# Unravelling long-term behaviour in historic geophysical data sets

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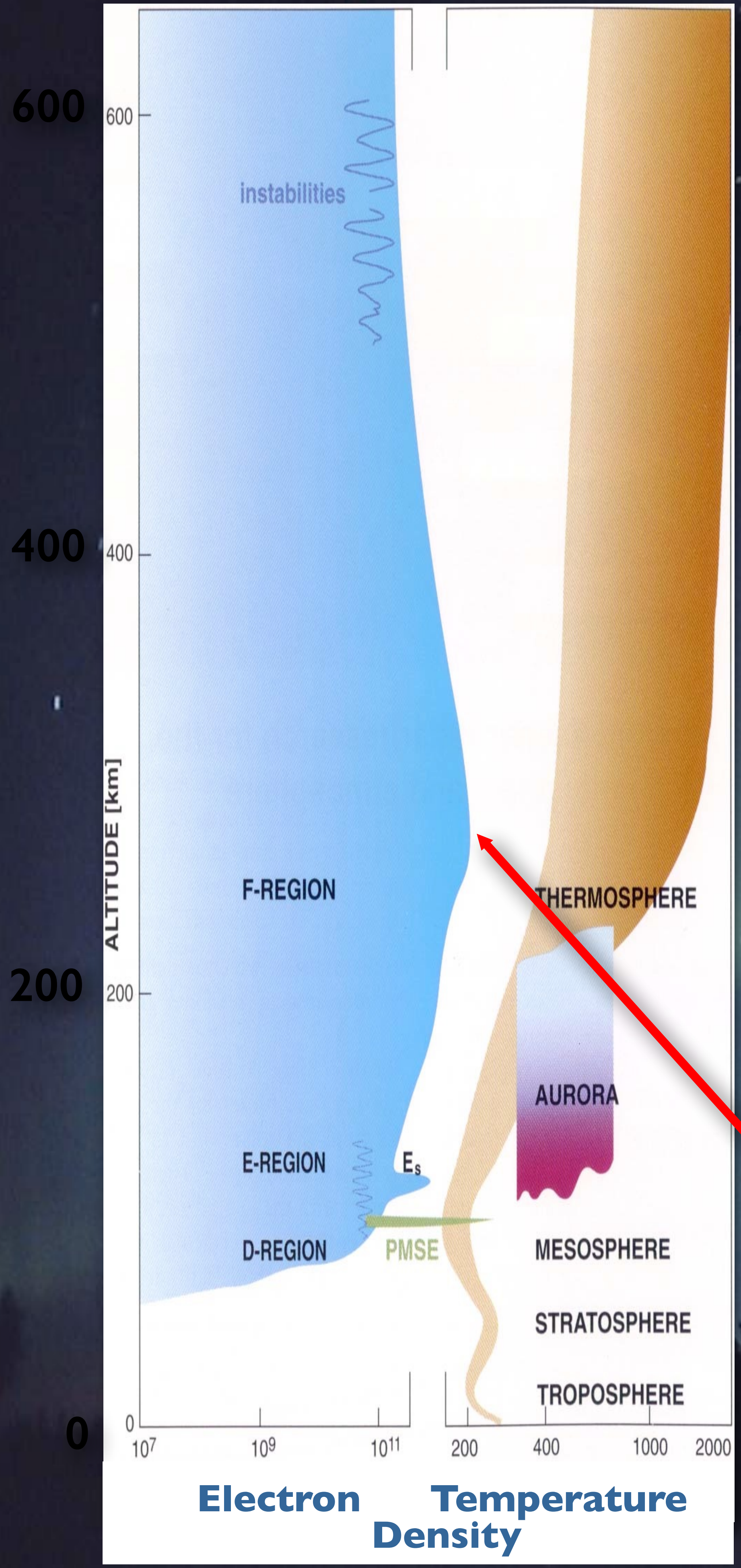
# Greenhouse high up?

- Model results, assuming doubling of $CO_2$ and $CH_4$:
- Stratopause cools by 8 K, stratosphere by 15 K.
  (Brasseur & Hitchman, 1988)
- Mesosphere and thermosphere cool by 10 K and 50 K, respectively.
  (Roble & Dickinson, 1989)
- F2-layer peak (hmF2) lowers by 15-20 km.
  (Rishbeth, 1990)
- Riometer absorption decreases.
  (Serafimov & Serafimova, 1992)
- Stratopause cools by 14 K, mesosphere by 8 K, thermosphere by 50 K.
  (Akmaev & Fomichev, 1998)




# Greenhouse Cooling

Doubling of [$CO_2$] and [$CH_4$]

**cools**

Mesosphere by **10 K** and
Thermosphere by **50 K**.

**Atmosphere shrinks.**

Layer of maximum electron density lowers by 15-20 km.

# Sodankylä Ionosonde

- Sodankylä ionosonde measurements began 1st August 1957.
- Until Nov 2005: 1 sounding per 30 min.
- Until Mar 2007: 1 sounding per 10 min.
- IPY (Apr ´07-Mar ´08): 1 sounding per minute.
- April 2008: we forgot to turn off IPY mode.
- Tody: close to 5 million ionograms.
- High data quality:
  first 800.000+ ionograms were analysed by the very same person!

# Empirical hmF2 Formulae

**Shimazaki** [1955]

$$hpF2 = \frac{1490}{M} - 176$$

**Bilitza, Sheikh, Eyfrig** [1979]

$$hmF2 = \frac{1490}{M - \Delta M} - 176$$

$$\Delta M = \frac{F1 \times F4}{x - F2} + F3$$

$$F1 = 0.00232 \times R + 0.222$$

$$F2 = 1.2 - 0.016 \exp(0.0239 \times R)$$

$$F3 = 0.00064 \times (R - 25)$$

$$F4 = 1 - \frac{R}{150} \exp\left(\frac{-\Phi^2}{1600}\right)$$

**Bradley, Dudeney** [1973], eq. (3)

$$hmF2 = a \times M^b$$

$$a = 1890 - \frac{355}{x - 1.4}$$

$$b = (2.5x - 3)^{-2.35} - 1.6$$

**Dudeney** [1974], eq. (56)

$$hmF2 = \frac{1490(M \times F)}{M - \Delta M} - 176$$

$$\Delta M = \frac{0.253}{x - 1.215} - 0.012$$

$$(M \times F) = M \sqrt{\frac{0.0196 M^2 + 1}{1.2967 M^2 - 1}}$$

$M = M(3000)F2$

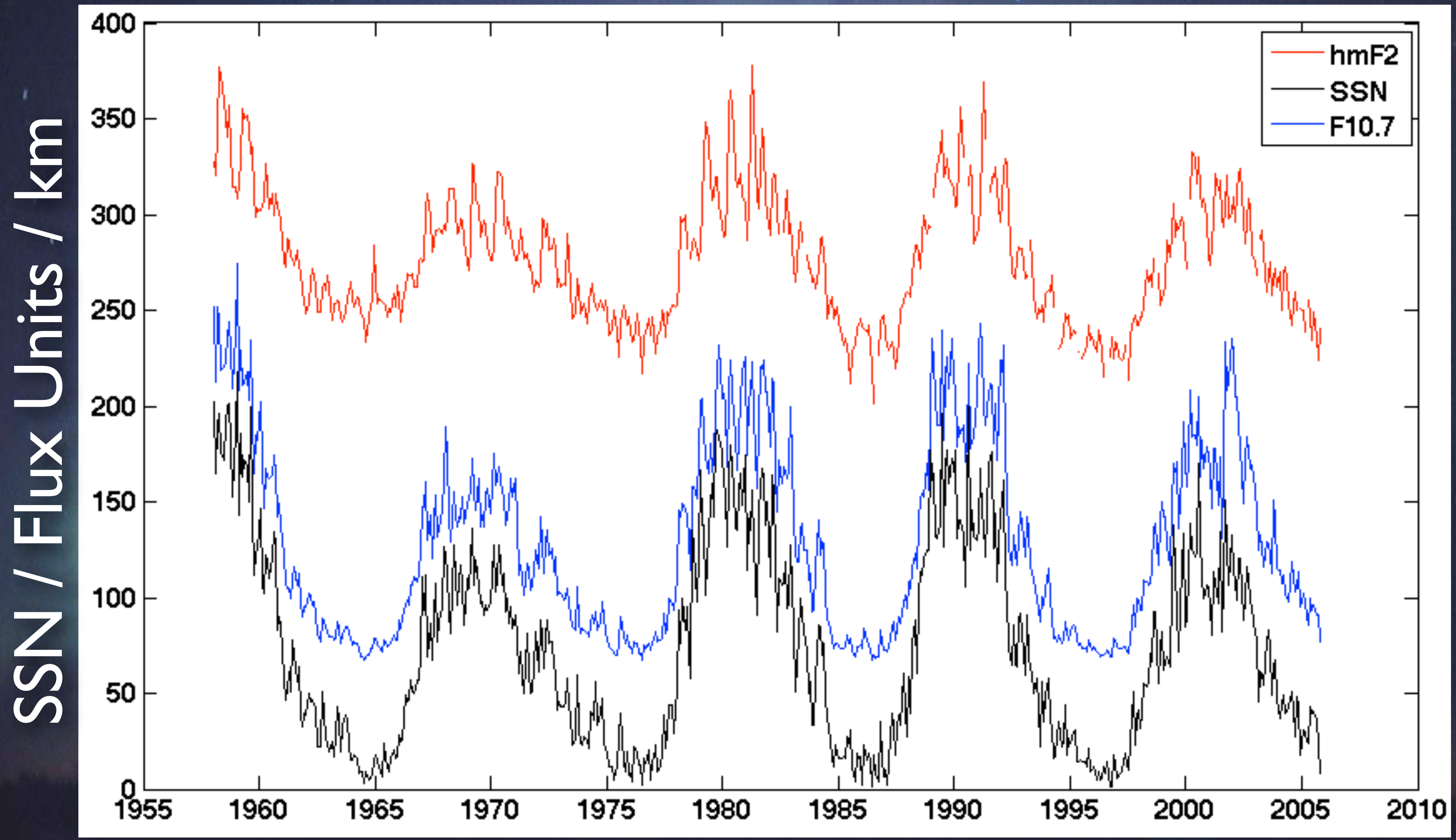
$x = foF2 / foE$

$\Delta M$ = Correction Term

$\Phi$ = Geomagnetic Latitude

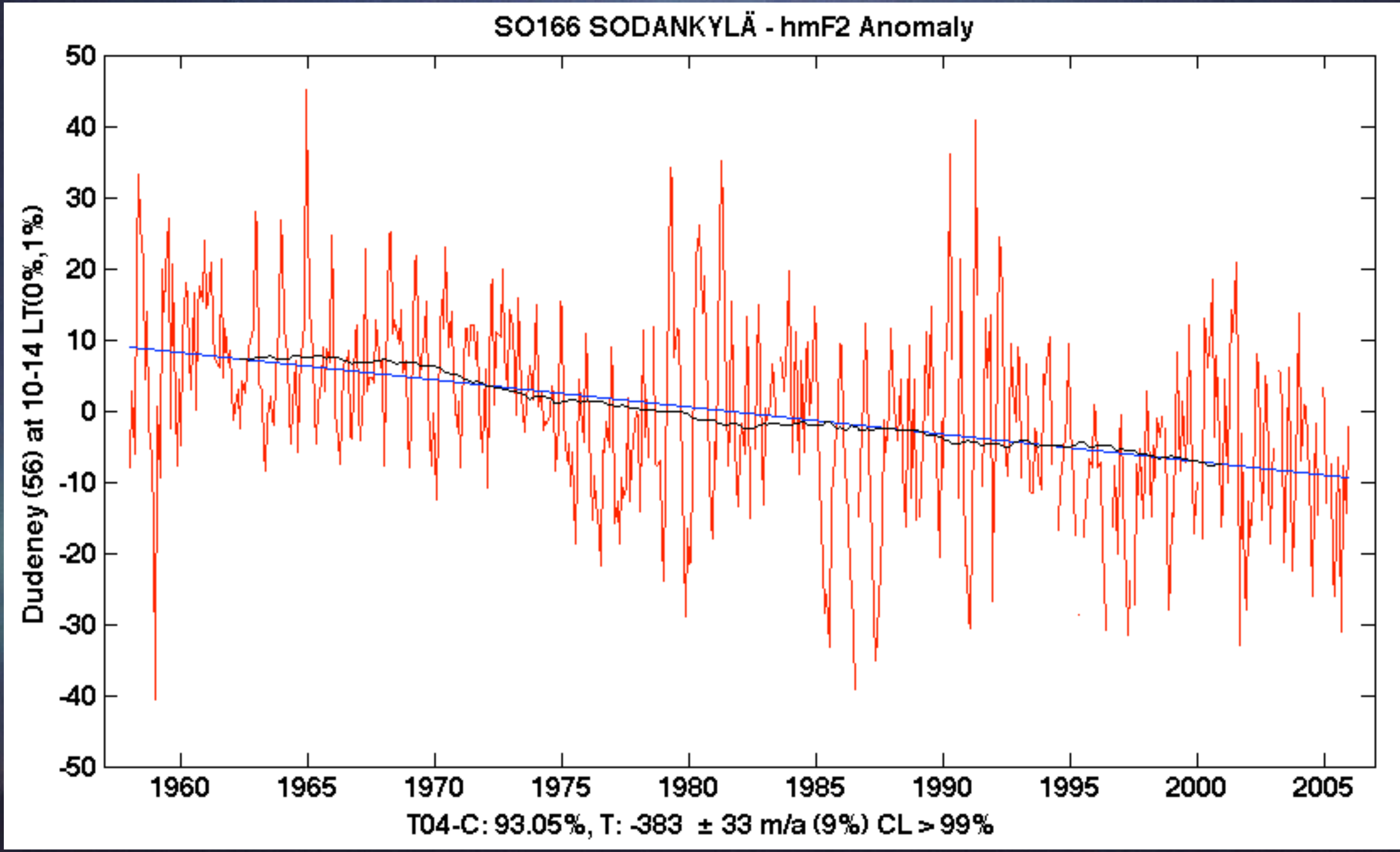
$R$ = Sunspot Number

# hmF2 & Solar Activity




Note: hmF2 computed using the empirical formula of Dudeney (eq. 56; 1974), which has been tested against true height at Sodankylä estimated during different periods of the time series using Titheridge's (1969) single-polynomial method.

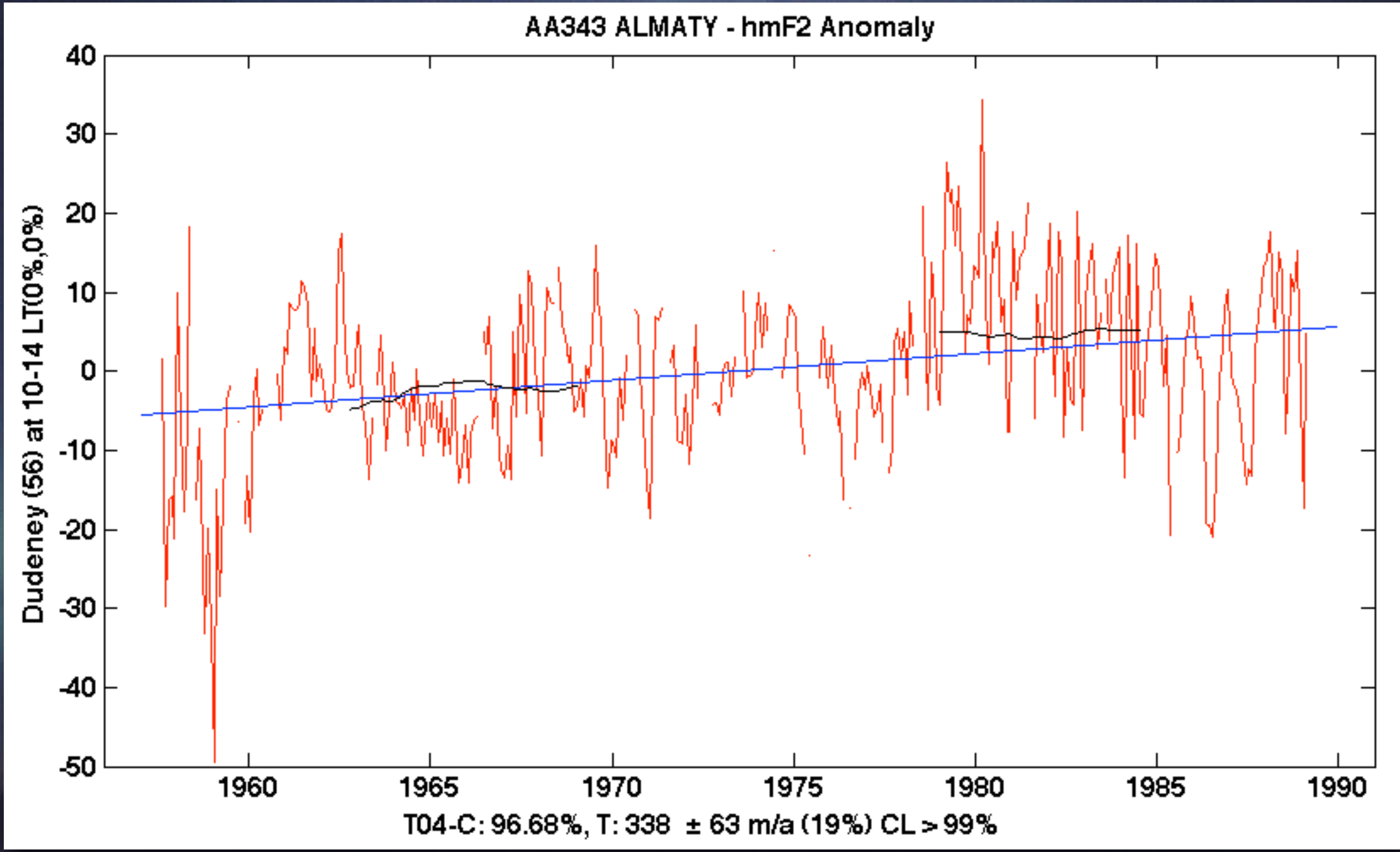
# Sodankylä hmF2 Trend



# Conclusion

- The enhanced greenhouse effect is clearly visible in the ionosphere.

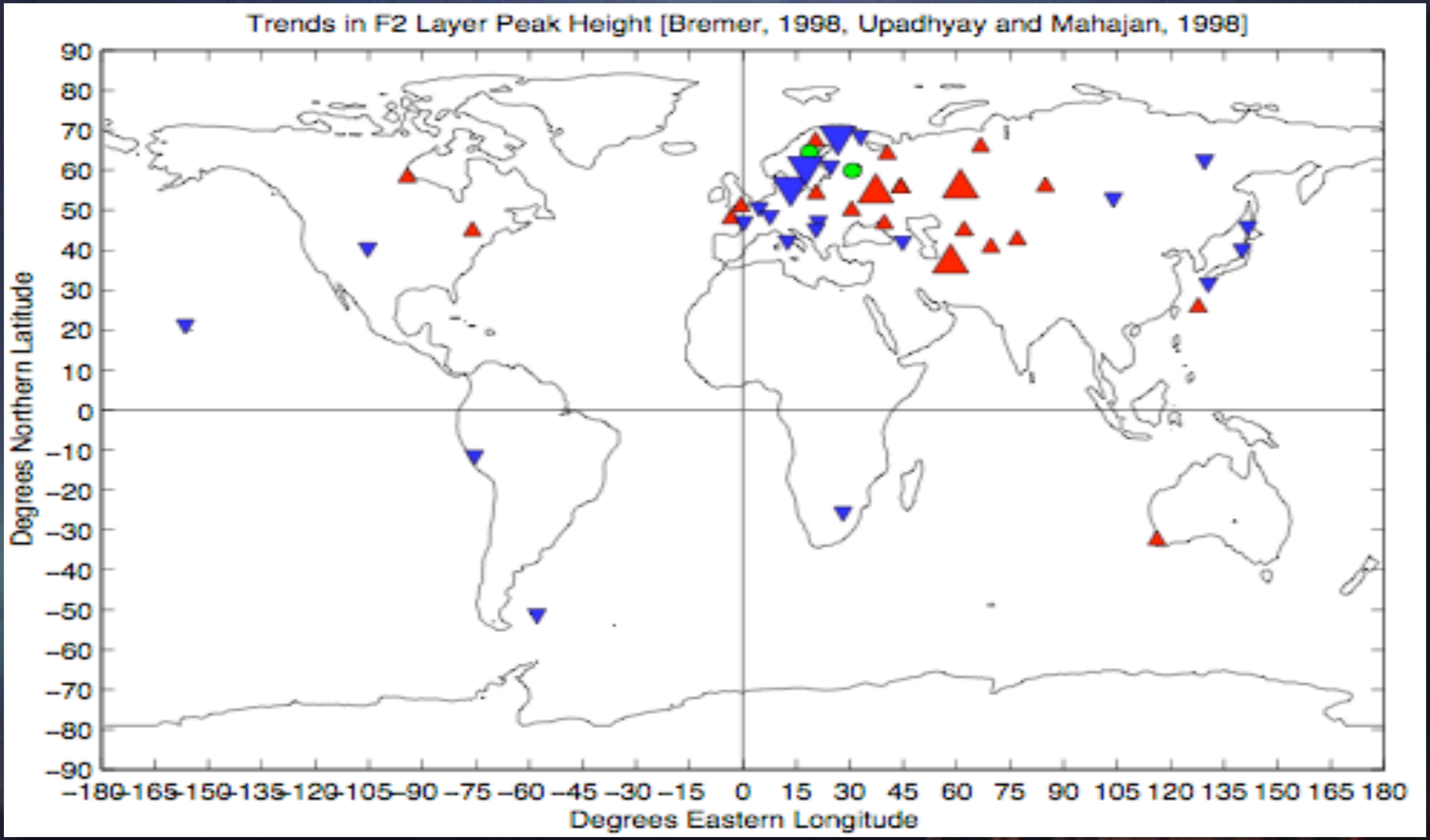
# Almaty hmF2


AA343 ALMATY - hmF2 Anomaly
Dudeney (56) at 10-14 LT(0%,0%)
40
30
20
10
0
-10
-20
-30
-40
-50
1960
1965
1970
1975
1980
1985
1990
T04-C: 96.68%, T: 338 ± 63 m/a (19%) CL > 99%

# Conclusion

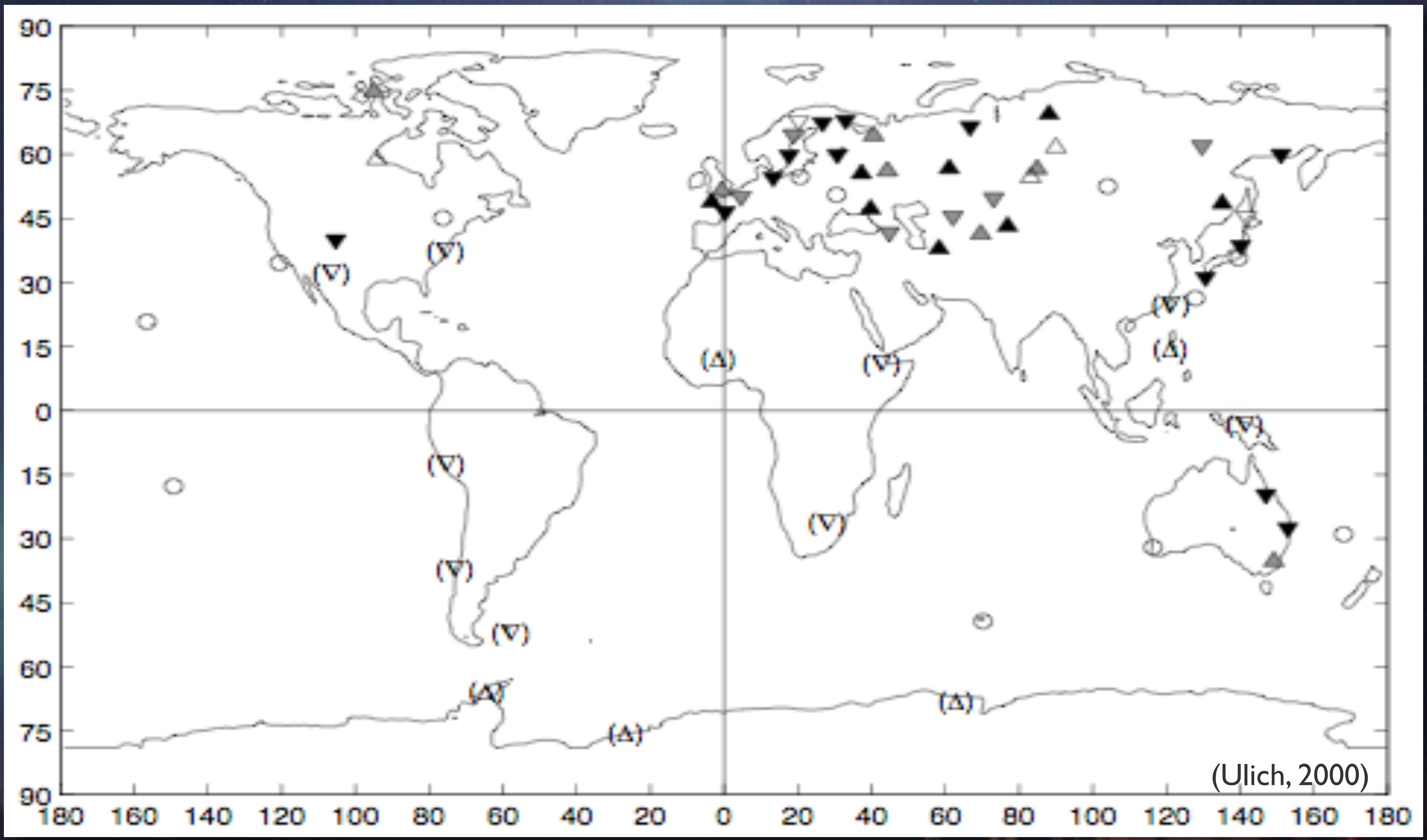
- Obviously, my data set is better than yours.
- The enhanced greenhouse effect is clearly visible in the ionosphere.

# hmF2 Trends



Trends in F2 Layer Peak Height [Bremer, 1998, Upadhyay and Mahajan, 1998]
Degrees Northern Latitude
Degrees Eastern Longitude
90
80
70
60
50
40
30
20
10
0
−10
−20
−30
−40
−50
−60
−70
−80
−90
−180 −165 −150 −135 −120 −105 −90 −75 −60 −45 −30 −15 0 15 30 45 60 75 90 105 120 135 150 165 180

# Global hmF2 Trends




(Ulich, 2000)

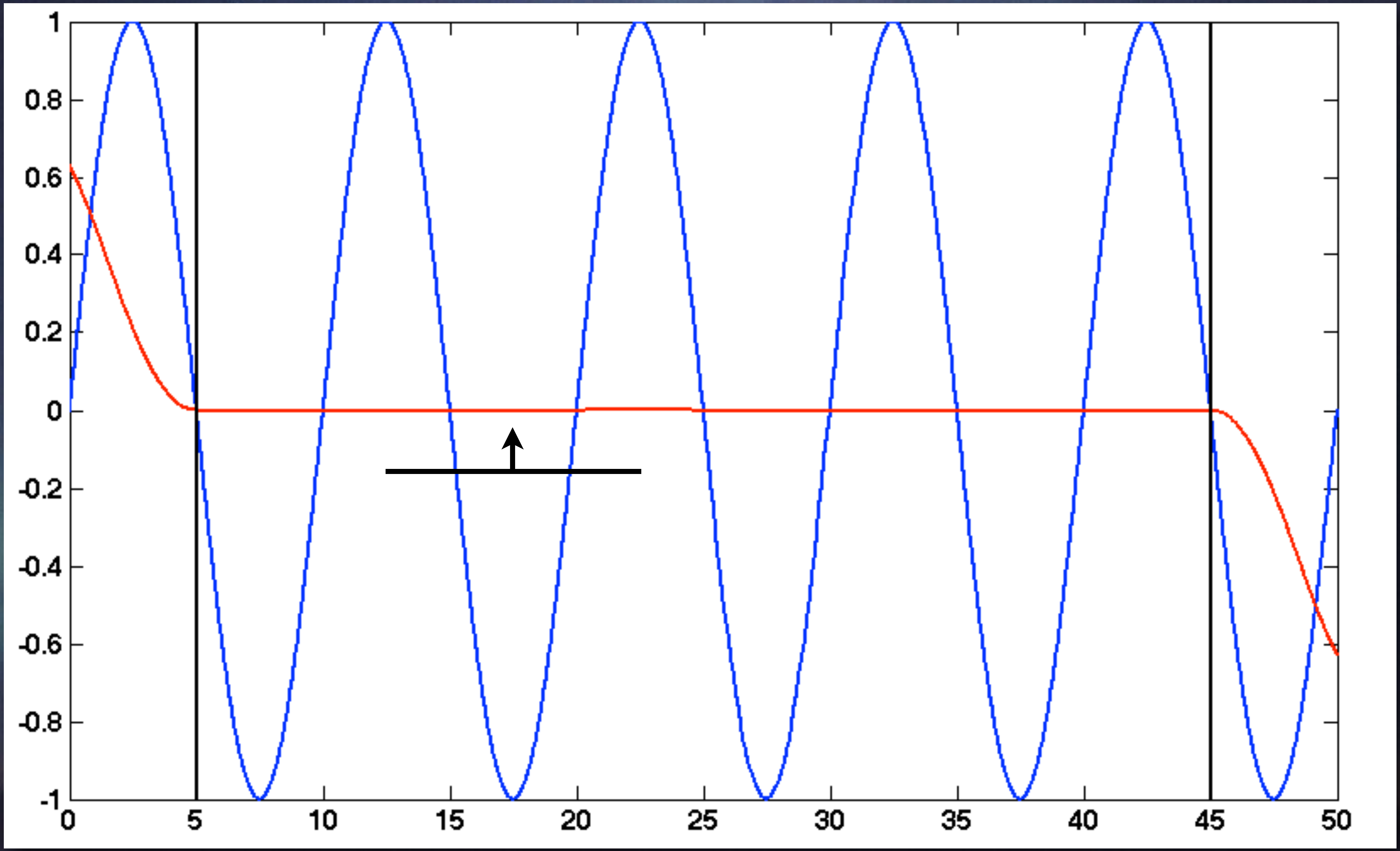
# Conclusion

- What the ... ???

# Problems

- Data resolution (h, 3-h, day, month(?), ...)
- Low-pass filtering or polynomial fitting...

# Running Mean Filter



Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=1  (first frame of 25 fps film)

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=2

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=4

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=8

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=16

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=32

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=64

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=128

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=256

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=512

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=1024 ≈ 41s

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.


Photo: 30s exposure, f/11, ISO250.

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Same film with background (average N=1024) subtracted.

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

Average: N=1  (first frame of 25 fps film)

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

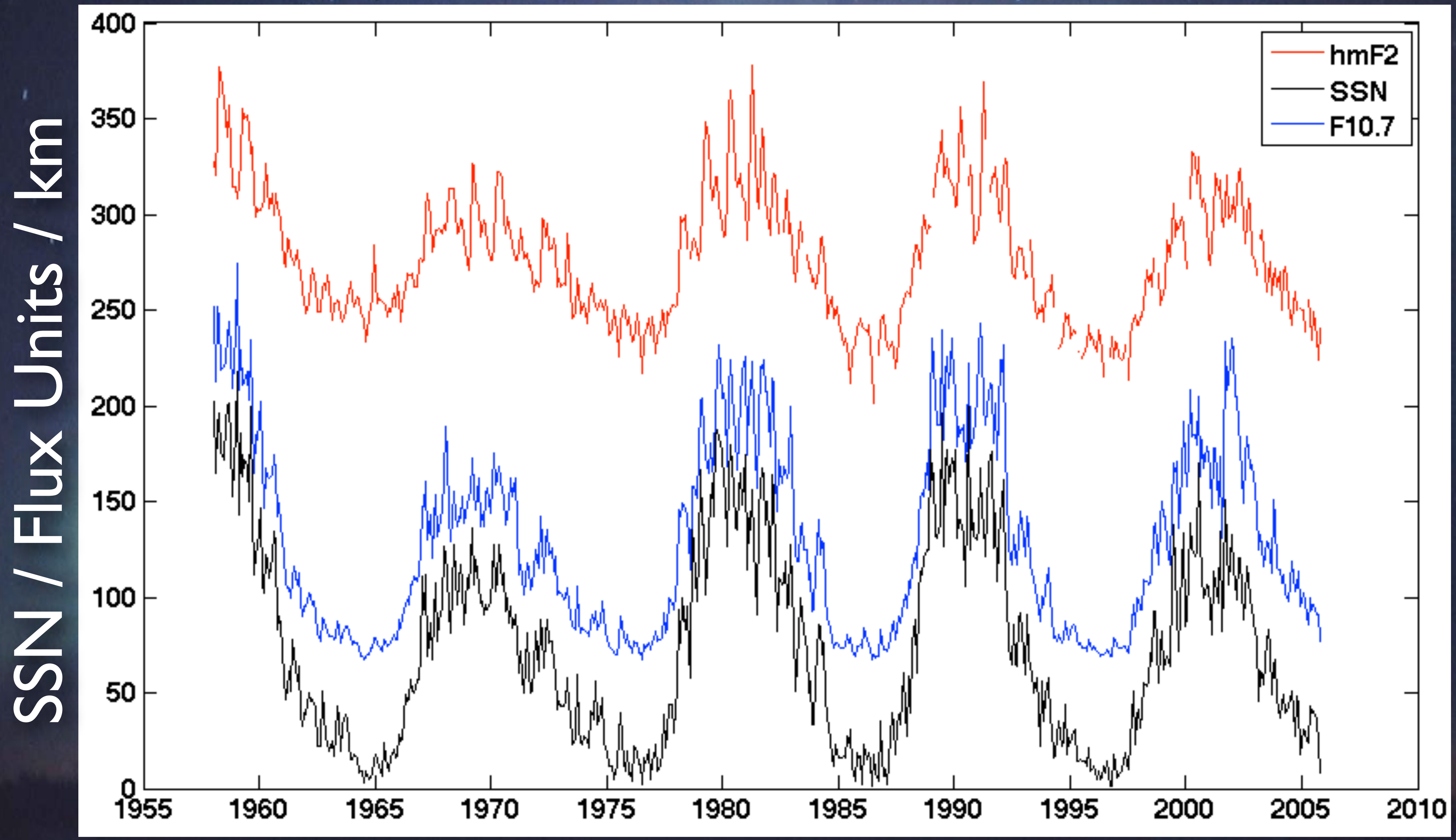
Photo: fast sweep, 41s.

Images: Thomas Ulich featuring participants of International Incoherent Scatter Radar School 2016, Sodankylä, Finland.

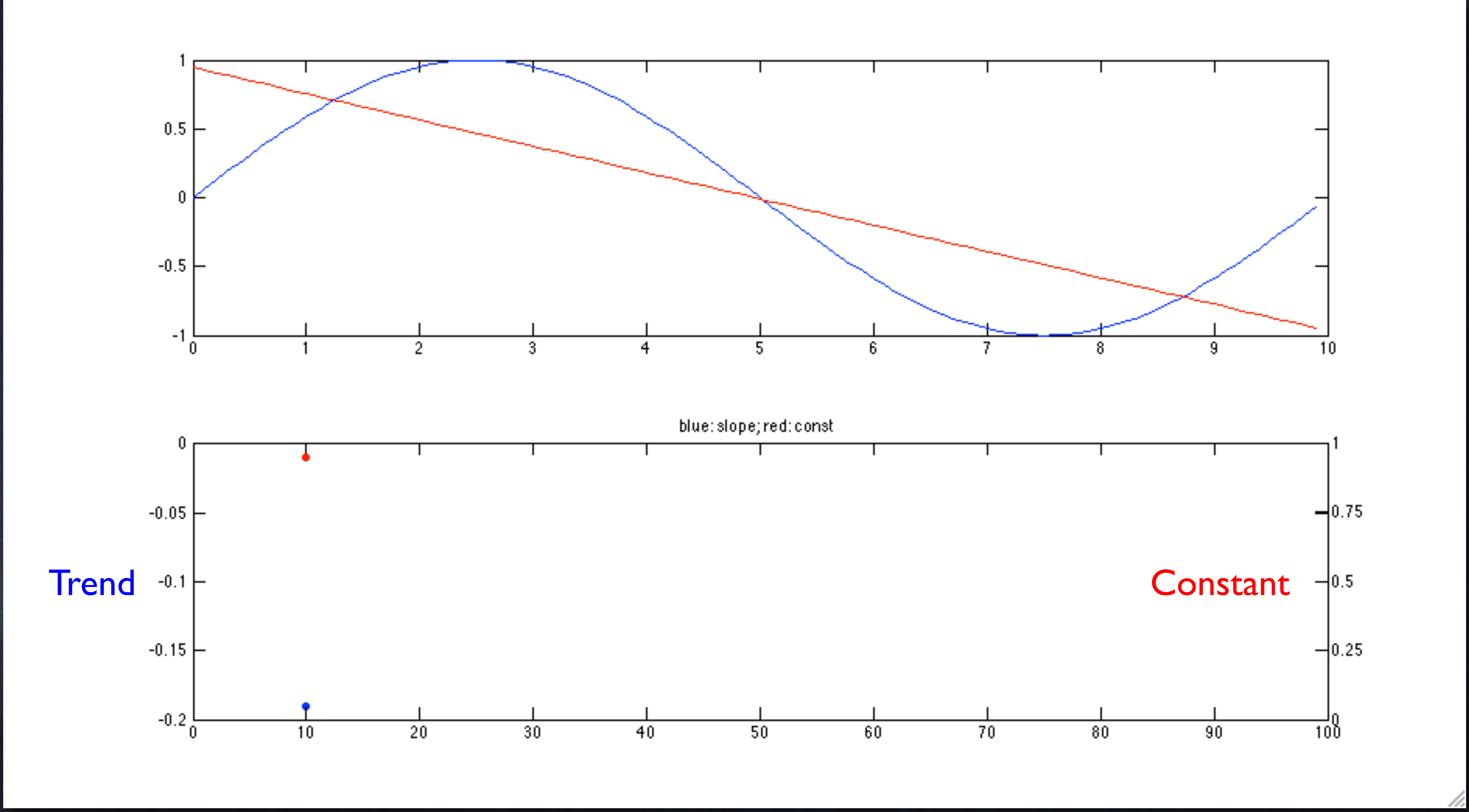
# Problems

- Data resolution (h, 3-h, day, month(?), ...)
- Low-pass filtering or polynomial fitting...
- Removal of underlying (cyclic) variability:
  - Choice of proxy (sinusoid, SSN, Group SSN, F10.7 (adj./obs.), Ly-α, Mg II, E10.7, ...)
  - Resolution of proxy: compatibility with data

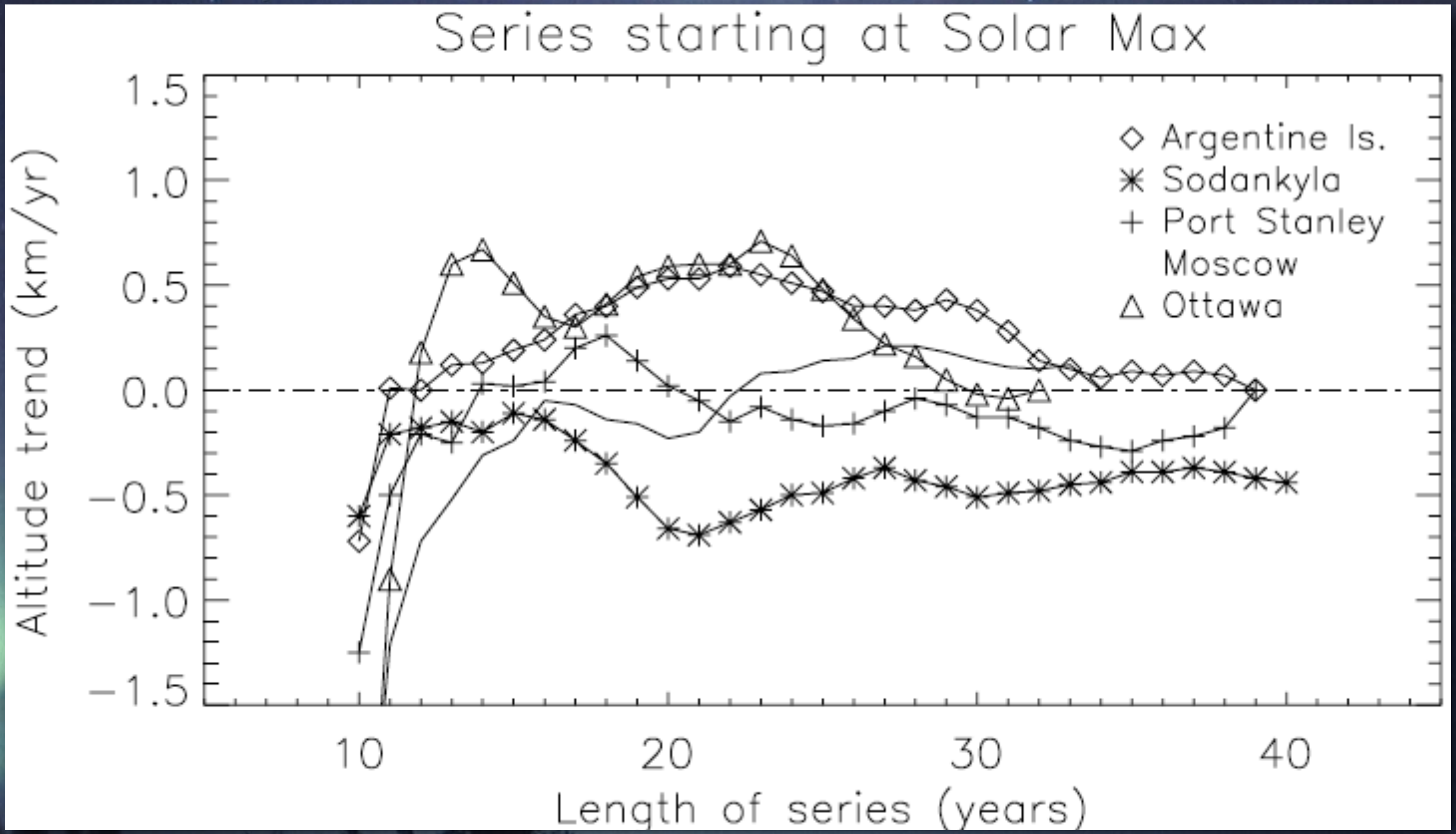
# hmF2 & Solar Activity



# Ringing



# Ringing



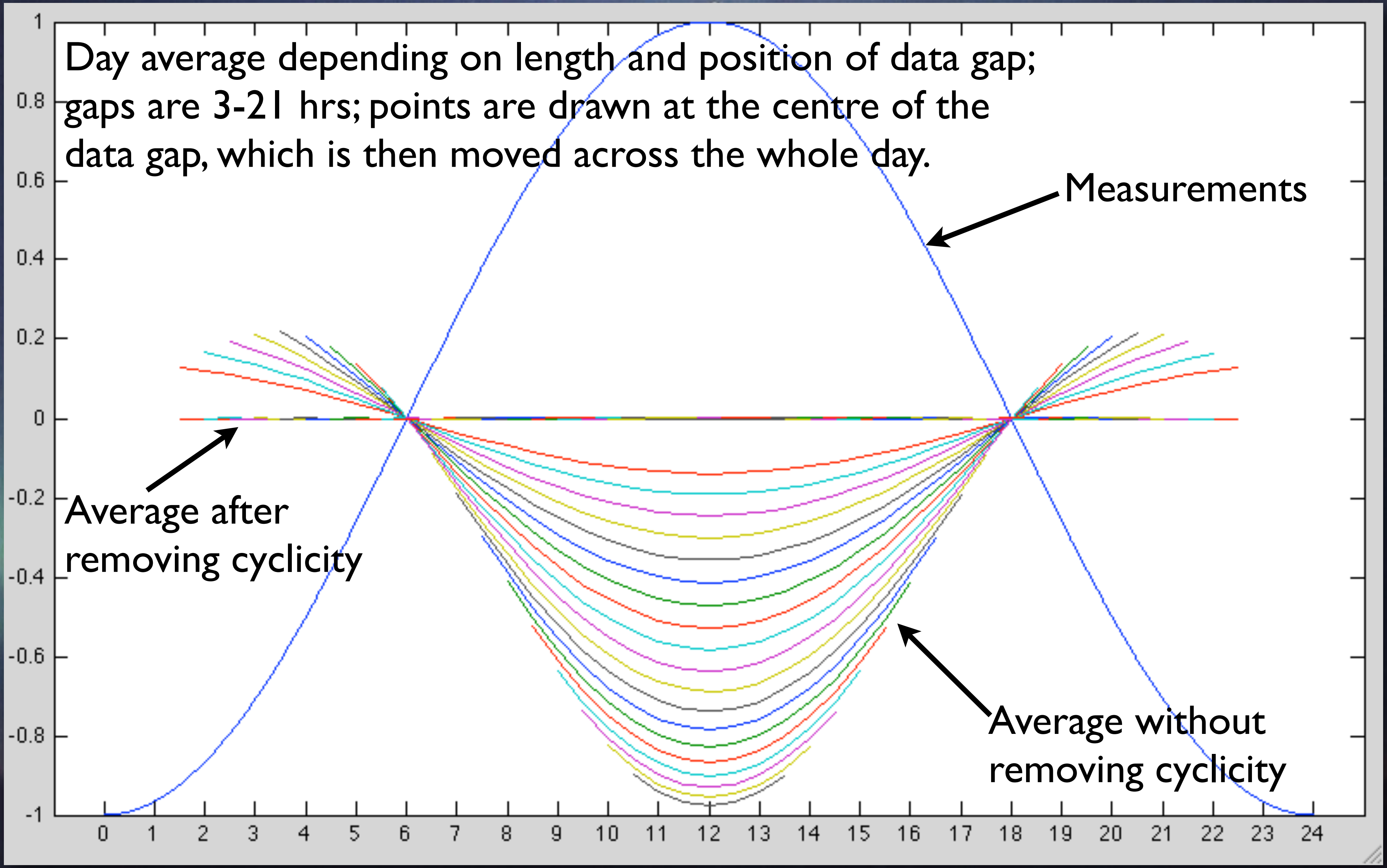

The ringing idea was first introduced by Jarvis et al., 2002. The plots shown here are from a follow-up paper by Clilverd et al., 2003.

# Problems

- Data resolution (h, 3-h, day, month(?), ...)
- Low-pass filtering or polynomial fitting...
- Removal of underlying (cyclic) variability: ...
- Data gaps

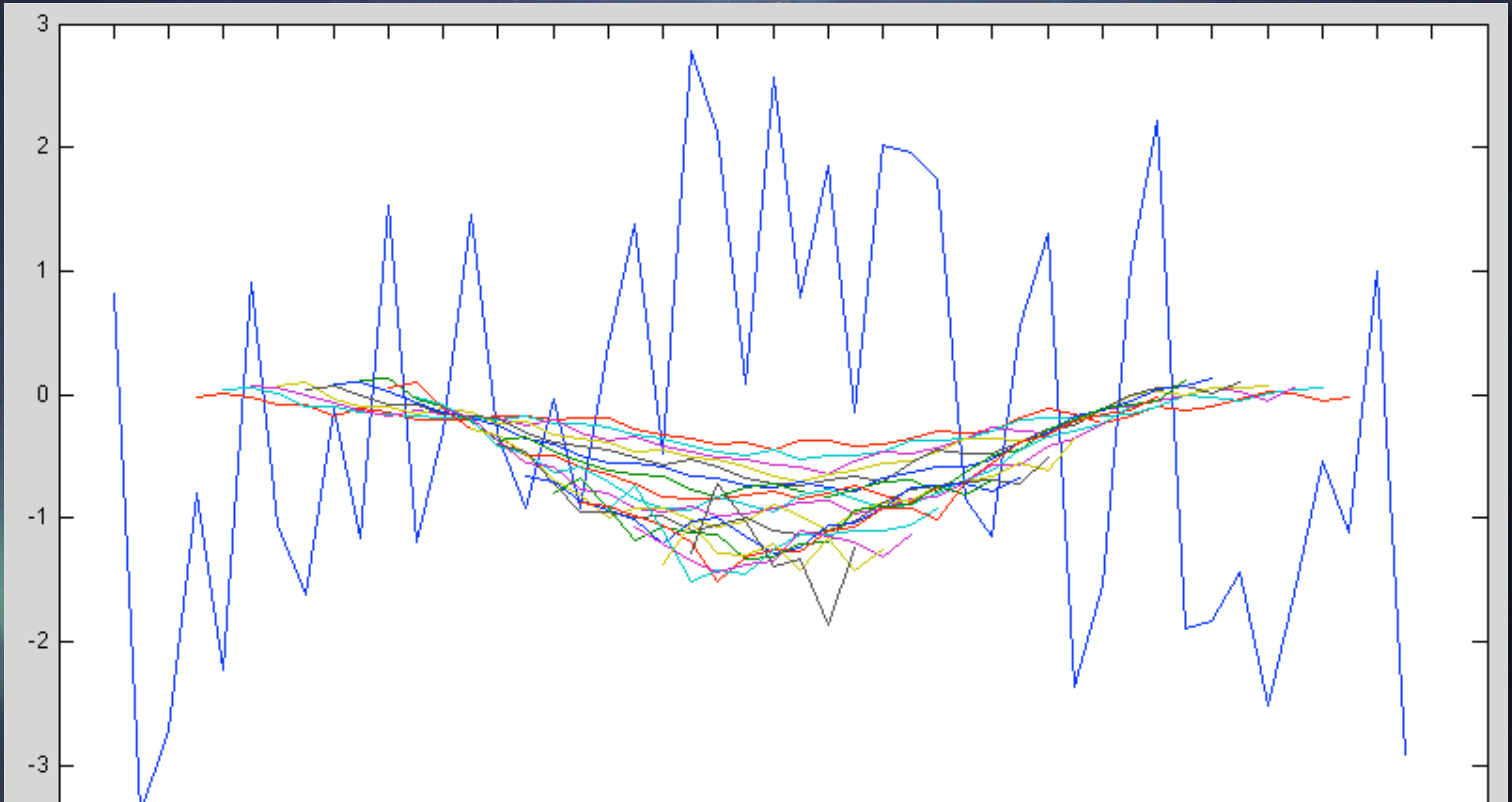
# Example: Data Gaps

# Data Gaps



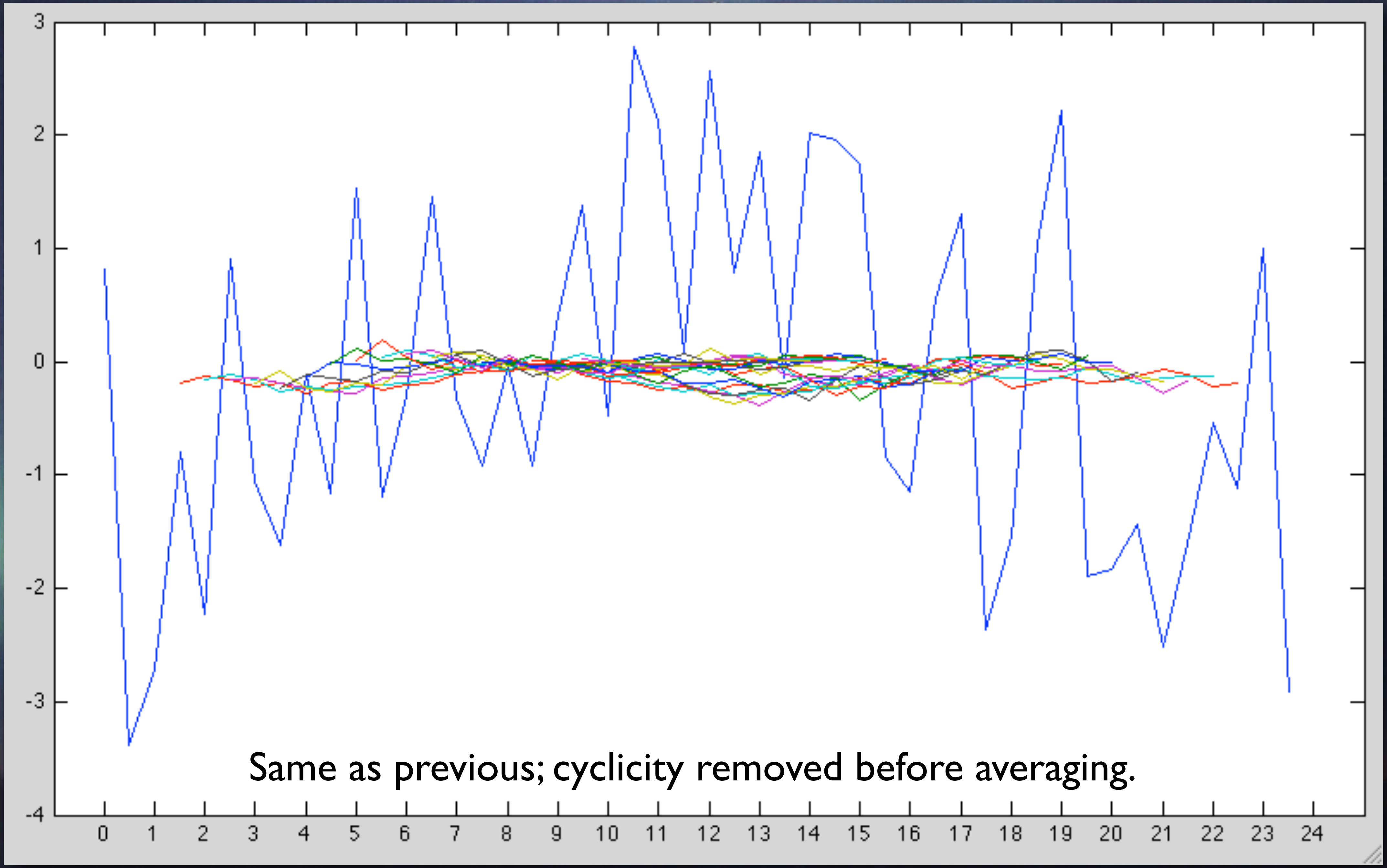
Day average depending on length and position of data gap; gaps are 3-21 hrs; points are drawn at the centre of the data gap, which is then moved across the whole day.
Measurements
Average after removing cyclicity
Average without removing cyclicity
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

# Data Gaps




Same as previous; cyclicity not removed before averaging.

# Data Gaps




Same as previous; cyclicity removed before averaging.

# Problems

- Data resolution (h, 3-h, day, month(?), ...)
- Low-pass filtering or polynomial fitting...
- Removal of underlying (cyclic) variability: ...
- Data gaps
- Measurement errors
- Mathematics of trend detection
  - stepwise or multi-parameter fit
  - error propagation

# Making models

- Base functions of the model(s) are, e.g.:

$m_i = \varepsilon_i$ -> measurement errors

$+ x_1$ -> constant

$+ x_2 t_i$ -> sampling times

$+ x_3 F_{10.7}(t_i)$ -> solar activity

$+ x_4 Ap(t_i)$ -> geomagnetic activity

$+ x_5 \sin(2\pi t_i)$

$+ x_6 \cos(2\pi t_i)$ -> annual variation

$+ x_7 \sin(4\pi t_i)$

$+ x_8 \cos(4\pi t_i)$ -> semi-annual variation

$+ \ldots$

# Modelling the data

The ionospheric property of interest is function of time and a number of other parameters. The model of the data is therefore

$$m(t) = \mathcal{F}(t, x_1, \ldots, x_M)$$

where

$$\mathcal{F}(t, x_1, \ldots, x_M) = \sum_{i=1}^{M} x_i \, f_i(t)$$

The actual measurements $m_i$ observed at time $t_i$ are equal to the model plus some measurement error $\varepsilon_i$

$$m_i = \mathcal{F}(t_i, x_1, \ldots, x_M) + \varepsilon_i$$

# Inverse problem I

This can be expressed as a matrix equation. Usually there are many more data points than unknowns $x_i$ and the problem is over-determined:

$$\begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{pmatrix} = \begin{pmatrix} f_1(t_1) & f_2(t_1) & \cdots & f_M(t_1) \\ f_1(t_2) & f_2(t_2) & \cdots & f_M(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(t_N) & f_2(t_N) & \cdots & f_M(t_N) \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

In other words:

$$\mathbf{m} = \mathbf{A} \cdot \mathbf{x} + \varepsilon$$

# Inverse problem II

Measurements and theory are weighted by the measurement errors:

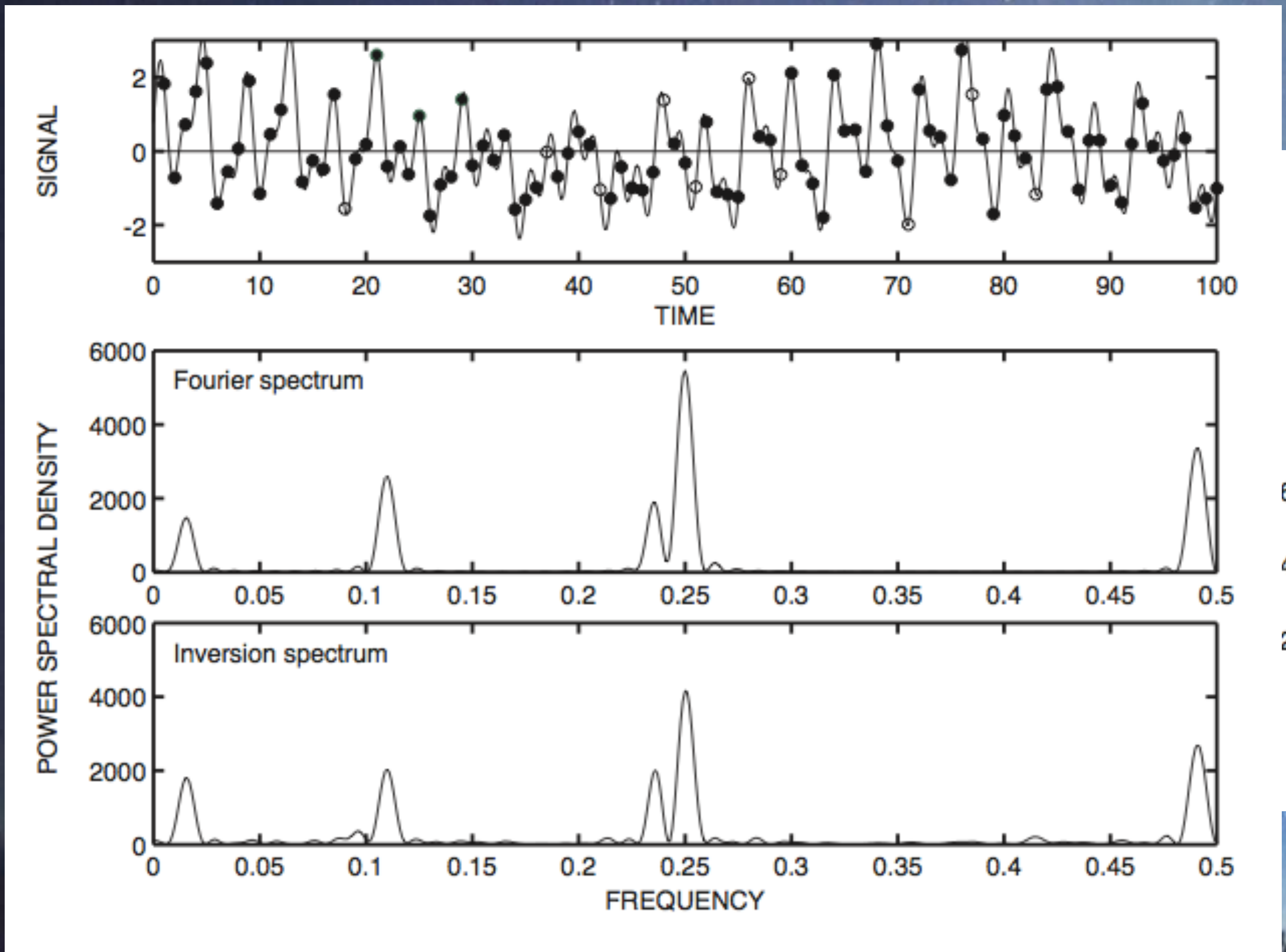
$$B_{ij} := \frac{A_{ij}}{\varepsilon_i} \quad \text{and} \quad b_i := \frac{m_i}{\varepsilon_i}$$

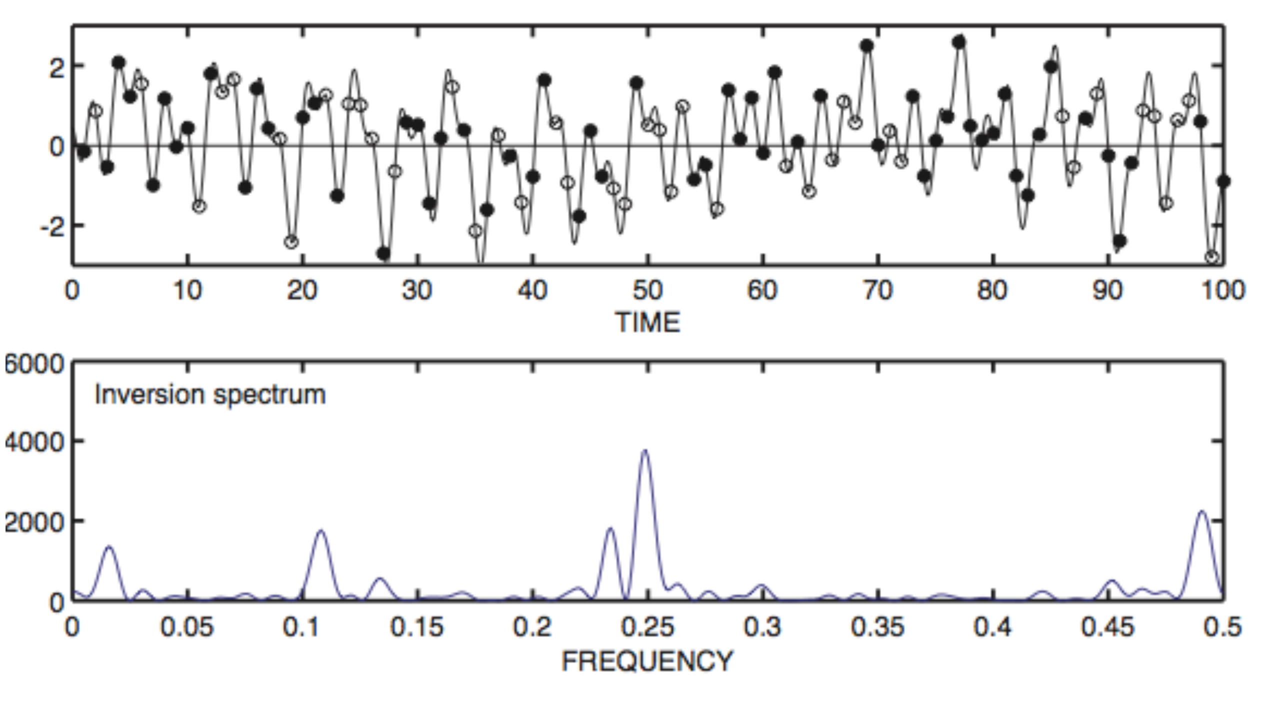
The solution is the vector **x**, which minimises the following expression:

$$\chi^2 = |\mathbf{B} \cdot \mathbf{x} - \mathbf{b}|^2$$

We are left with a general least squares problem. Solving this results in the most probable solution for **x**.

# Signal Spectrum by Stochastic Inversion



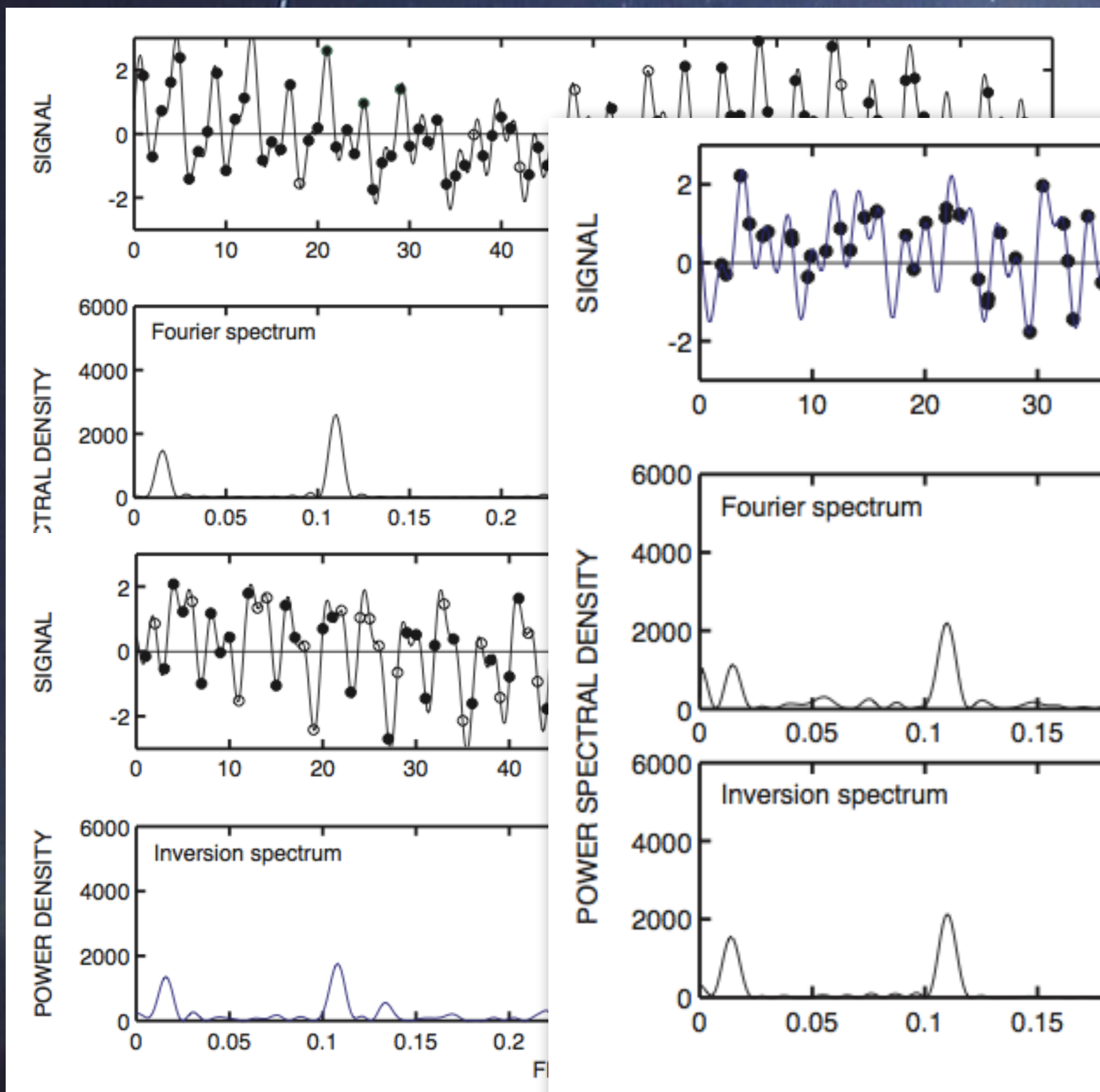



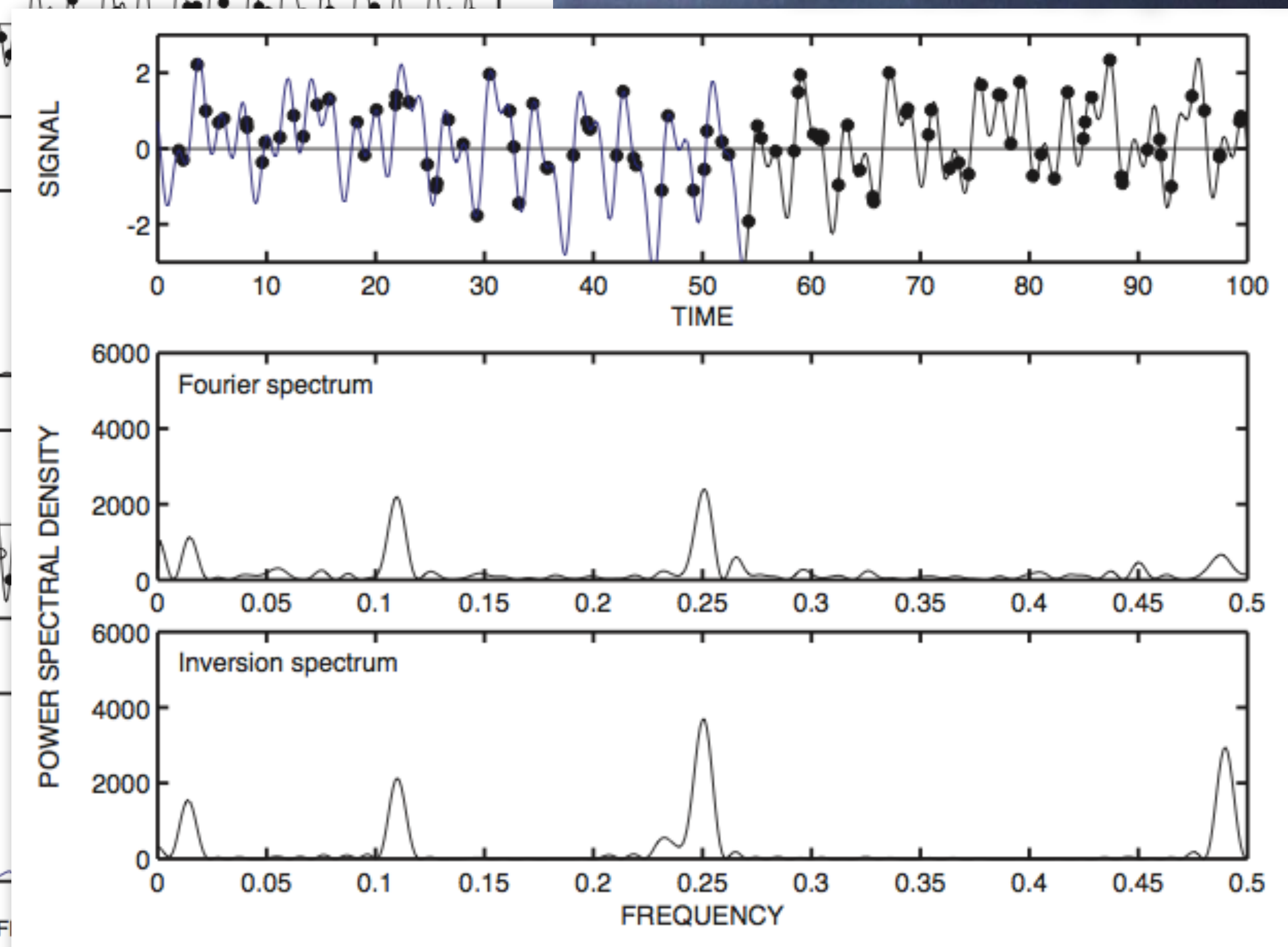

Left: 100 pts for Fourier, 90 for inversion.
Above: 59 pts.

T. Nygrén and Th.Ulich, Calculation of signal spectrum by means of stochastic inversion, Ann. Geophys., 28, 1409-1418, 2010.

# Signal Spectrum by Stochastic Inversion

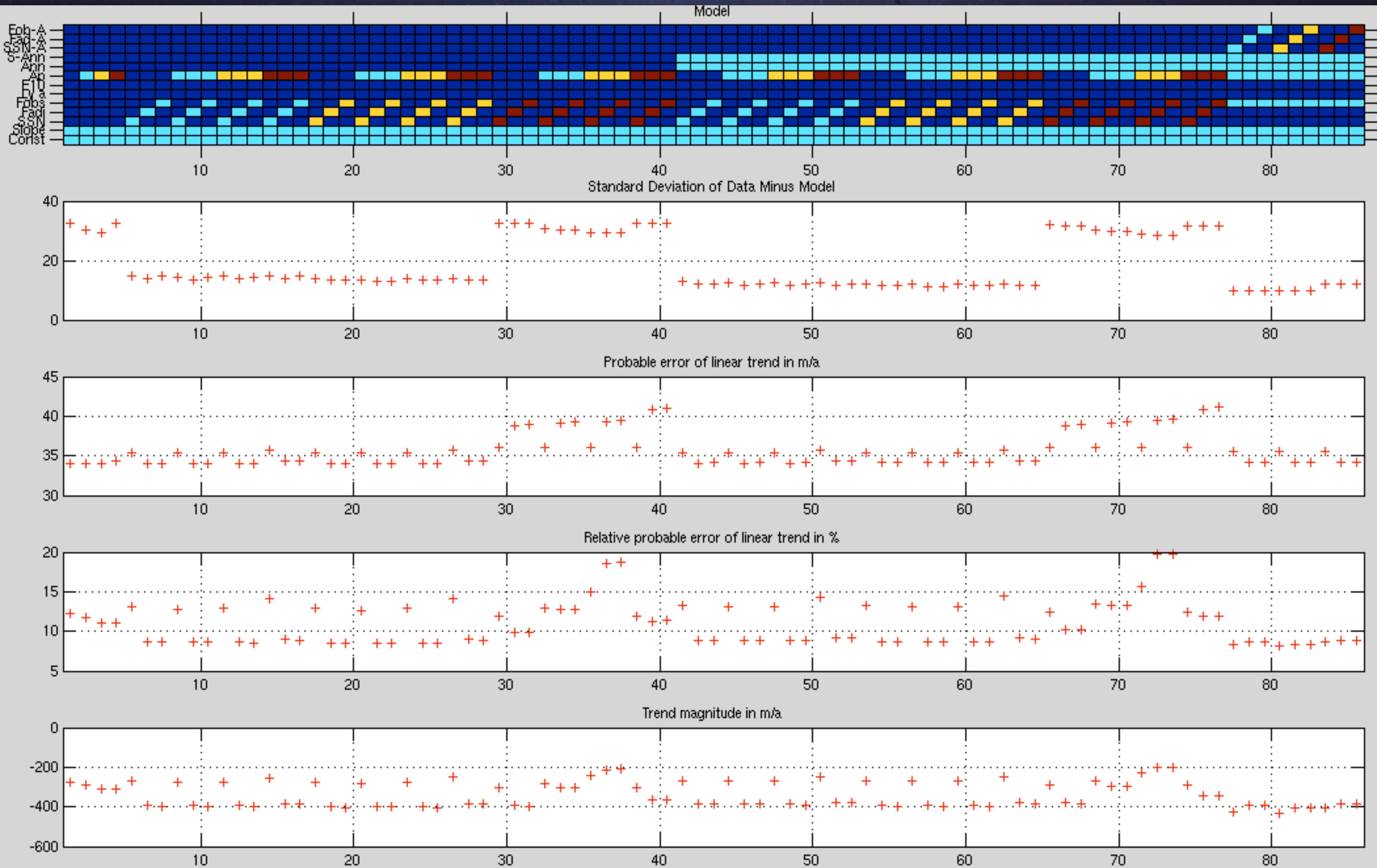
Below: random sampling intervals.



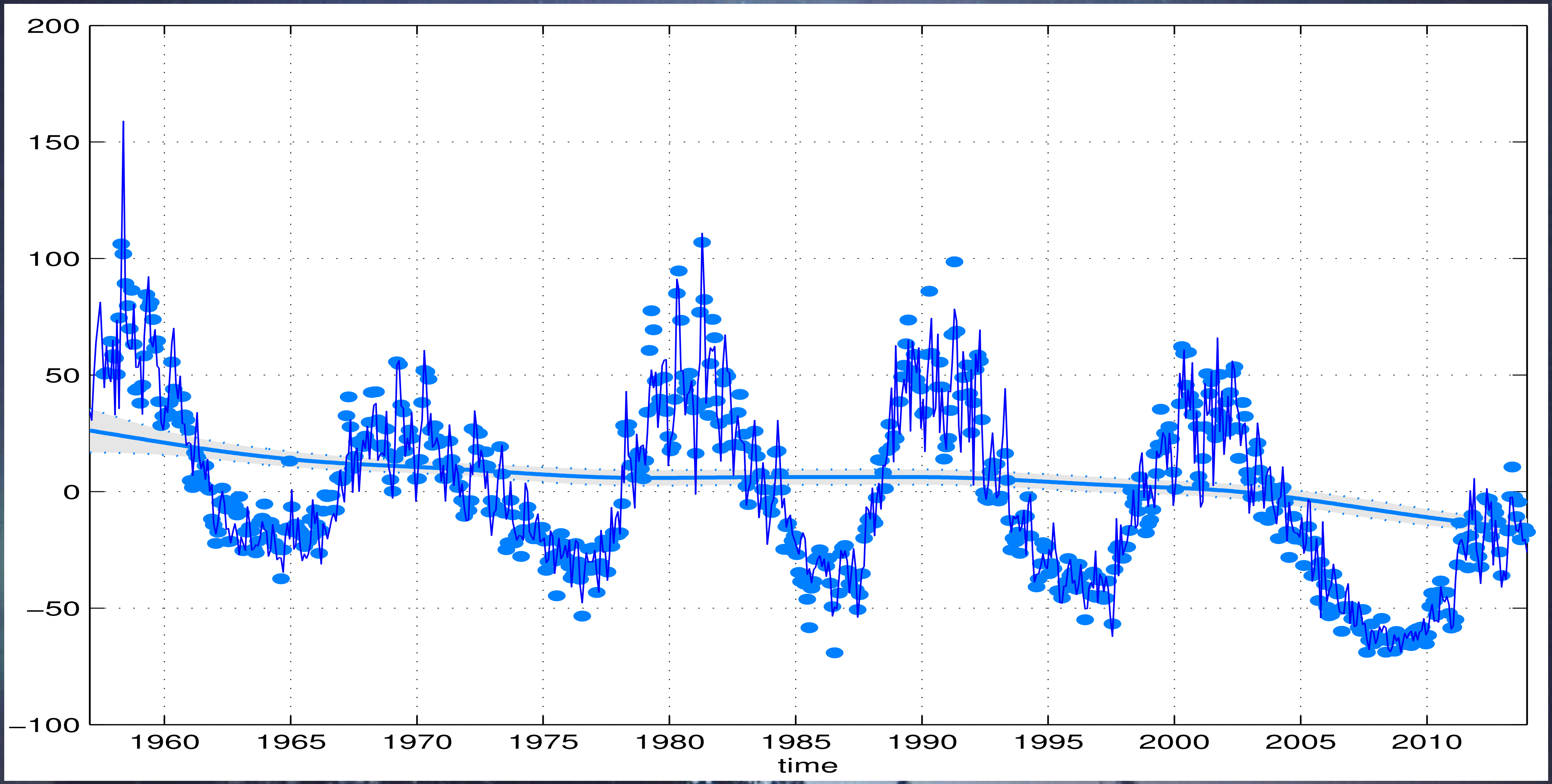




T. Nygrén and Th.Ulich, Calculation of signal spectrum by means of stochastic inversion, Ann. Geophys., 28, 1409-1418, 2010.

# Sodankylä F2-layer peak height hmF2



# Dynamic Linear Model




New approach using a dynamic linear model based on constant, trend, annual & semi-annual wave, as well as F10.7cm radio fluxes. Here, hmF2 is based on the same Dudeney (1974) computation as earlier plots.
(Roininen, Ulich, and Laine, Cambridge (UK) Trend Workshop 2014)

# Conclusion

- This is pointless?

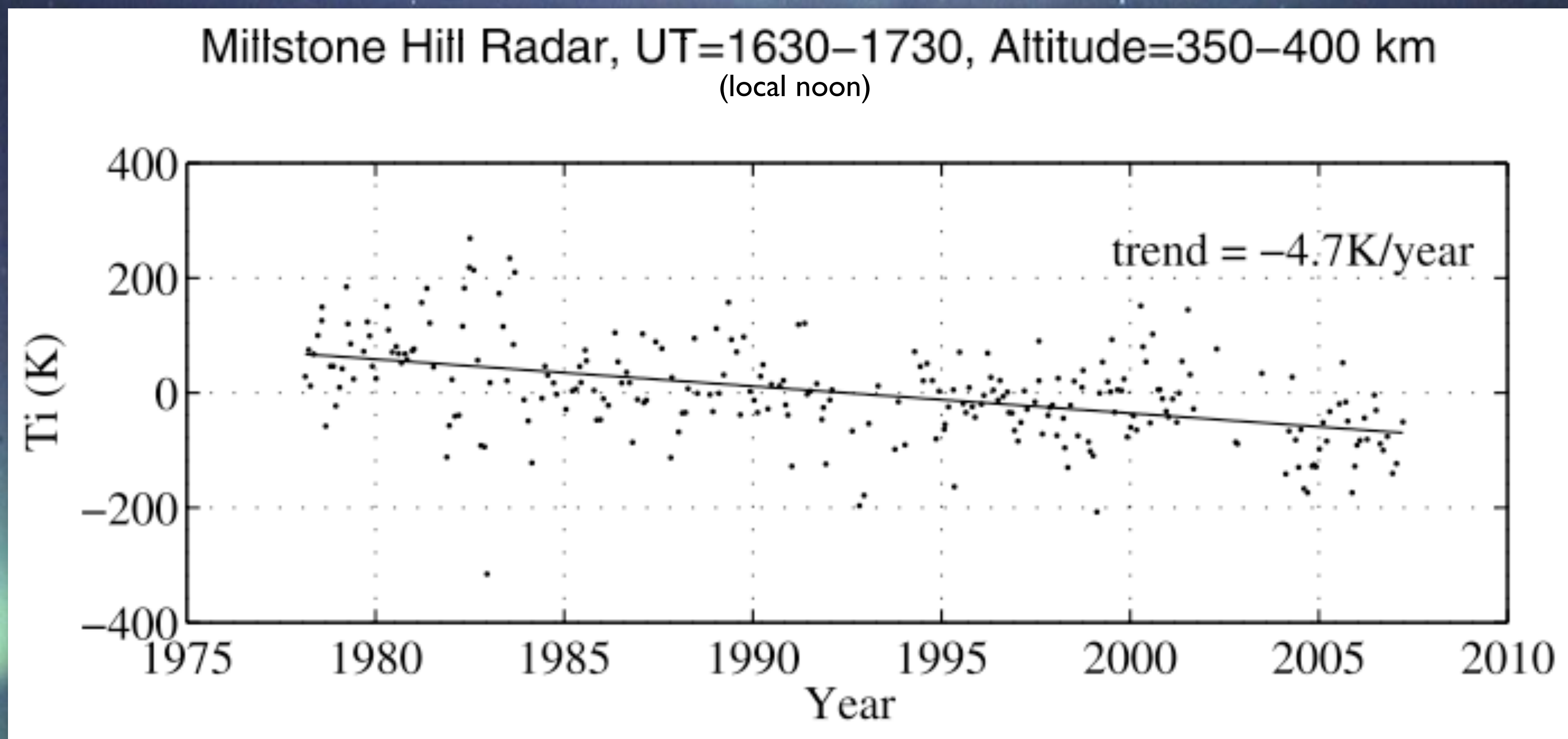
  I don’t think so...

  ...yet!

# Trends in other Observations

| Height in km | Method | Parameter | Trend per Year | Reference |
|---|---|---|---|---|
| | | | | |
| 75 | Sounding rocket | Temperature | -0.6 K | Kokin and Lysenko, 1994 |
| 70 | Sounding rocket | Temperature | -0.7 K | Golitsyn et al., 1996 |
| 60-70 | Lidar | Temperature | -0.4 K | Hauchecorne et al., 1991 |
| 60 | Sounding rocket | Temperature | -0.4 K | Golitsyn et al., 1996 |
| 60 | Sounding rocket | Temperature | -0.33 K | Keckhut et al., 1999 |
| 50-60 | Lidar | Temperature | -0.25 K | Aikin et al., 1991 |
| 50 | Sounding rocket | Temperature | -0.25 K | Golitsyn et al., 1996 |
| 40 | Sounding rocket | Temperature | -0.1 K | Golitsyn et al., 1996 |
| 30-60 | Sounding rocket | Temperature | -0.17 K | Dunkerton et al., 1998 |
| 30-50 | Sounding rocket | Temperature | -0.17 K | Keckhut et al., 1999 |
| 30 | Sounding rocket | Temperature | -0.1 K | Golitsyn et al., 1996 |
| 25 | Sounding rocket | Temperature | -0.1 K | Golitsyn et al., 1996 |
| 25 | Sounding rocket | Temperature | -0.11 K | Keckhut et al., 1999 |

# Direct F-Region Temperature




**Long-term temperature trends in the ionosphere above Millstone Hill**

J. M. Holt[1] and S. R. Zhang[1]

GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L05813, doi:10.1029/2007GL031148, 2008

# Conclusion

(the last one, I promise!)

- Definitely, there's long-term change in the ionosphere and thermosphere!
- The enhanced greenhouse effect is probably a part of it.
- Other (unknown?) processes are involved.
- Solution in modelling?
- We don't understand what's going on.
- Student exercise: Find out!

# Conclusion

(I lied to you!)

Ionsondes, originally deployed for monitoring ionospheric conditions for HF radio communication and for studying short-term events, are becoming useful in an environmental context.

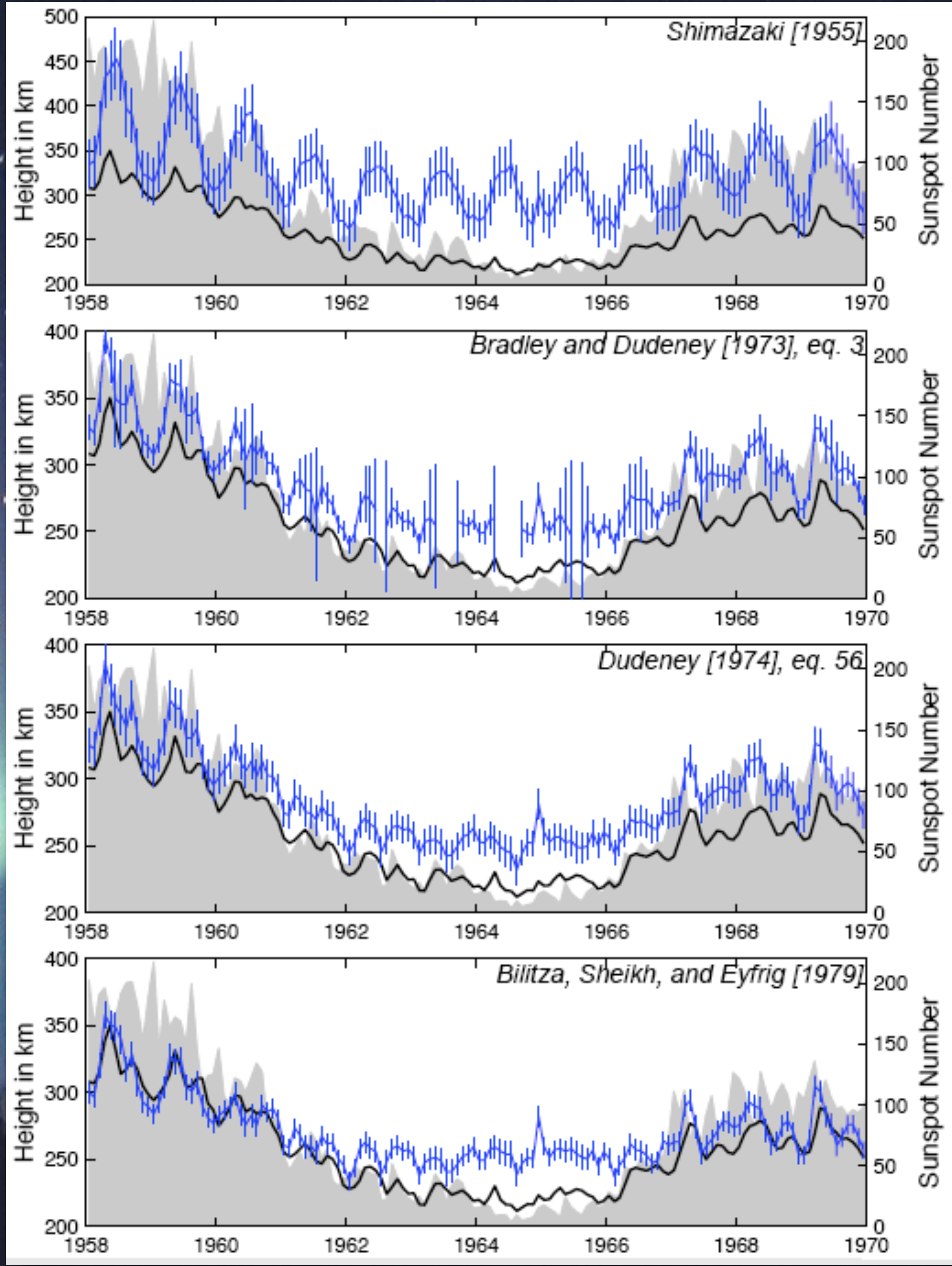
They provide long-term measurements of our environment!

Do not discontinue atmospheric observations at a time of climate change!

Photo: Thomas Ulich, Finland.


Kiitos!

# hmF2
# at
# Sodankylä




(hmF2: height of maximum electron density)

# Ionospheric Trends

| Team | hmF2 | foF2 | foE |
|---|---|---|---|
| Bremer, 1992 | - | | |
| Givishvili et al., 1995 | | o | o |
| Ulich & Turunen, 1997 | - | | |
| Bencze & Poor, 1997 | - | | |
| Chandra et al., 1997 | | o | |
| Mahajan & Shastri, 1997 | | o | |
| Upadhyay & Mahajan, 1998 | o | o | |
| Jarvis et al., 1998 | - (+) | | |
| Bremer, 1998 | o | | |
| Foppiano et al., 1999 | o | - | |
| Sharma et al., 1999 | | - | |
| Ulich, 2000 | o | | |
| Mikhailov & Marin, 2000 | geomag. | geomag. | |
| Mikhailov & Marin, 2001 | geomag. | geomag. | |
| Alfonsi et al., 2001 | | - | |
| Ortiz de Adler et al., 2002 | - | | |
| Hall & Cannon, 2002 | - | - | |
| Danilov, 2002 | | - | |
| Bencze, 2002 | - (+) | | |
| Mikhailov et al., 2002 | geomag. | geomag. | |
| Alfonsi et al., 2002 | | - | |
| Clilverd et al., 2003 | - | | |
| Mikhailov & de la Morena, 2003 | | | geomag. |
| Danilov, 2003 | | - | |
| Xu et al., 2004 | o | o | o |
| Cannon et al., 2004 | - | - | |

# F2 Region

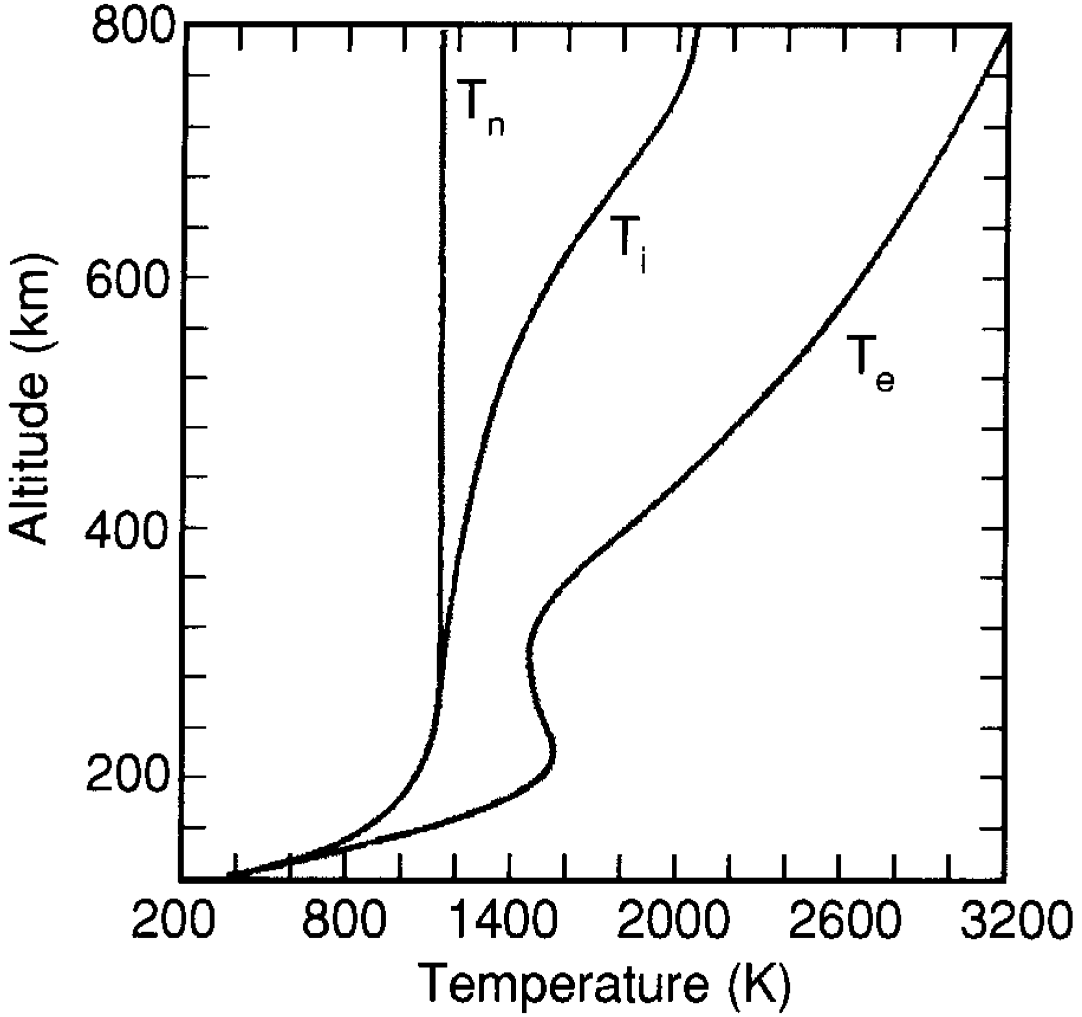
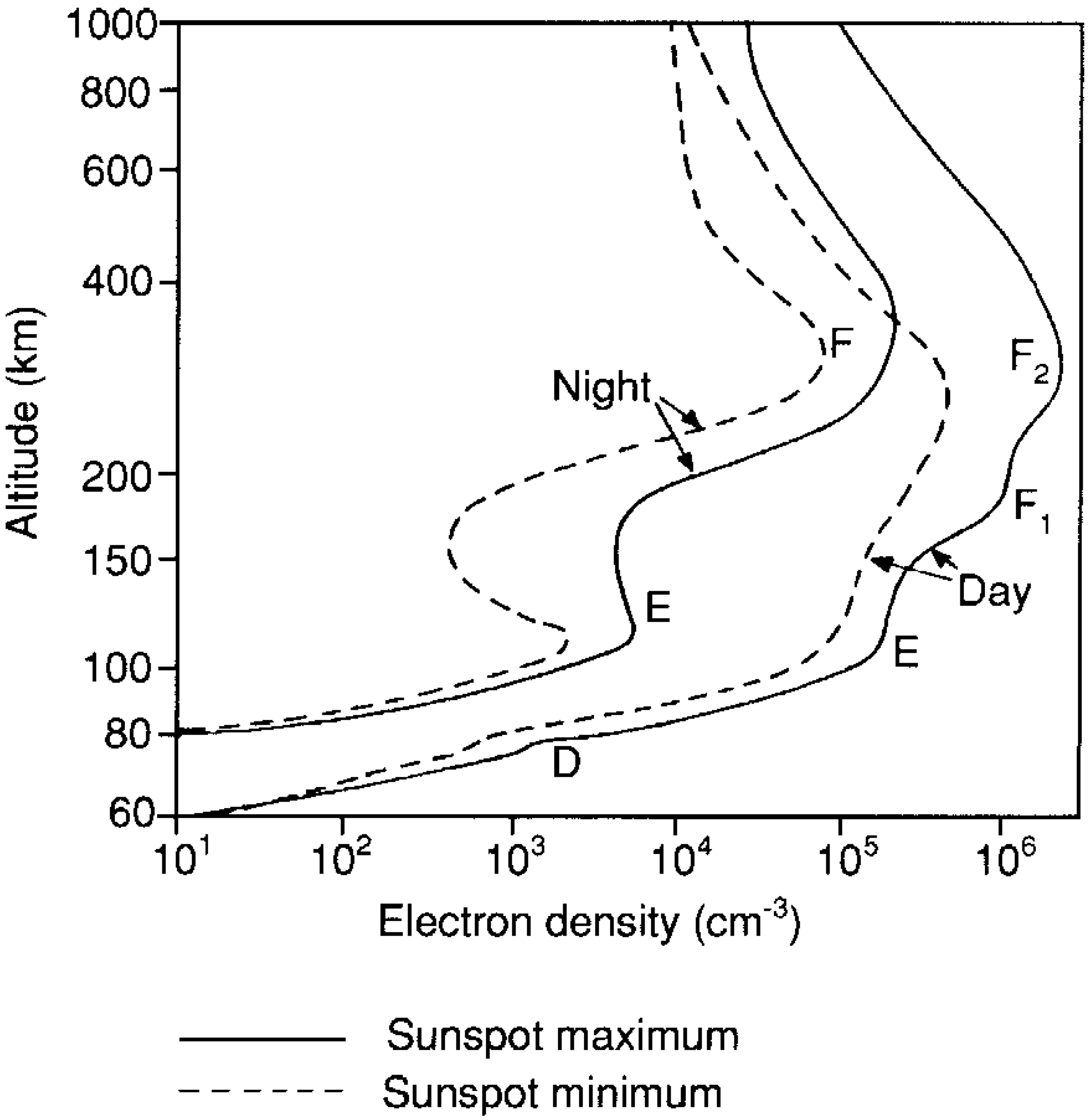
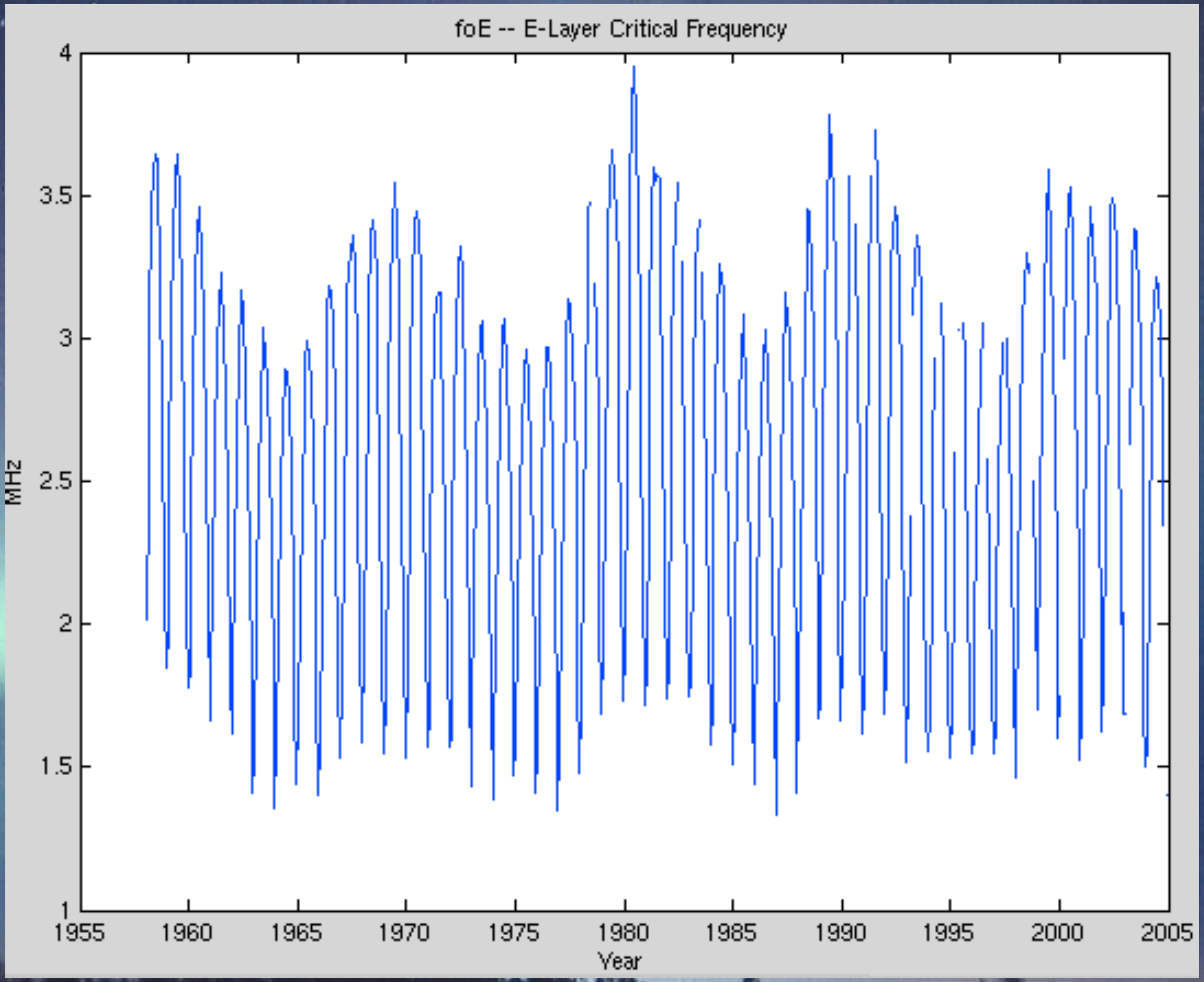


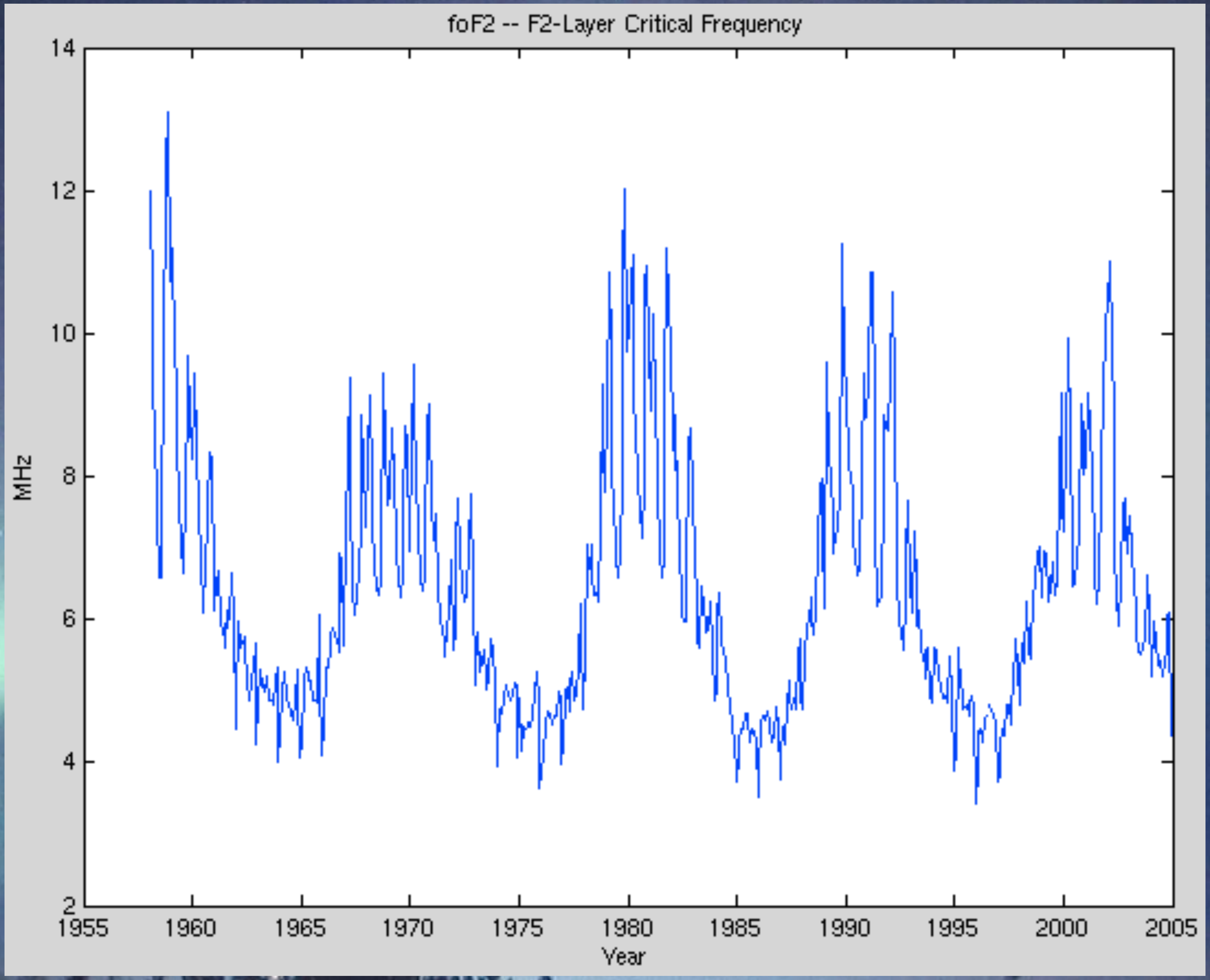

**Figure 10.12** Calculated ionospheric temperature profiles. (From *Roble*.[9])



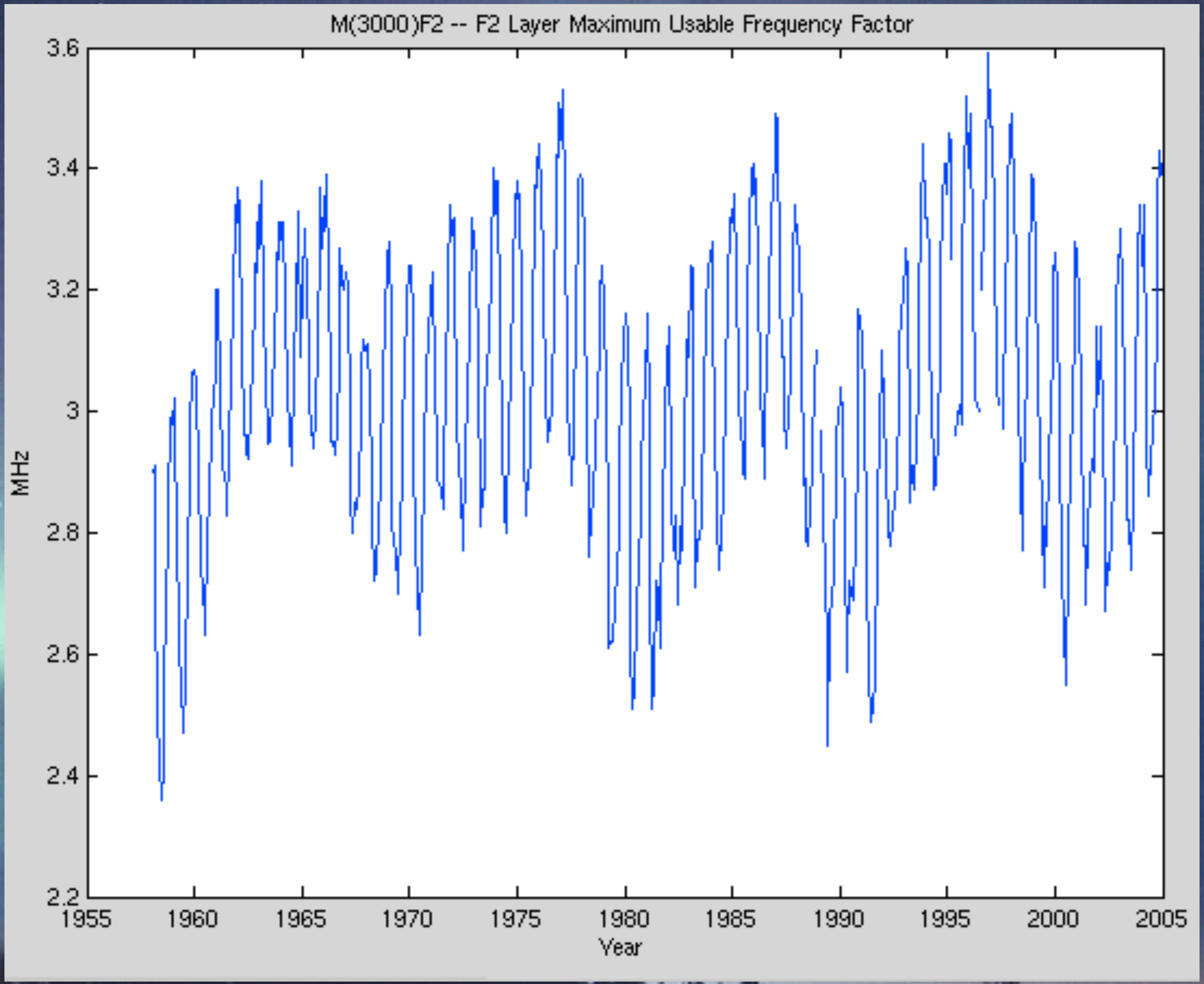
# E.g.: foE at Sodankylä



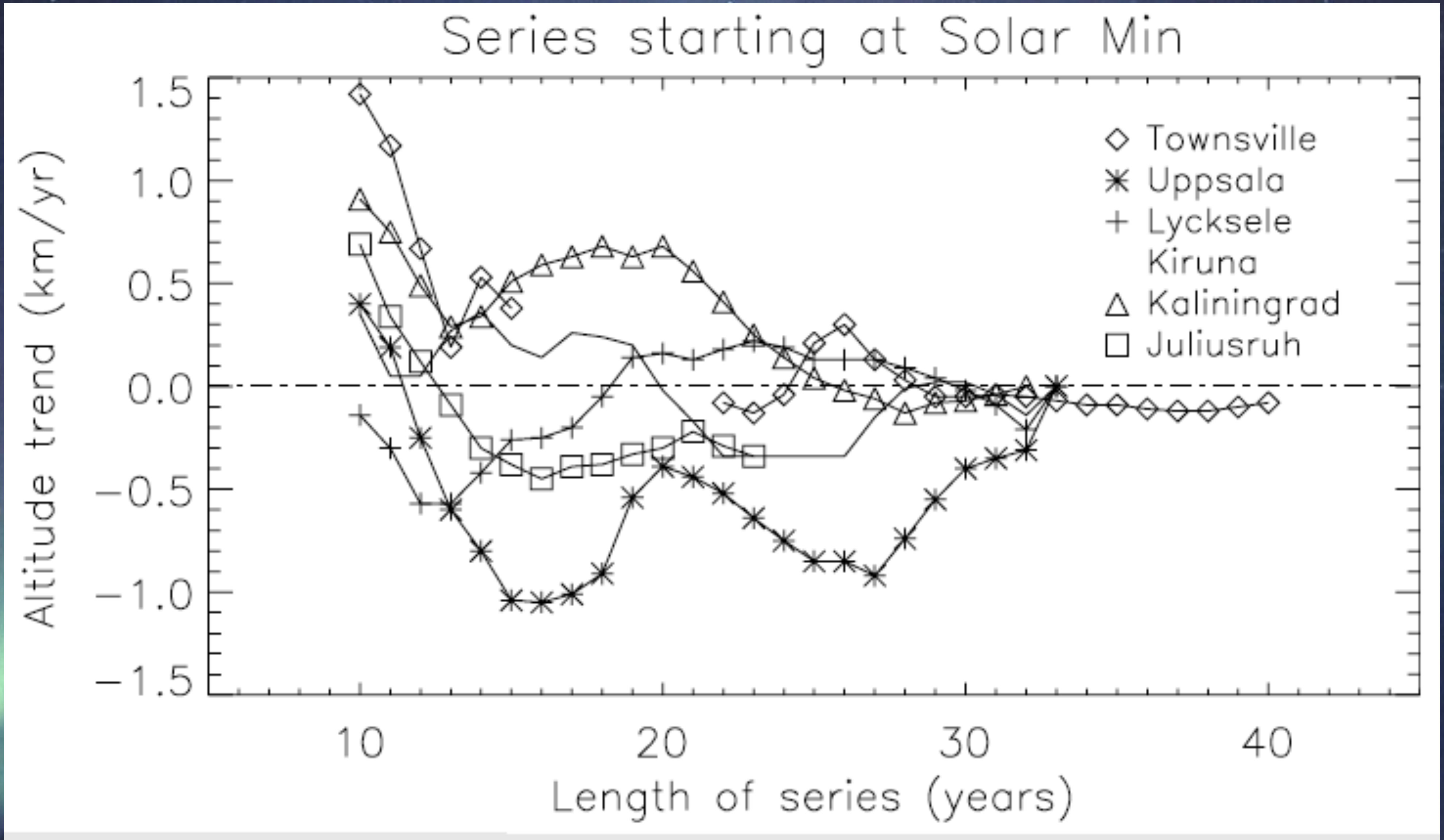
# Example: foF2 at Sodankylä



# E.g.: M(3000)F2 at Sodankylä

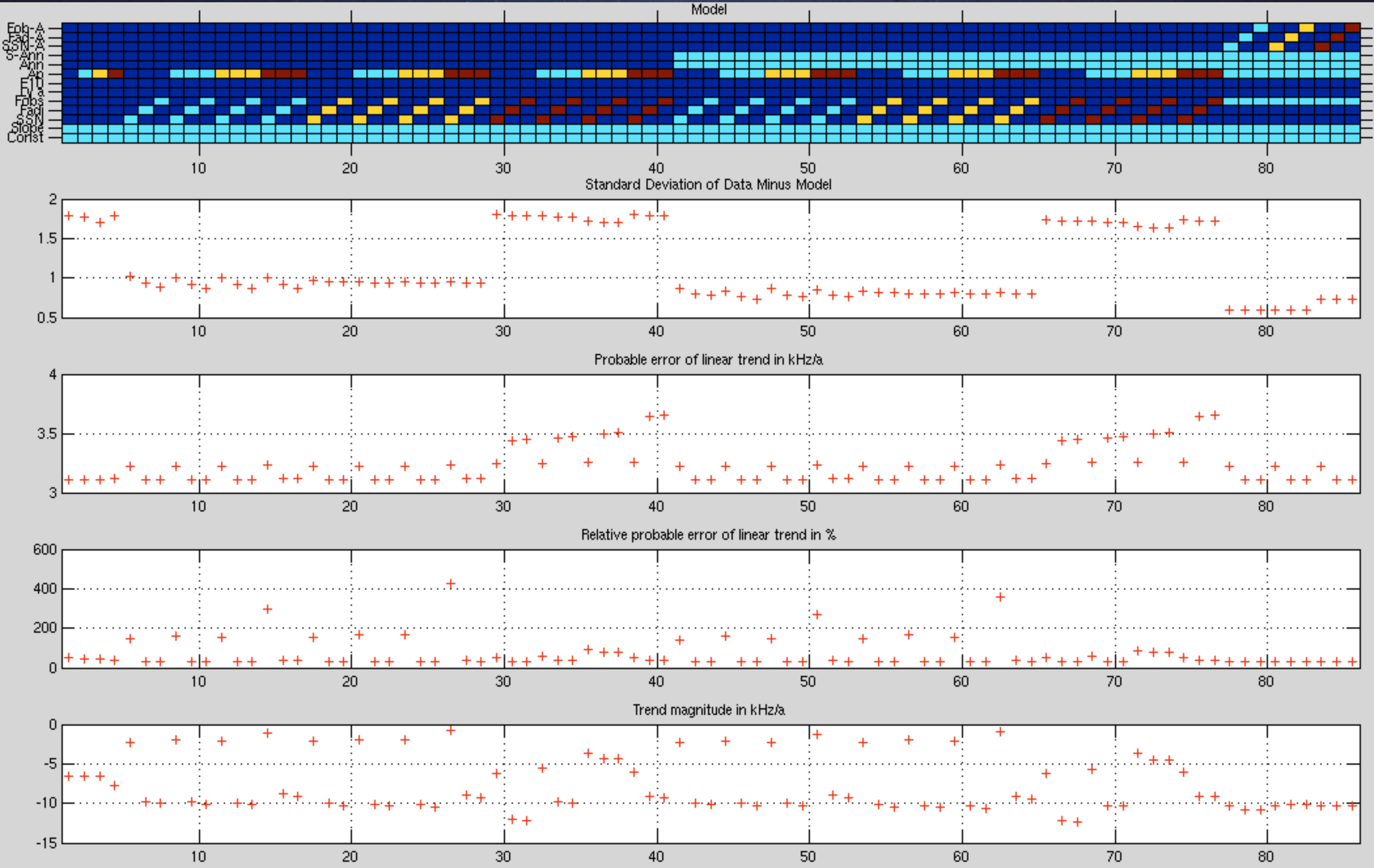


# Ringing




From Clilverd et al., 2003.

# Sodankylä F2-layer critical frequency foF2



# The Problem

**“It should be stressed that different authors use different approaches to extract long-term trends from the ionospheric observations and the success of analysis depends to a great extent on the method used. The useful ‘signal’ is very small and the ‘background’ is very noisy, so special methods are required to reveal a significant trend in the observed [...] variations.”**

[*Mikhailov & Marin, Ann. Geophys. 18, 653, 2000*]

# Problems I

- Type of data used by various authors:
  - –hourly
  - –3-hourly (like geomagnetic indices)
  - –daily (selection of hours or averages of a certain time of day like noon (10-14LT))
  - –monthly (unphysical, months are of different length)
  - –seasonal (averages of certain months)
  - –annual

# Problems II

- Low-pass filtering:
  - often a running mean filter is employed to reveal underlying overall behaviour
  - some authors use polynomial fits instead
- How can we ensure that the filtered data still represent the physical behaviour of the observed property?
- "Smooth to death" - one can make anything correlate.

# Known components

- The temporal behaviour of a given ionospheric time series contains a number of known components:
  - solar activity variations (11/22-year cycles)
  - geomagnetic activity
  - annual (seasonal) cycle
  - semi-annual cycle
  - (non-linear) combinations of the above

# Unknown components

- The known components have to be removed from the time series in order to reveal the unknown components, which make up the variation of the residual:
  - –changes of atmospheric chemical composition (greenhouse effect)
  - –changes of Earth's magnetic field
  - –changes of thermospheric winds (dynamics)
  - –measurement errors
  - –what else???

# Problems III

- How to remove solar activity?
  - Sinusoid
  - Sunspot Number
  - Group Sunspot Number
  - F10.7 radio fluxes
    - observed
    - adjusted to 1 A.U.
  - solar Lyman $\alpha$ index
  - E10.7 proxy from Solar2000
  - lower resolution than ionospheric data, i.e. smoothed versions

# Problems IV

- What about geomagnetic activity?
  - Which index? aa, ak, Ak, ap, Ap, K, Kp?
  - Which representation? 3-hourly? Smoothed?
  - Some argue that including geomagnetic activity leads to increased noise only.

# Problems V

- Which method should be used for trend determination?
  - –multi-step removal of known components
  - –single-step removal by multi-parameter fitting
- The error propagation through the multi-step method is very difficult to estimate.
- What to do with gaps in the data? This is crucial for historic geophysical data sets.

# Problems VI

- What are the errors $\varepsilon_i$ of the measurements?
  - –accuracy of foE 0.05 MHz ?
  - –accuracy of foF2 0.10 MHz ?
  - –accuracy of M(3000)F2 0.05 ?
- … but these are only scaling requirements, errors are likely to be larger!

# Problems VII (hmF2)

- The height of the F2-layer peak (hmF2) is (usually) not routinely scaled.
- It can be estimated empirically.
- Several methods have been derived in the past.
- Which method should be used?
- The method depends on the ionosonde location.

# Problems VIII

- What about the errors $\varepsilon_i$ of the F2-layer peak height hmF2?
  - need to compute error propagation through the empirical expressions
  - tedious, but not difficult
  - fortunately the computer takes care of the actual calculations

# Problems IX

- Without removal of cyclic components, the observed trend depends upon the phase of the cyclic components in the data.
- Removal helps, but due to noise, the cyclic components cannot be removed entirely.
- Even after reducing the data, the trend might still depend upon the phase.

# Is this hopeless?

- I don't think so.

# Is this hopeless?

- I don’t think so -- yet.

# Is this hopeless?

- I don’t think so -- yet.
- But indeed, “special methods are required…”
- Identical reference data given to many teams to compare methods: paper submitted.
- Work continues…

# Solution

# Solution

$4^{th}$ IAGA/ICMA/CAWSES Workshop on

Long-Term Changes and Trends in the Atmosphere

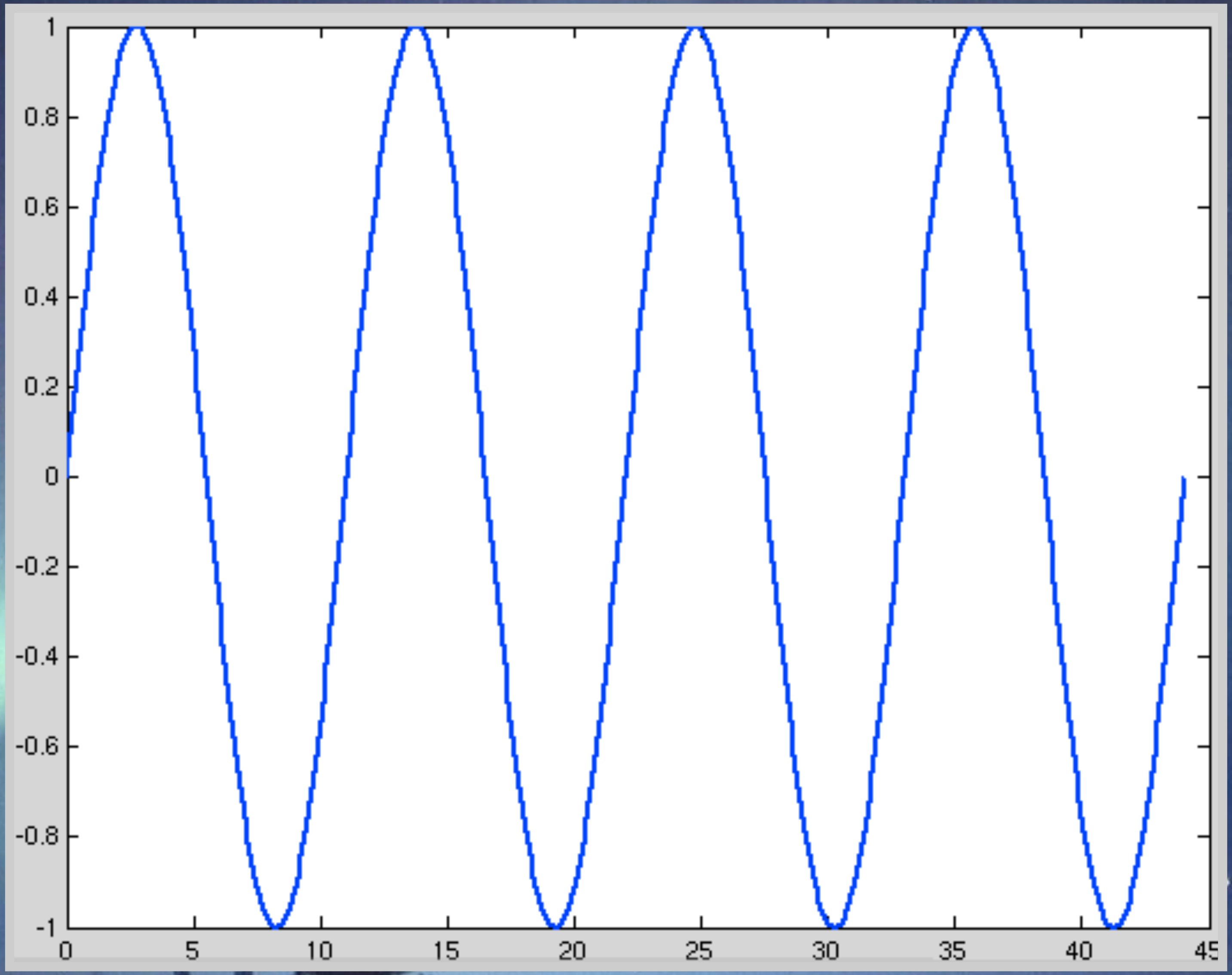
Sodankylä, Finland

$4^{th}$ to $8^{th}$ September 2006

**Welcome!**


Thank you!

# E.g.: 11-year running mean



# Errors of unknowns

The inverse problem can be solved by means of Singular Value Decomposition (SVD). Any matrix can be expressed as a product of two orthogonal matrices and a diagonal matrix:

$$\mathbf{B} = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T$$

Orthogonal means:

$$\mathbf{V}^T \cdot \mathbf{V} = \mathbf{1}$$

Therefore:

$$\mathbf{x} = \mathbf{V} \cdot \mathbf{W}^{-1} \cdot (\mathbf{U}^T \cdot \mathbf{b})$$

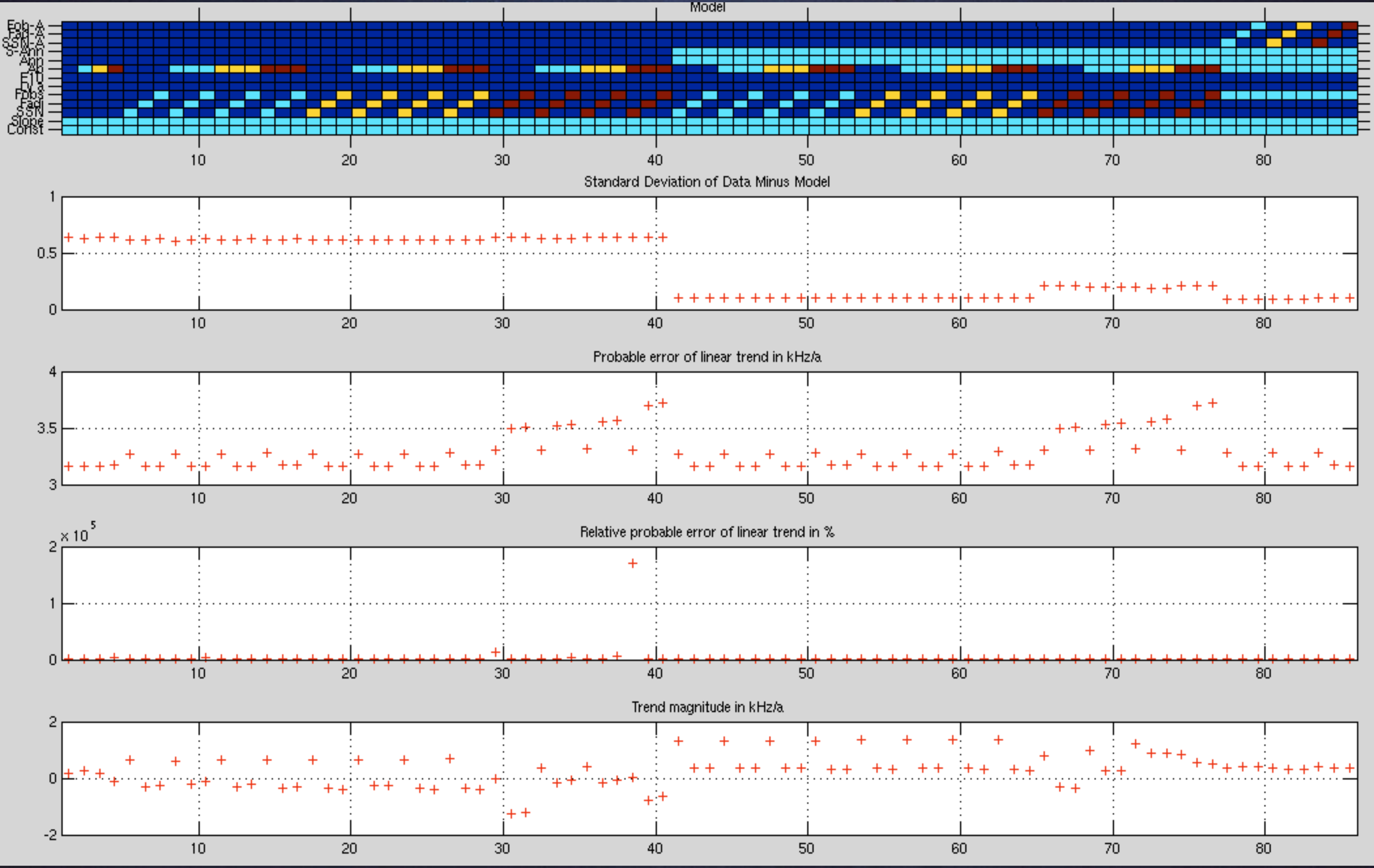
The errors of the unknowns are already contained in this solution:

$$\varepsilon^2(x_i) = \sum_{i=1}^{M} \left( \frac{V_{ji}}{w_{ii}} \right)$$

# No problem, let's try!

- For the critical frequencies, let's choose 1 MHz as the error for all measurements.
- We have a computer, so let's try all(?) possible models!

# Sodankylä E-layer critical frequency foE



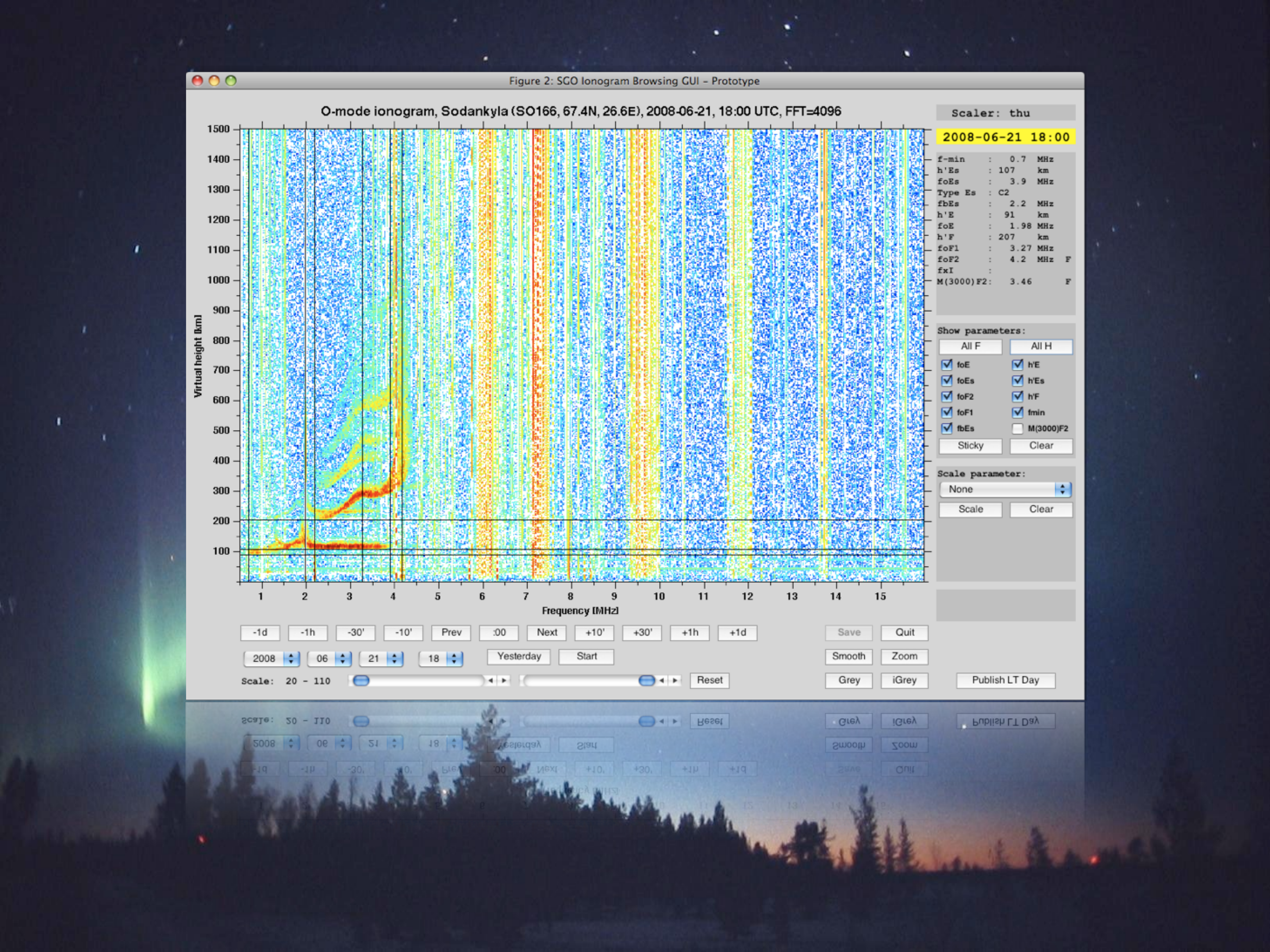
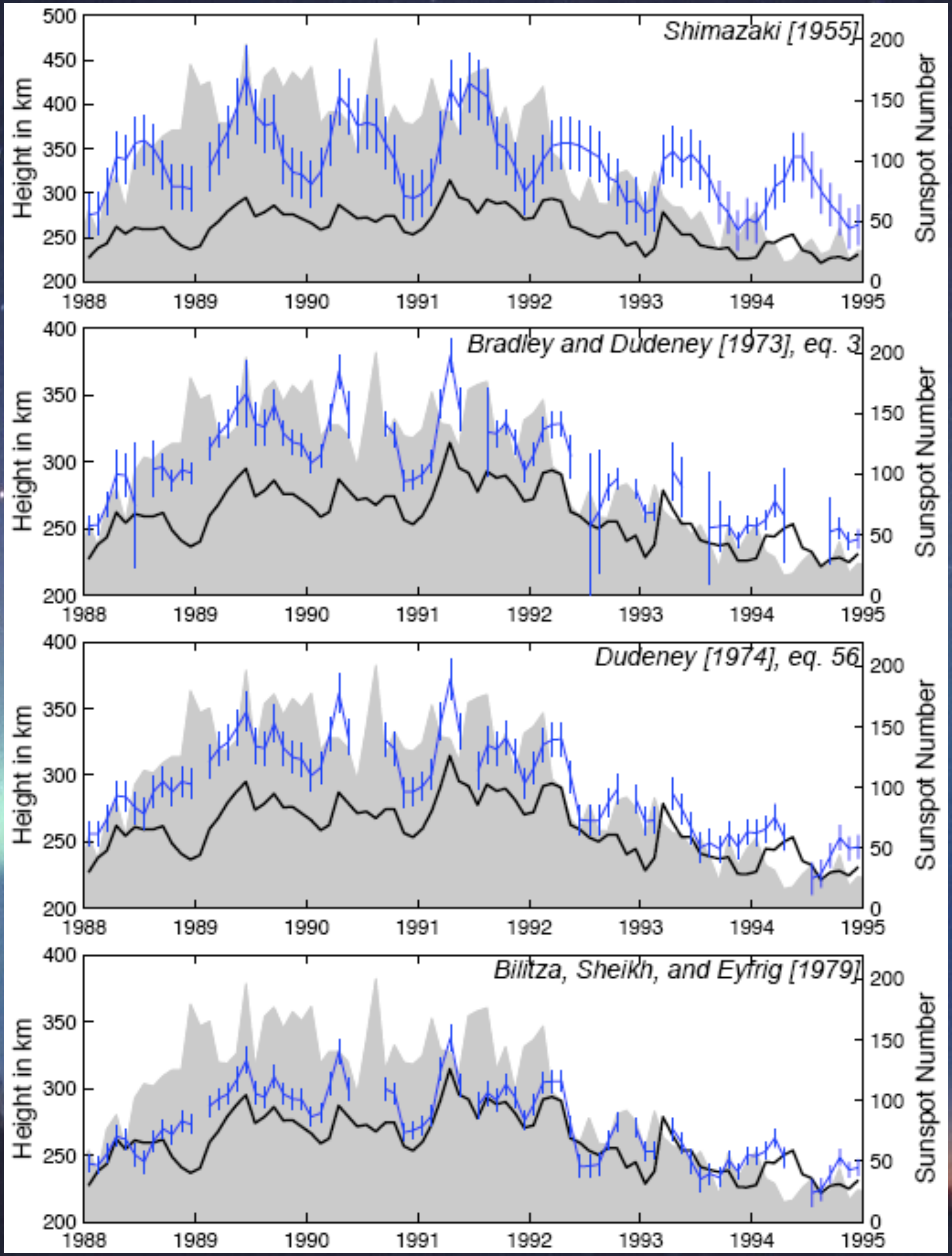
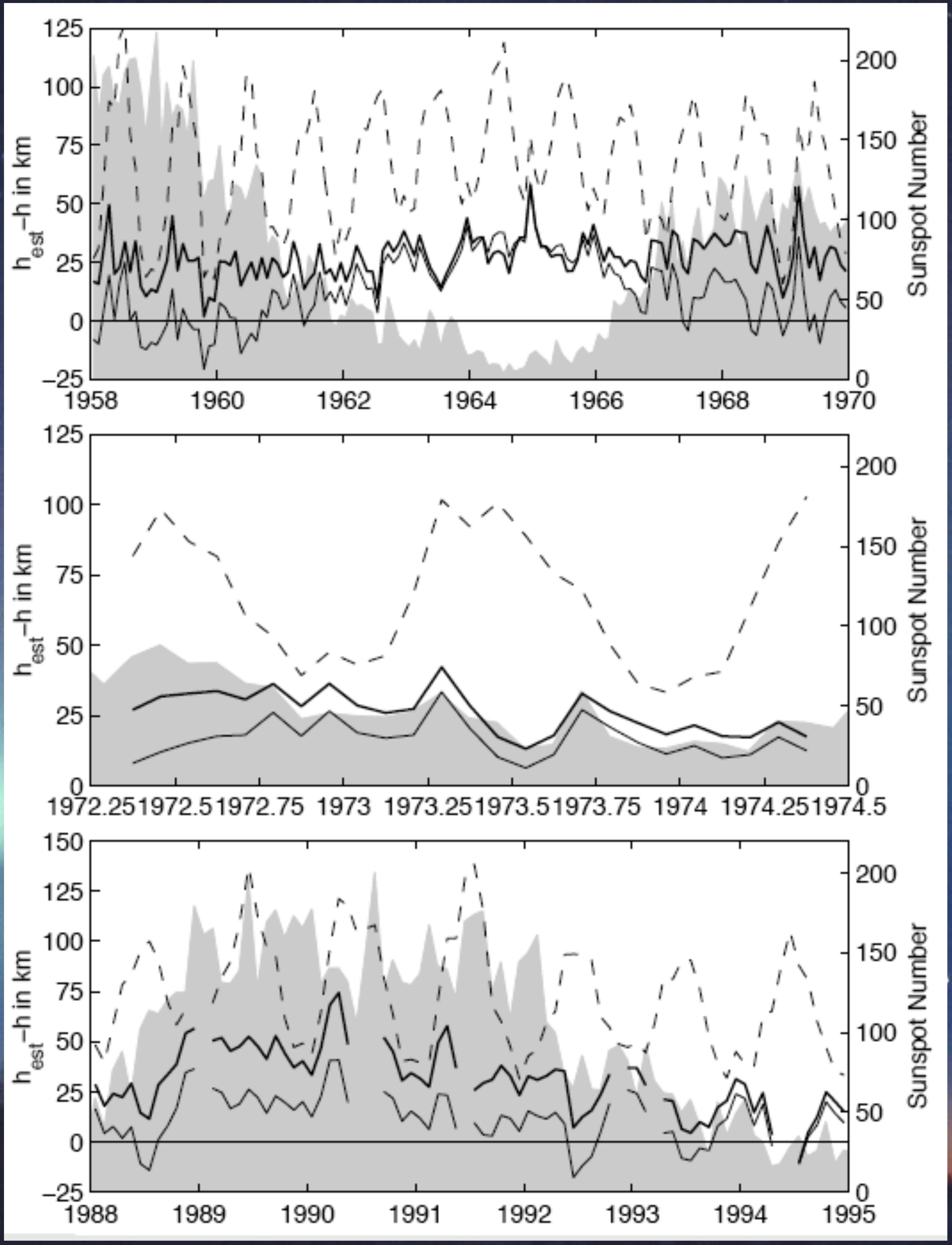


Figure 2: SGO Ionogram Browsing GUI - Prototype
O-mode ionogram, Sodankyla (SO166, 67.4N, 26.6E), 2008-06-21, 18:00 UTC, FFT=4096
Virtual height [km]
Frequency [MHz]
Scaler: thu
2008-06-21 18:00
f-min : 0.7 MHz
h'Es : 107 km
foEs : 3.9 MHz
Type Es : C2
fbEs : 2.2 MHz
h'E : 91 km
foE : 1.98 MHz
h'F : 207 km
foF1 : 3.27 MHz
foF2 : 4.2 MHz F
fxI :
M(3000)F2: 3.46 F
Show parameters:
All F
All H
foE
foEs
foF2
foF1
fbEs
h'E
h'Es
h'F
fmin
M(3000)F2
Sticky
Clear
Scale parameter:
None
Scale
Clear
-1d -1h -30' -10' Prev :00 Next +10' +30' +1h +1d
2008 06 21 18
Yesterday
Start
Scale: 20 - 110
Reset
Save
Quit
Smooth
Zoom
Grey
iGrey
Publish LT Day

# hmF2 at Sodankylä

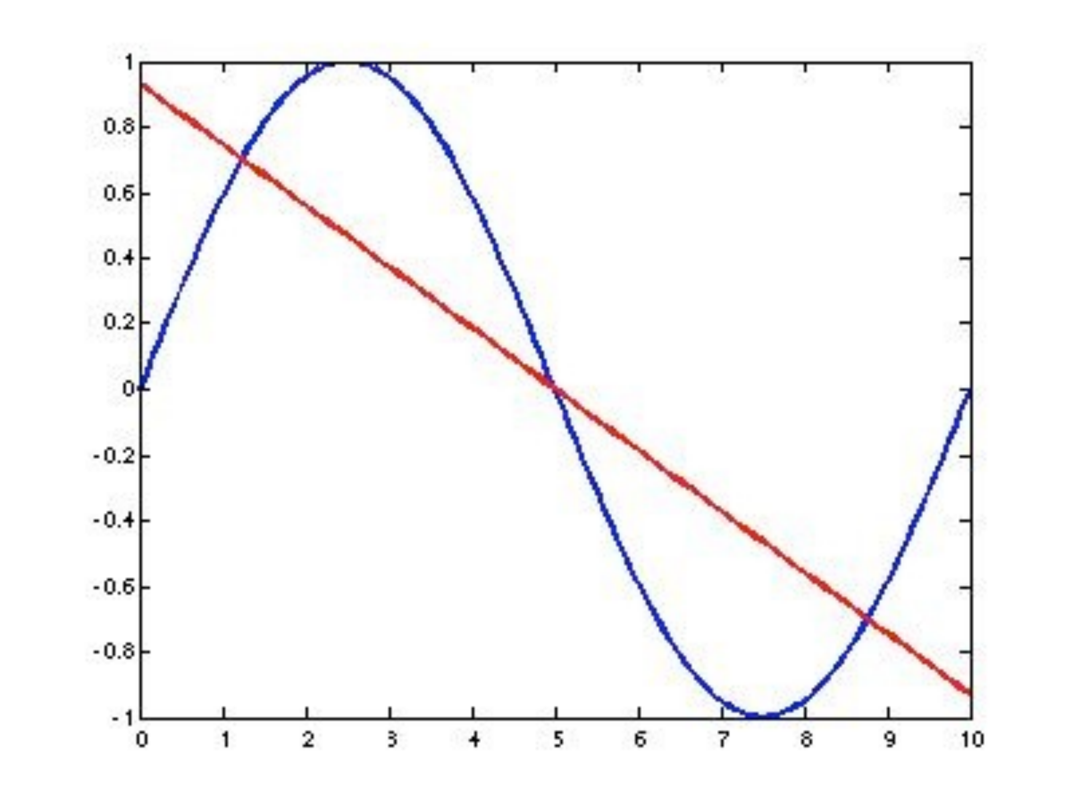


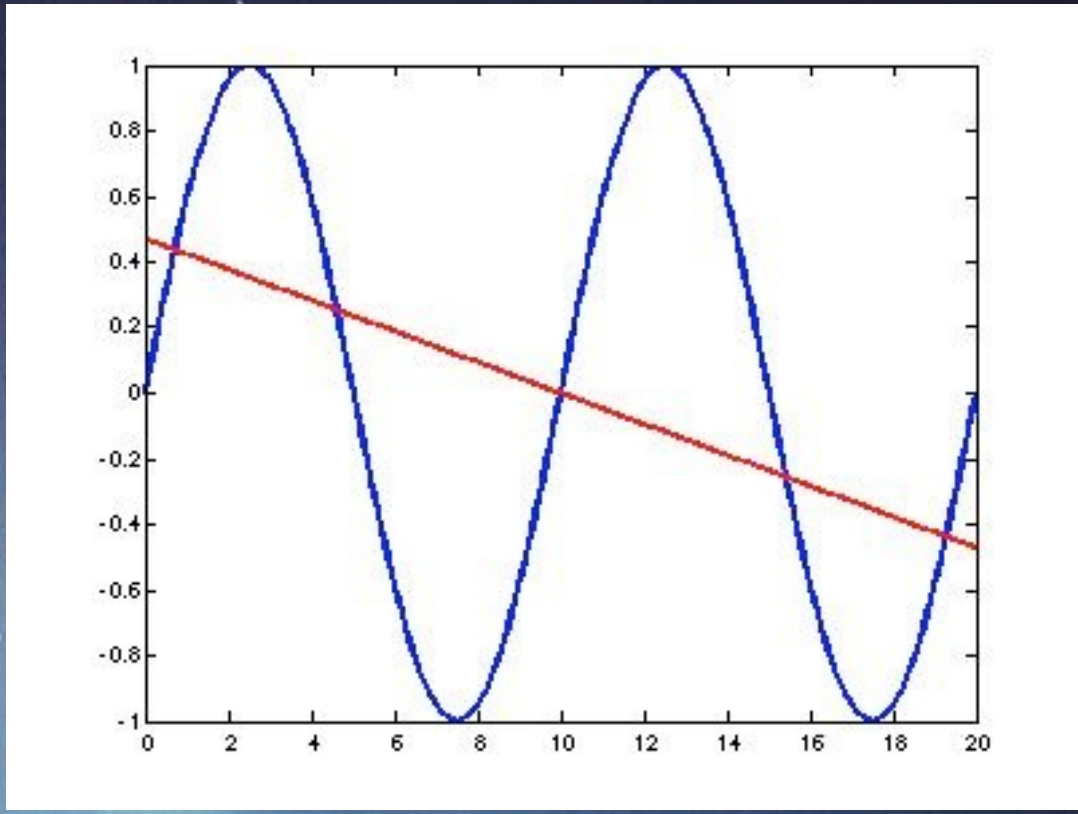
# hmF2 minus real-height at Sodankylä

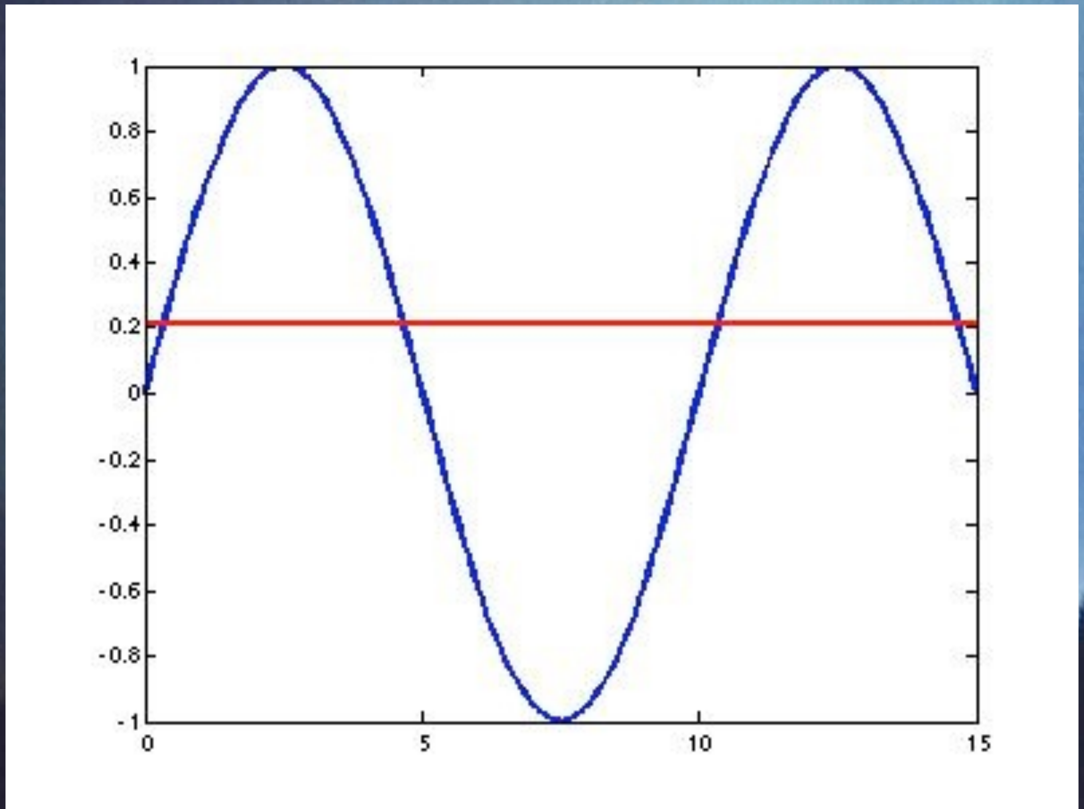


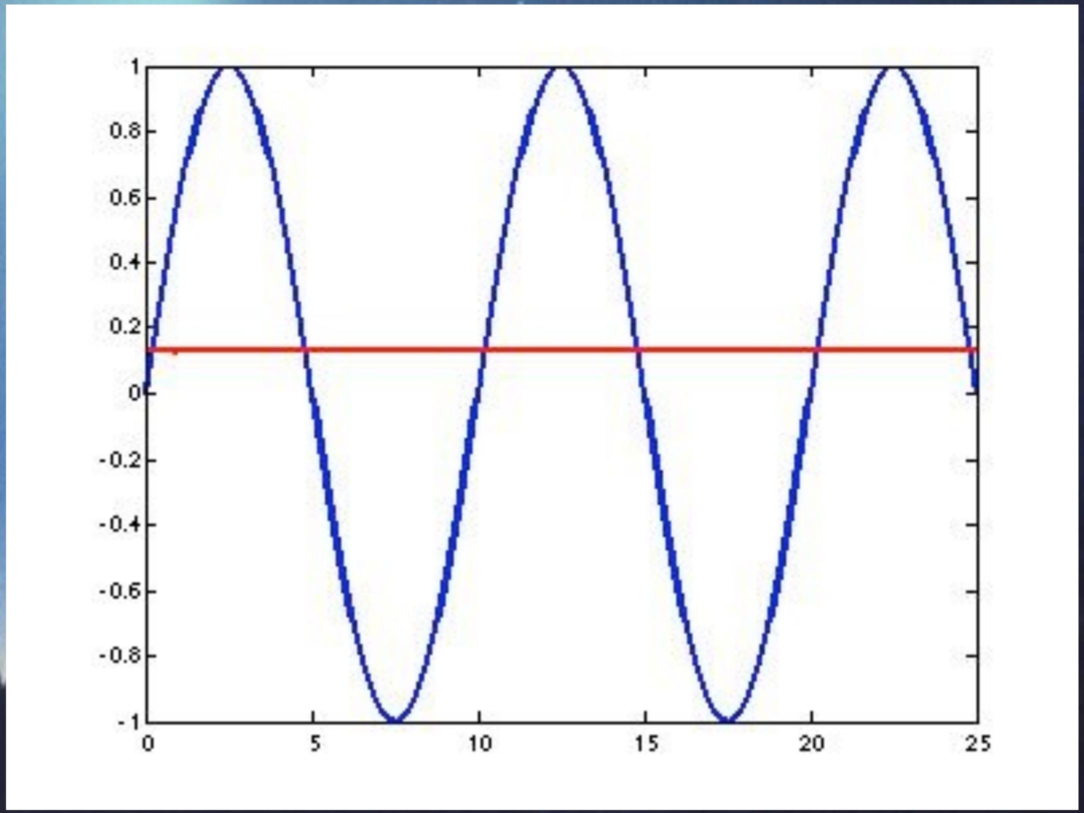
# No problem, let’s try!

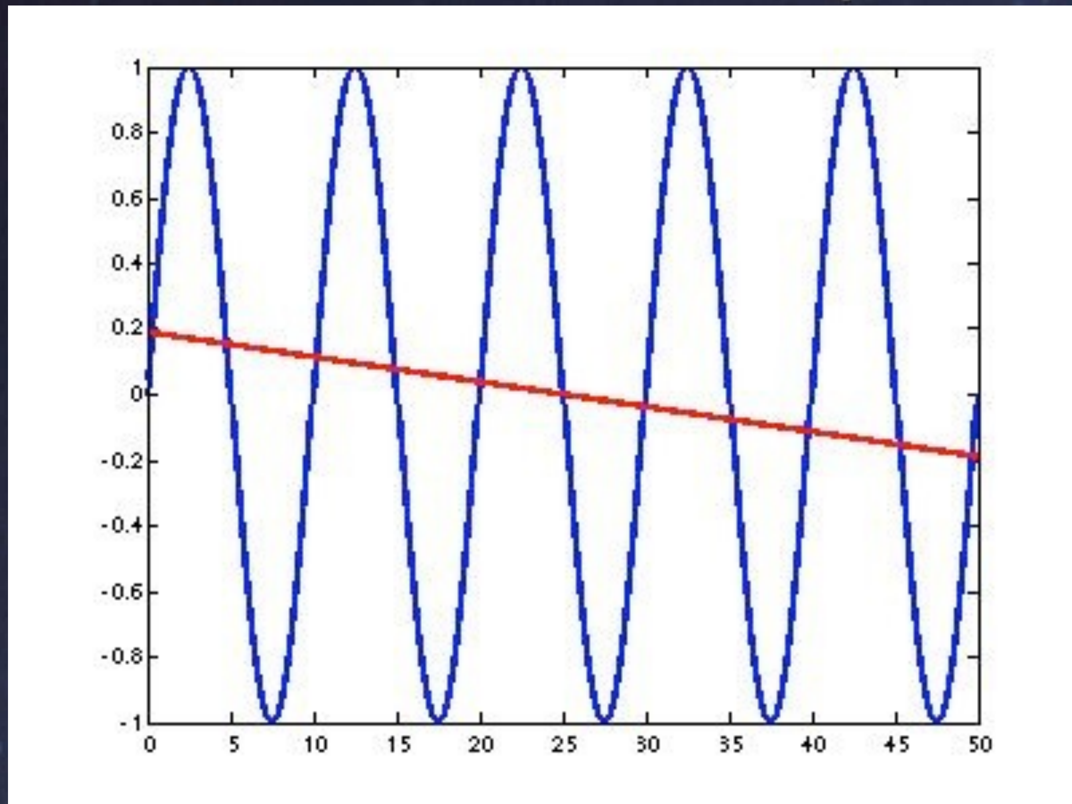
- For the ionosonde of Sodankylä (and this will most likely be different for your ionosonde station!), the method of Dudeney (1974, eq. 56) is the most appropriate.
- Here I’ve been more optimistic with the errors.

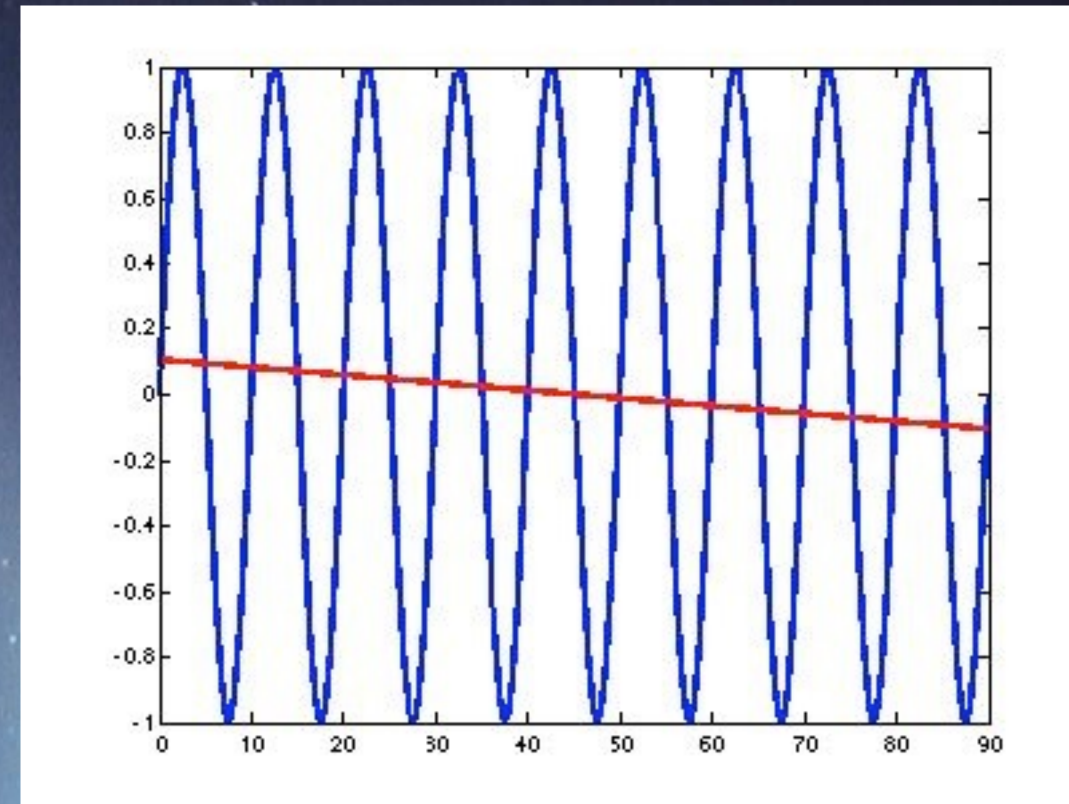

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 1 2 3 4 5 6 7 8 9 10

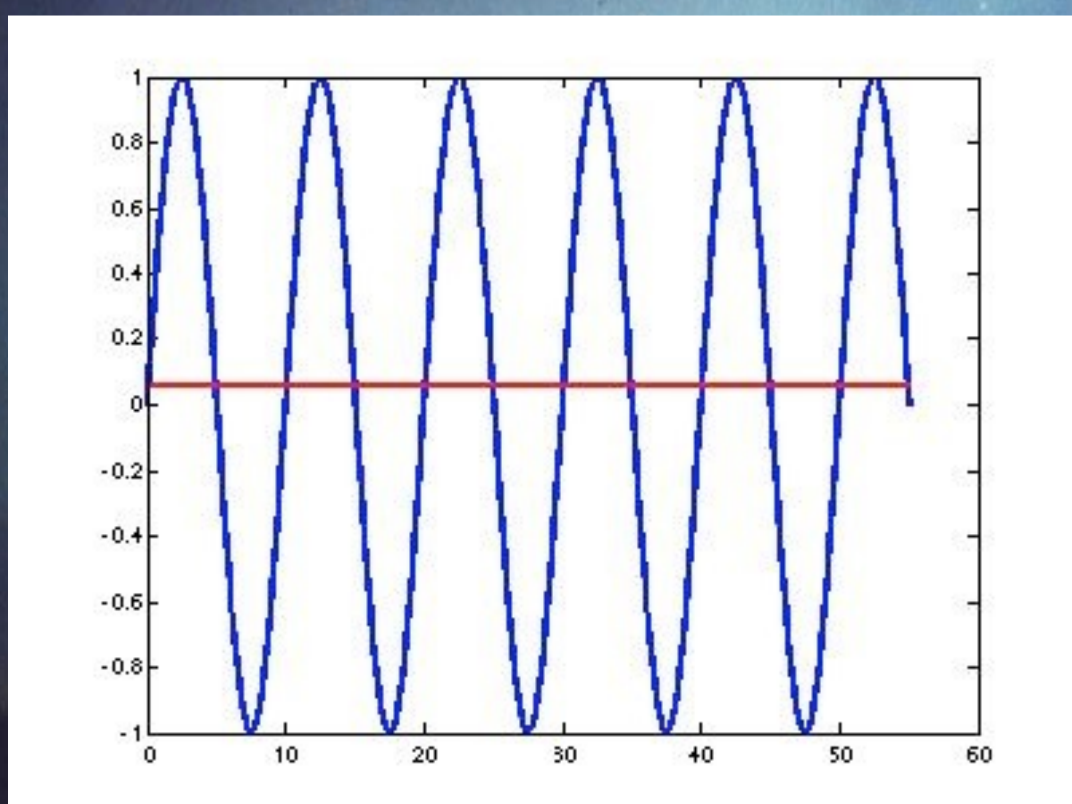

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 2 4 6 8 10 12 14 16 18 20

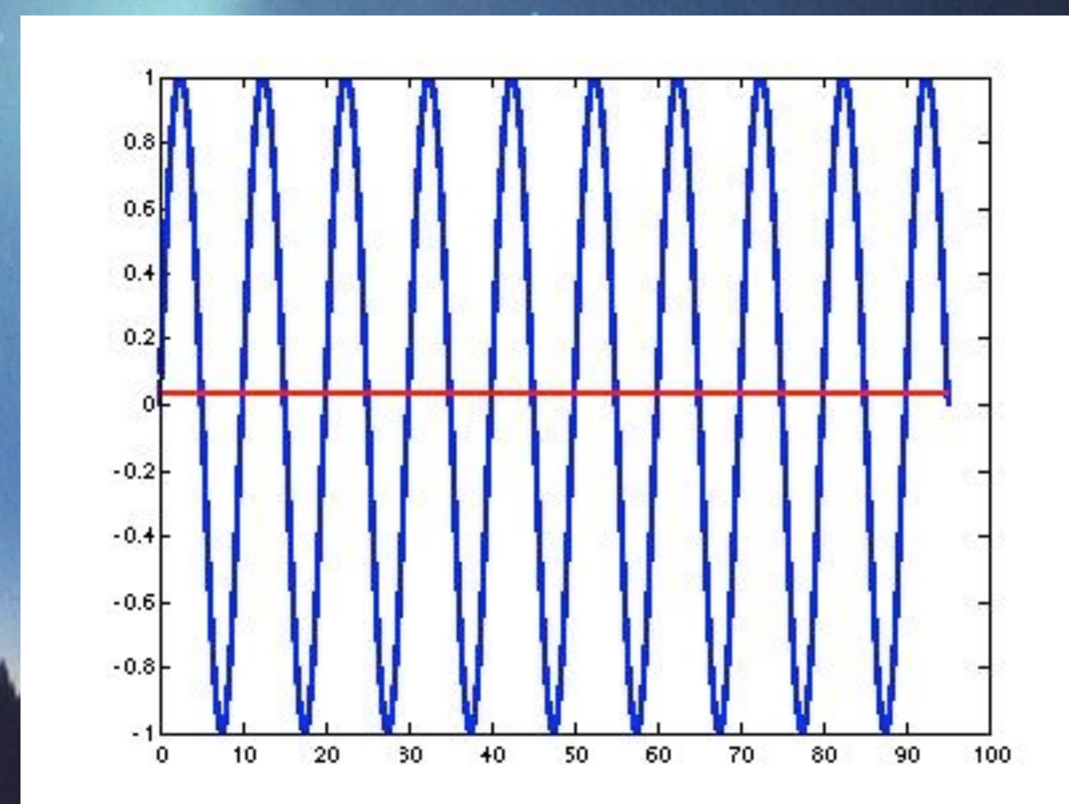

1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 5 10 15


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 5 10 15 20 25


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 5 10 15 20 25 30 35 40 45 50


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 10 20 30 40 50 60 70 80 90


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 10 20 30 40 50 60


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 10 20 30 40 50 60 70 80 90 100

# Solutions I

- Indeed, “special methods are required...”
- Be careful when using (low-pass) filters!
- Observed F10.7 radio fluxes seem to be a good proxy for solar activity (maybe E10.7 is better?).
- A single-step multi-parameter fit does the job best.

# Solutions II

- Errors are information, too, and thus should be included in the fit. What are reasonable error limits for the data at hand?
- For hmF2, it is crucial to find an empirical method, which works for the specific ionosonde site (location). This needs testing of the formulae against real-height analysis!

# Solutions III

- Using variance of residual for searching for a good model looks like a good idea, but increasing number of base functions (degrees of freedom) will at some point always lead to a better fit, but it might not make sense physically.
- The Ringing Method can be used to predict the length of the time series needed to derive a trend.
- Moreover, it can be used for quality control!