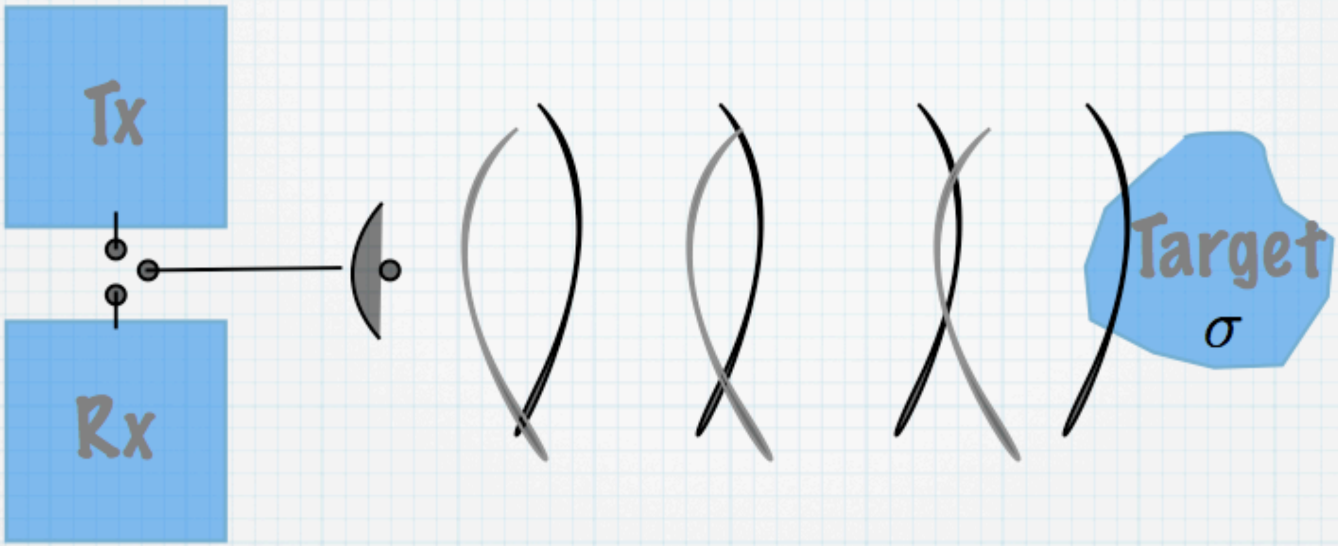
Introduction to Phased Arrays

Craig Heinselman

"Equations are the devil's sentences"
- **Stephen Colbert**

Basic Radar



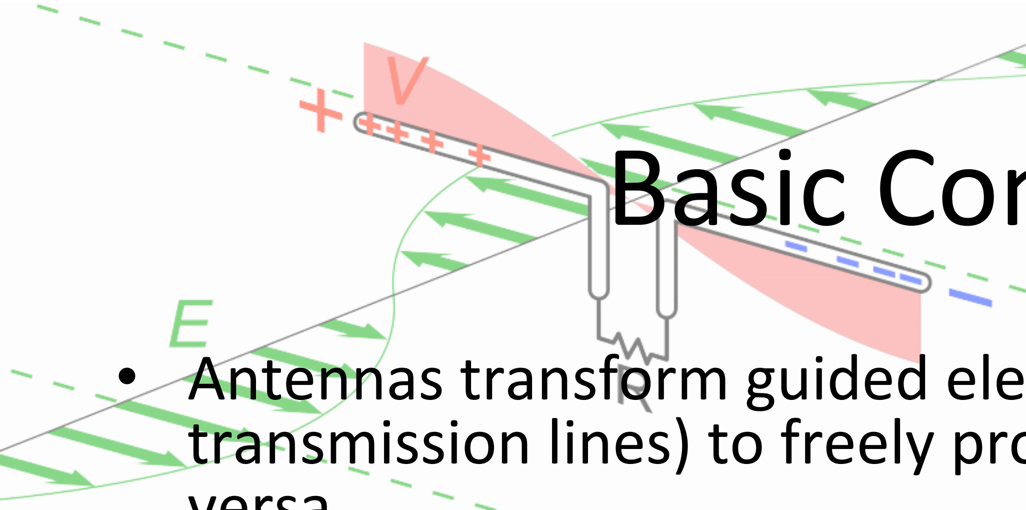
Target has a cross section (isotropic) [m^2]

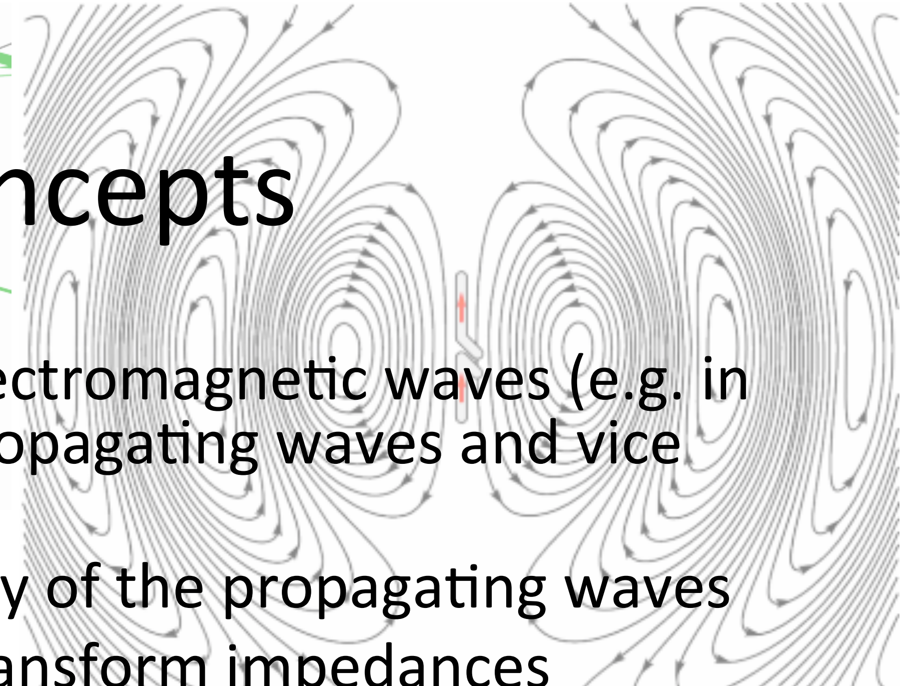
$$P_{inc} = P_t \frac{G_{tx}}{4\pi R^2} \quad \mathrm{W/m^2}$$ Power incident on target

$$P_{scat} = P_{inc}\sigma_{radar} \quad \mathrm{W}$$ Scattered power

$$P_{rec} = P_{scat}\frac{A_{eff}}{4\pi R^2} \quad \mathrm{W}$$ Received power

$$= P_t \frac{G_{tx} A_{eff} \sigma_{radar}}{16\pi^2 R^4} \quad \mathrm{W}$$ Radar equation





Basic Concepts

- Antennas transform guided electromagnetic waves (e.g. in transmission lines) to freely propagating waves and vice versa
- Antennas also direct the energy of the propagating waves
- Properly designed antennas transform impedances between transmission lines (typically 50Ω or 75Ω) and free space ($Z_0=\mu_0 c_0 \simeq 377\Omega$)
- Polarization of the propagating waves are specified/ controlled
- Orbital Angular Momentum of propagating waves may be of interest
- Basic relationship, $A_{eff} = \lambda^2 G/4\pi$, so antenna gain and effective aperture reflect the same thing
- Antenna calculations typically start with a current element…

Hertzian Dipole

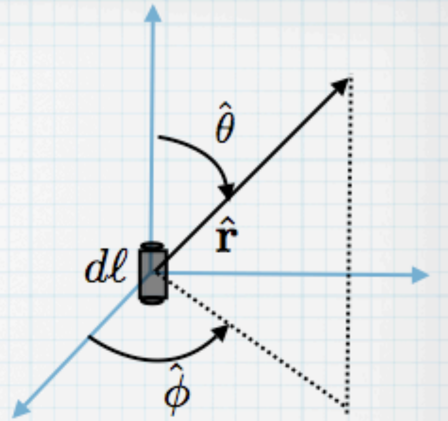
far field — near field

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{j z_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{j z_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

Spherically expanding wavefront



$$z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \omega\sqrt{\mu_0\epsilon_0}$$

For $r \gg \lambda$, keep terms only linear in r - **far field approximation.**

$$E_\theta \perp H_\phi \perp \hat{\mathbf{r}} \qquad \frac{E_\theta}{H_\phi} = z_0$$

Power flow represented by Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \qquad \langle P_r \rangle = \frac{1}{2}\Re\{P_r\} = \frac{1}{2}\Re\{\mathbf{E} \times \mathbf{H}\} \cdot \hat{\mathbf{r}} \quad \mathrm{W/m^2}$$

Hertzian Dipole (2)

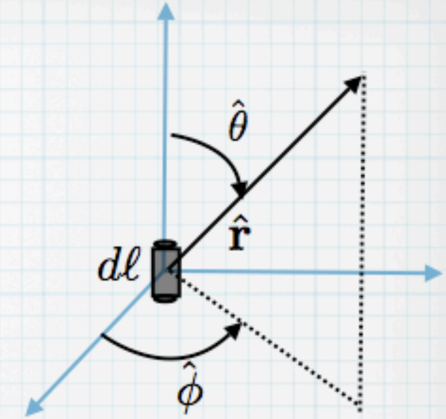
far field near field

$$H_\phi = I d\ell \sin\theta \frac{1}{4\pi}\left[\frac{jk_0}{r} + \frac{1}{r^2} + 0\right] e^{j(\omega t - k_0 r)}$$

$$E_r = I d\ell \cos\theta \frac{j z_0}{2\pi k_0}\left[0 + \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

$$E_\theta = I d\ell \sin\theta \frac{j z_0}{4\pi k_0}\left[\frac{k_0^2}{r} - \frac{jk_0}{r^2} + \frac{1}{r^3}\right] e^{j(\omega t - k_0 r)}$$

Spherically expanding



Directivity pattern:

$$D(\theta,\phi) = \frac{\text{Power Density Radiated In } (\theta,\phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta,\phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\}\cdot\hat{\mathbf{r}} = I^2 z_0 (d\ell)^2 k_0^2 \sin^2\theta \frac{1}{32\pi^2 r^2} \ \mathrm{W/m^2}$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^{\pi} \langle P_r \rangle r^2 \sin\theta d\theta = z_0 \frac{\pi}{3}\left(\frac{I d\ell}{\lambda}\right)^2 \ \mathrm{W}$$

$$P_{total} = \frac{1}{2} I^2 R_{rad}$$

$$D(\theta,\phi) = \frac{3}{2}\sin^2\theta$$

Directivity Patterns for Dipoles

Hertzian Dipole

$$D(\theta, \phi) = \frac{3}{2} \sin^2 \theta$$

$$\text{HPBW} = 90^\circ$$

Half-Wave Dipole

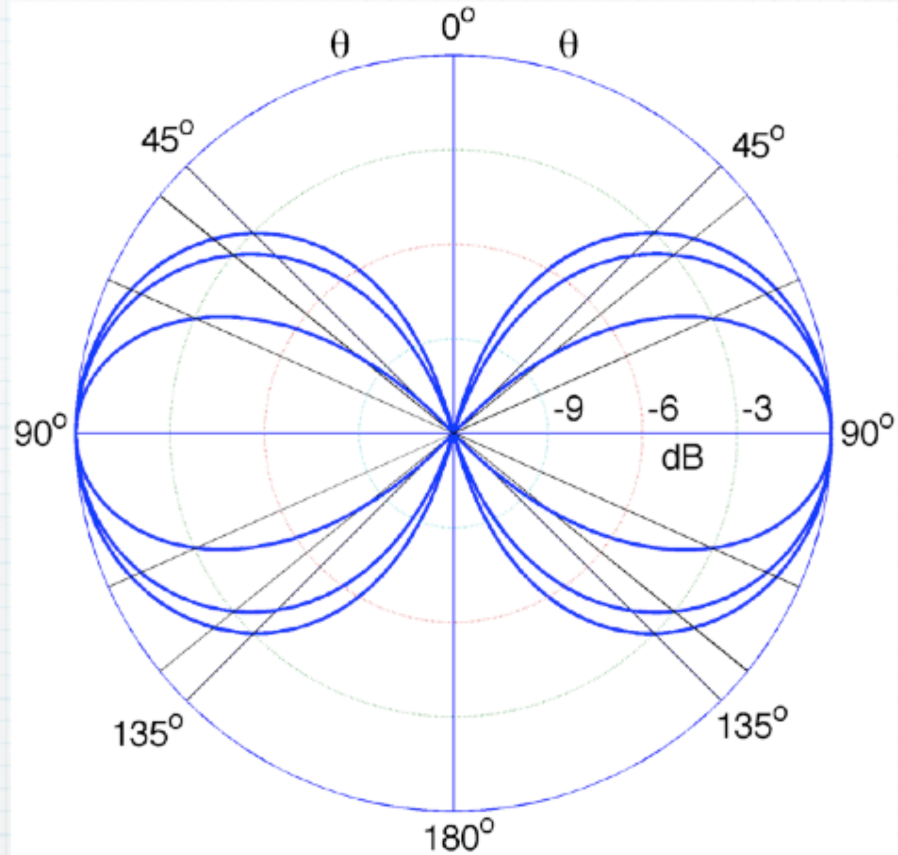
$$D(\theta, \phi) = 1.64 \left[\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right]^2$$

$$\text{HPBW} \approx 78^\circ$$

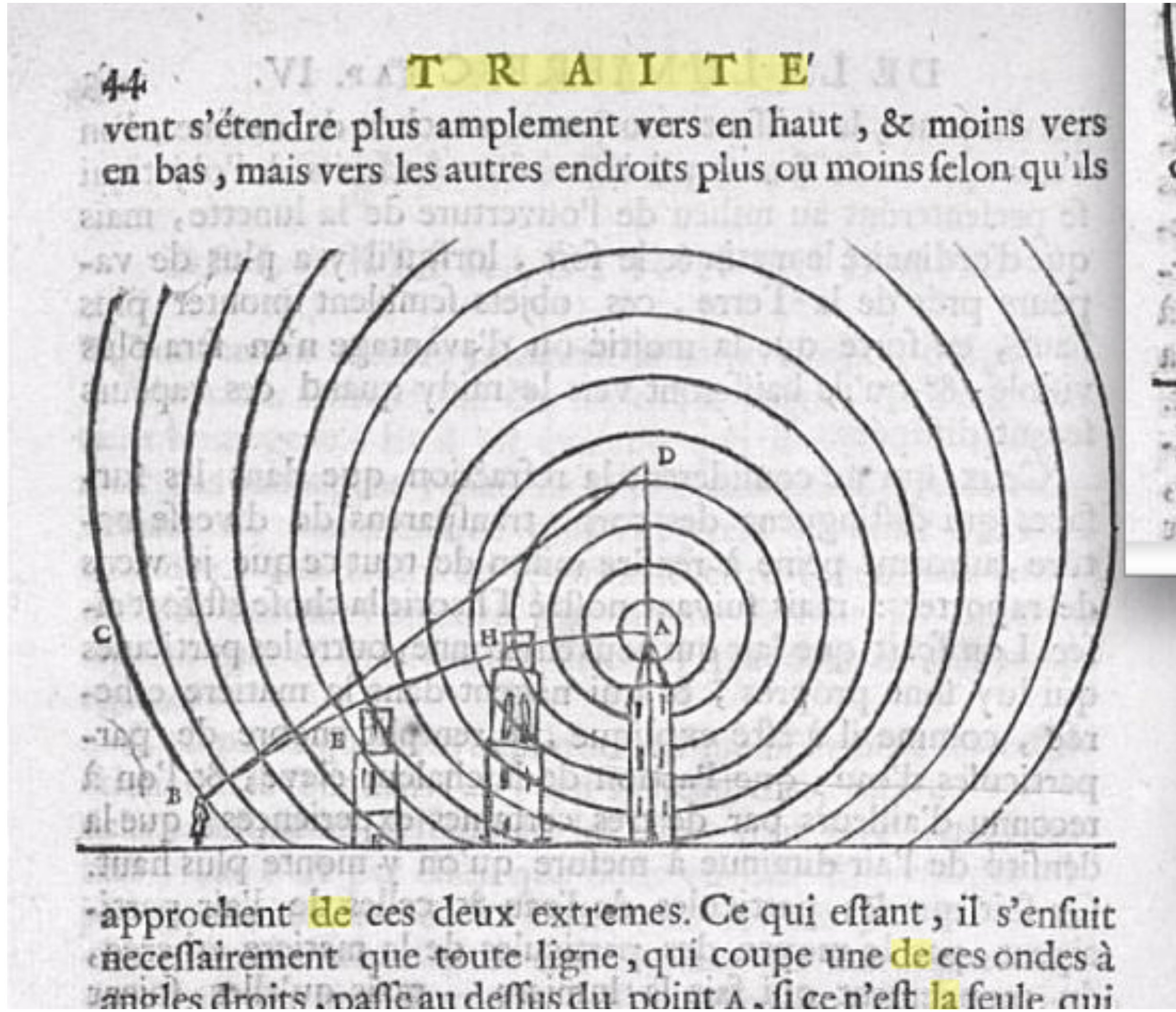
Full-Wave Dipole

$$D(\theta, \phi) = 2.41 \left| \frac{\cos(\pi \cos \theta) - 1}{\sin \theta} \right|^2$$

$$\text{HPBW} \approx 48^\circ$$



Christiaan Huygens

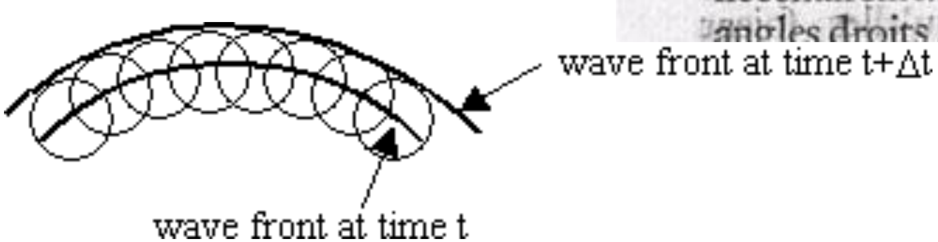


44 TRAITE

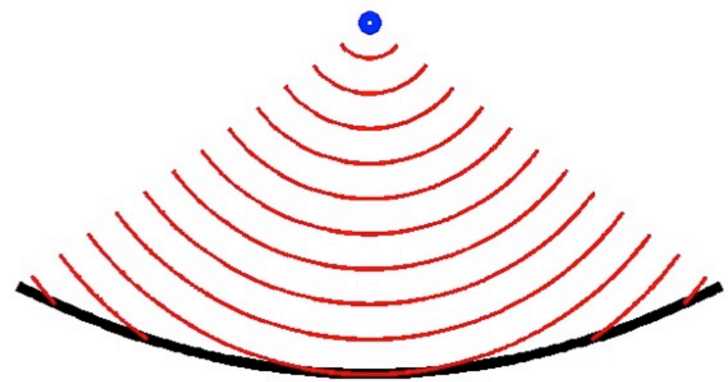
vent s'étendre plus amplement vers en haut, & moins vers en bas, mais vers les autres endroits plus ou moins ſelon qu'ils

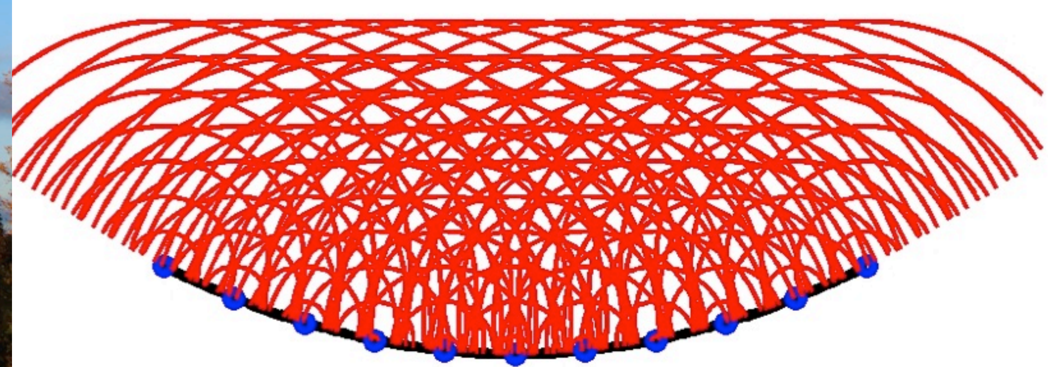
approchent de ces deux extremes. Ce qui eſtant, il s'enſuit neceſſairement que toute ligne, qui coupe une de ces ondes à angles droits, paſſe au deſſus du point A, ſi ce n'eſt la ſeule qui

Traitė de la Lumiere (Treatise on Light) completed in 1678, published in Leyden in 1690



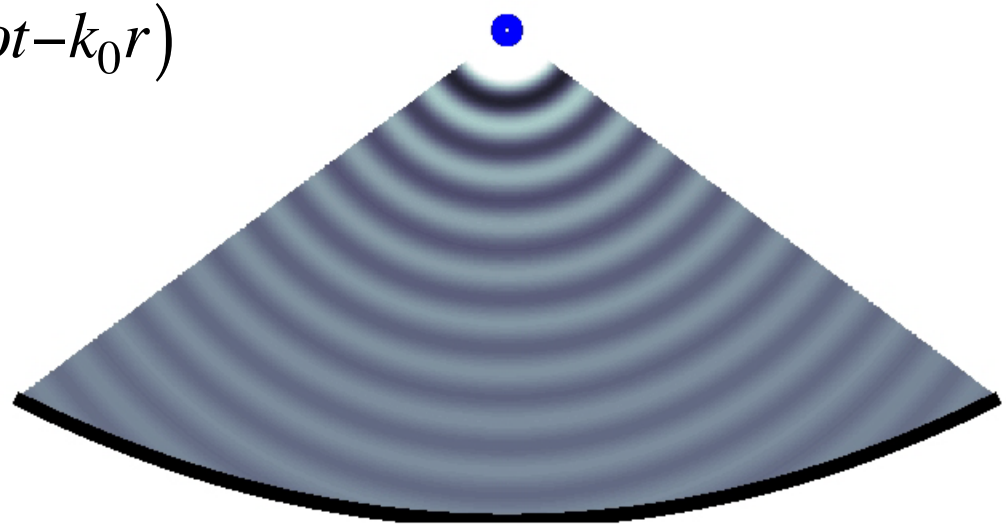
Dish Antennas





https://heinselslug.smugmug.com/Professional/Incoherent-Scatter-Radars/

$$E_\theta \propto \frac{1}{r} \mathrm{e}^{j(\omega t - k_0 r)}$$



What is a Phased Array?

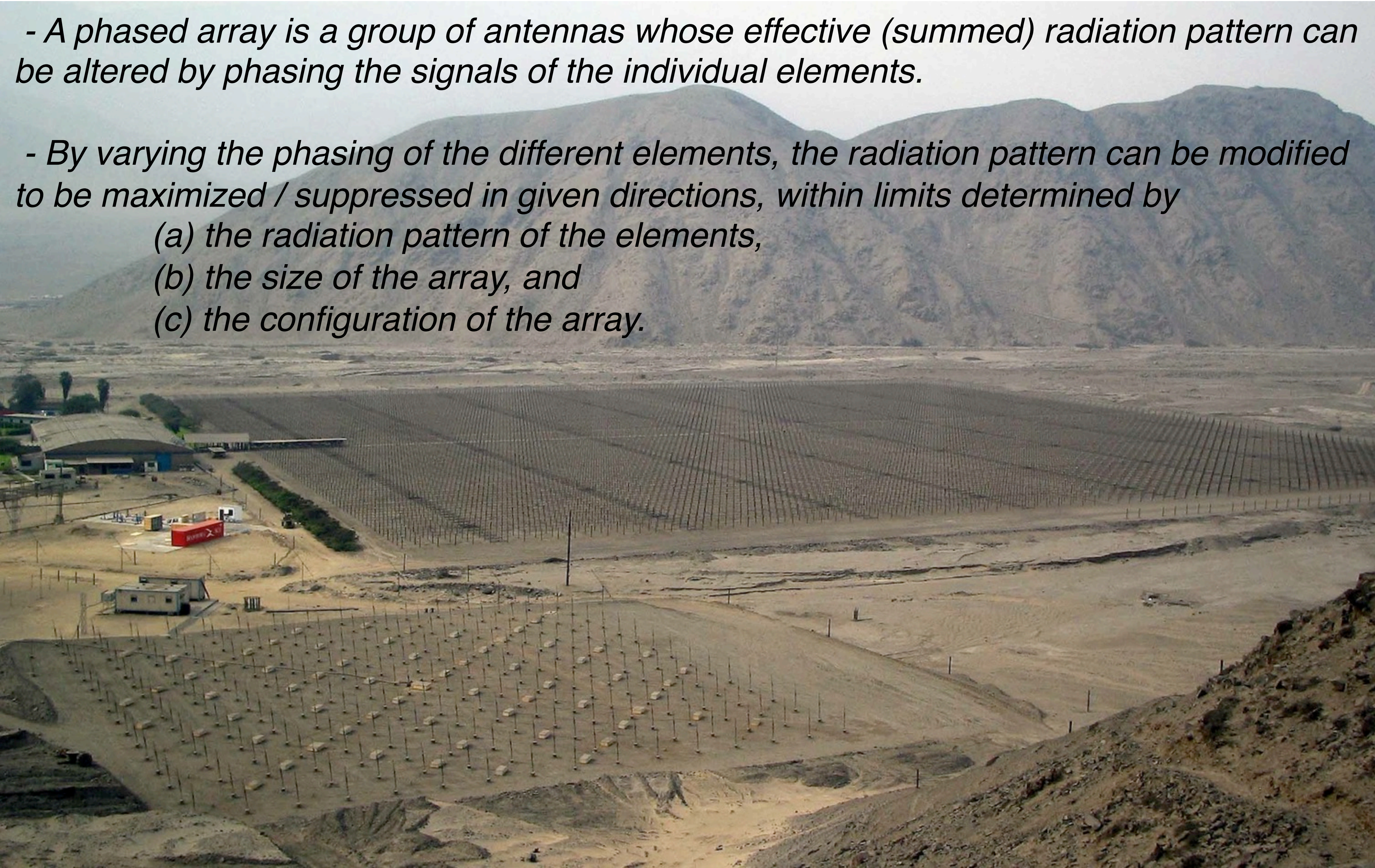
- *A phased array is a group of antennas whose effective (summed) radiation pattern can be altered by phasing the signals of the individual elements.*

- *By varying the phasing of the different elements, the radiation pattern can be modified to be maximized / suppressed in given directions, within limits determined by*

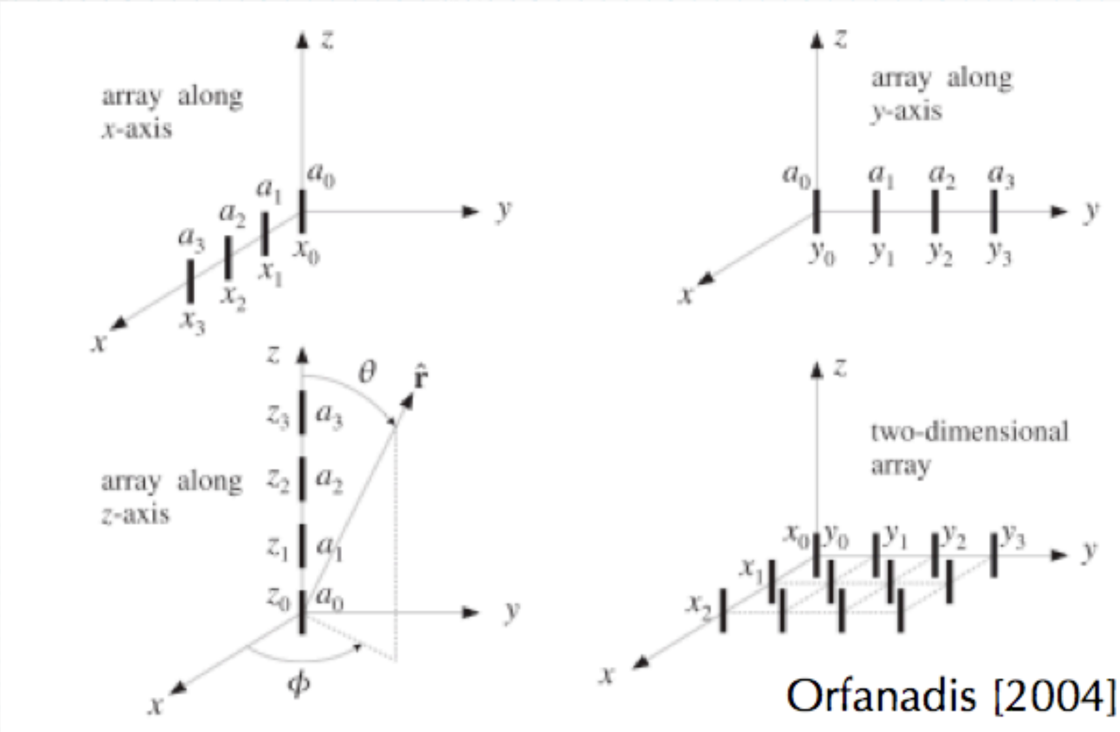
  (a) the radiation pattern of the elements,

  (b) the size of the array, and

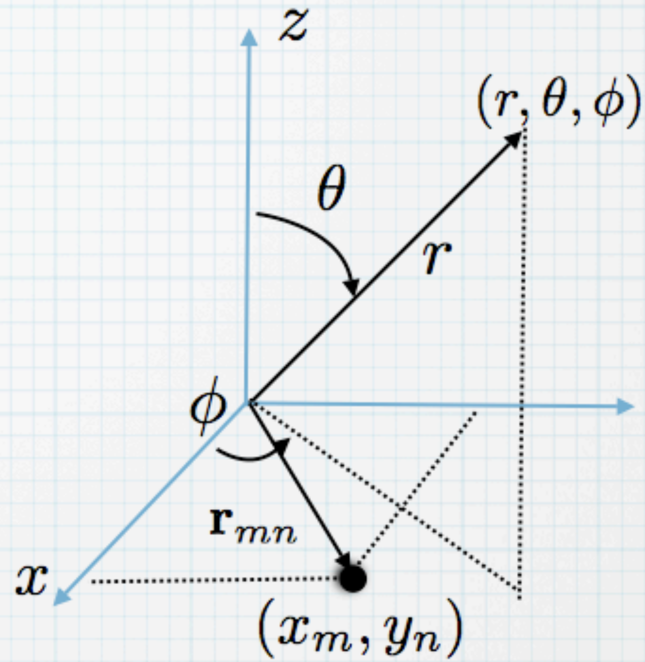
  (c) the configuration of the array.



Antenna Arrays



Orfanadis [2004]

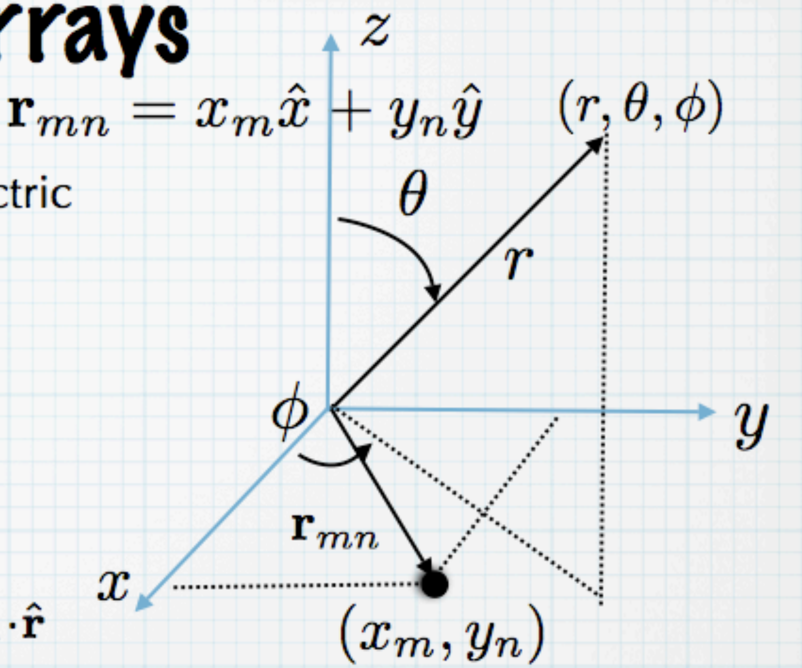


$$\mathbf{r}_{mn} = x_m \hat{x} + y_n \hat{y}$$

Assumptions:

1. Far field
   - parallel rays, 1/r amplitude dependence
2. No mutual coupling between elements (will discuss later)
3. A "reference" element radiates from the origin
4. All elements/radiators are identical, max radiation in z direction (broadside)

Antenna Arrays



Reference element at origin will produce a vector electric field at point (r, θ, ϕ)

$$\mathbf{E}_{00} = I_{00}(E_\theta\hat{\theta} + E_\phi\hat{\phi})$$

Constant

Fields due to *mn*th element is:

$$\mathbf{E}_{mn} = I_{mn}(E_\theta\hat{\theta} + E_\phi\hat{\phi})e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

$$= I_{mn}(E_\theta\hat{\theta} + E_\phi\hat{\phi})e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

Total vector field at (r, θ, ϕ)

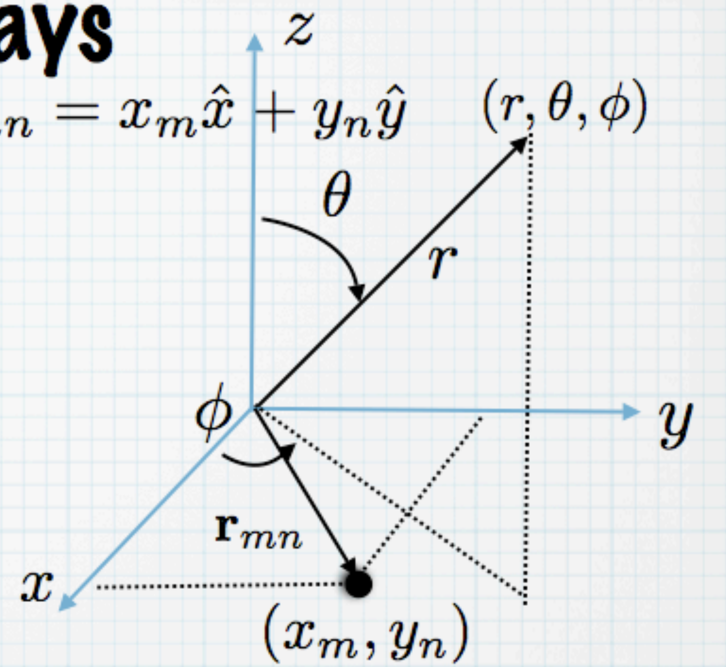
$$\mathbf{E} = (E_\theta\hat{\theta} + E_\phi\hat{\phi})\sum_m\sum_n I_{mn}e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

Element Factor

Array Factor

Antenna Arrays

$$\mathbf{r}_{mn} = x_m\hat{x} + y_n\hat{y}$$



$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk\mathbf{r}_{mn}\cdot\hat{\mathbf{r}}}$$

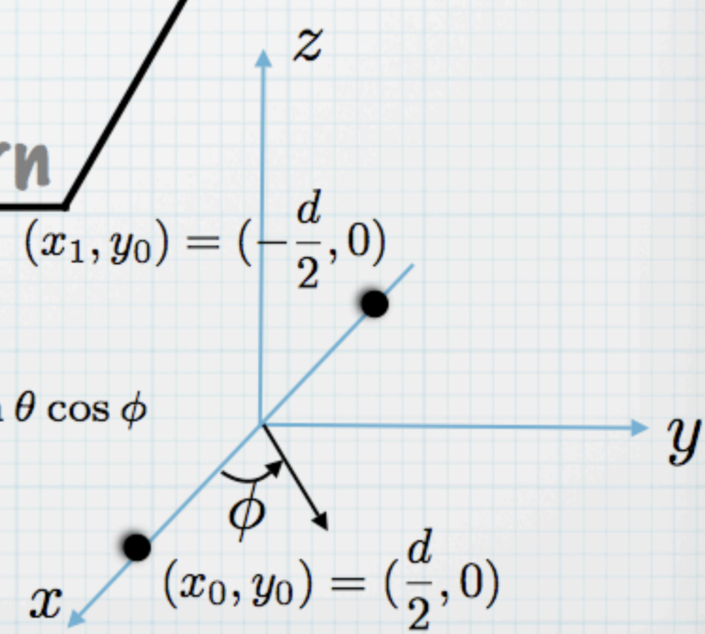
Poynting vector

$$\mathbf{P} = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\} = \frac{1}{2z_0}|\mathbf{E}|^2\hat{\mathbf{r}}$$

$$= \frac{1}{2z_0}(|E_\theta|^2 + |E_\phi|^2)|F_{array}|^2\hat{\mathbf{r}}$$

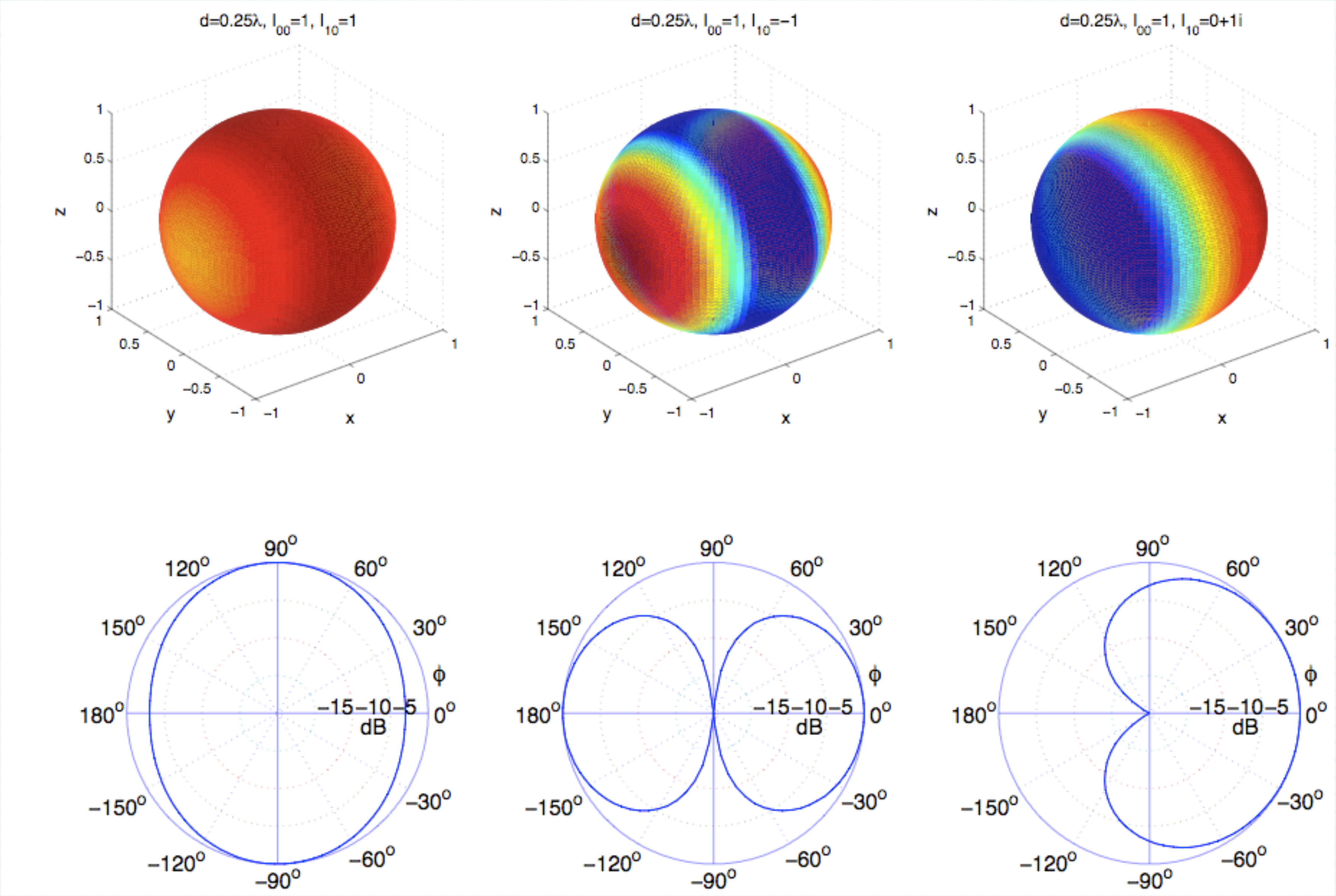
Element Pattern　　Array Pattern

Simple Two Element Array



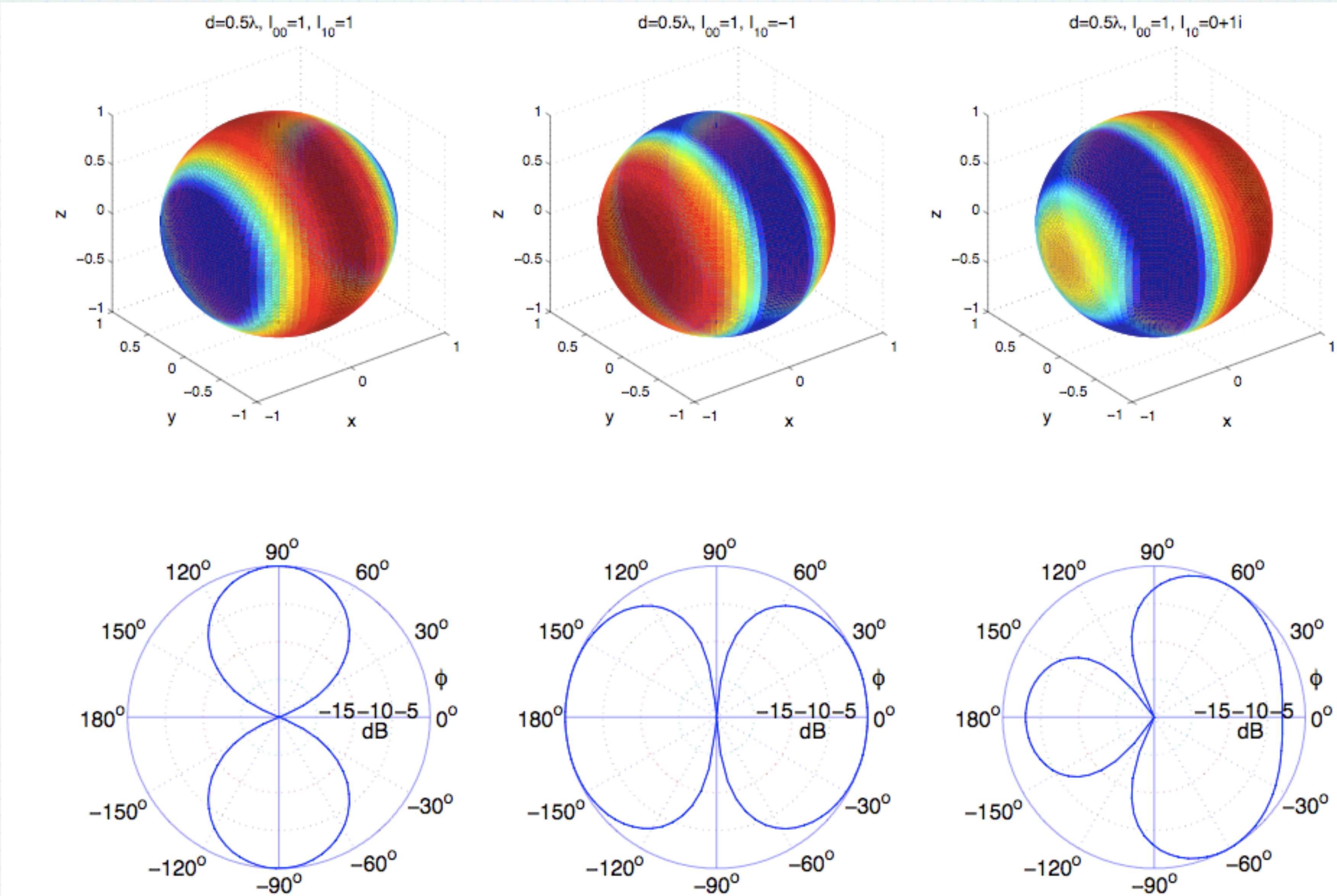
$$F_{array} = I_{00}e^{jk(d/2)\sin\theta\cos\phi} + I_{10}e^{-jk(d/2)\sin\theta\cos\phi}$$

Two-element Array



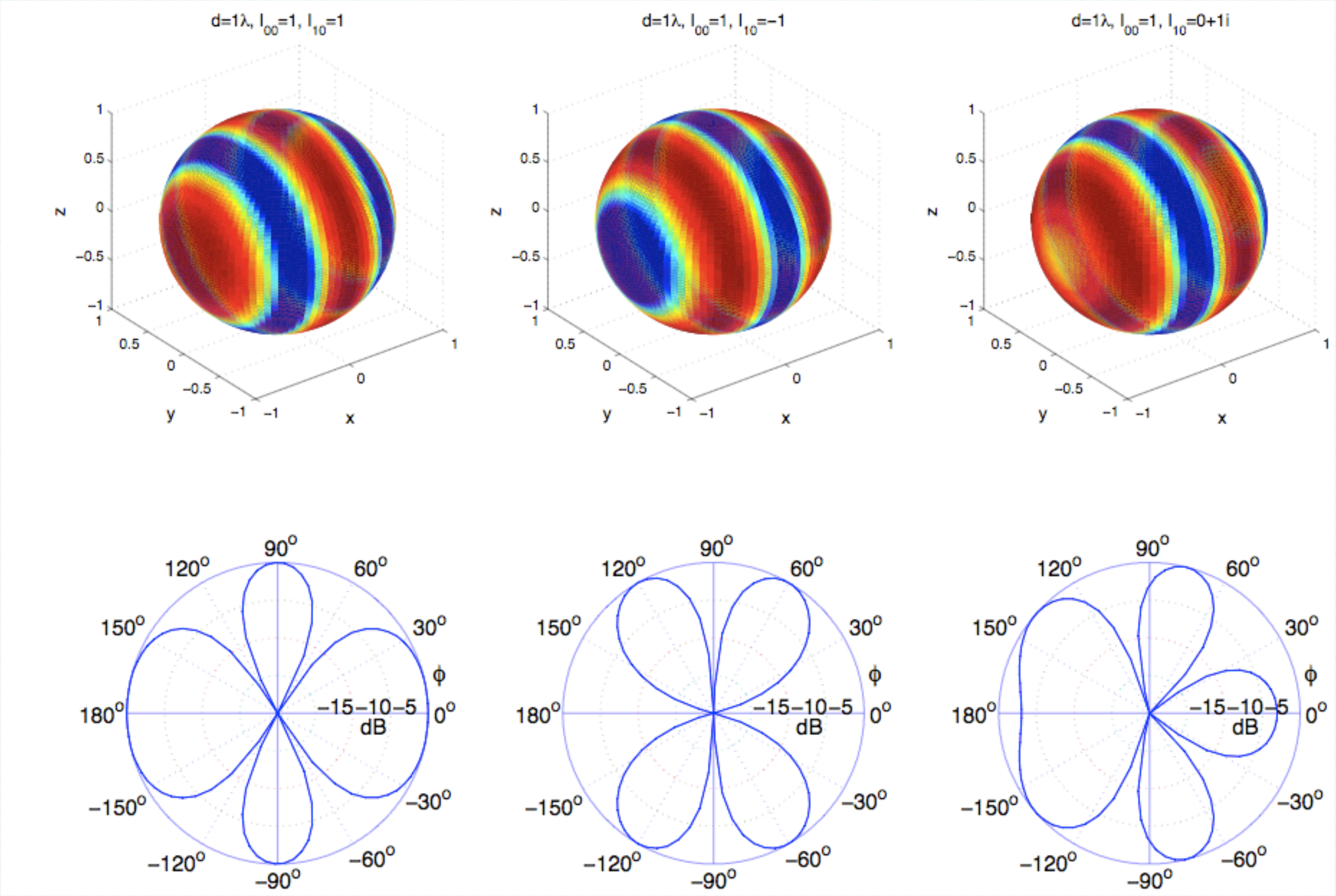
Element spacing: λ/4

Two-element Array



Element spacing: λ/2

Two-element Array



Element spacing: λ

Array Steering

$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

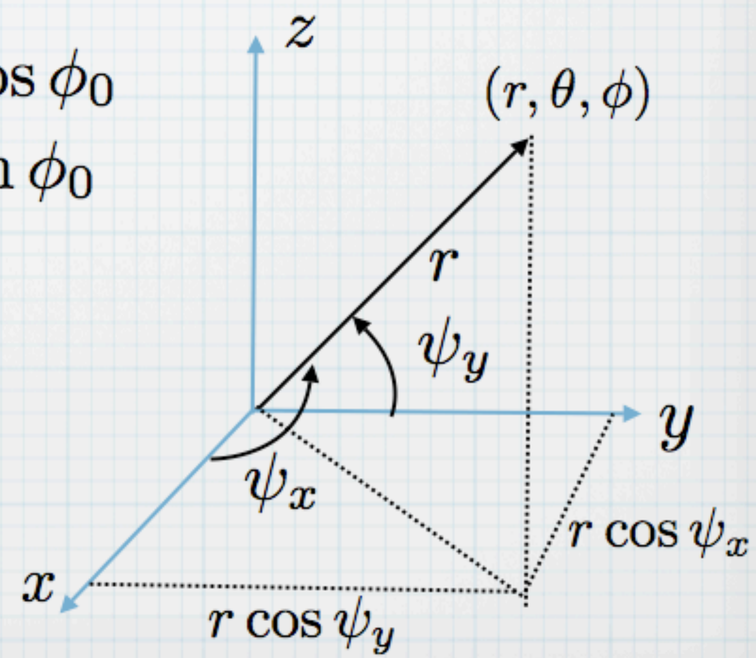
If the element constants have no phase angles, beam maximum will be in direction:

$$x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi = 0 \longrightarrow \theta = 0$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin\theta_0\cos\phi_0 + y_n \sin\theta_0\sin\phi_0 + (\psi_m + \psi_n) = 0$$

$$\psi_m = -kx_m \sin\theta_0\cos\phi_0$$
$$\psi_n = -ky_n \sin\theta_0\sin\phi_0$$



Direction cosines:

$$\cos\psi_{x0} = \sin\theta_0\cos\phi_0$$
$$\cos\psi_{y0} = \sin\theta_0\sin\phi_0$$

Array Steering

$$F_{array}(\theta,\phi) = \sum_m \sum_n I_{mn} e^{jk(x_m \sin\theta\cos\phi + y_n \sin\theta\sin\phi)}$$

$$F_{array} = \sum_{m,n} I_{mn} e^{jkx_m(\cos\psi_x - \cos\psi_{x0})} e^{jky_n(\cos\psi_y - \cos\psi_{y0})}$$

Say we want to point in direction (θ_0, ϕ_0)

$$x_m \sin\theta_0 \cos\phi_0 + y_n \sin\theta_0 \sin\phi_0 + (\psi_m + \psi_n) = 0$$

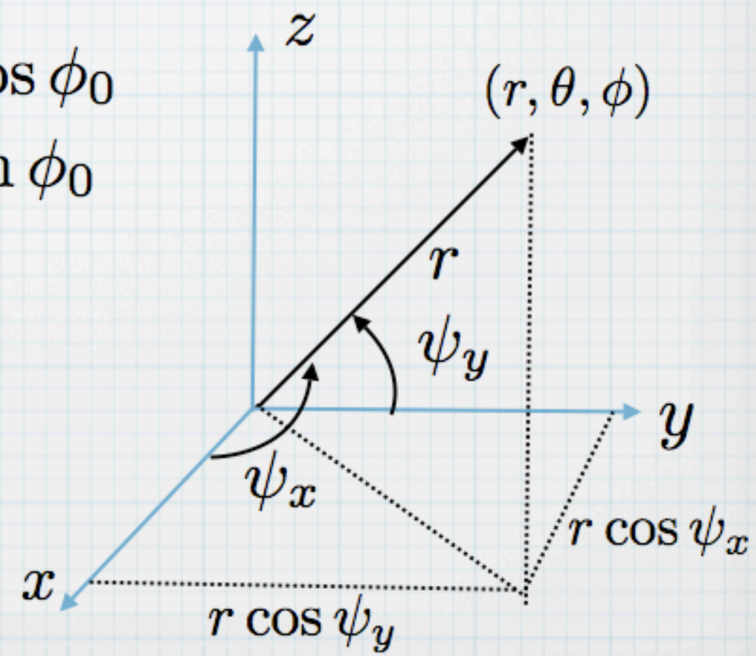
$$\psi_m = -kx_m \sin\theta_0 \cos\phi_0$$

$$\psi_n = -ky_n \sin\theta_0 \sin\phi_0$$

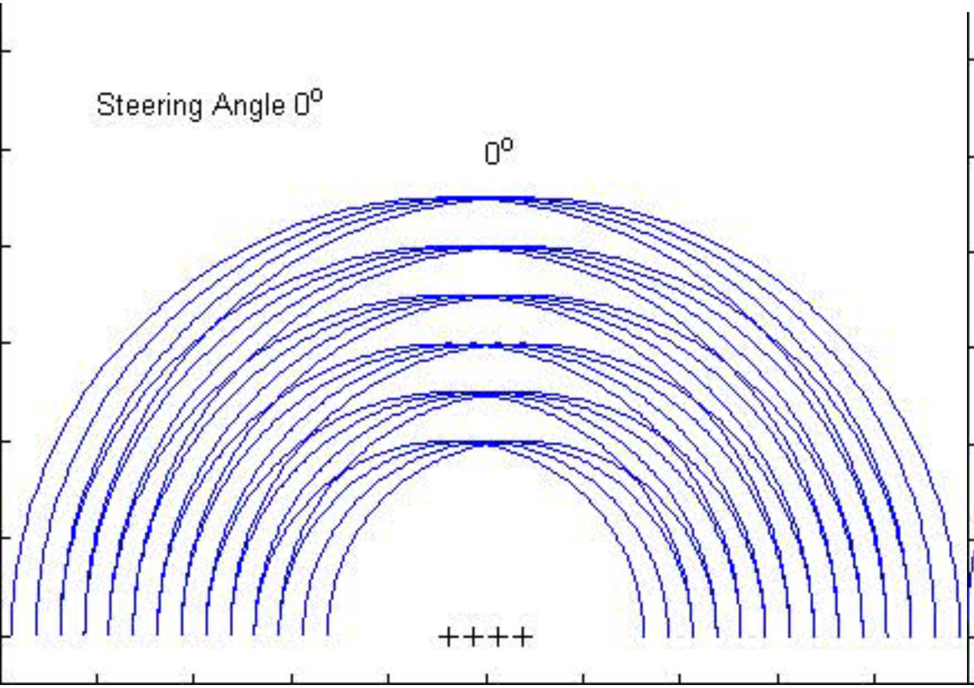
Direction cosines:

$$\cos\psi_{x0} = \sin\theta_0 \cos\phi_0$$

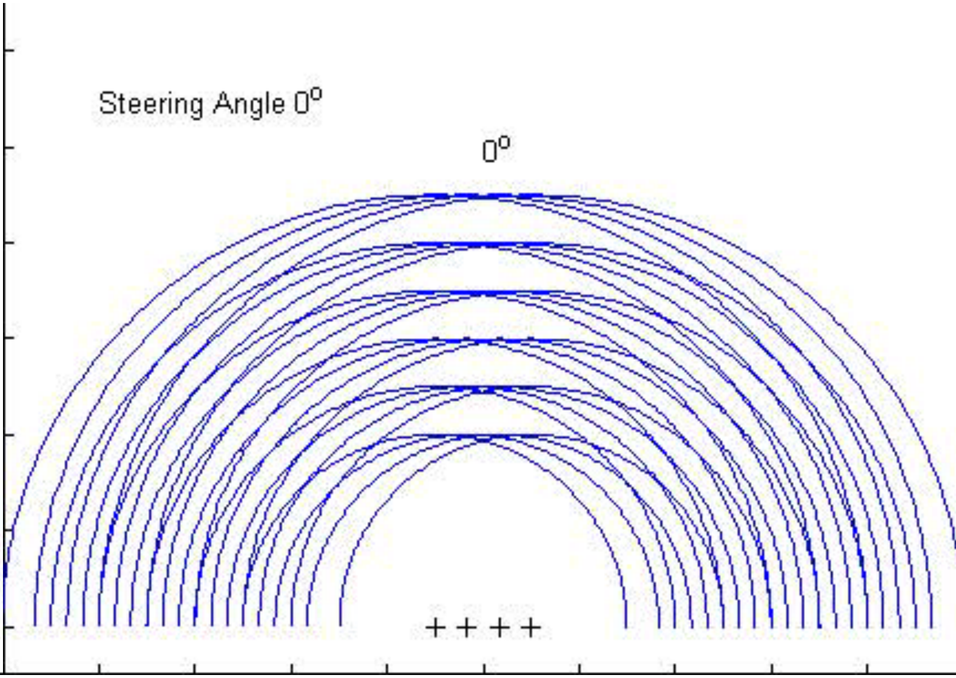
$$\cos\psi_{y0} = \sin\theta_0 \sin\phi_0$$



Grating Lobes

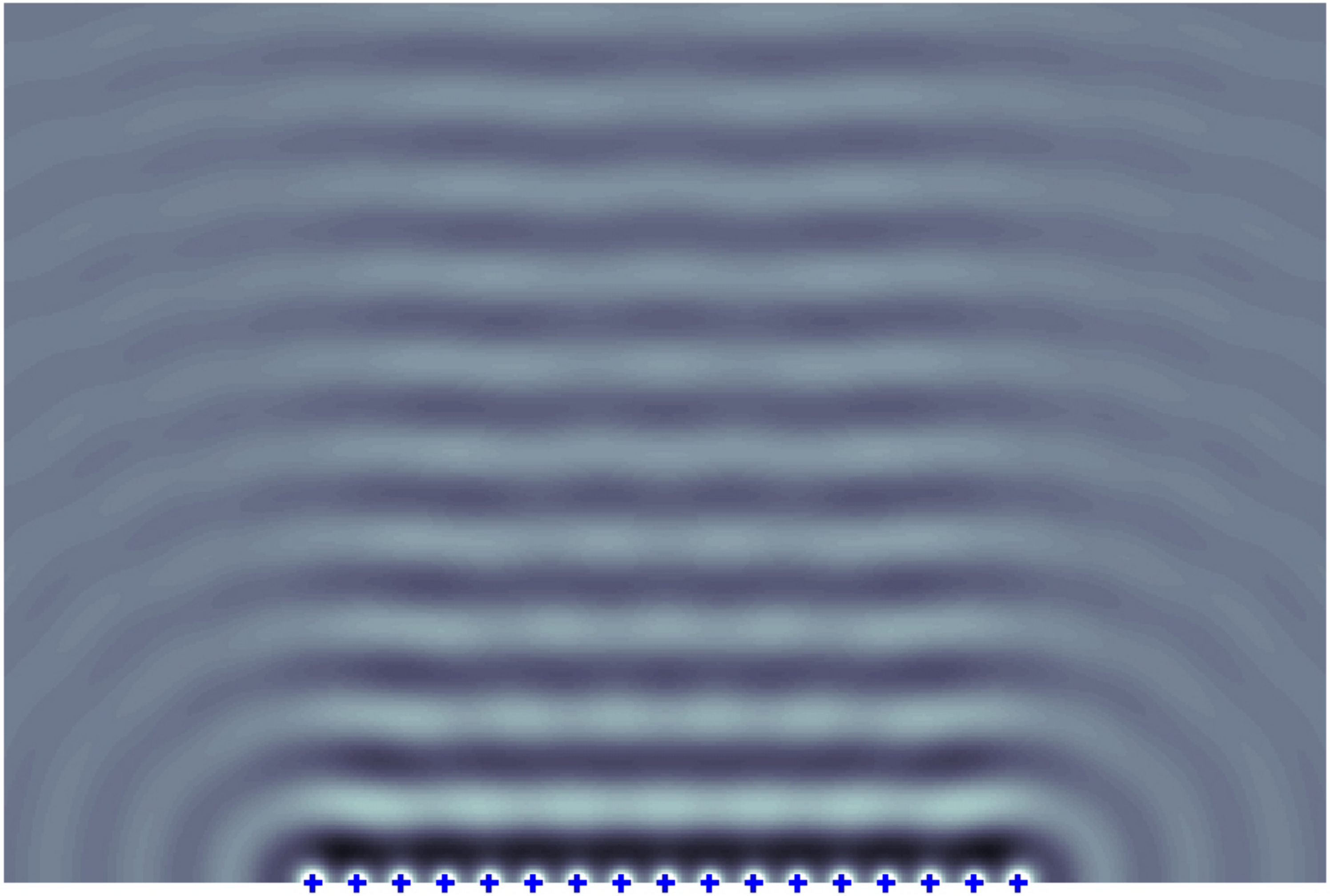


Element Spacing 0.50λ

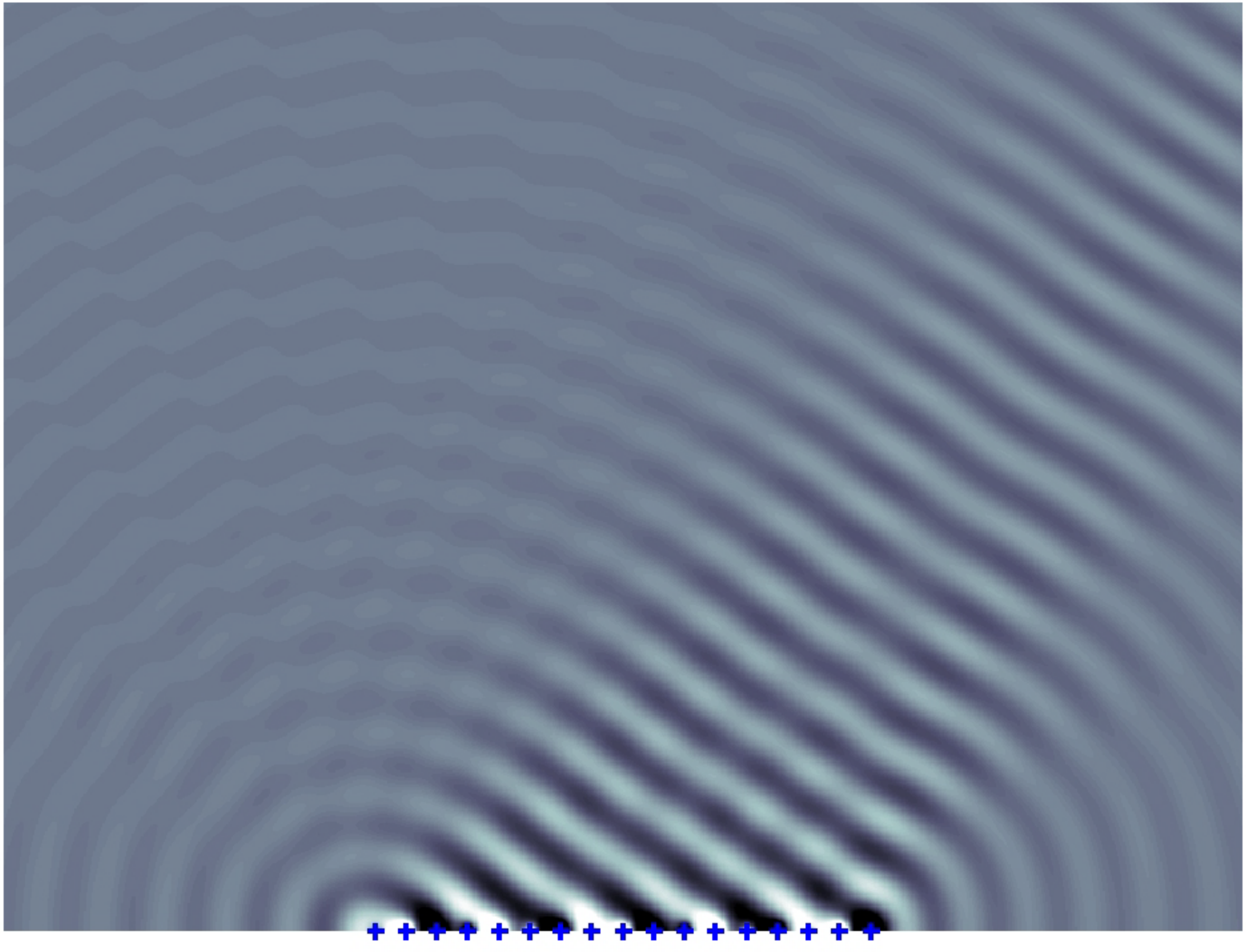


Element Spacing 0.67λ

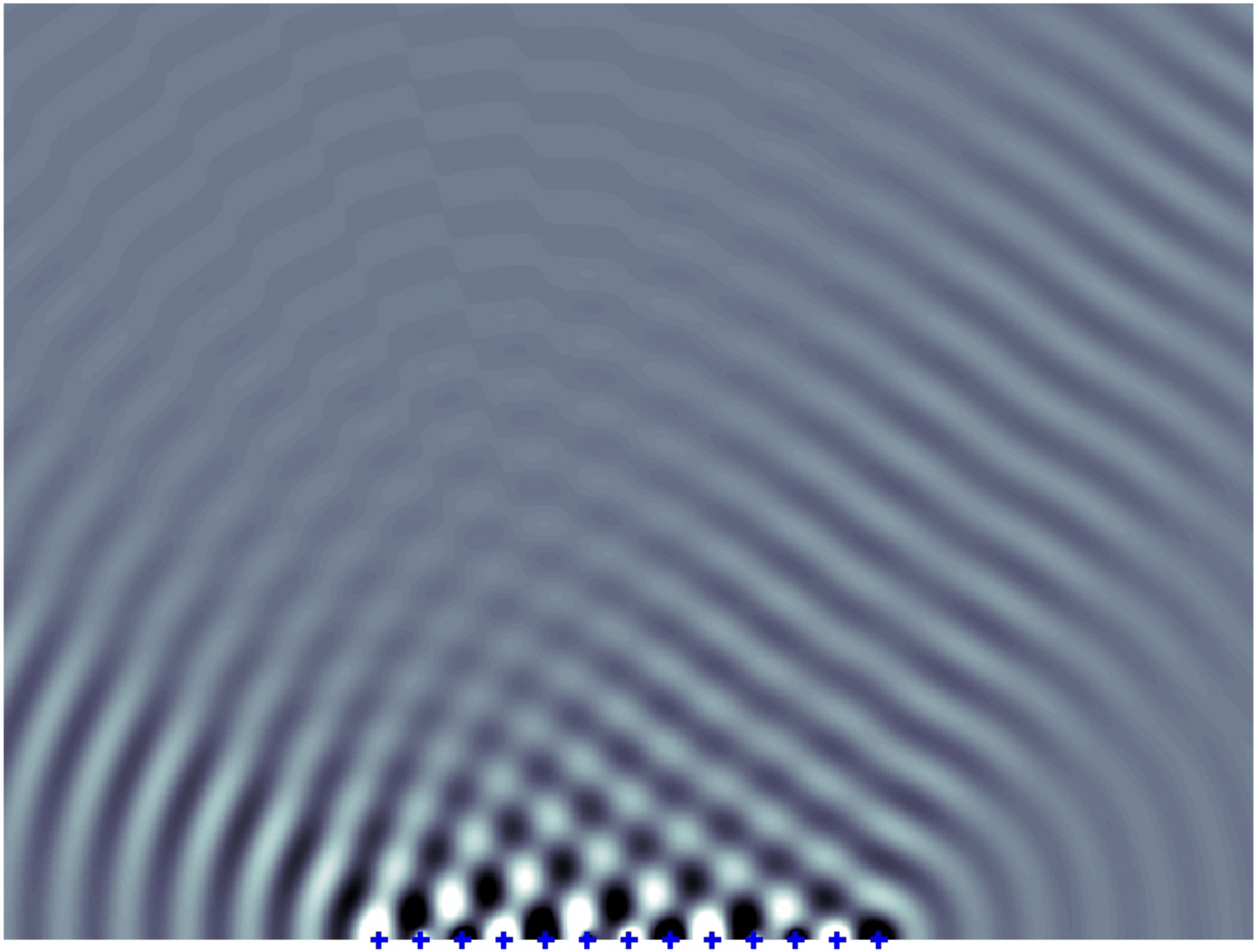
Phased Array, $\lambda/2$ spacing



Phased Array, $\lambda/2$ spacing



Phased Array, $2\lambda/3$ spacing



Directive Gain of Antenna Array

Recall:

$$D(\theta,\phi) = \frac{\text{Power Density Radiated In } (\theta,\phi) \text{ Direction}}{\text{Average Power Density}} = 4\pi R^2 \frac{\text{Power Density In } (\theta,\phi)}{\text{Total Power Radiated}}$$

$$\langle P_r \rangle = \frac{1}{2}\Re\{\mathbf{E}\times\mathbf{H}\}\cdot\hat{\mathbf{r}} = \frac{1}{2z_0}|\mathbf{E}|^2|F_{array}|^2 = P_{el}|F_{array}|^2$$

$$P_{total} = \int_0^{2\pi} d\phi \int_0^{\pi} P_{el}|F_{array}|^2 r^2 \sin\theta d\theta$$

$$D(\theta,\phi) = 4\pi r^2 \frac{P_{el}|F_{array}|^2}{\int_0^{2\pi} d\phi \int_0^{\pi} P_{el}|F_{array}|^2 r^2 \sin\theta d\theta}$$

If element pattern is much broader than array pattern,

Element pattern doesn't matter.

$$D(\theta,\phi) = 4\pi r^2 \frac{|F_{array}|^2}{\int_0^{2\pi} d\phi \int_0^{\pi} |F_{array}|^2 r^2 \sin\theta d\theta}$$

The Fourier Analogy

$$
\begin{aligned}
F_{array} &= \sum_m I_m e^{jkdm(\cos\psi_x - \cos\psi_{x0})} \\
&= \sum_m I_m e^{jm\gamma} \\
&= \sum_m I_m z^m
\end{aligned}
$$

Array factor can be interpreted as DFT of weighting factors

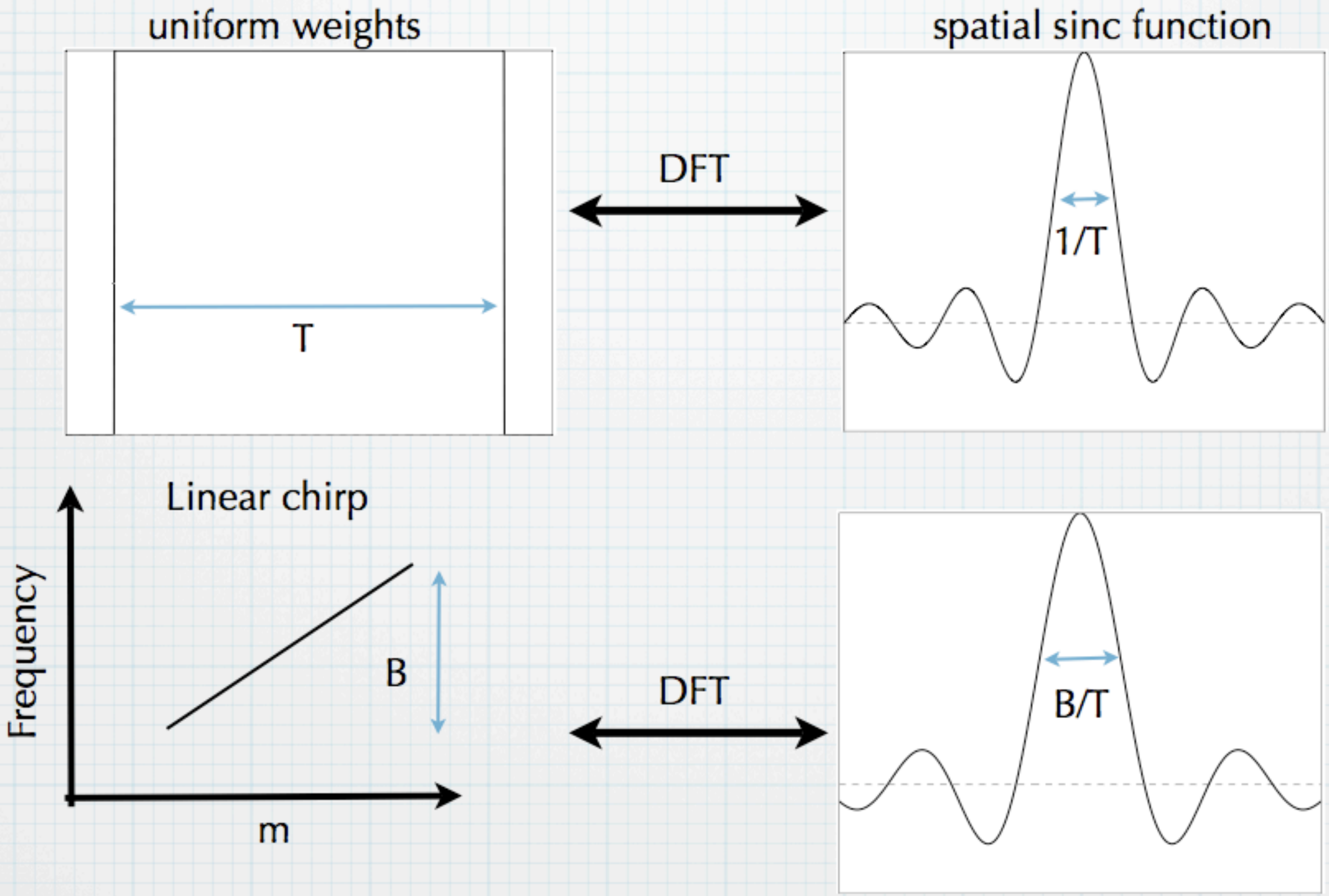
Array factor in spatial z domain

$$I_m = \frac{1}{2\pi}\int_{-\pi}^{\pi} F_{array}(\gamma) e^{-j\gamma m} d\gamma$$

Inverse DFT - principle of many array design methods (analogous to FIR filter design)

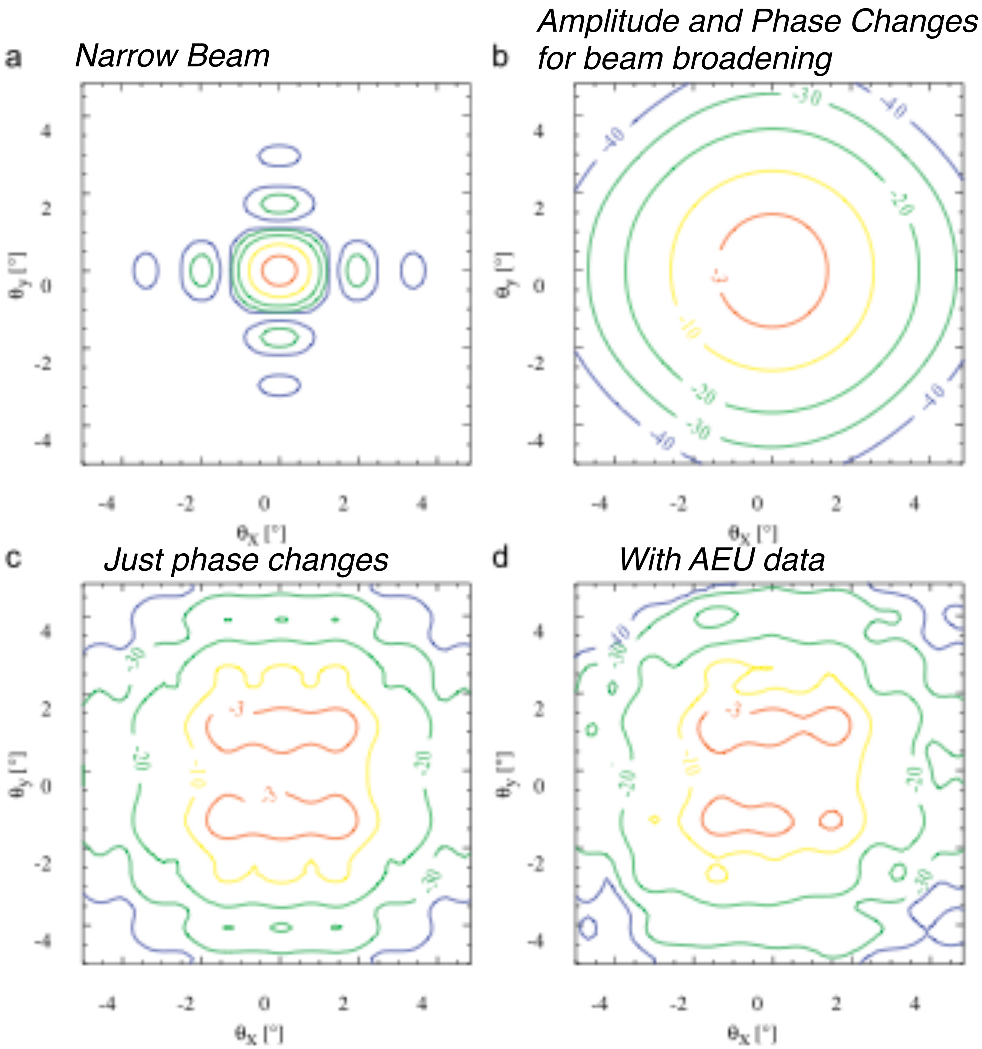
The Fourier Analogy (2)

One application - beam broadening



Tx/Rx Beam Pattern Control

Chau et al., [2009]



- Determine which meteors in the narrow beam are coming from sidelobes (~15 %)
- Increase number of large cross-section meteor detections

Method of Moments (mutual coupling)

RADIO SCIENCE, VOL. 46, RS2012, doi:10.1029/2010RS004518, 2011

A review on array mutual coupling analysis

C. Craeye[1] and D. González-Ovejero[1]

Received 8 September 2010; revised 14 December 2010; accepted 6 January 2011; published 8 April 2011.

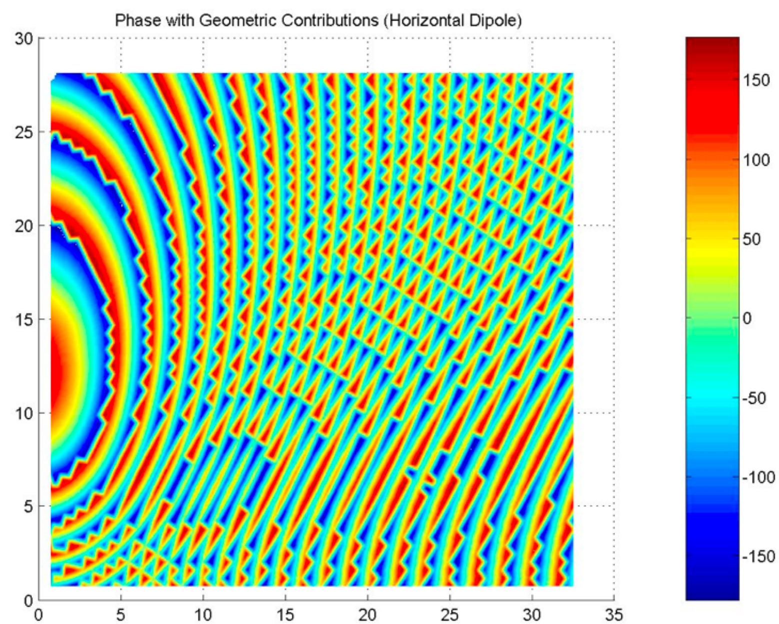
[1] An overview about mutual coupling analysis in antenna arrays is given. The relationships between array impedance matrix and embedded element patterns, including beam coupling factors, are reviewed while considering general-type antennas; approximations resulting from single-mode assumptions are pointed out. For regular arrays, a common Fourier-based formalism is employed, with the array scanning method as a key tool, to explain various phenomena and analysis methods. Relationships between finite and infinite arrays are described at the physical level, as well as from the point of view of numerical analysis, considering mainly the method of moments. Noise coupling is also briefly reviewed.

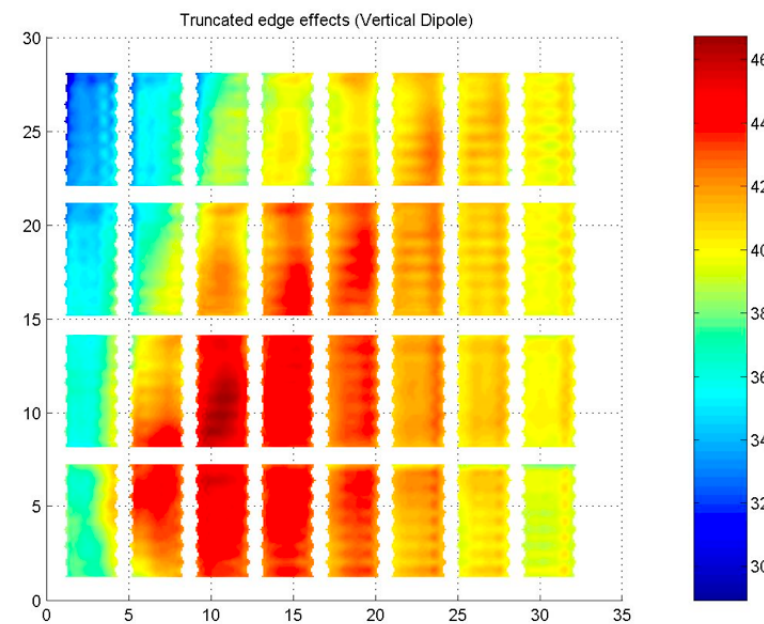
Citation: Craeye, C., and D. González-Ovejero (2011), A review on array mutual coupling analysis, *Radio Sci.*, *46*, RS2012, doi:10.1029/2010RS004518.

Method of Moments (NEC)

Significant errors are introduced if mutual coupling contributions are neglected.
Standard MoM code scales poorly, making it impractical to model a full array.
Great research project for engineering-minded student: develop sparse MoM code for an entire array!

136 PROCEEDINGS OF THE IEEE, VOL. 55, NO. 2, FEBRUARY, 1967

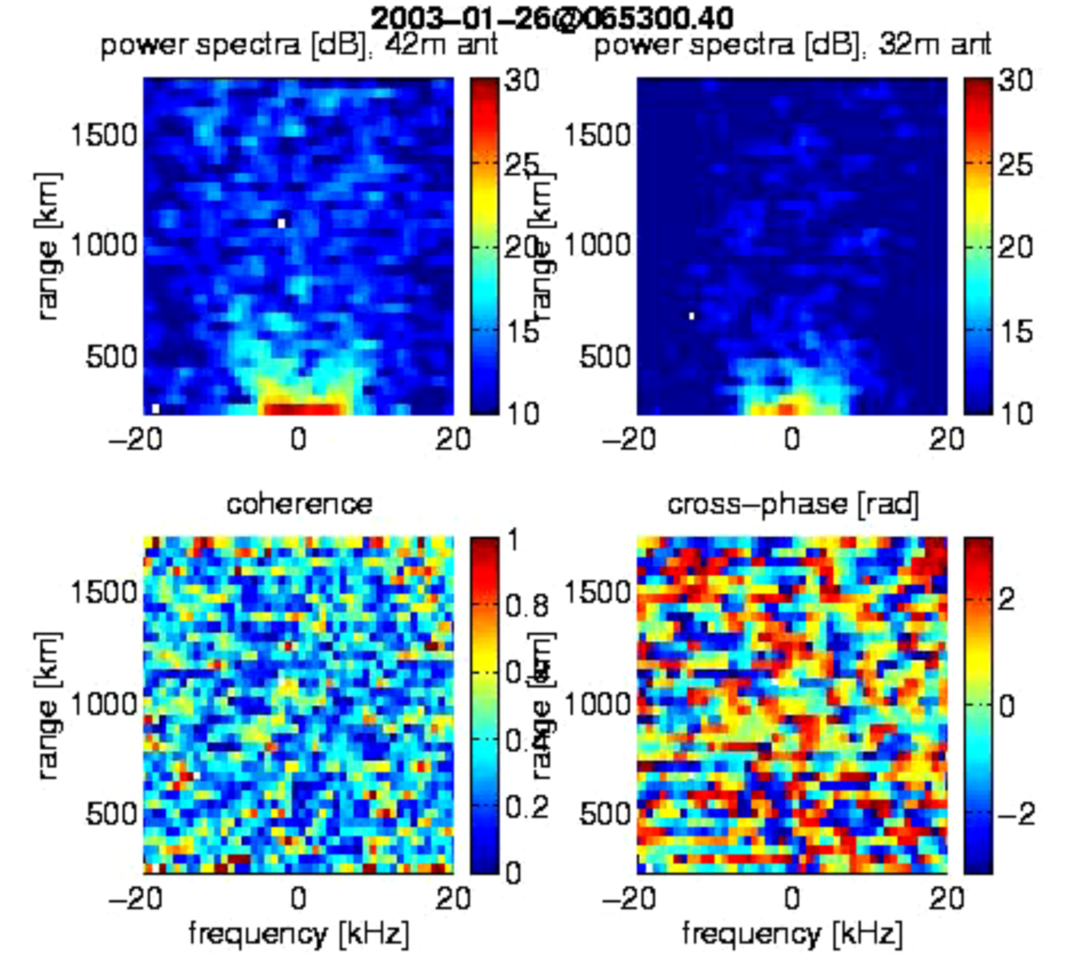




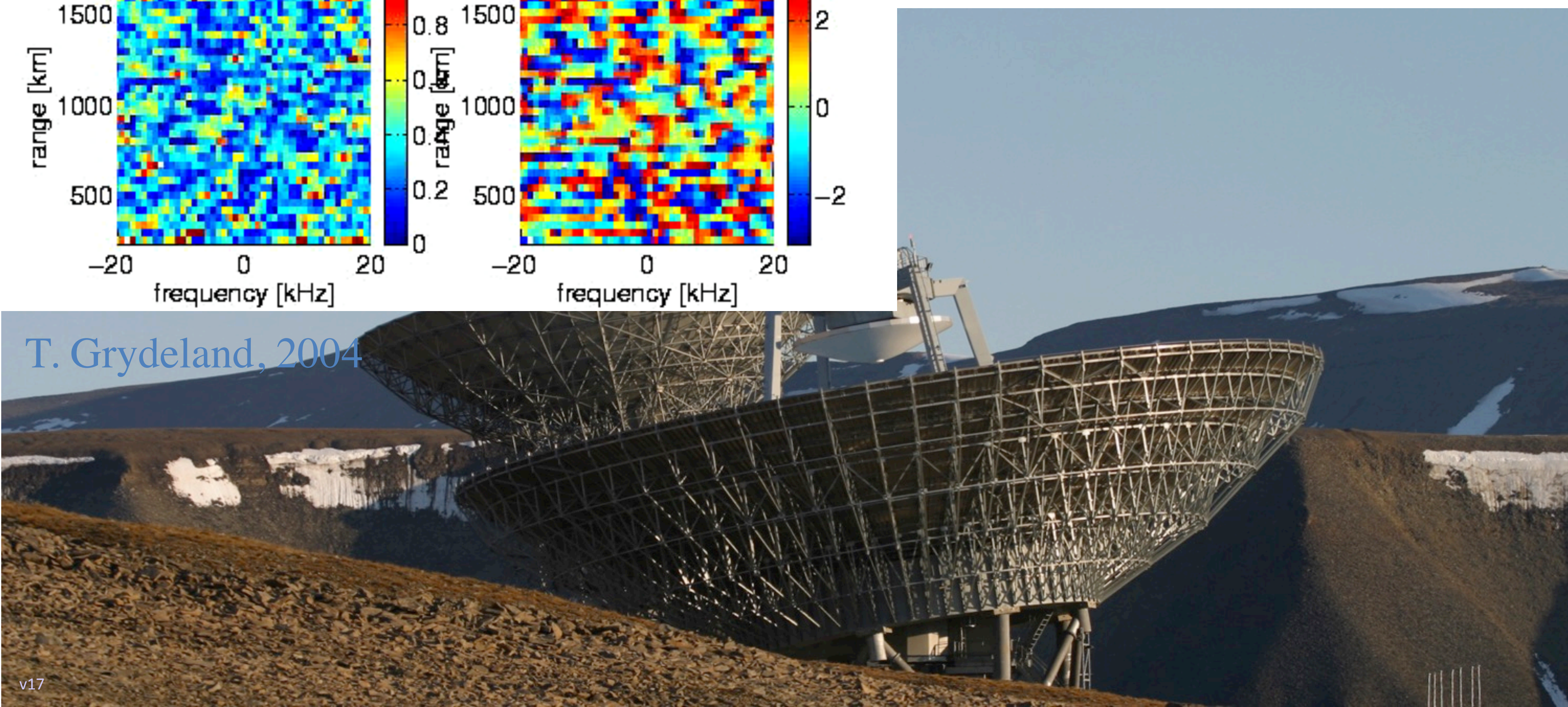
THE USE of high-speed digital computers not only

specified. This paper deals only with analysis.

etry



NEIAL Interferometry
ESR



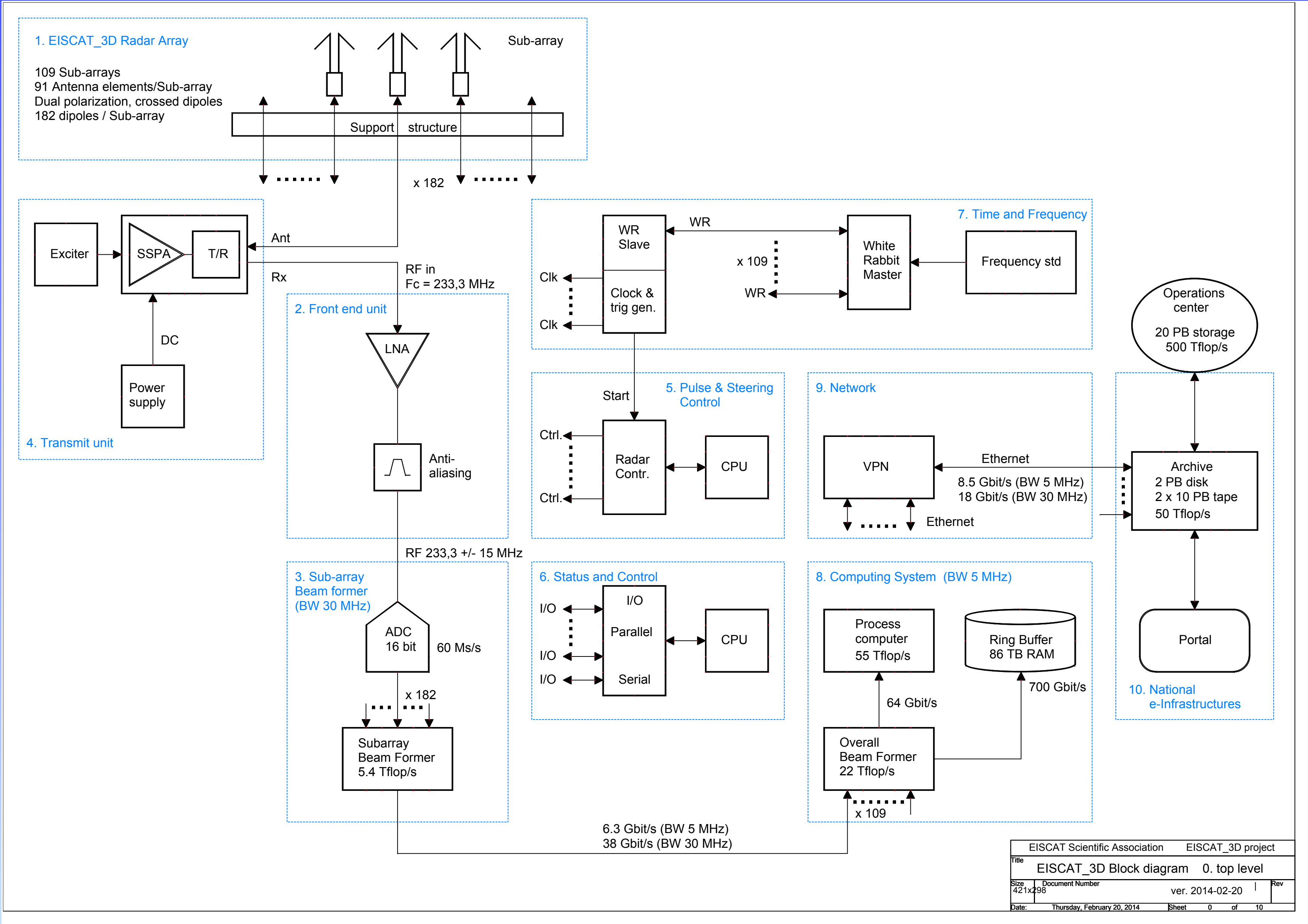
T. Grydeland, 2004

What are the Measurement Improvements

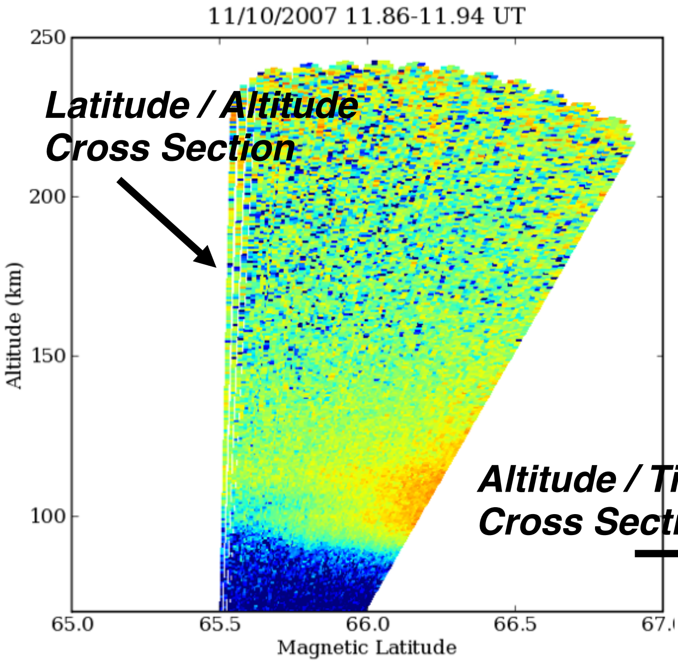
- Inertia-less antenna pointing
  - Pulse-to-pulse beam positioning
  - Supports great flexibility in spatial sampling
  - Helps remove spatial/temporal ambiguities
  - Eliminates need for predetermined integration (dish antenna dwell time)
  - Opens possibilities for in-beam imaging through interferometry

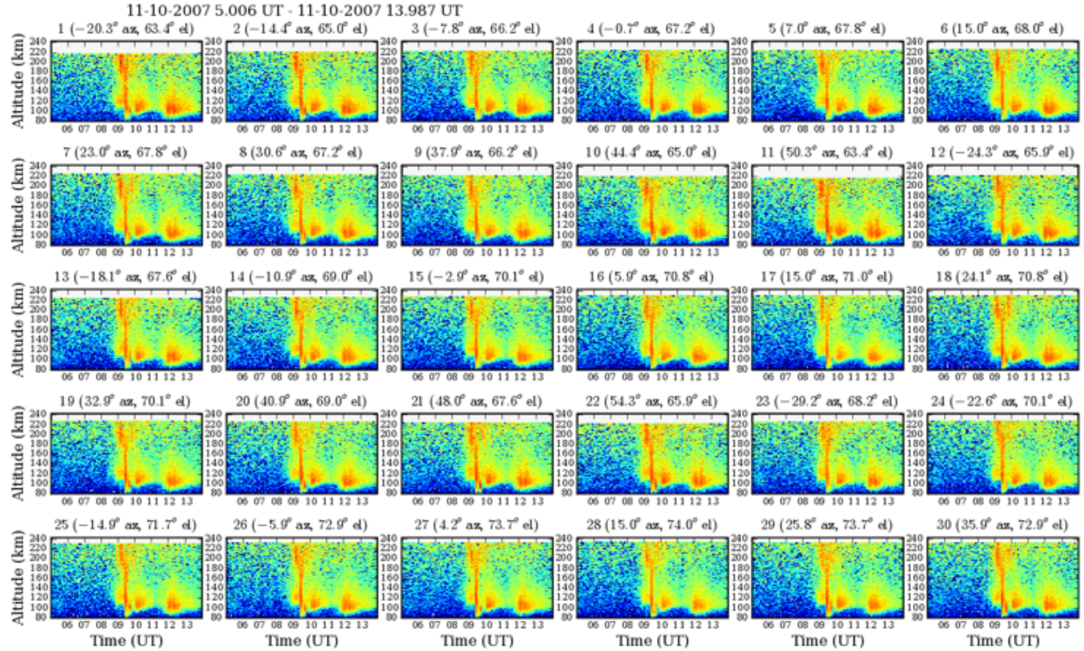
EISCAT Scientific Association

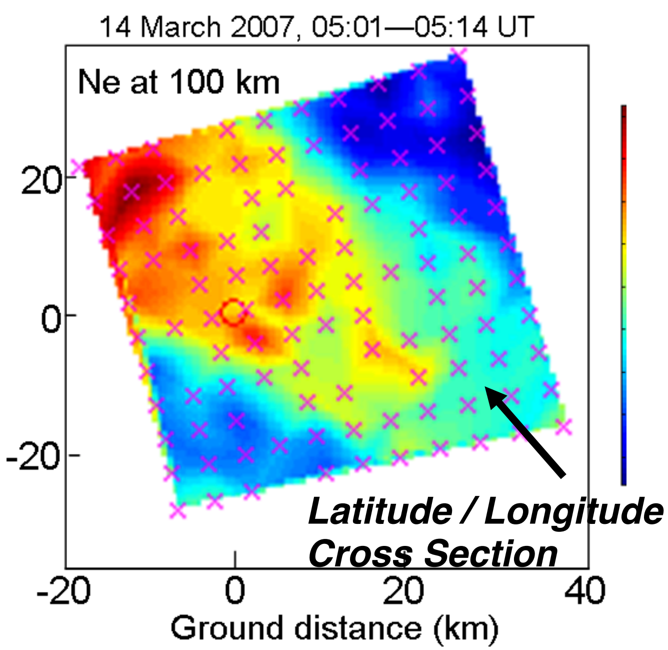
Significant Data Processing

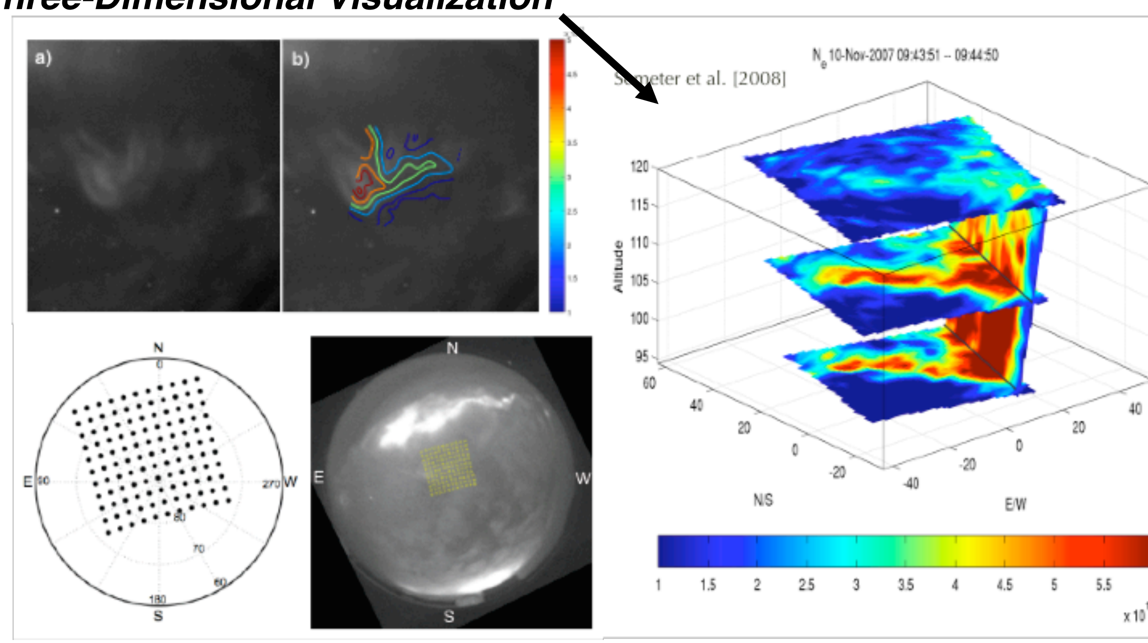


PFISR: Images of the Aurora in 4-Dimensions (3-D images v. time)









Get ready for more fun!