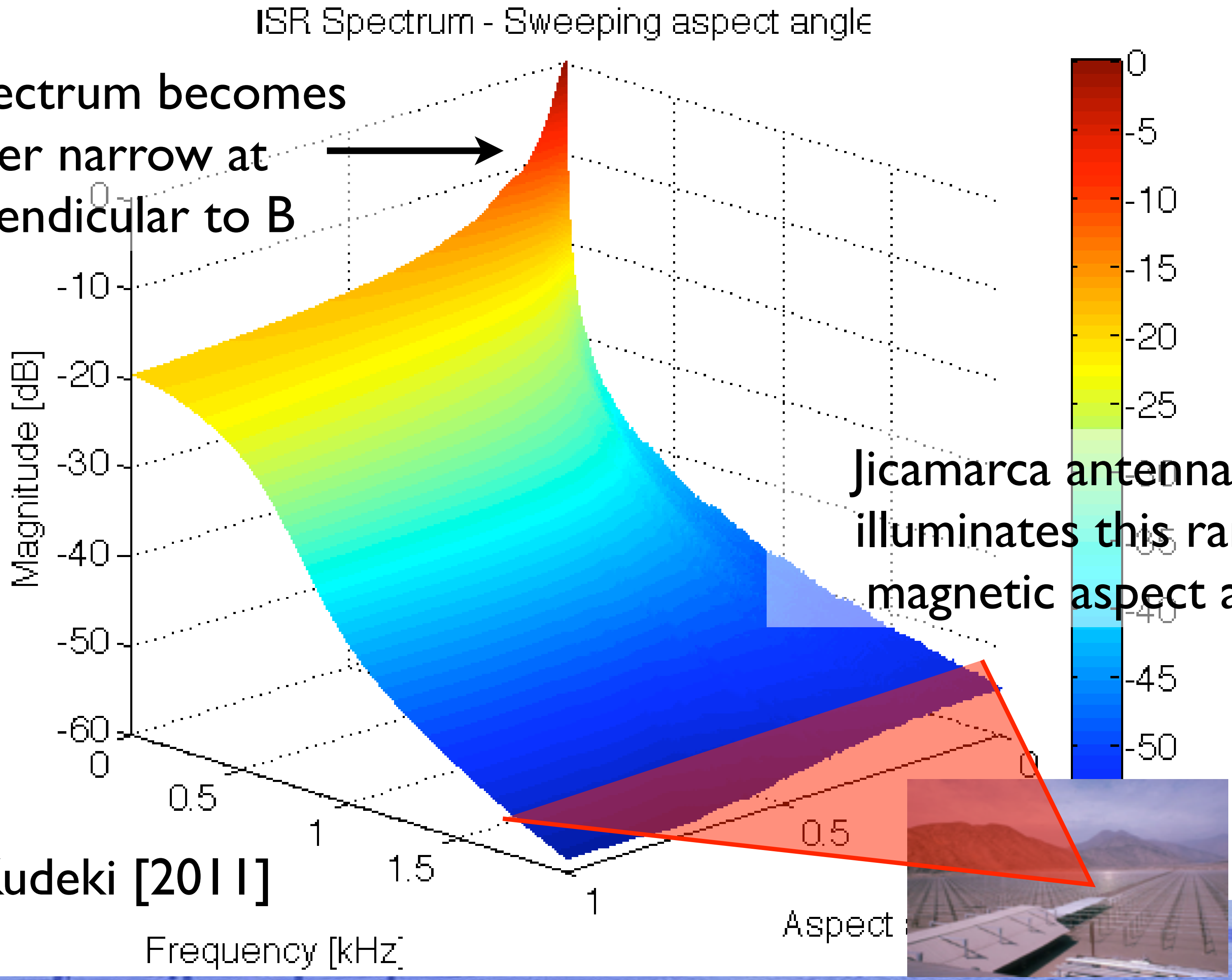
Database of single-electron ACFs and Gordeyev integrals



- Before, we built a library of single-electron ACFs (and corresponding Gordeyev integrals).
- The library spans different values of Ne, Te, B, and α.
- As the electron ACFs are independent of the plasma configuration, there is no need to develop another library but to parametrize it in order to use it for ISR applications.



Collisional IS Spectrum



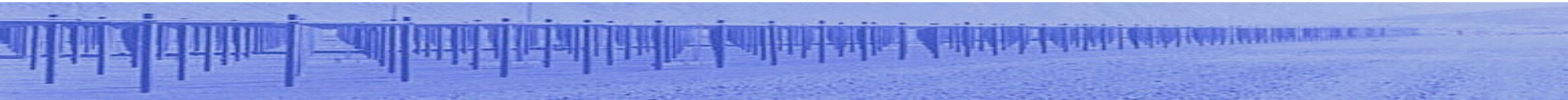
The spectrum becomes super narrow at perpendicular to B

Jicamarca antenna beam illuminates this range of magnetic aspect angles

Milla & Kudeki [2011]

Conclusions

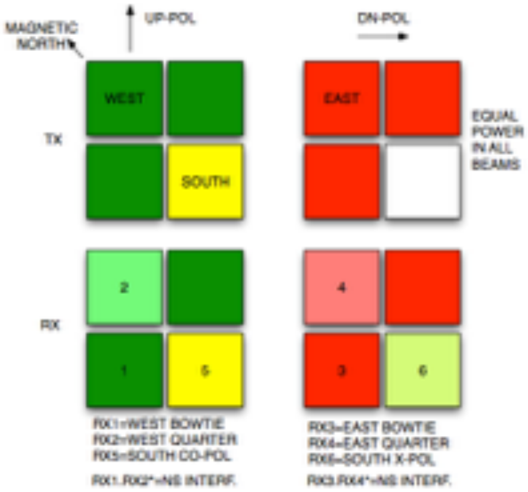
- Coulomb collision effects (as modeled by the Fokker-Planck equation with Spitzer coefficients) on the ion motion can be approximated as a Brownian motion process for H^+, He^+, and O^+ (ionospheric) plasmas.
- In the case of the electrons, Brownian motion does not capture all the details of the electron ACFs because the electron displacement distributions are not Gaussian. The approximation is not appropriate in this case.
- Electron displacement statistics are independent of the plasma configuration, therefore, electron ACFs are the same for H^+, He^+, and O^+ plasmas. We expect the same to happen in multiple-ion component plasmas.

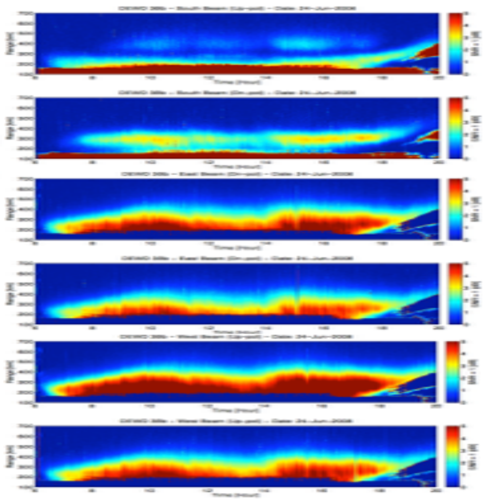


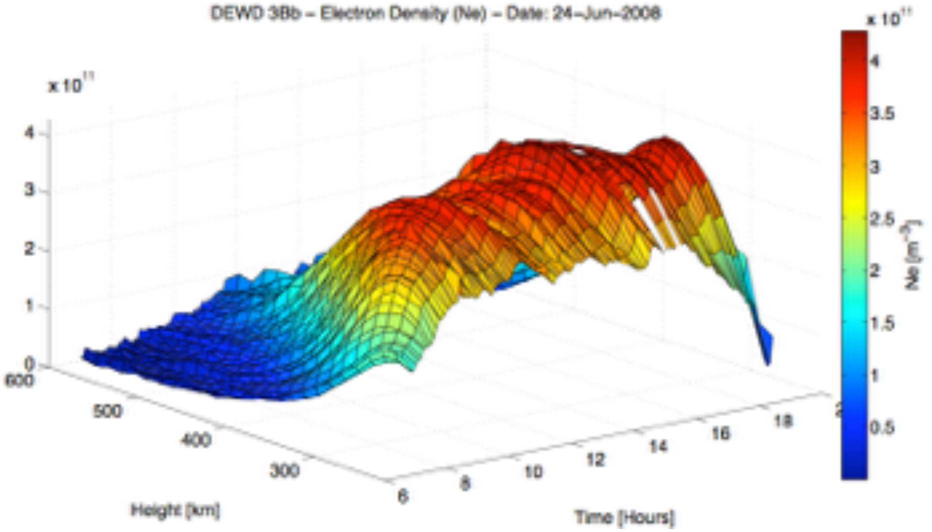
Future Work

ISR radar experiments and data analysis

- Validation of the collisional ISR spectrum model with radar experiments still needs to be done at Jicamarca.
- Multi-beam radar experiments to measure perpendicular-to-B and off perpendicular ISR data from the topside ionosphere.
- Analysis of radar data and inversion of ionospheric parameters (Densities, temperatures, and drifts).







More on the future

Another test for the collisional theory will be UHF ISR observations at the magnetic equator.