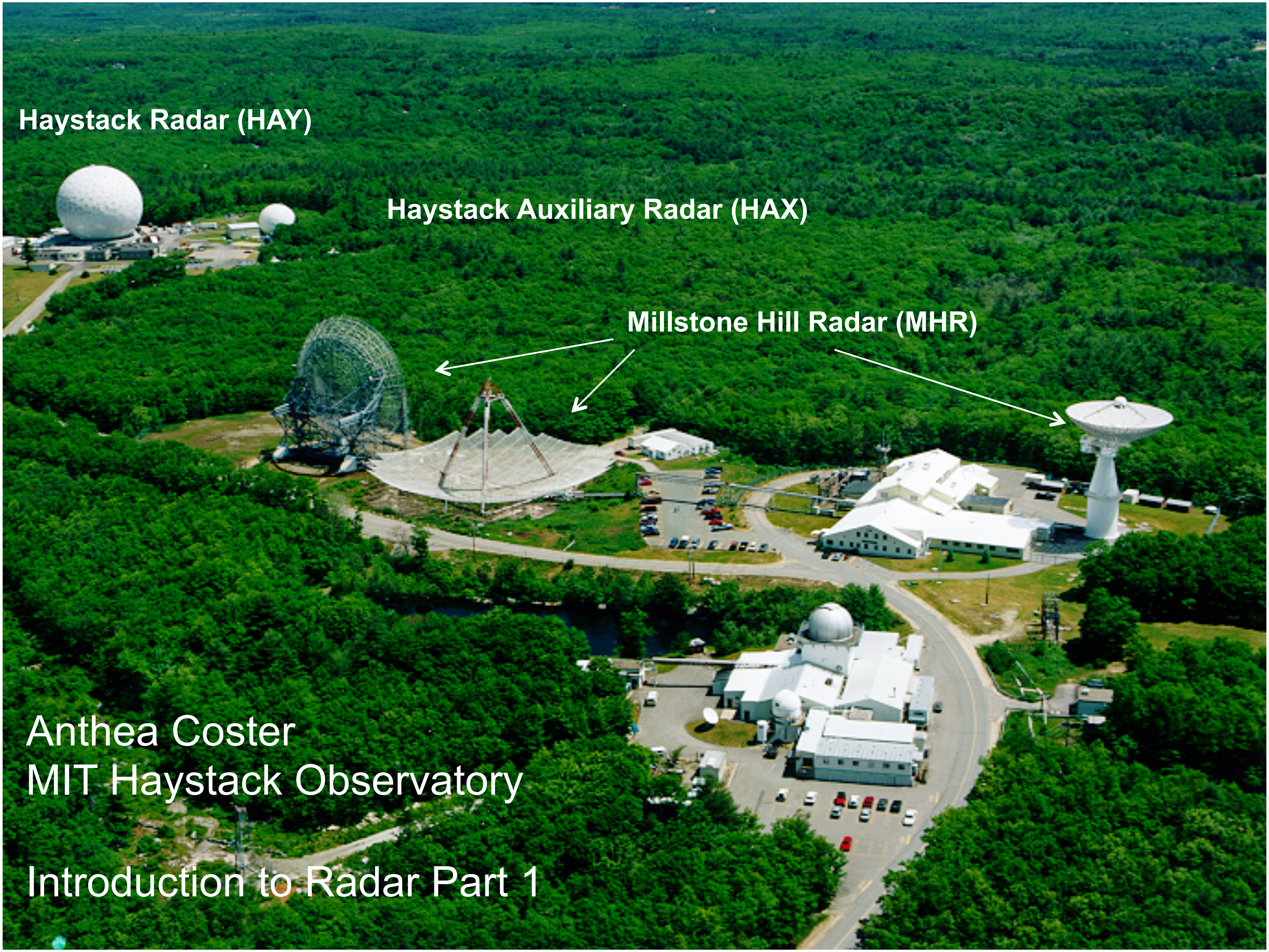
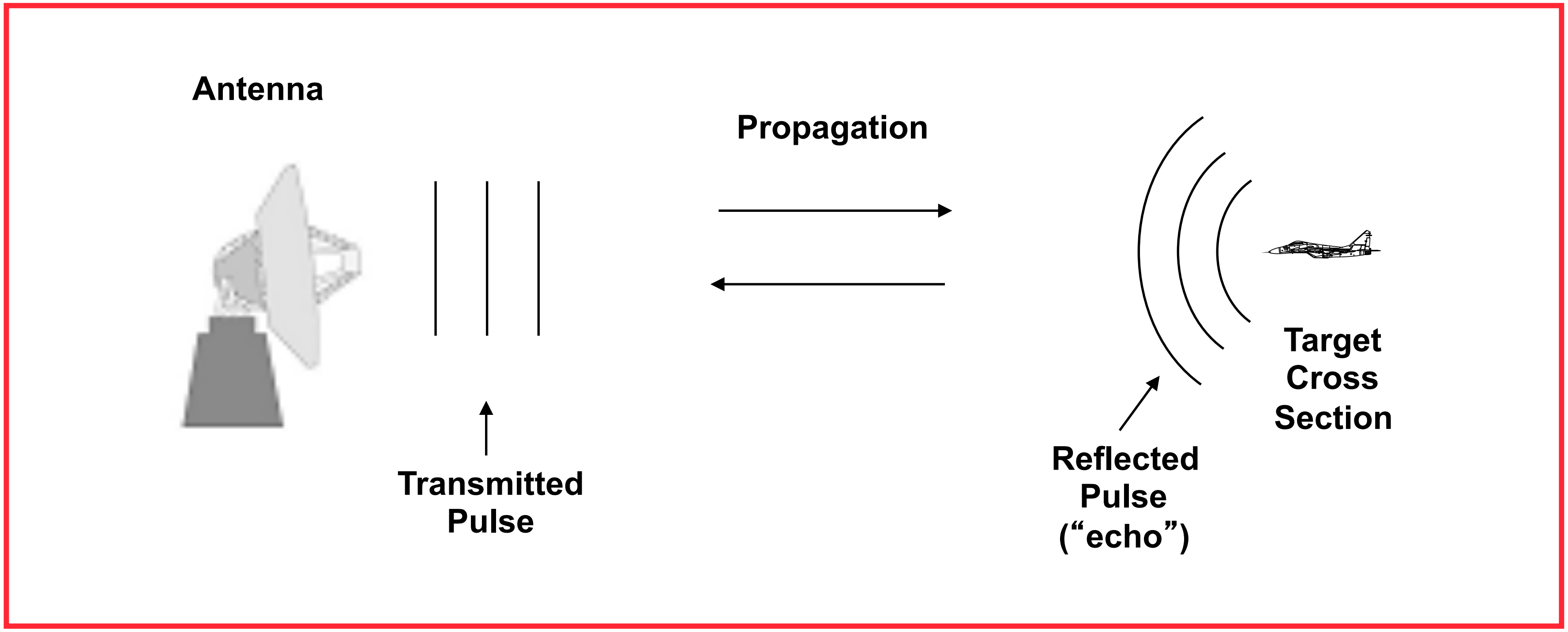


Introduction to Radar Part 1

Anthea Coster
MIT Haystack Observatory

RADAR
RAdio Detection And Ranging



Radar observables:
- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

OUTLINE

RADAR –definition

Basic principles of radio waves:

- properties of waves
- amplitude phase coherent/destructive interference
- polarization
- Doppler

Antennas

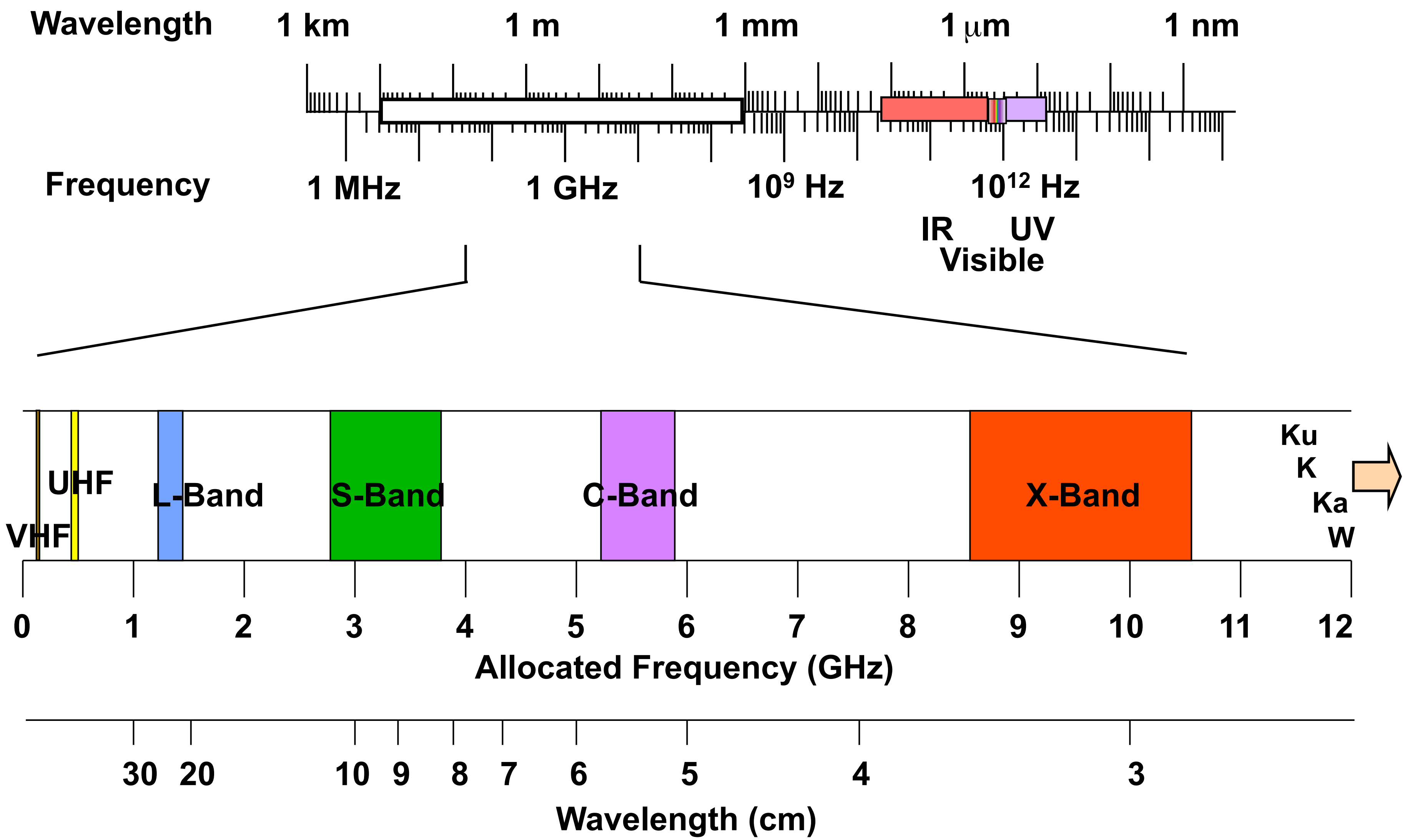
Radar Equation

Hard Targets versus Soft Targets

Signal Processing

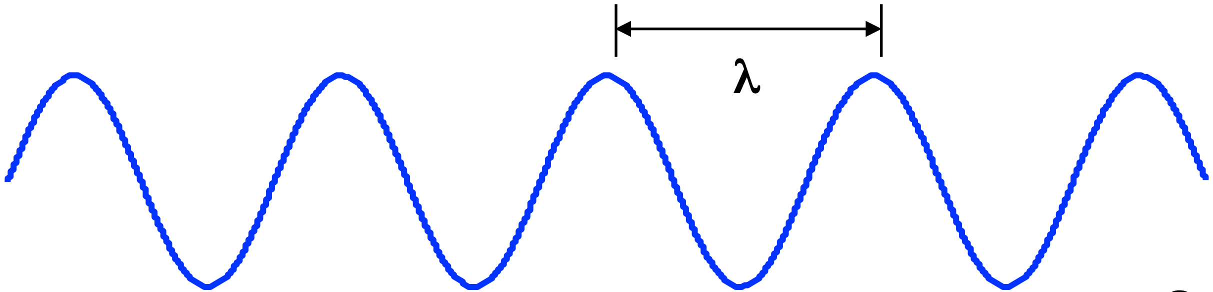
correlation versus convolution

Radar Frequency Bands



Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c

$c = 3 \times 10^8$ m/sec

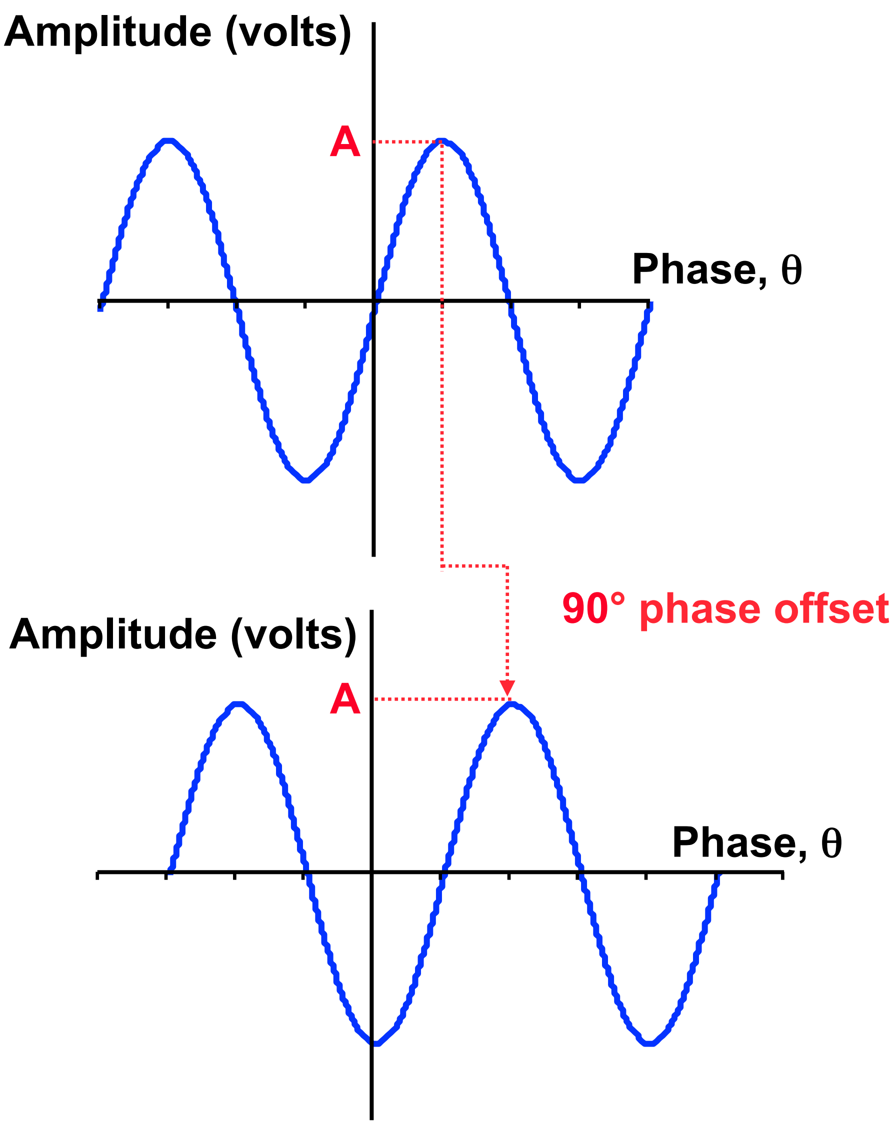
$= 300{,}000{,}000$ m/sec

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

Frequency	Wavelength
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

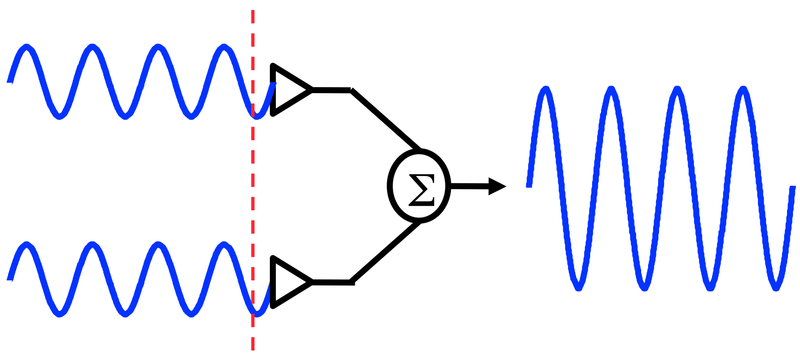
Properties of Waves
Phase and Amplitude



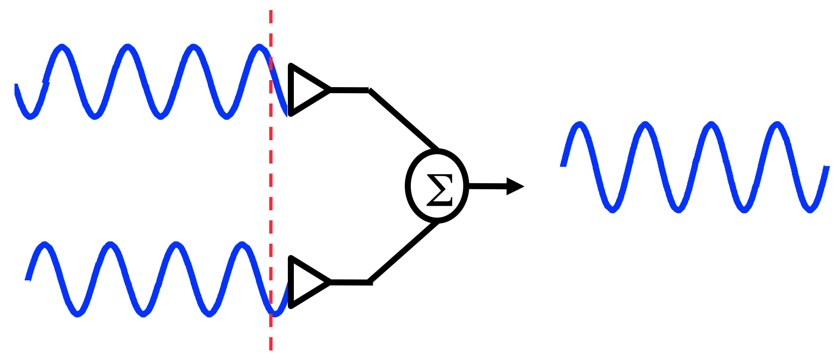
$$A\sin(\theta)$$

$$A\sin(\theta - 90^{o})$$

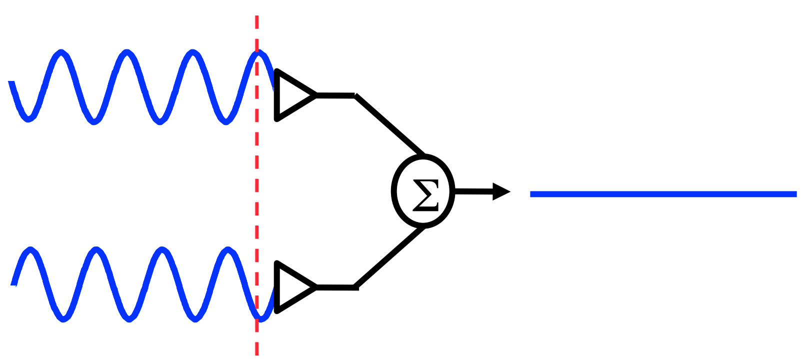
Properties of Waves
Constructive vs. Destructive Addition



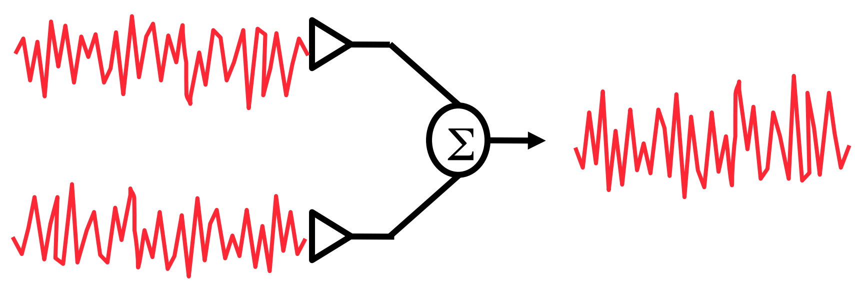
Constructive
(in phase)



Partially Constructive
(somewhat out of phase)



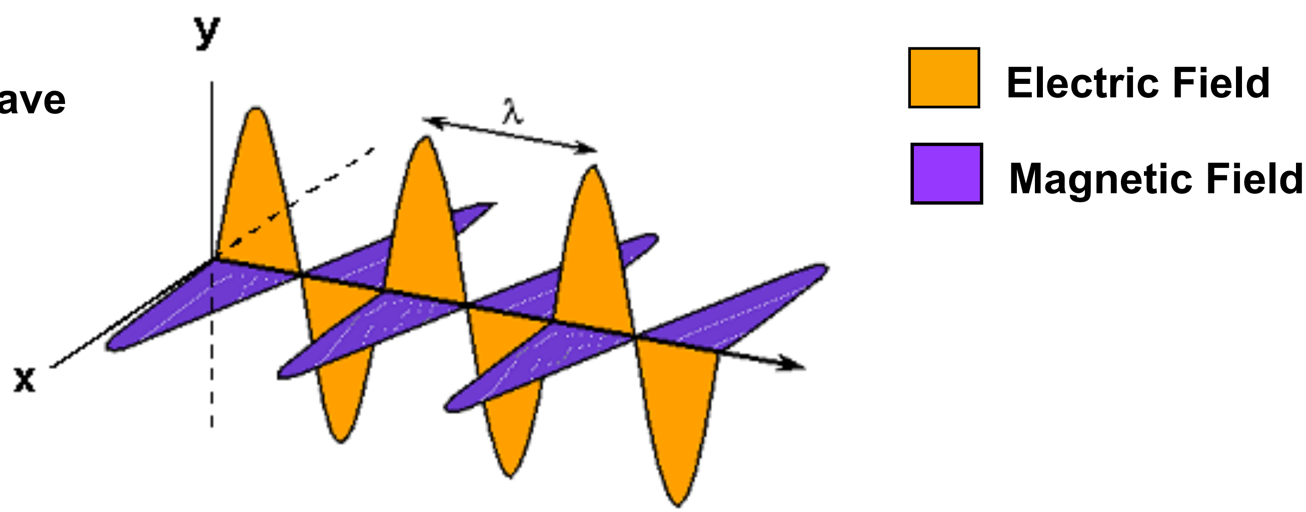
Destructive
(180° out of phase)



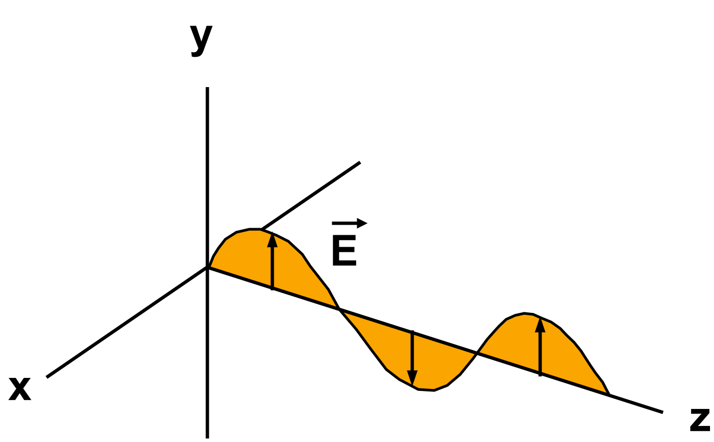
Non-coherent signals
(noise)

Polarization

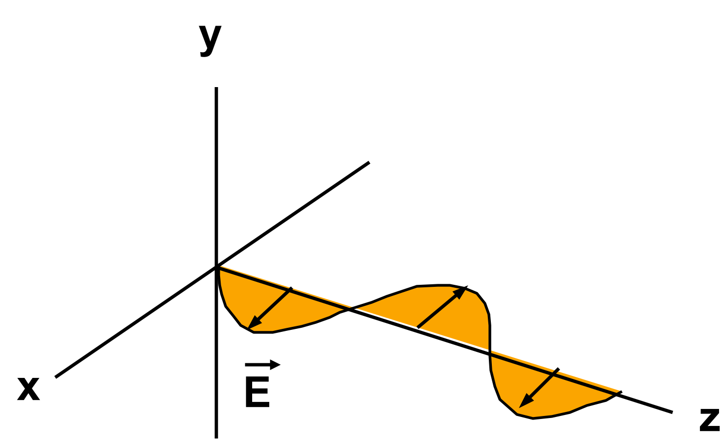
Electromagnetic Wave



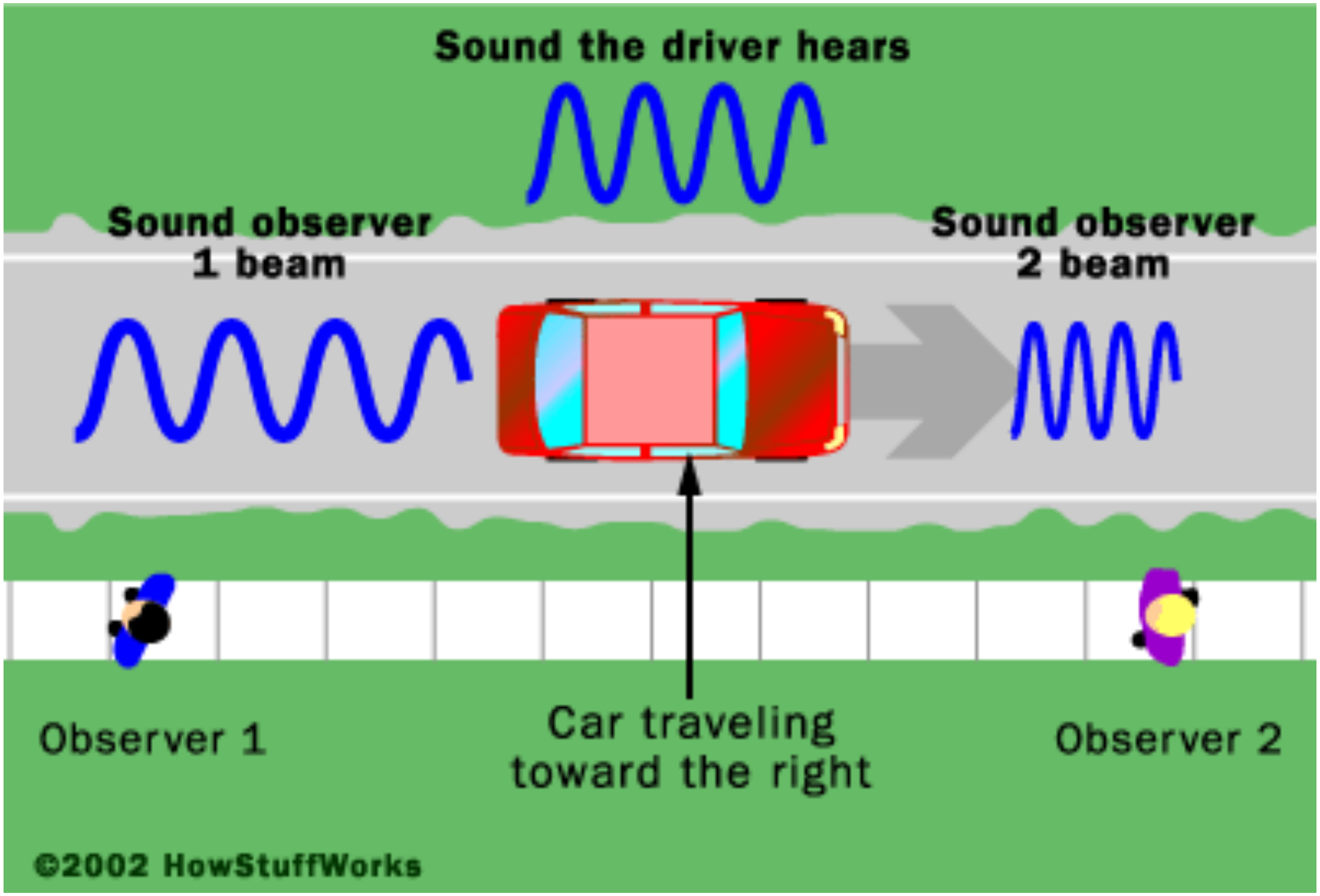
Vertical Polarization



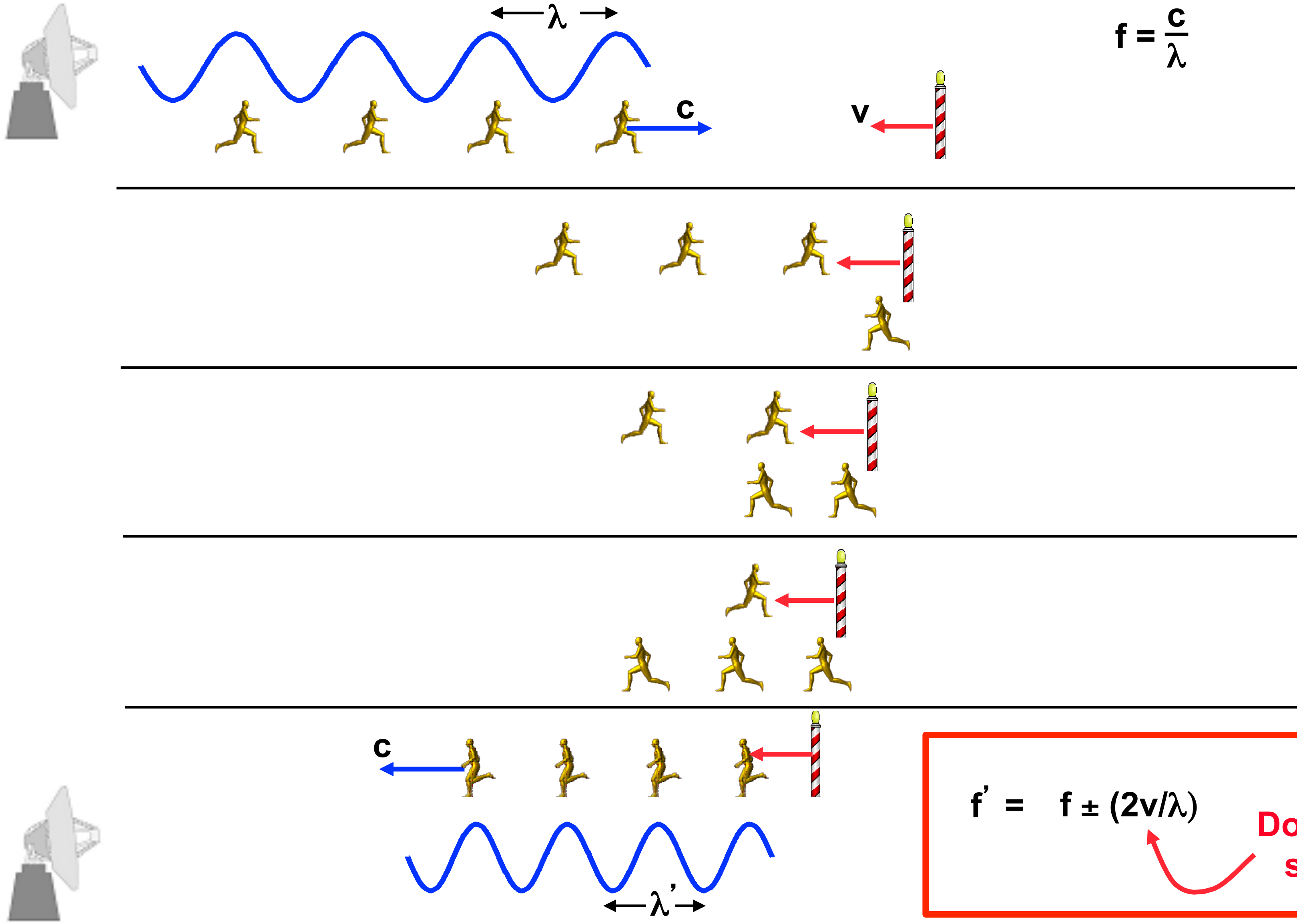
Horizontal Polarization



Doppler Effect



Doppler Shift Concept



$$f = \frac{c}{\lambda}$$

$$f' = f \pm (2v/\lambda)$$

Doppler shift

OUTLINE

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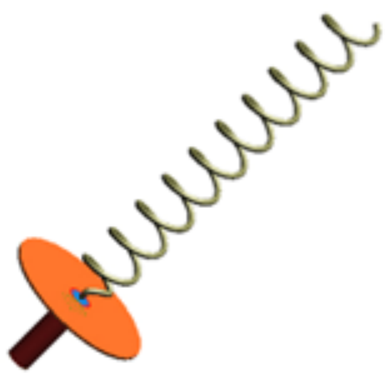
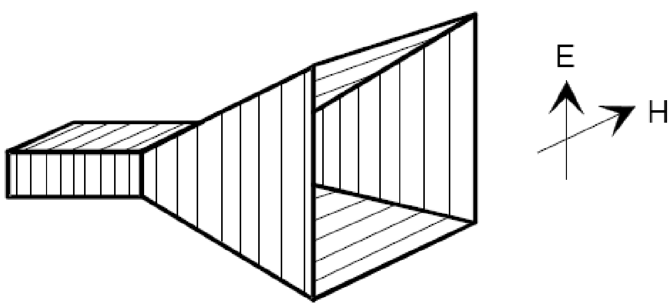
Antennas

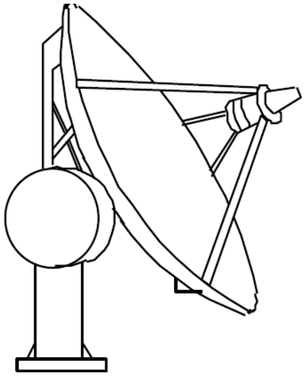
- Four primary functions of an antenna for radar applications
  - Impedance transformation (free-space intrinsic impedance to transmission-line characteristic impedance)
  - Propagation-mode adapter (free-space fields to guided waves)
  - Spatial filter (radiation pattern – direction-dependent sensitivity)
  - Polarization filter (polarization-dependent sensitivity)



Helical antenna



Horn antenna



Parabolic reflector antenna

Impedance transformer

•Intrinsic impedance of free-space, $\eta_o \equiv E/H$ is

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\,\pi \cong 376.7\,\Omega$$

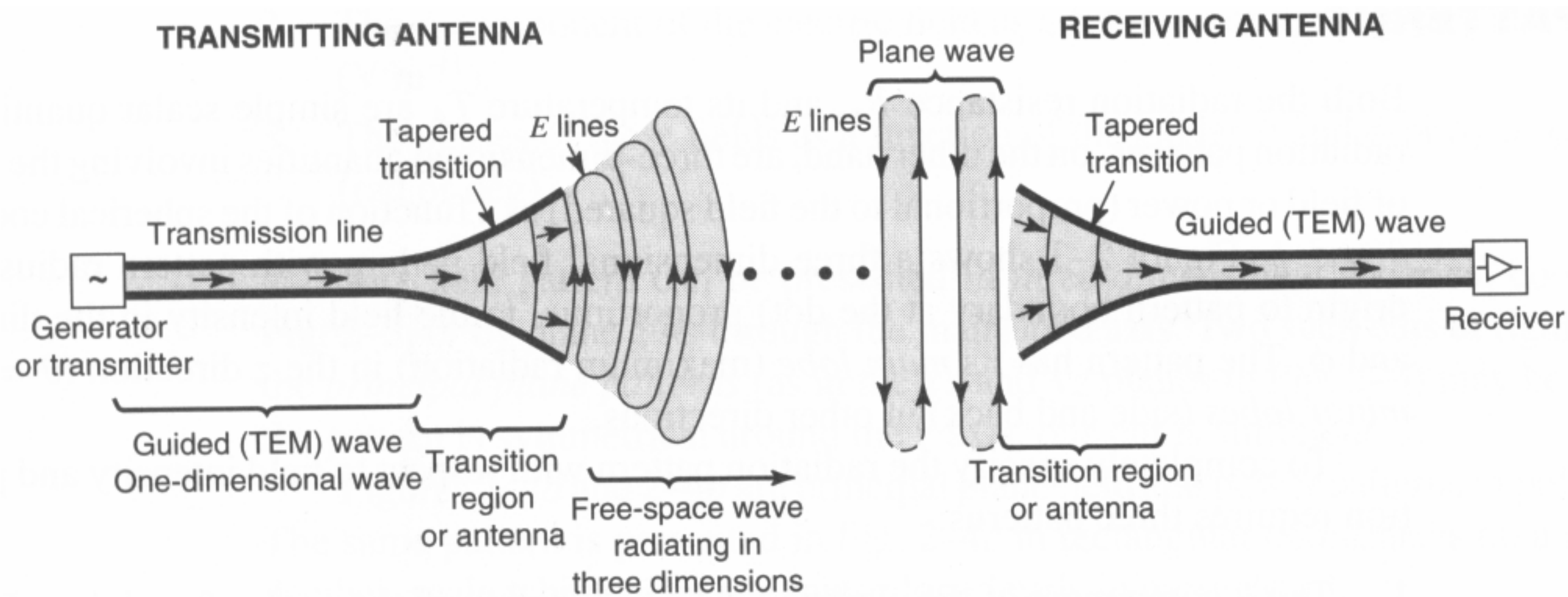
•Characteristic impedance of transmission line, $Z_o = V/I$

•A typical value for Z_o is 50 Ω.

•Clearly there is an impedance mismatch that must be addressed by the antenna.

Propagation-mode adapter

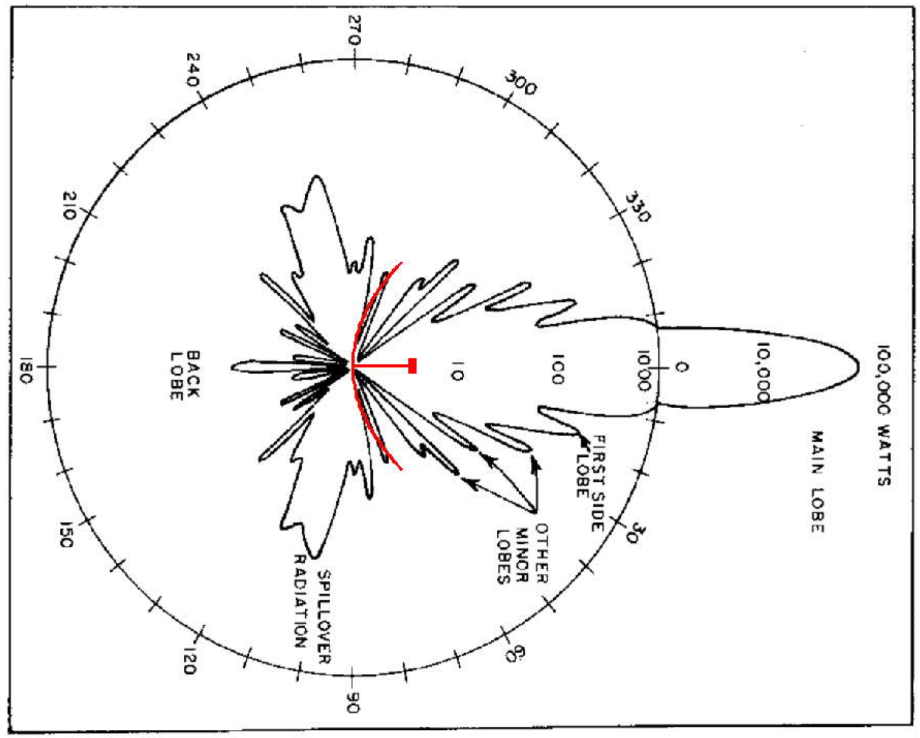
- During both transmission and receive operations the antenna must provide the transition between these two propagation modes.



Spatial filter (also Polarization Filter)

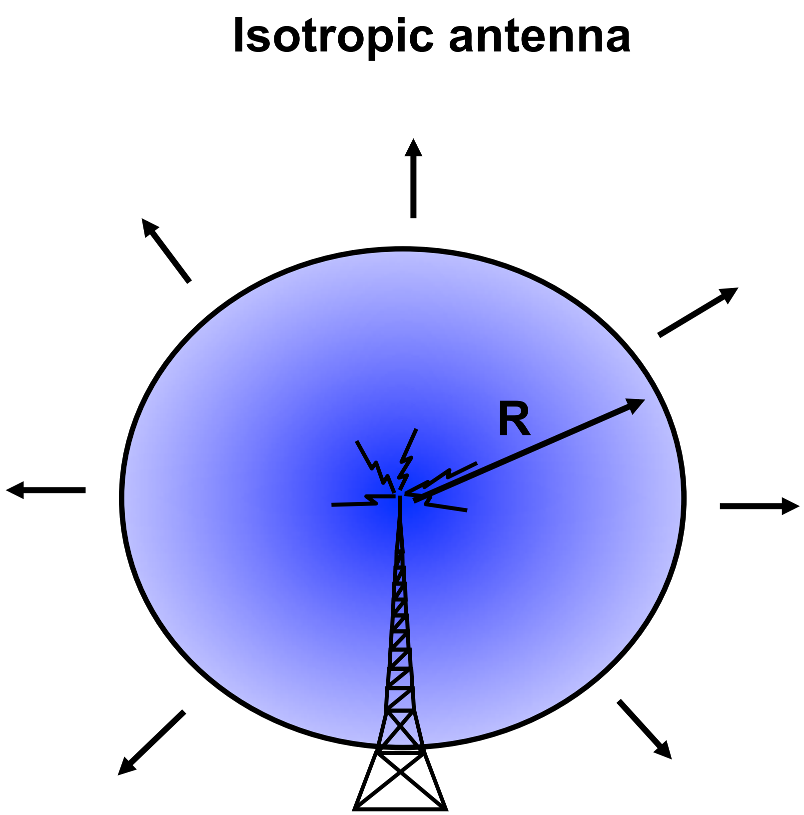
- **Antennas have the property of being more sensitive in one direction than in another which provides the ability to spatially filter signals from its environment.**

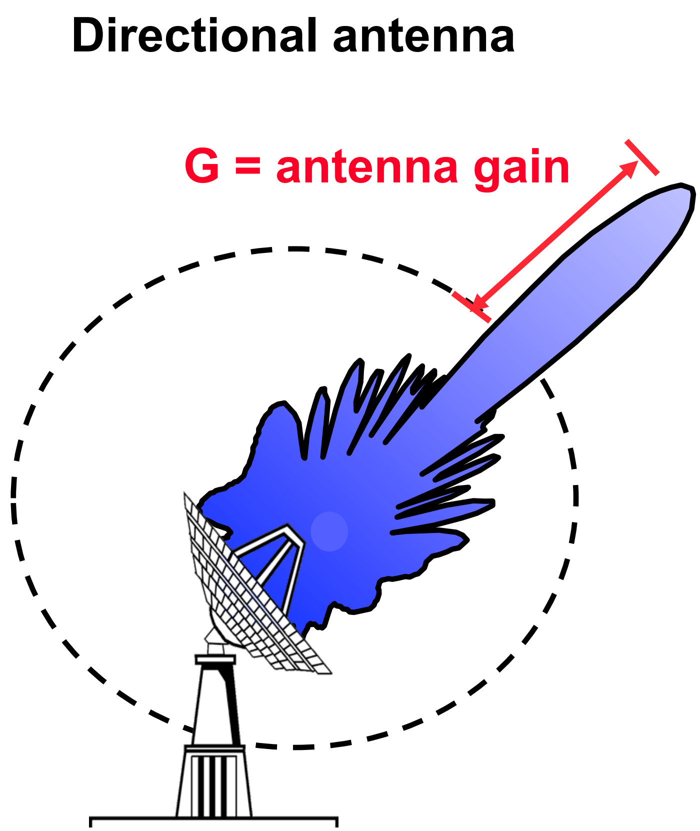
Directive antenna.



Radiation pattern of directive antenna.

Antenna Gain





Write equation for Gain on blackboard

$$D = \frac{S_{out}}{P \big/ 4\pi R^2}$$

$$A_{eff} = \frac{P_{RX}}{S_{inc}}$$

$$G = \frac{4\pi A_{eff}}{\lambda^2}$$

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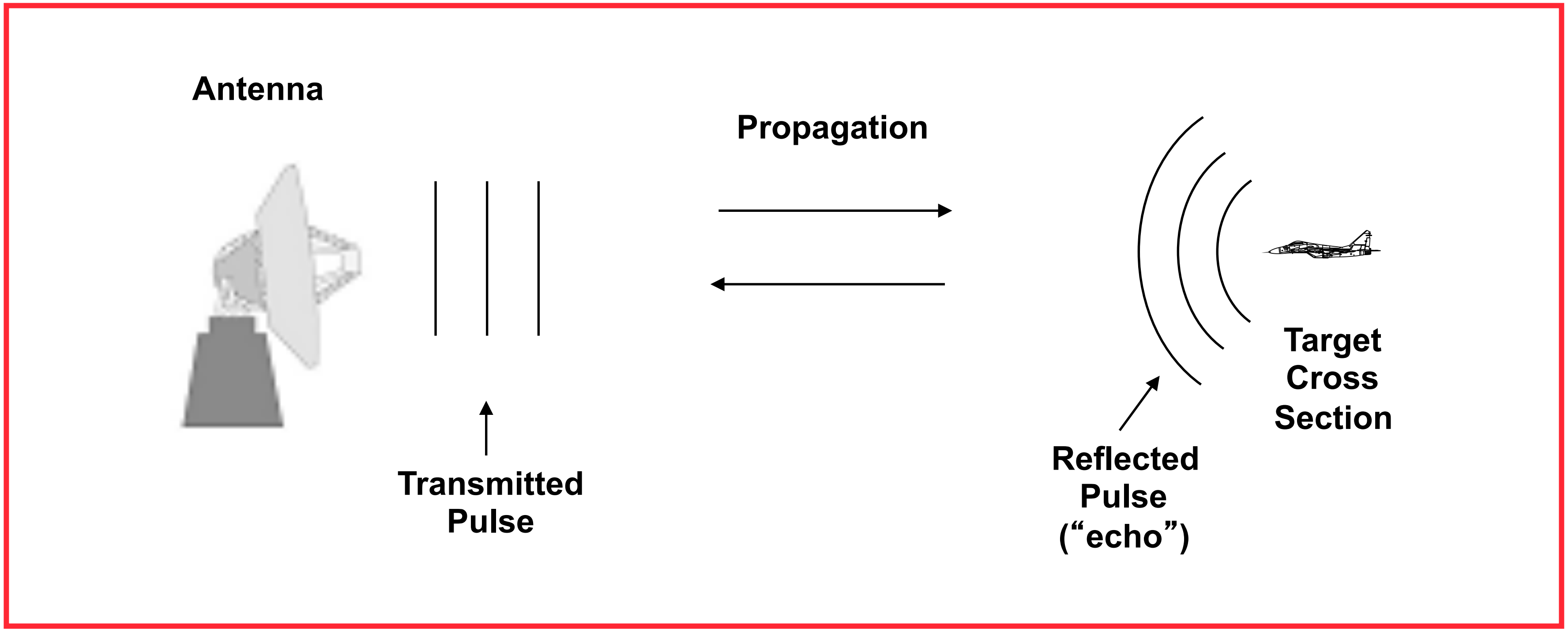

Radar observables:

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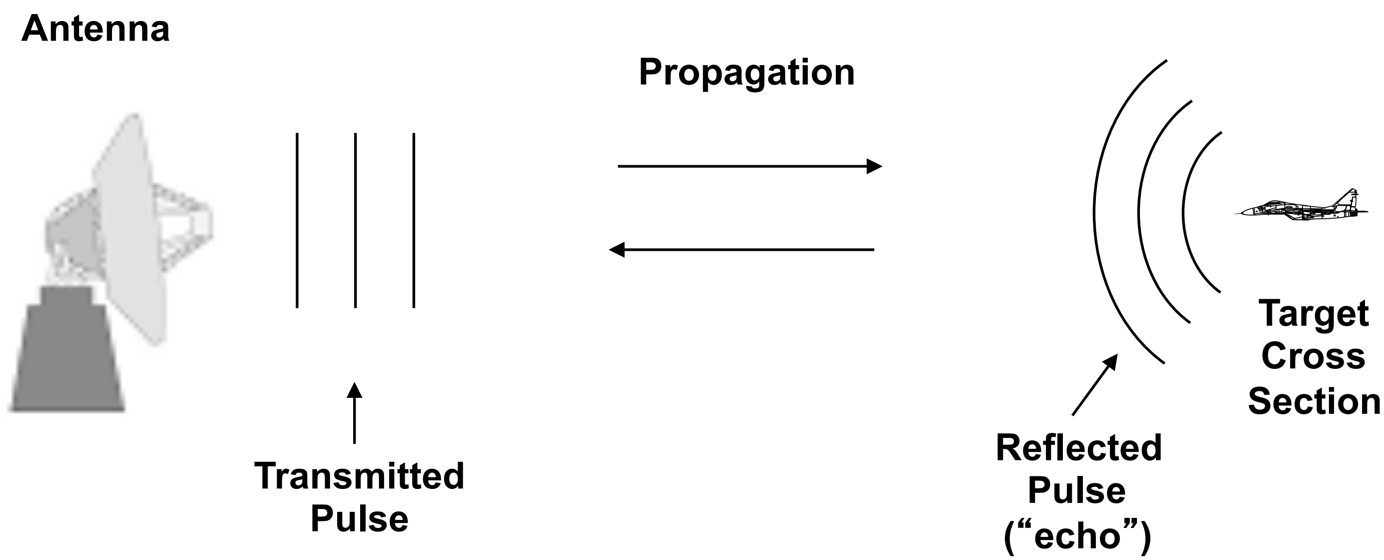
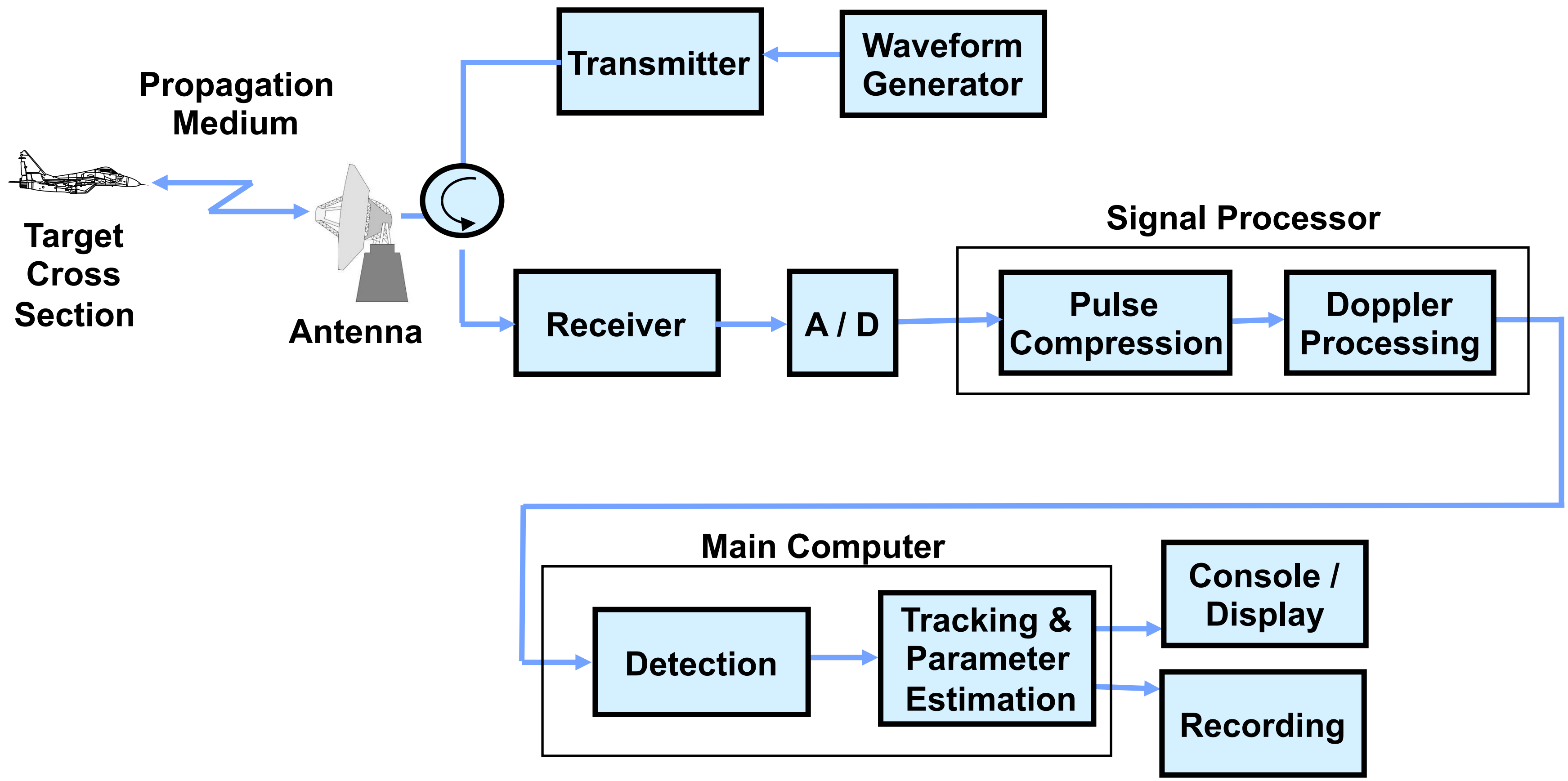
RADAR
RAdio Detection And Ranging

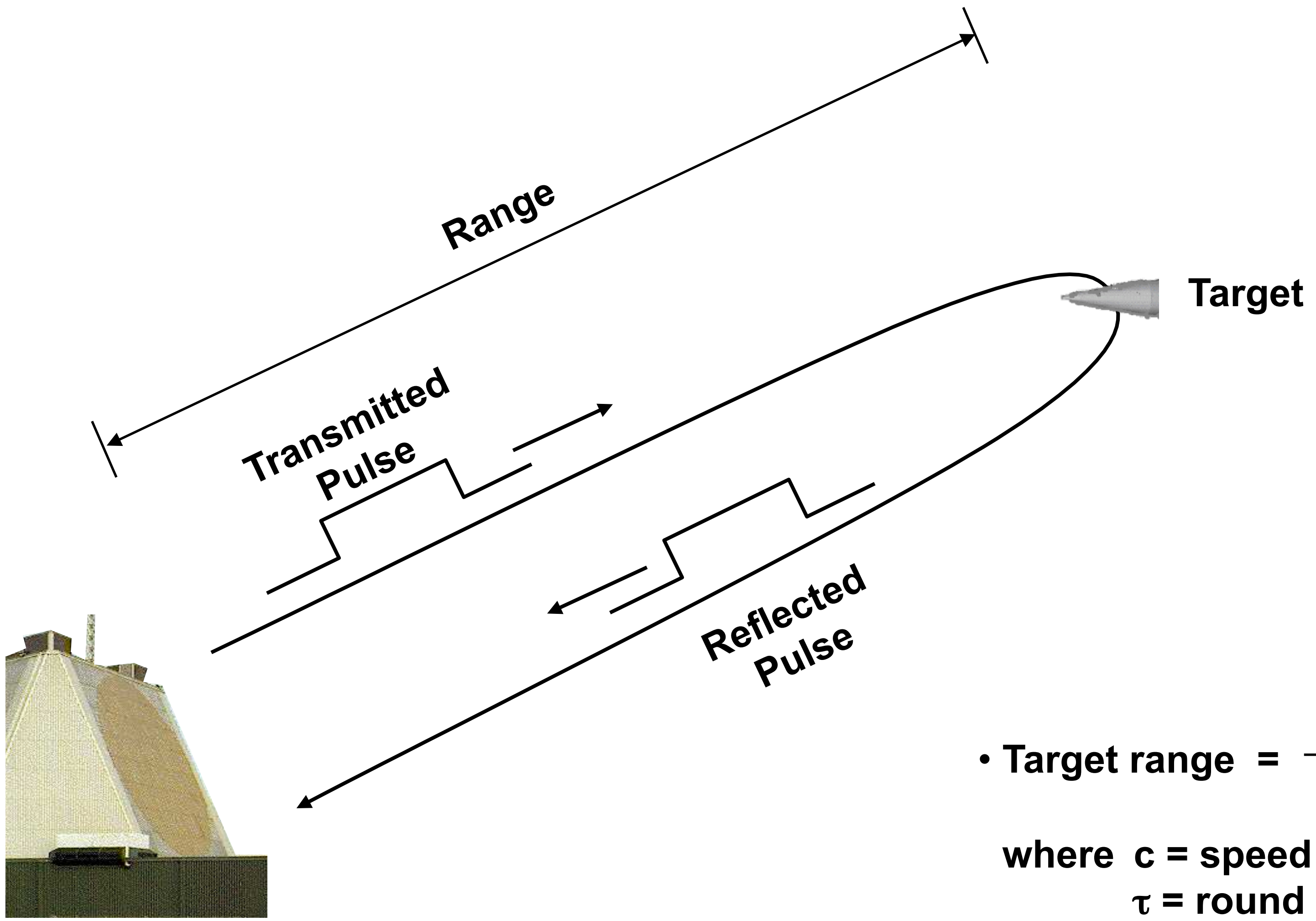


Radar observables:
- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

Radar Block Diagram



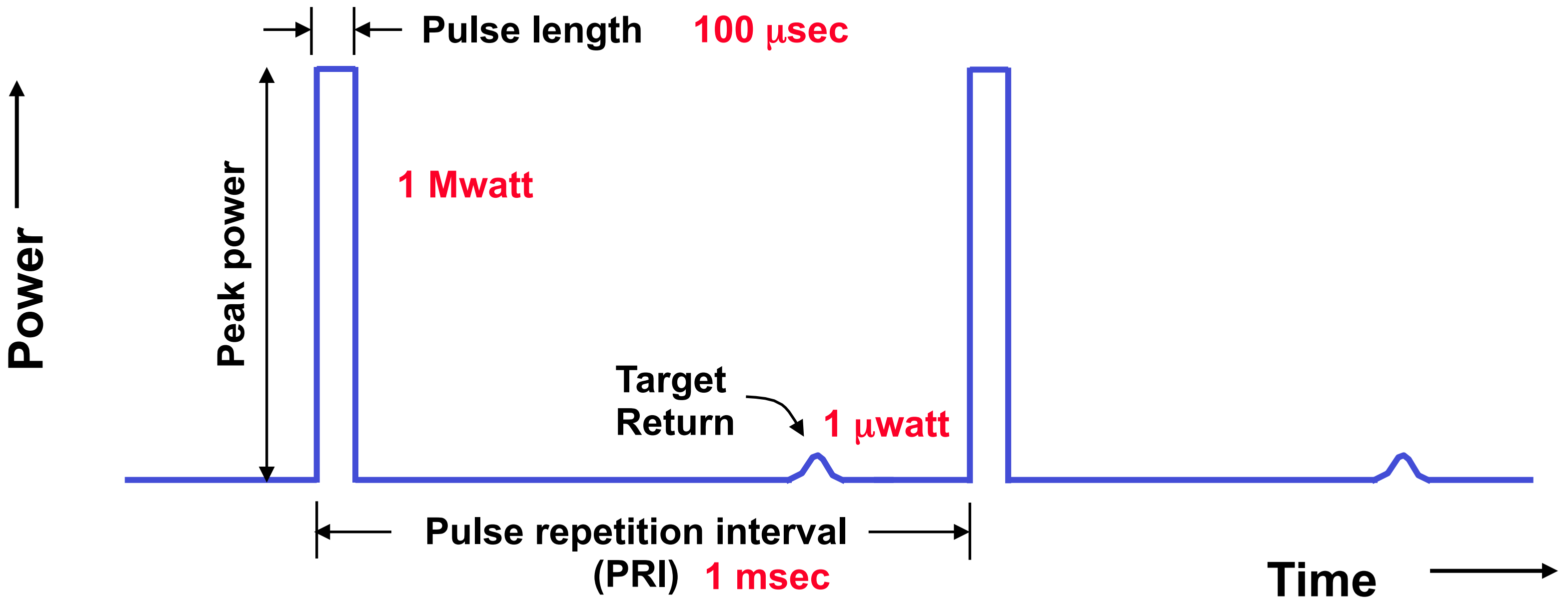
Radar Range Measurement



- Target range $= \dfrac{c\tau}{2}$

where c = speed of light
τ = round trip time

Pulsed Radar
Terminology and Concepts

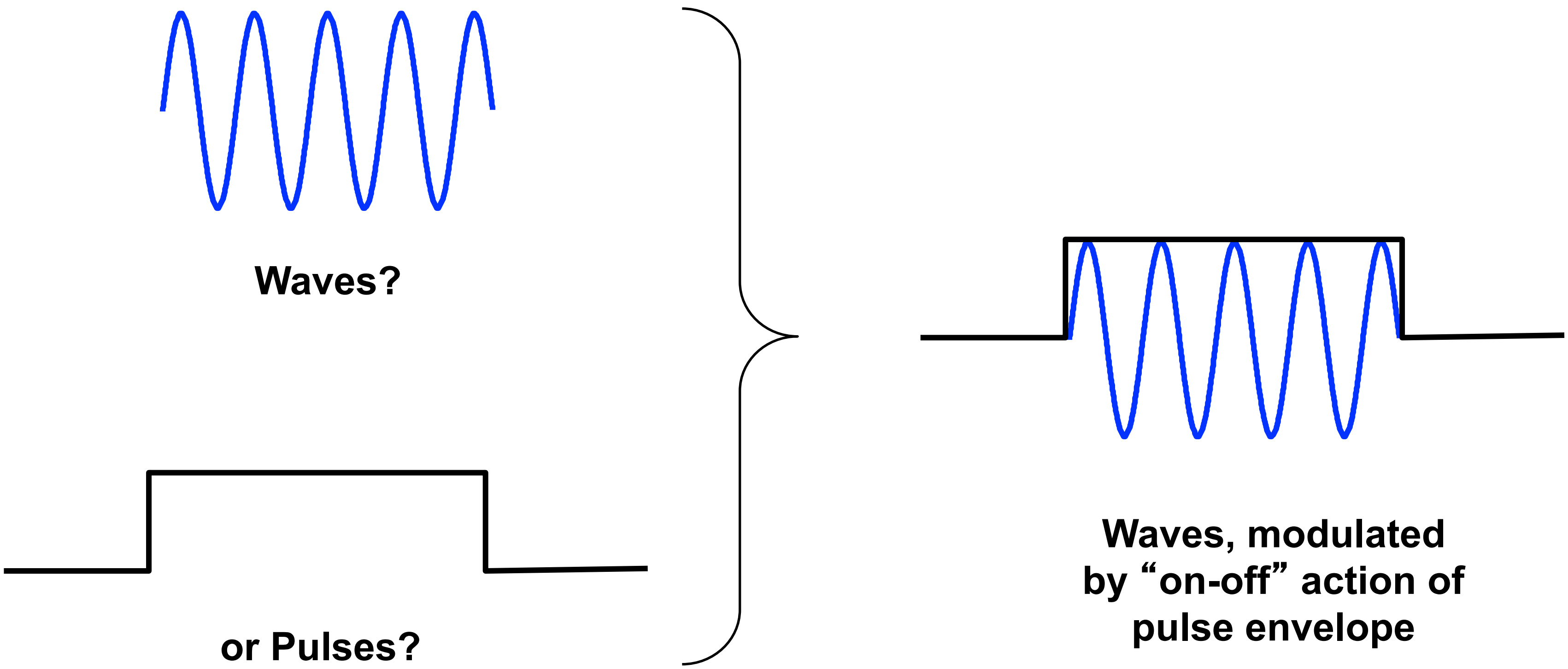


Duty cycle =	$\frac{\text{Pulse length}}{\text{Pulse repetition interval}}$	10%
Average power = Peak power * Duty cycle		100 kWatt
Pulse repetition frequency (PRF) = 1/(PRI)		1 kHz

Continuous wave (CW) radar: Duty cycle = 100% (always on)

Radar Waveforms

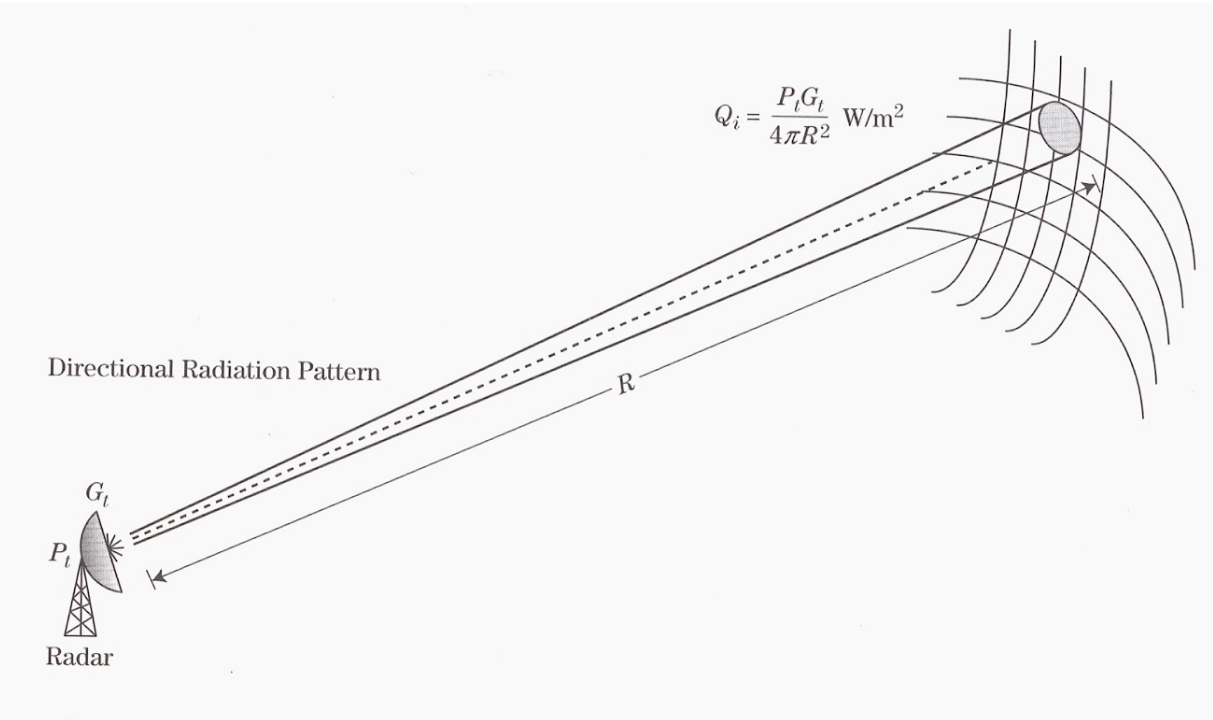
What do radars transmit?



The Radar Equation: Monostatic Version

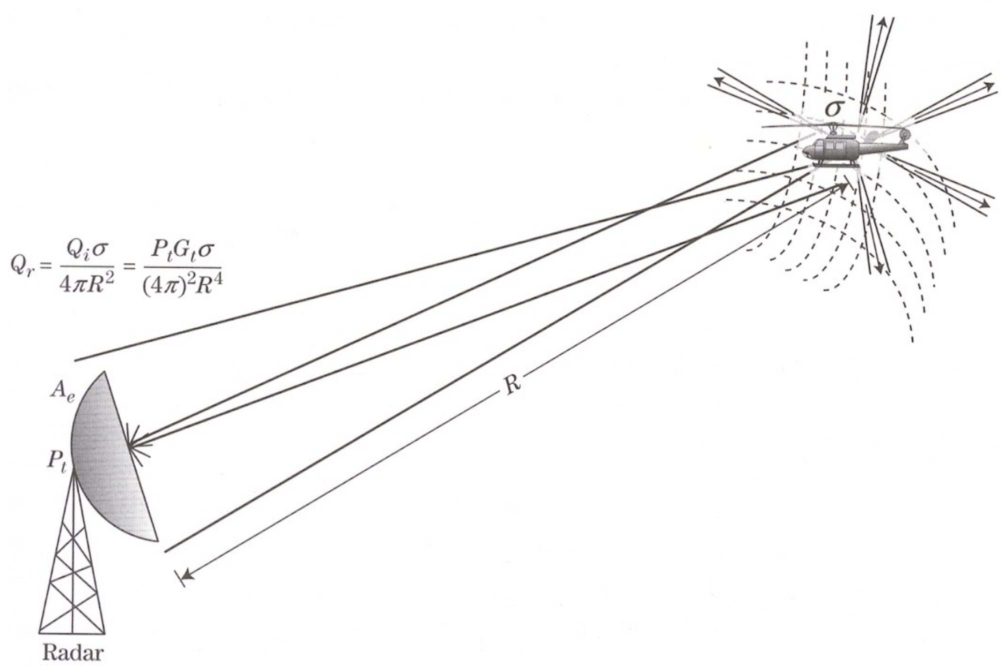
Power density at range R (directional): $$\frac{P_t\ G}{4\pi R^2}$$

1. P_t = Transmit Power
2. G = Gain of Antenna
3. $\frac{1}{4\pi R^2}$ Spread Factor



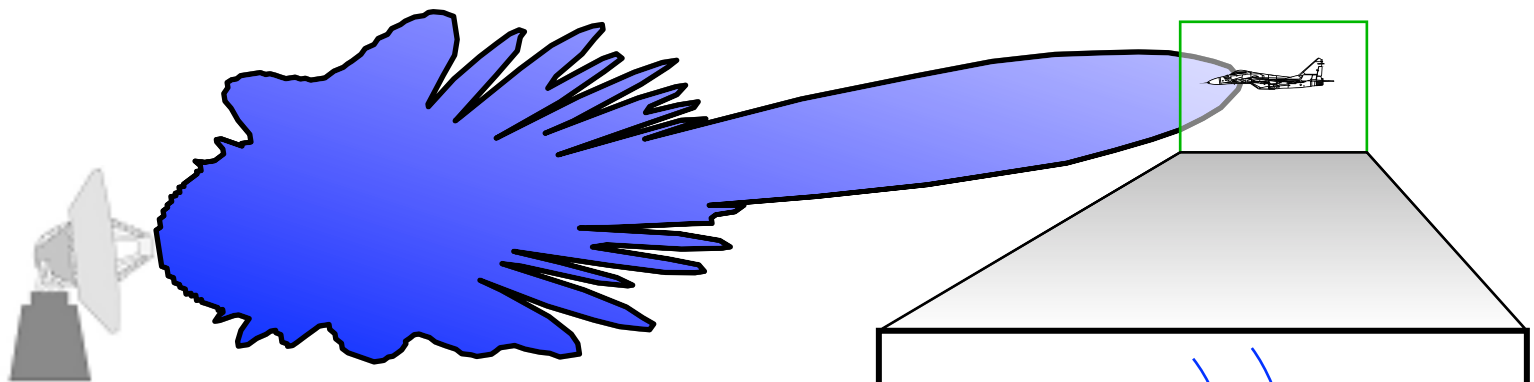
The Radar Equation: Monostatic Version

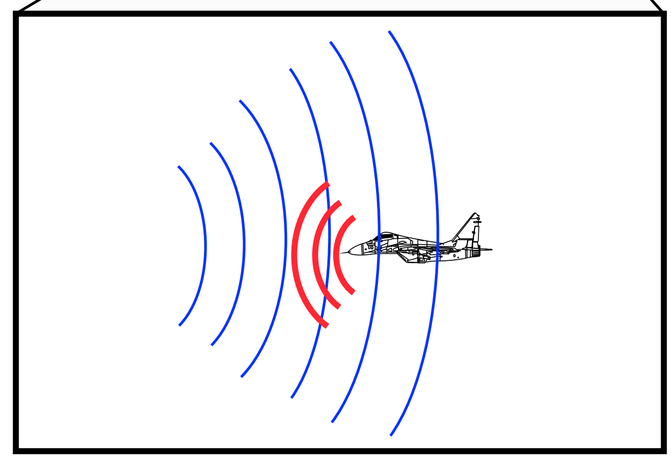
Reradiated power density at Rx: $$\frac{P_t\ G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}$$



1. P_t = Transmit Power
2. G = Gain of Antenna
3. $\frac{1}{4\pi R^2}$ = Spread Factor
4. σ = radar cross section (m^2)
5. $\frac{1}{4\pi R^2}$ = Spread Factor

Radar Cross Section (RCS)





RCS

Incident Power Density (Watts/m^2) x σ (m^2) = Reflected Power (Watts)

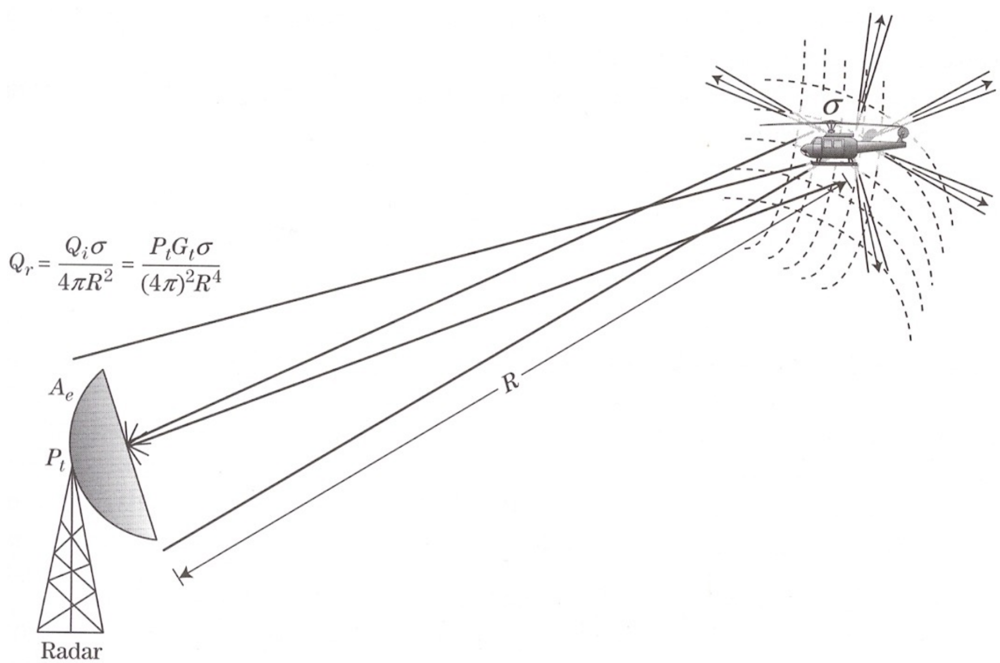
Radar Cross Section (RCS, or s) is the *effective* cross-sectional area of the target as seen by the radar

measured in m^2, or dBm^2

The Radar Equation: Monostatic Version

Total Received Power at Rx:

$$P_r = \frac{P_t\ G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t\ G\ A_e\ \sigma}{(4\pi)^2 R^4}$$



1. P_t = Transmit Power
2. G = Gain of Antenna
3. $\frac{1}{4\pi R^2}$ = Spread Factor
4. σ = radar cross section (m^2)
5. $\frac{1}{4\pi R^2}$ = Spread Factor
6. Ae = effective collecting area

The Radar Equation: Monostatic Version

Total received power: $$P_r = \frac{P_t\ G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t\ G\ A_e\ \sigma}{(4\pi)^2 R^4}$$

Use gain/area relation -

The Radar Equation: $$P_r = P_t \frac{A_e^2}{4\pi\lambda^2 \boxed{R^4}}\ \sigma$$

Hard vs Soft Radar Targets

Generalize radar equation for one or more scatterers, distributed over a volume:

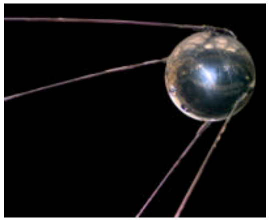
$$P_r = \int P_t \frac{A_e^2}{4\pi\lambda^2 R^4} \sigma(\vec{x})\ dV_s$$

First case: single scatterer ("hard target") at single point in space:

$$\int \sigma(\vec{x})\ dV_s = \sigma_{target} \equiv \sigma$$

Hard target radar equation:

$$P_r = P_t \frac{A_e^2}{4\pi\lambda^2 R^4} \sigma$$

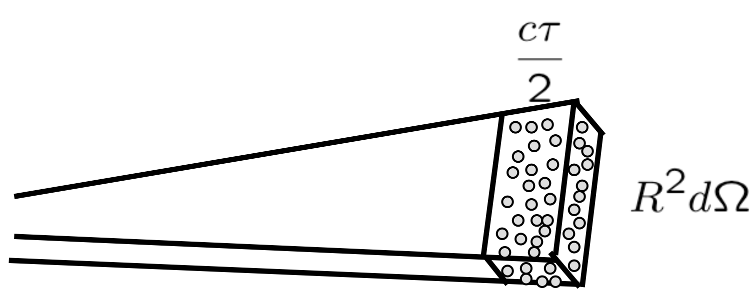


Sputnik 1 (1957-10-04)

Distributed Targets

$$\int \sigma(\vec{x})\ dV_s = \int_0^{2\pi} \int_0^{\pi} \sigma(\vec{x})\ \frac{c\tau}{2}\ R^2 d\Omega$$

$$\int \sigma(\vec{x})\ dV_s = \frac{c\tau}{2} \int_0^{2\pi} \int_0^{\pi} \sigma(\vec{x})\ R^2 \sin\theta\ d\theta\ d\phi$$



Assume volume is filled with identical, isotropic scatters

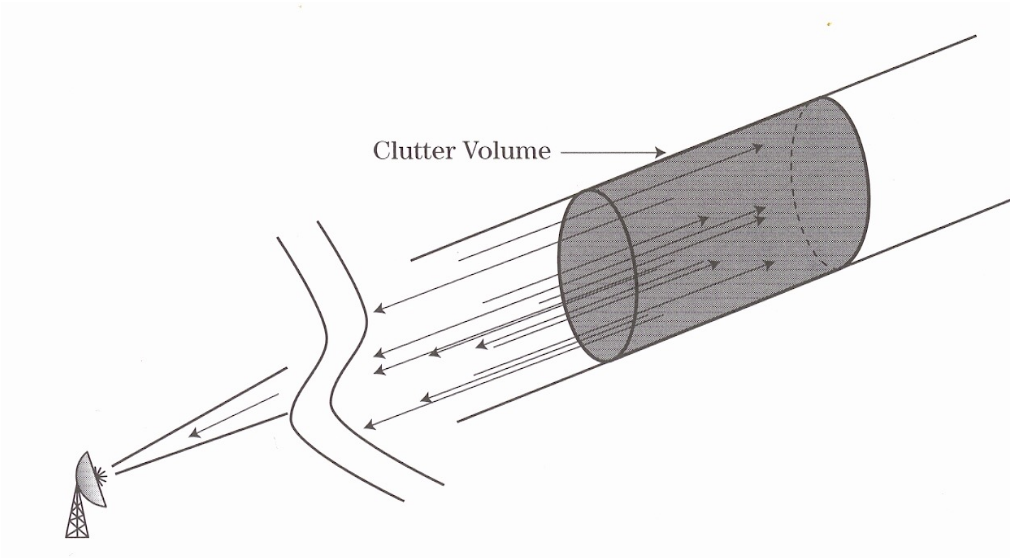
$$\int \sigma(\vec{x})\ dV_s = \frac{c\tau}{2} R^2 \sigma$$

Distributed Scatterers

$$P_r = P_t \, \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^4} \, \sigma \, \frac{c\tau}{2} R^2$$

The "soft target" Radar Equation

$$P_r = P_t \, \frac{c\rho_a^2 A^2 \tau}{8\pi\lambda^2 \boxed{R^2}} \, \sigma$$



OUTLINE

Basic principles of radio waves:
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- Doppler

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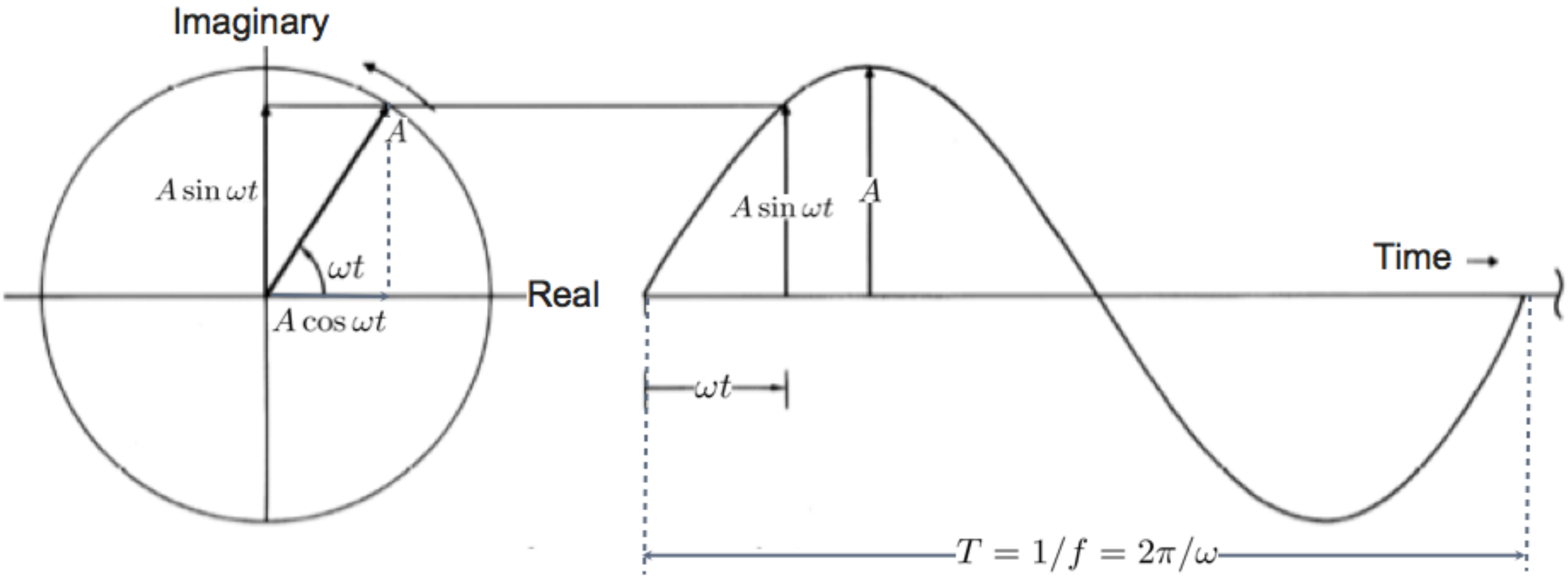
Euler identity and the complex plane



ω is the "angular velocity" (radians/s) of the spinning arrow

f is the number of complete rotations (2π radians) in one second (1/s or Hz)

We need a signal that tells us **how fast** and in **which direction** the arrow is spinning. This signal is the complex exponential. Invoking the Euler identity,

$$s(t) = Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t = I + jQ$$

I = in-phase component

Q = in-quadrature component

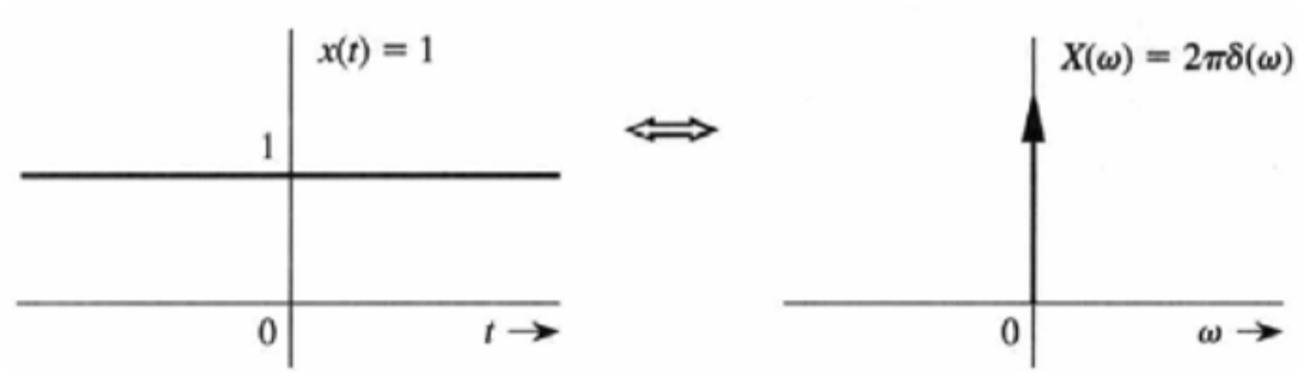
Exponentials are eigenfunctions of linear, time-invariant systems!

Essential mathematical operations

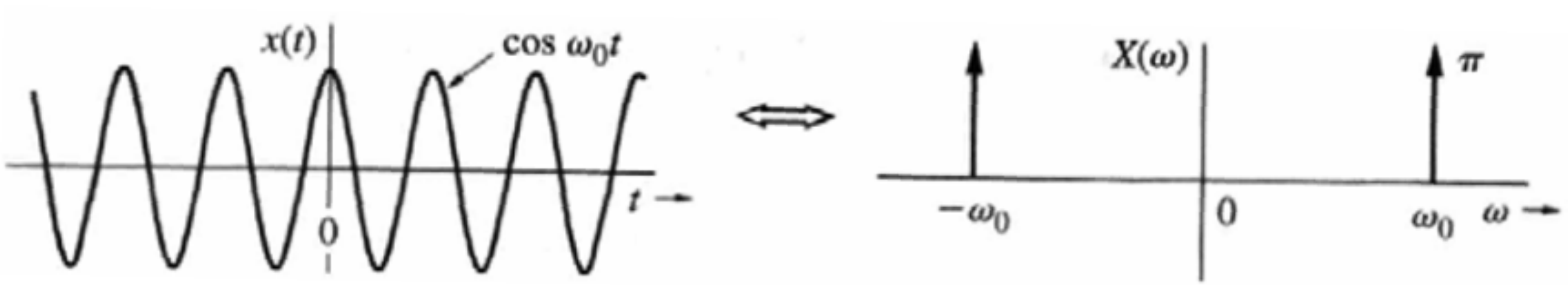
<u>Fourier Transform:</u> Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Harmonic Functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$



$$\sin \omega_0 t \Longleftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$e^{j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega - \omega_0)$$

Another Example

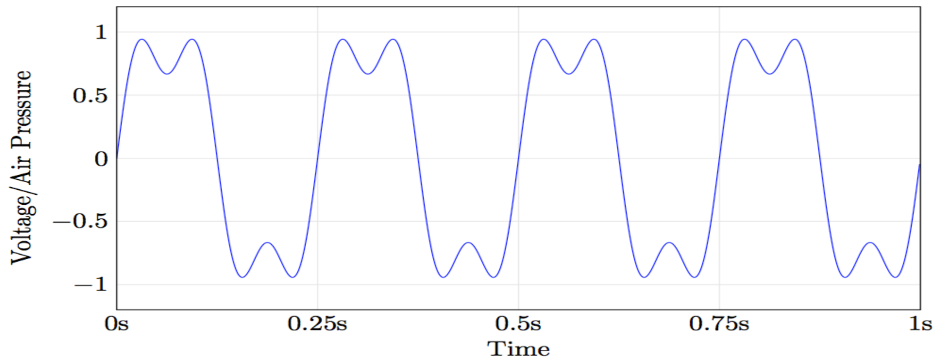


Figure 4: 4Hz + 12Hz Sin Wave.

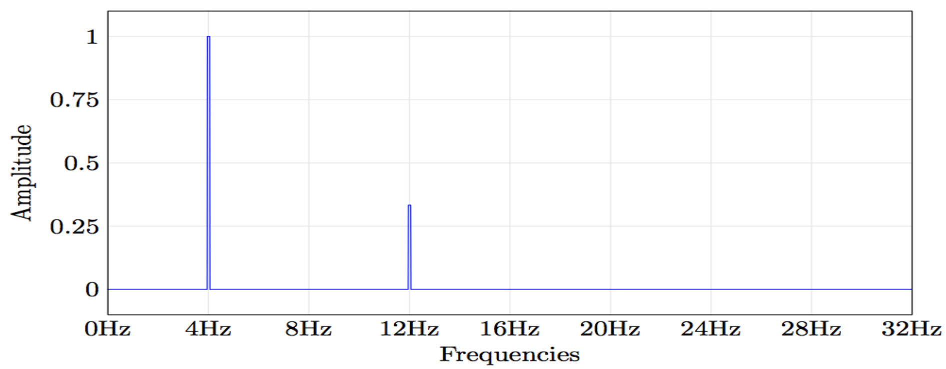


Figure 5: Frequency Domain of 4Hz + 12Hz Sin Waves.

Fourier Transform Properties

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Convolution	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
Differentiation	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$

Determining the Doppler Spectrum

1. Doppler spectrum is measured at a particular range gate (e.g. at $r = \dfrac{c\Delta t}{2}$)

2. Must process a <u>time series of discrete samples</u> of echo $E_r(t)$ at intervals of the pulse period T_r

3. Analyze the sampled signal using (fast) Fourier Transform methods:

$$E(mT_r) = \frac{1}{M}\sum_{m=0}^{M-1} F(kf_0)\cos\left[2\pi k f_0 m T_r\right]$$

$$F(kf_0) = \sum_{m=0}^{M-1} E_r(mT_r)\cos\left[2\pi k f_0 m T_r\right]$$

M = # of samples
f_0 = frequency resolution

4. Frequency components (radial velocities) occur at discrete intervals, with M intervals separated by intervals of $1/MT_r = f_D$

Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(t-\tau) d\tau \qquad f(t) * g(t) \Longleftrightarrow F(f)G(f)$$

Correlation: A measure of the degree to which two functions look alike at a given offset.

$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau) g(t+\tau) d\tau \qquad f(t) \circ g(t) \Longleftrightarrow F^*(f)G(f)$$

End of Day One - RADAR