Introduction to ISR Signal Processing

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Why study ISR?

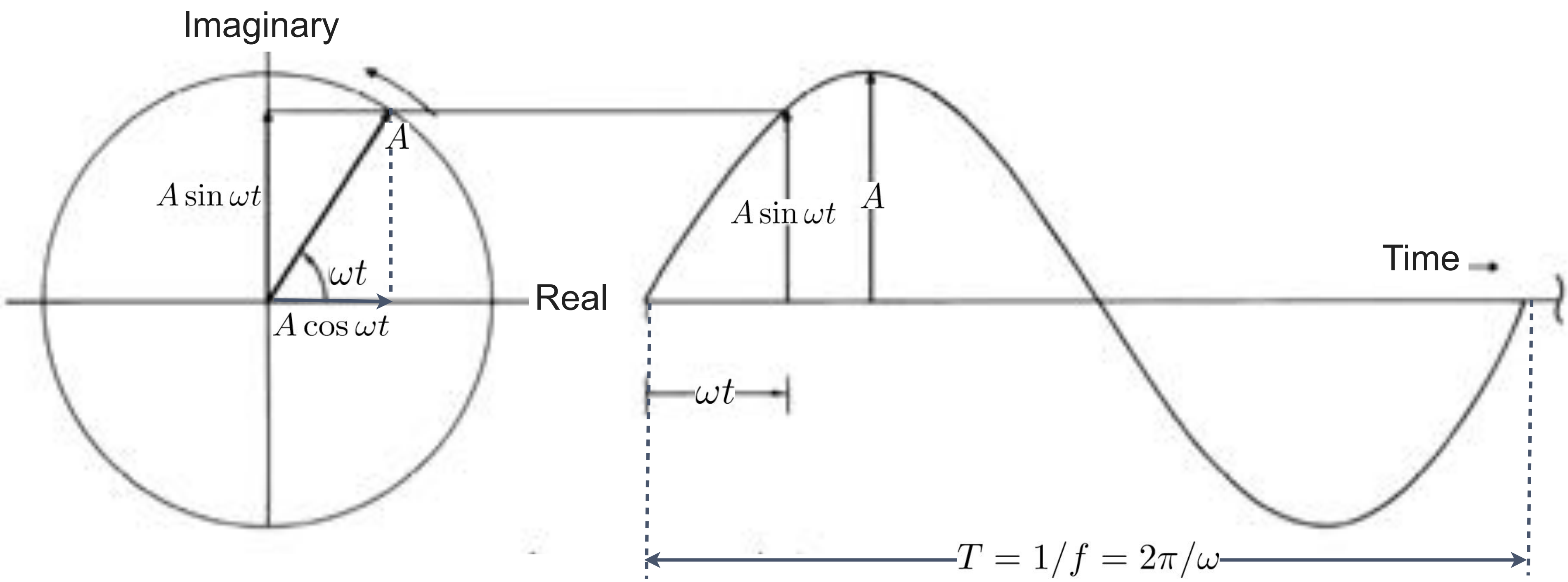
Requires that you learn about a great many useful and fascinating subjects in substantial depth, including:

- Plasma physics
- Radar
- Coding (information theory)
- Electronics (Power, RF, DSP)
- Signal Processing
- Inverse theory

Outline

- Mathematical toolbox
- Review of basic radar concepts
- Ionospheric Doppler spectrum
- Range resolution and matched filtering
- I/Q demodulation
- Autocorrelation function (ACF) and Power Spectral Density (PSD)

Euler identity and the complex plane



ω is the "angular velocity" (radians/s) of the spinning arrow

f is the number of complete rotations (2π radians) in one second (1/s or Hz)

We need a signal that tells us **how fast** and in **which direction** the arrow is spinning. This signal is the complex exponential. Invoking the Euler identity,

$$s(t) = Ae^{j\omega t} = A\cos\omega t + jA\sin\omega t = I + jQ$$

I = in-phase component

Q = in-quadrature component

Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of harmonic functions (i.e., complex exponentials)

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega \iff F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau) g(\tau - t) d\tau \qquad f(t) * g(t) \iff F(f)G(f)$$

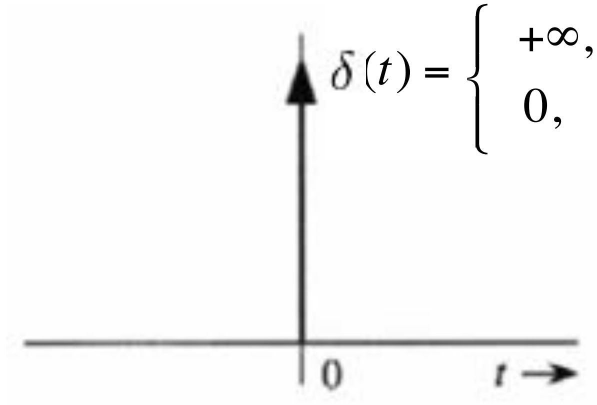
Correlation: A measure of the degree to which two functions look alike at a given offset.

$$f(t) \circ g(t) = \int_{-\infty}^{+\infty} f^*(\tau) g(t + \tau) d\tau \qquad f(t) \circ g(t) \iff F^*(f)G(f)$$

Autocorrelation, Convolution, Power Spectral Density, Wiener-Khinchin Theorem

$$R_{uu} = u(t) \circ u(t) = u(t) * u^*(-t) \qquad R_{uu} \iff |U(f)|^2$$

Dirac Delta Function



$$\delta(t) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$\delta(t)$ is defined by the property that for all continuous functions

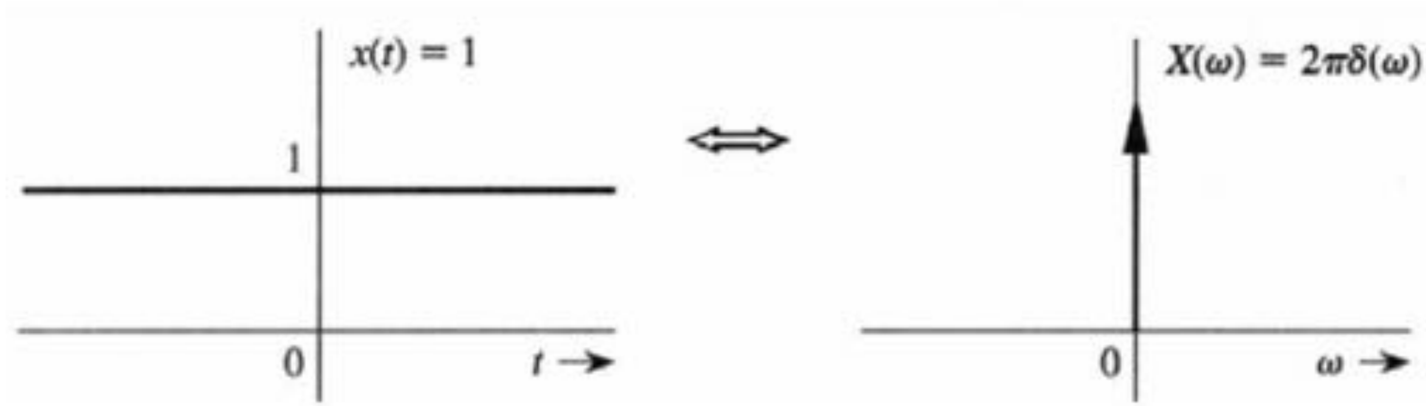
$$f(0) = \int_{-\infty}^{+\infty} \delta(t) f(t) dt$$

$$f(t - T) = f(t) * \delta(t - T)$$

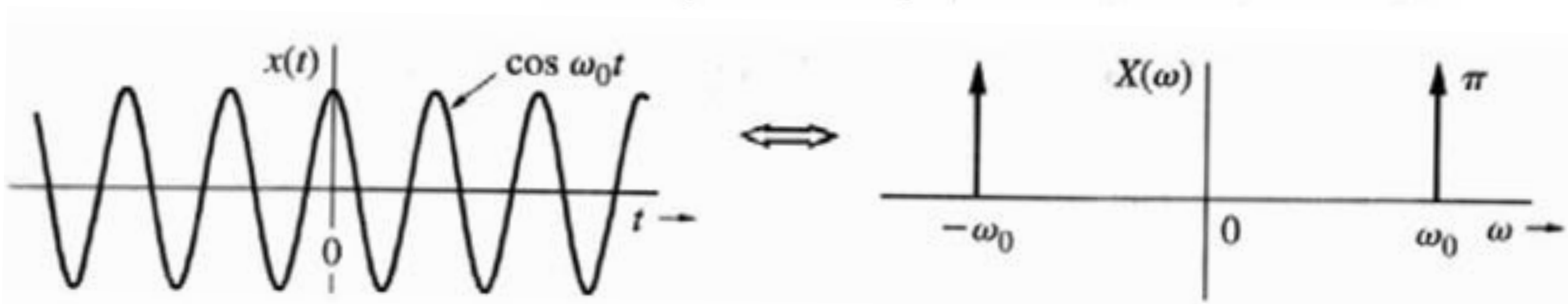
The Fourier Transform of a train of delta functions is a train of delta functions.

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

Harmonic Functions



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

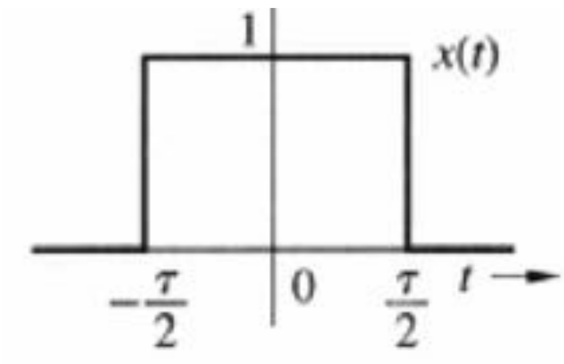


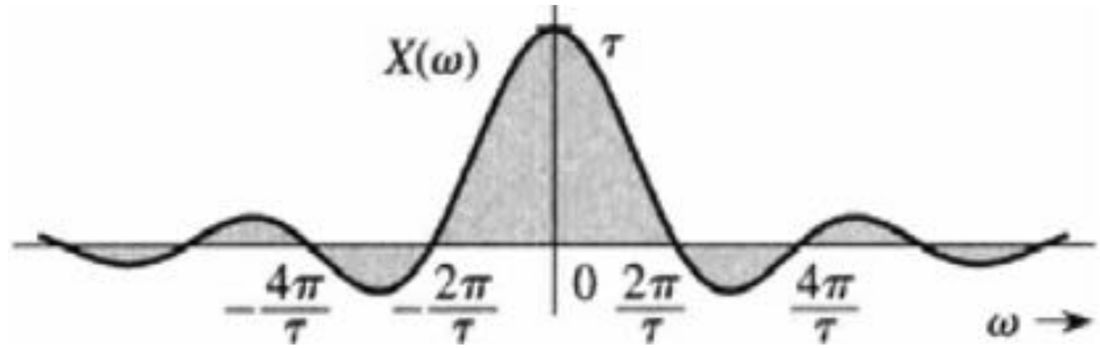
$$\sin \omega_0 t \quad \Longleftrightarrow \quad j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$e^{j\omega_0 t} \quad \Longleftrightarrow \quad 2\pi\delta(\omega - \omega_0)$$

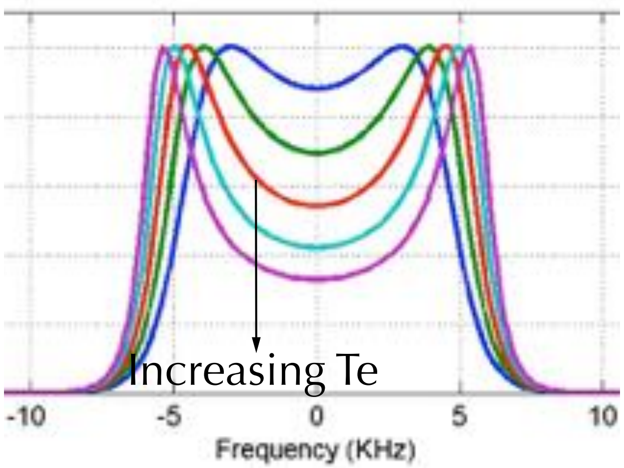
Gate function

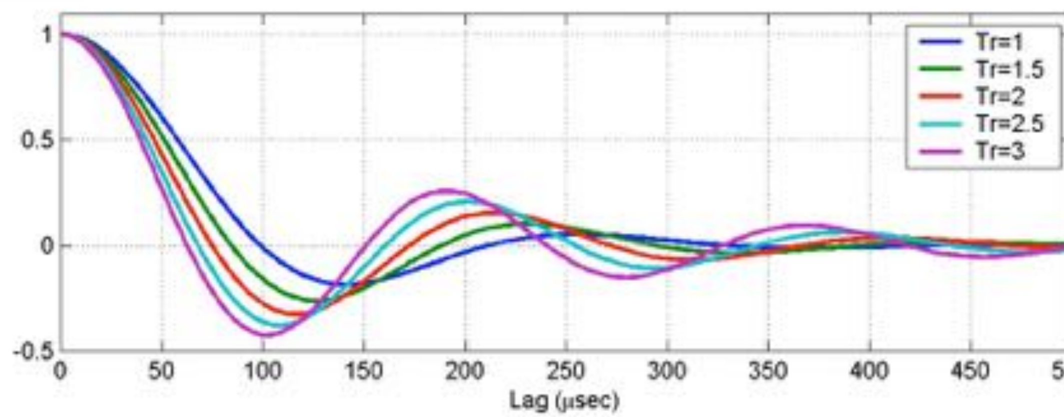
$$\text{rect}(t/\tau) = \begin{cases} 1 & \text{for} - \tau/2 < t < \tau/2 \\ 0 & \text{otherwise} \end{cases} \iff \tau \text{ sinc}\left(\frac{\omega\tau}{2}\right)$$





ISR spectrum $\iff$ Autocorrelation function (ACF)



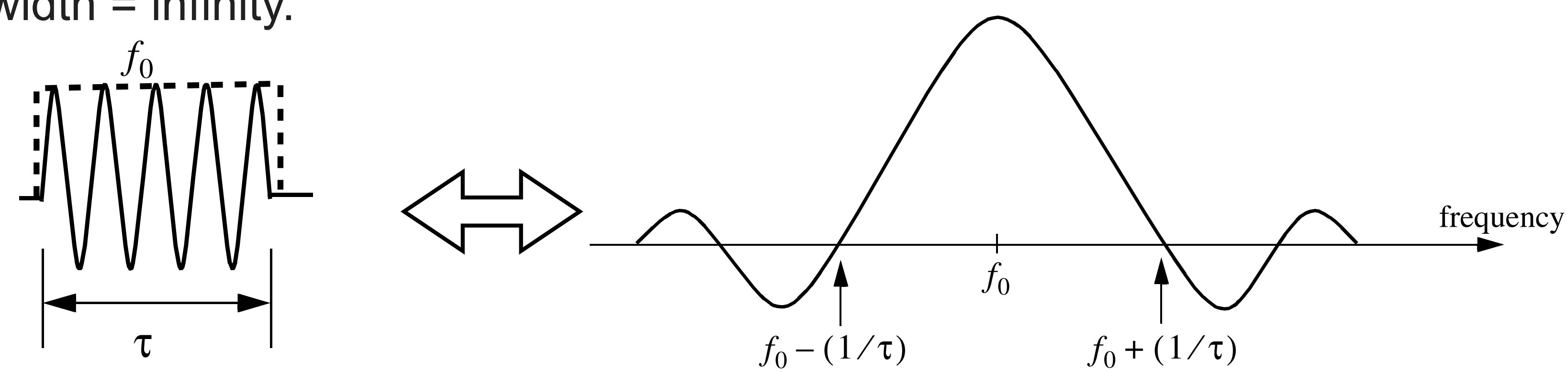


For low Te, the ISR ACF looks like a sinc function. For high Te the ACF becomes more oscillatory and looks more like a cosine (power concentrated at the Doppler frequency corresponding to the ion-acoustic wave speed.

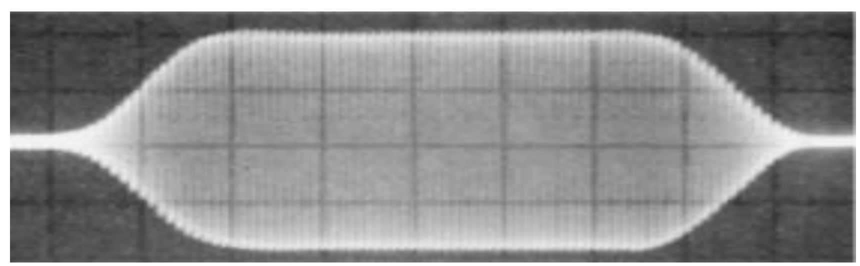
How it all hangs together.

- Duality:
  - Gate function in the time domain represents amplitude modulation
  - Gate function in the frequency domain represents filtering

- Limiting cases:
  - Gate function approaches delta function as width goes to 0 with constant area
  - A constant function in time domain is a special case of harmonic function where frequency = 0.
  - A constant function in time domain is a special case of a gate function where width = infinity.



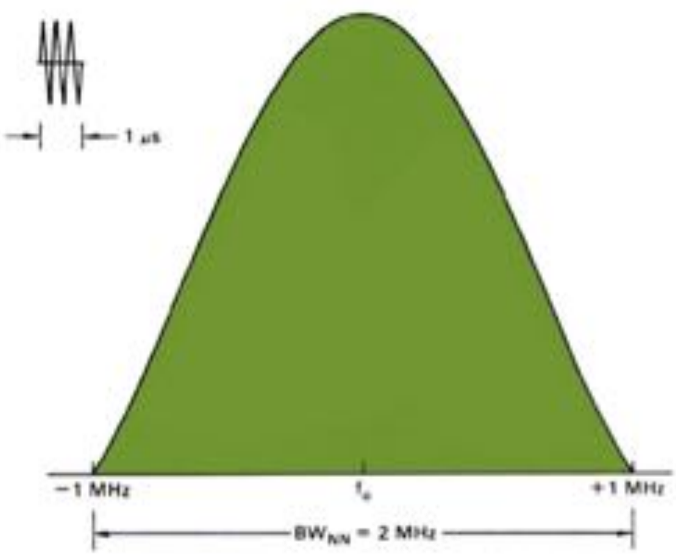
How many cycles are in a typical ISR pulse?
PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs ⇨ 215,520 cycles!



Bandwidth of a pulsed signal

Spectrum of receiver output has sinc shape, with sidelobes half the width of the central lobe and continuously diminishing in amplitude above and below main lobe

A 1 microsecond pulse has a 3 dB bandwidth of 1 MHz



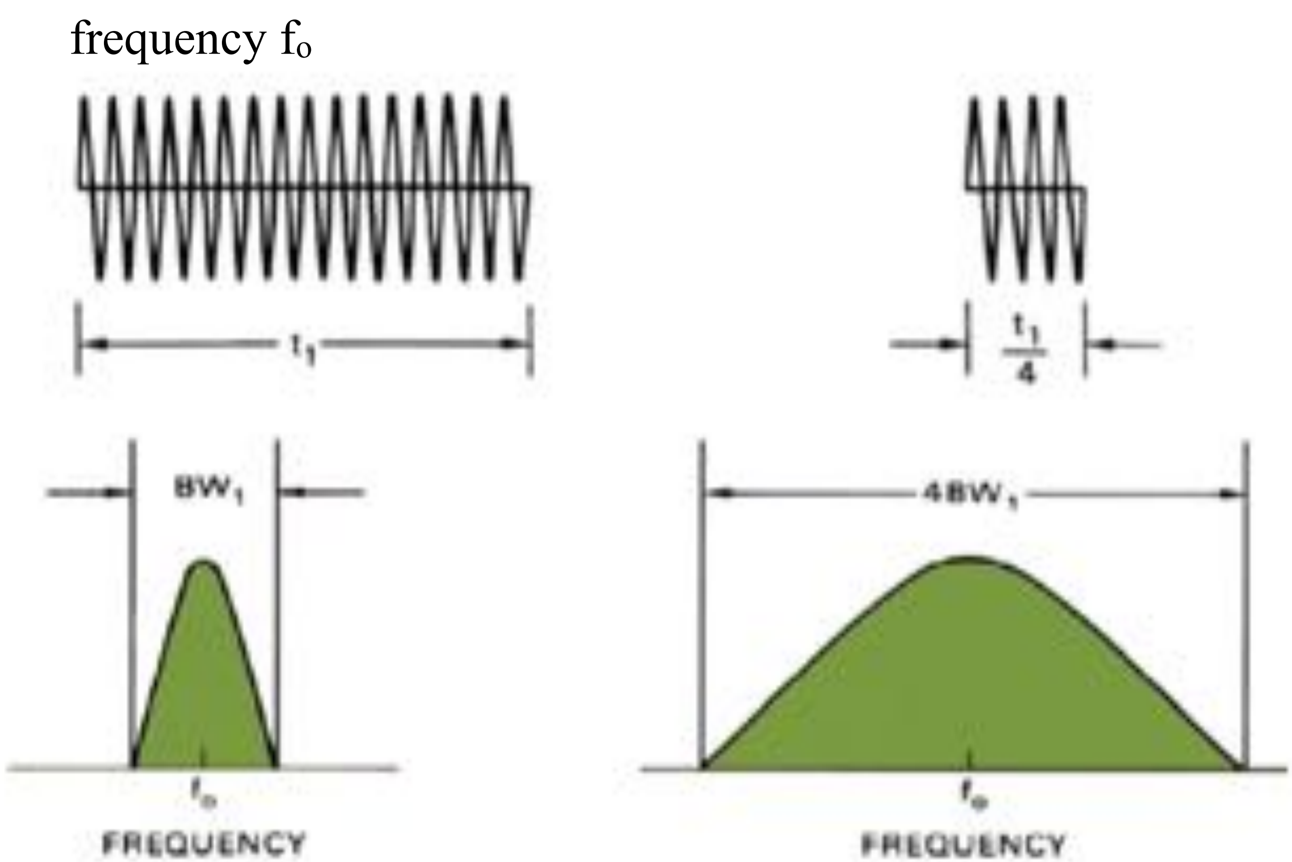
Two possible bandwidth measures:

"null to null" bandwidth $B_{\mathrm{nn}} = \dfrac{2}{\tau}$

"3dB" bandwidth $B_{3\mathrm{dB}} = \dfrac{1}{\tau}$

Unless otherwise specified, assume bandwidth refers to 3 dB bandwidth

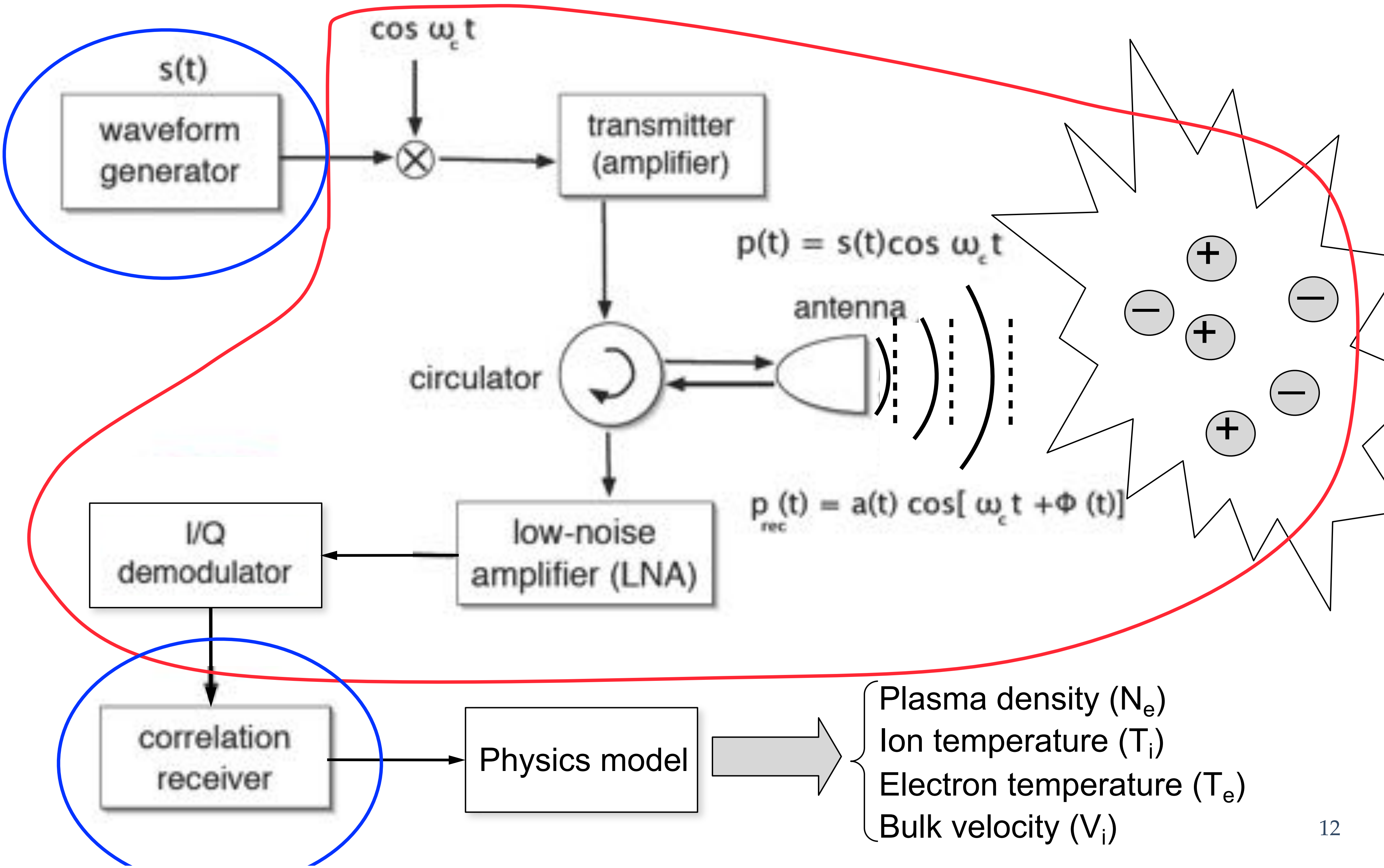
Pulse-Bandwidth Connection



Shorter pulse ⟺ Larger bandwidth

Faster sampling rate ⟺ Larger bandwidth

Components of a Pulsed Doppler Radar



- Plasma density (N_e)
- Ion temperature (T_i)
- Electron temperature (T_e)
- Bulk velocity (V_i)

The deciBel (dB)

The relative value of two quantities expressed on a logarithmic scale

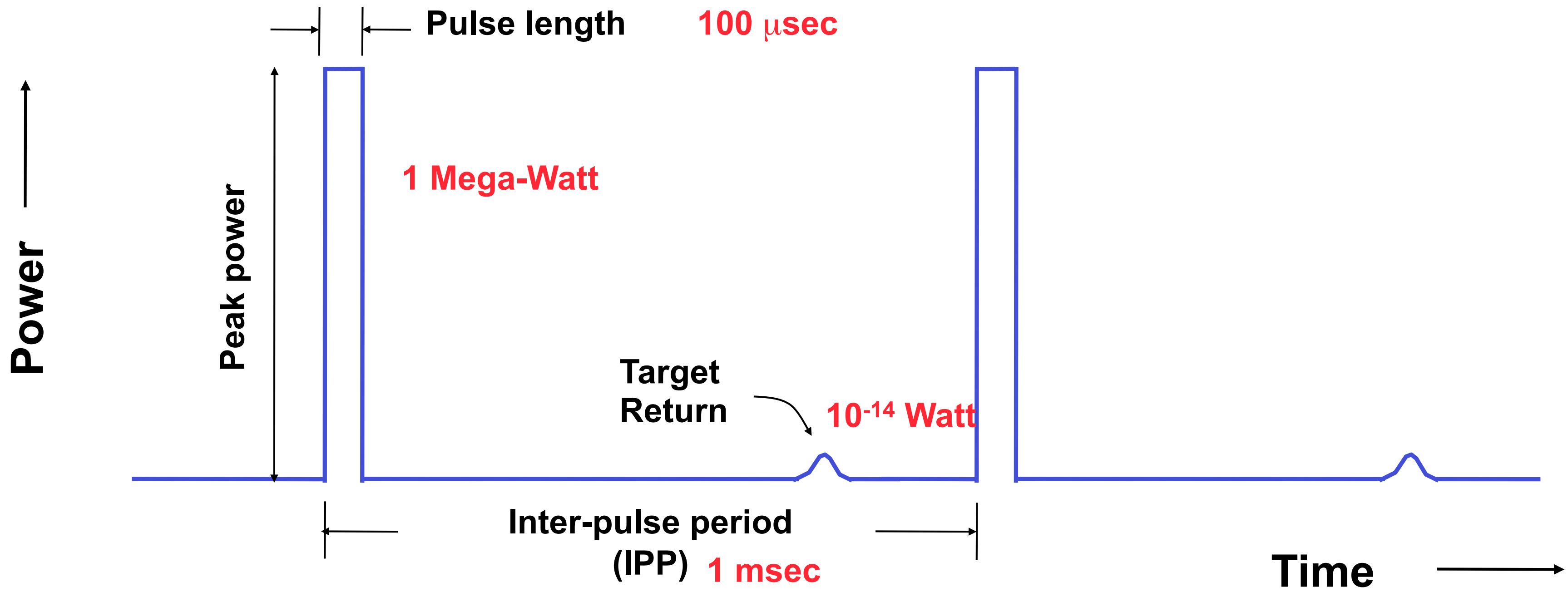
$$SNR = 10 \log_{10} \frac{P_1}{P_2} = 20 \log_{10} \frac{V_1}{V_2} \qquad (\text{Power} \propto \text{Voltage}^2)$$

Factor of:	Scientific Notation	dB
0.1	10^{-1}	-10
0.5	$10^{0.3}$	-3
1	10^{0}	0
2	$10^{0.3}$	3
10	10^{1}	10
100	10^{2}	20
1000	10^{3}	30
1,000,000	10^{6}	60

Other forms used in radar:

dBW	dB relative to 1 Watt
dBm	dB relative to 1 mW
dBsm	dB relative to 1 m^2 of radar cross section
dBi	dB relative to isotropic radiation

Pulsed Radar



Duty cycle = $\frac{\text{Pulse length}}{\text{Pulse repetition interval}}$ 10%

Average power = Peak power * Duty cycle 100 kWatt

Pulse repetition frequency (PRF) = 1/(IPP) 1 kHz

Continuous wave (CW) radar: Duty cycle = 100% (always on)

Doppler Frequency Shift

Transmitted signal: $\cos(2\pi f_o t)$

After return from target: $$\cos\left[2\pi f_o\left(t+\frac{2R}{c}\right)\right]$$

To measure frequency, we need to observe signal for at least one cycle.
So we will need a model of how *R* changes with time. Assume constant velocity:

$$R = R_o + vt$$

Substituting:

$$\cos\left[2\pi\left(f_o+\underbrace{f_o\frac{2v}{c}}_{-f_D}\right)t+\underbrace{\frac{2_o R_o}{c}}_{\text{constant}}\right]$$

$$f_D = -2f_o\left(\frac{v}{c}\right) = -2\left(\frac{v}{\lambda_o}\right) \propto \frac{\text{line-of-sight velocity}}{\text{radar wavelength}}$$

By convention, positive Doppler shift $\Longleftrightarrow$ Target and radar are "closing"

Two key concepts

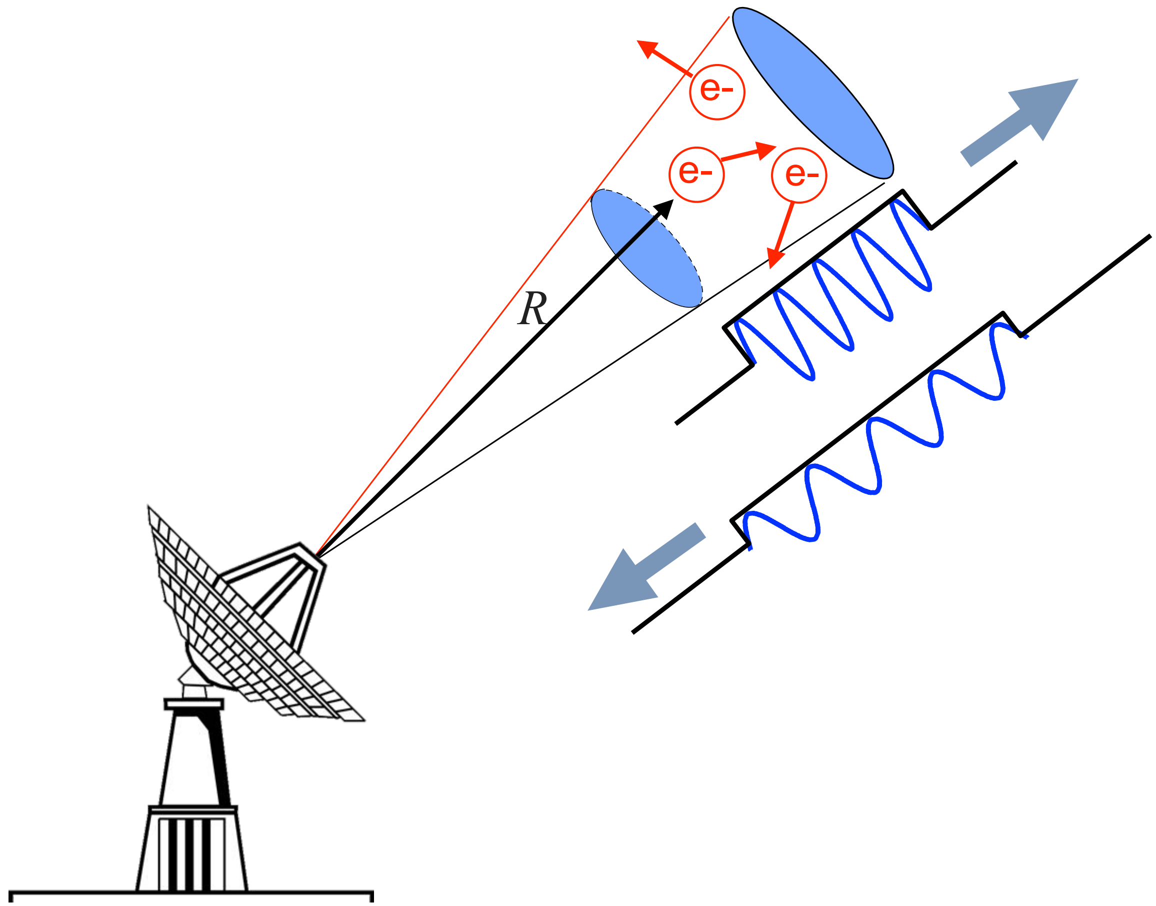
Two key concepts:

Distant ⟺ Time

$$R = c\Delta t/2$$

Velocity ⟺ Frequency

$$v = -f_D \lambda_o/2$$



A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

Two key concepts

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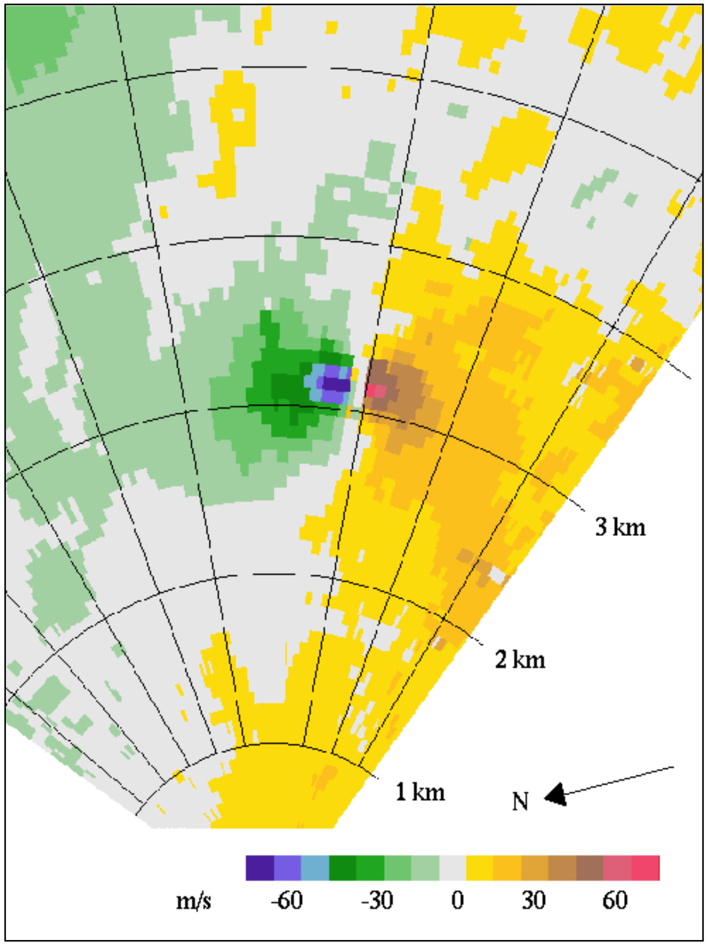

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Concept of a "Doppler Spectrum"

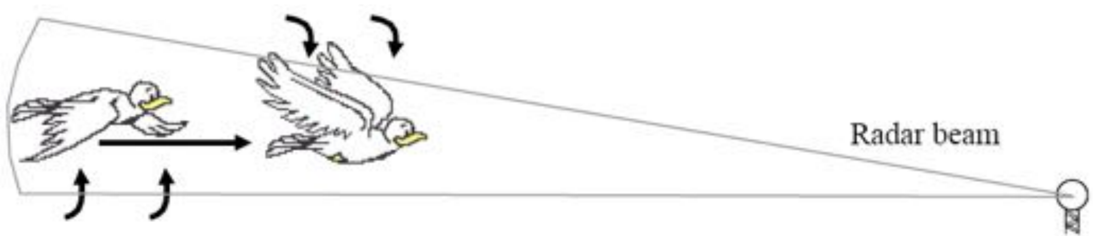
Two key concepts:

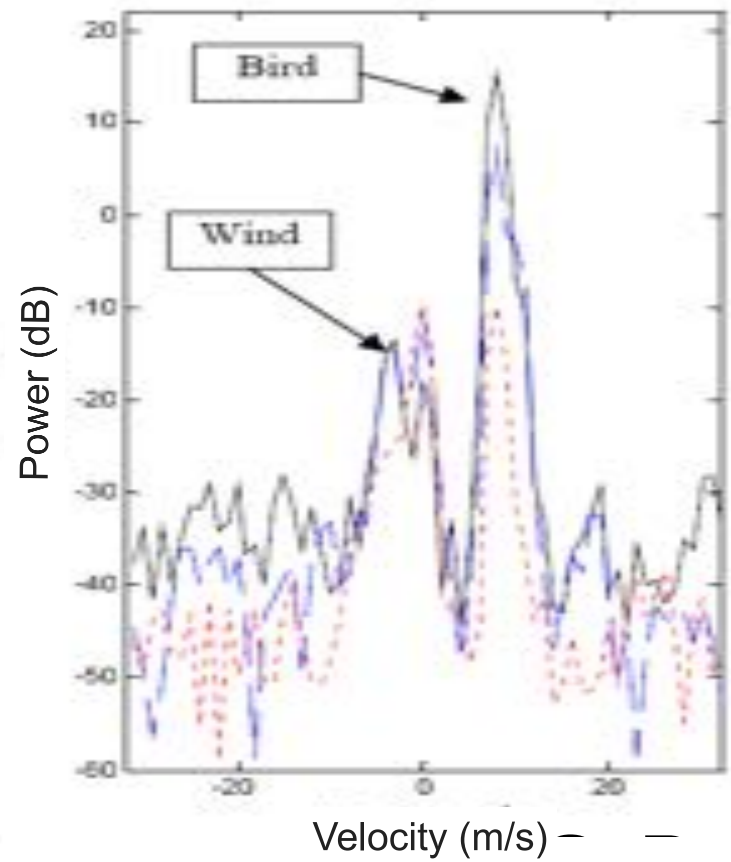
Distant ⟺ Time

$R = c\Delta t/2$

Velocity ⟺ Frequency

$v = -f_D \lambda_o/2$





If there is a distribution of targets moving at different velocities (e.g., electrons in the ionosphere) then there is no single Doppler shift but, rather, a Doppler spectrum.

What is the Doppler spectrum of the ionosphere at UHF (λ_o of 10 to 30 cm)?

Longitudinal Modes in a Thermal Plasma

Simple dispersion relation

$$f = c/\lambda$$

$$\omega = 2\pi f$$

$$k = 2\pi/\lambda$$

$$\omega = ck$$

k = wave number = “spatial frequency”

Ion-acoustic

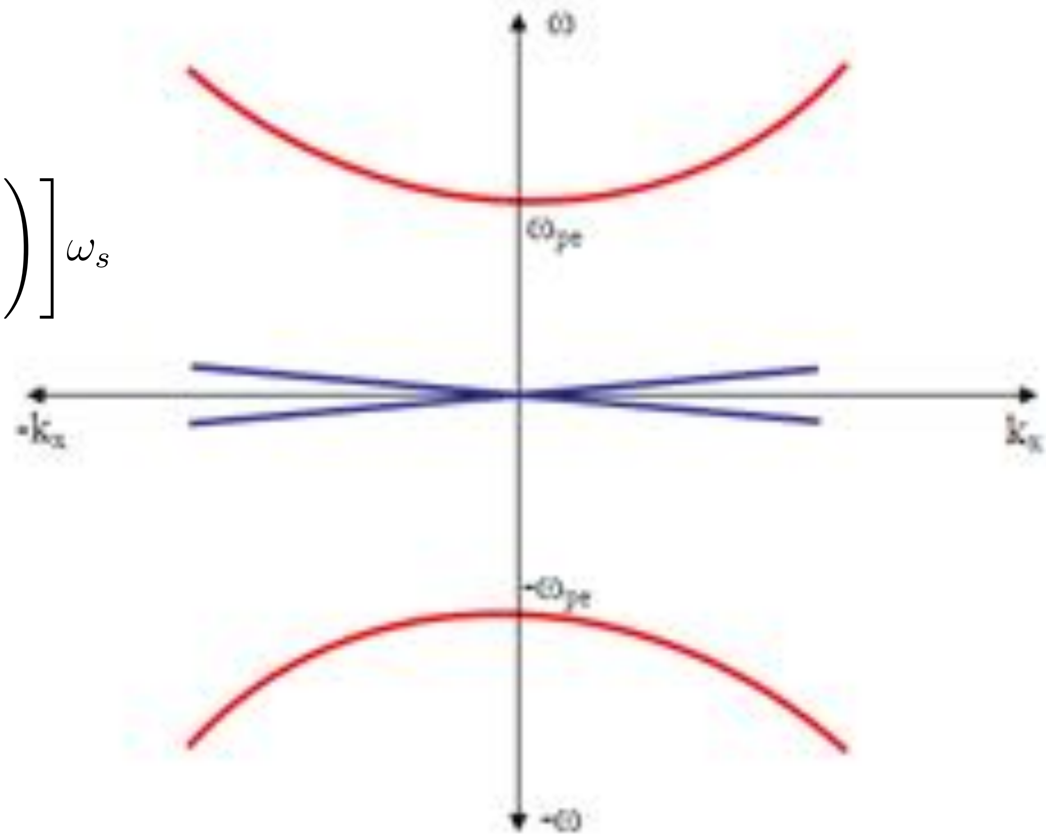
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the}\lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Incoherent Scatter Radar

Particle-in-cell (PIC):

$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

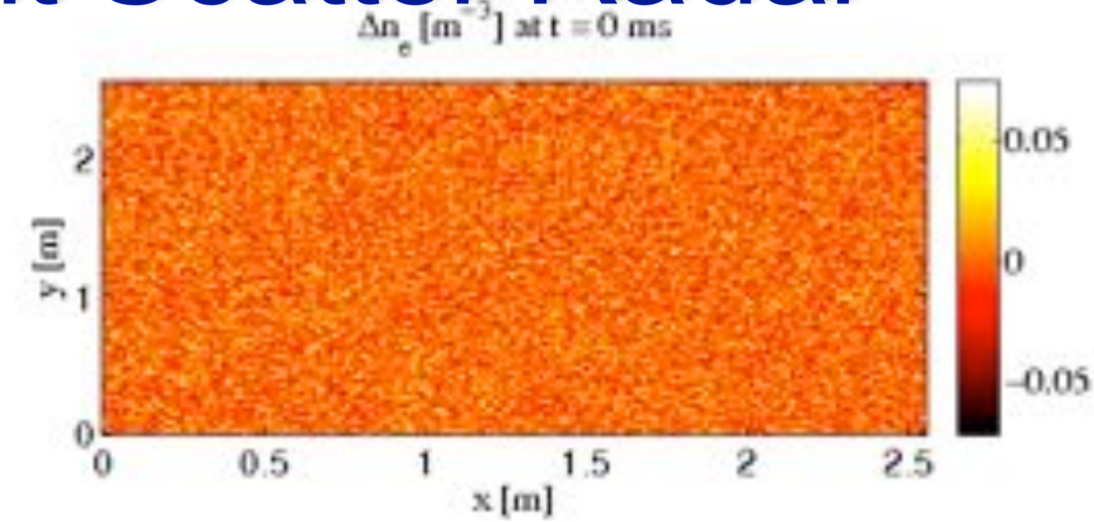
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

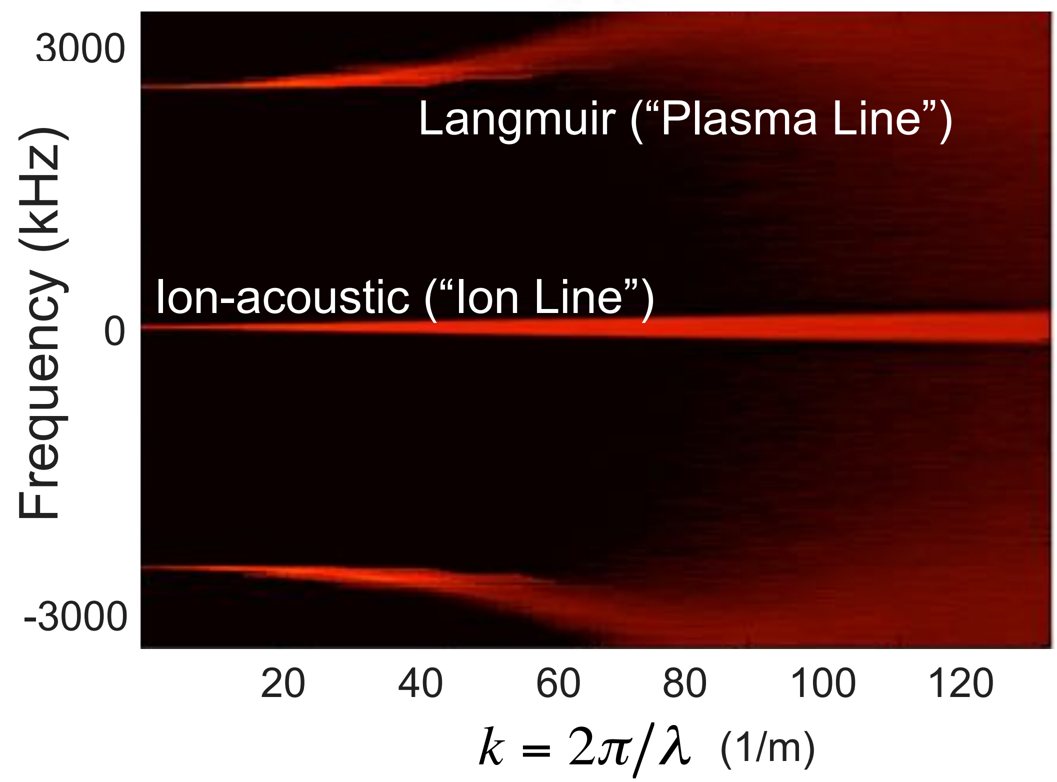
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

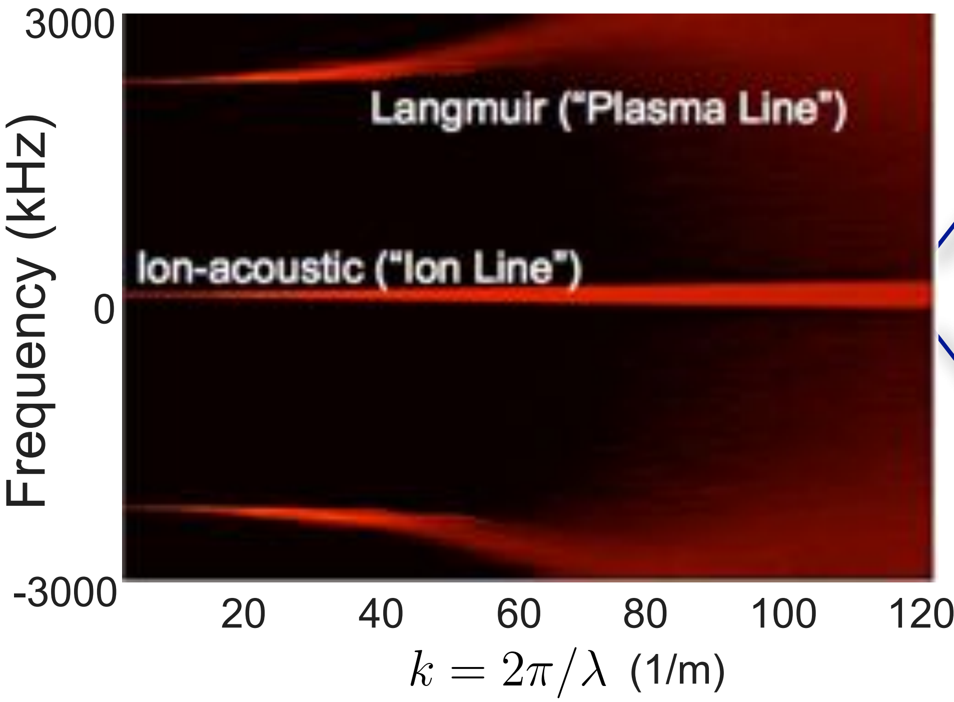
Simple rules yield complex behavior

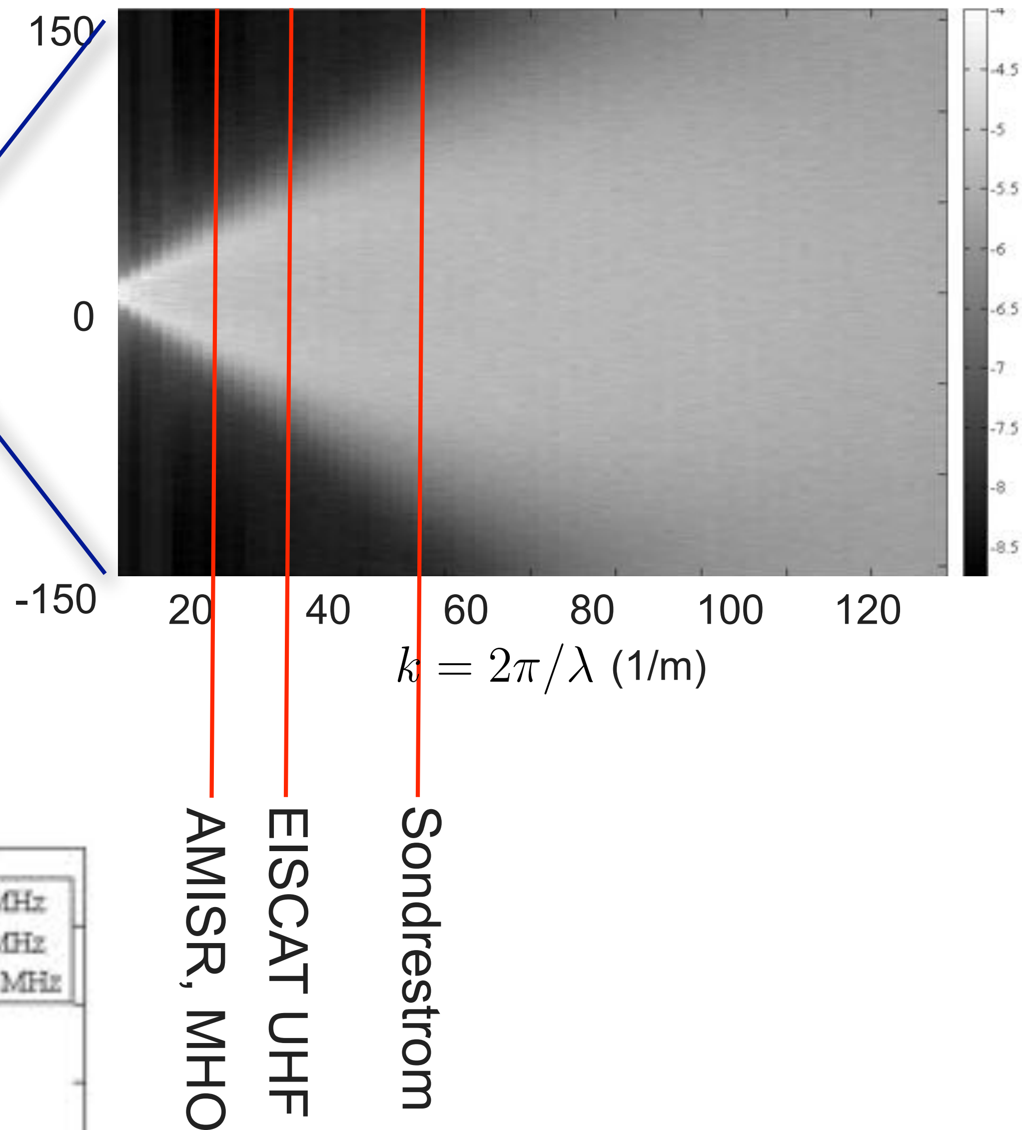


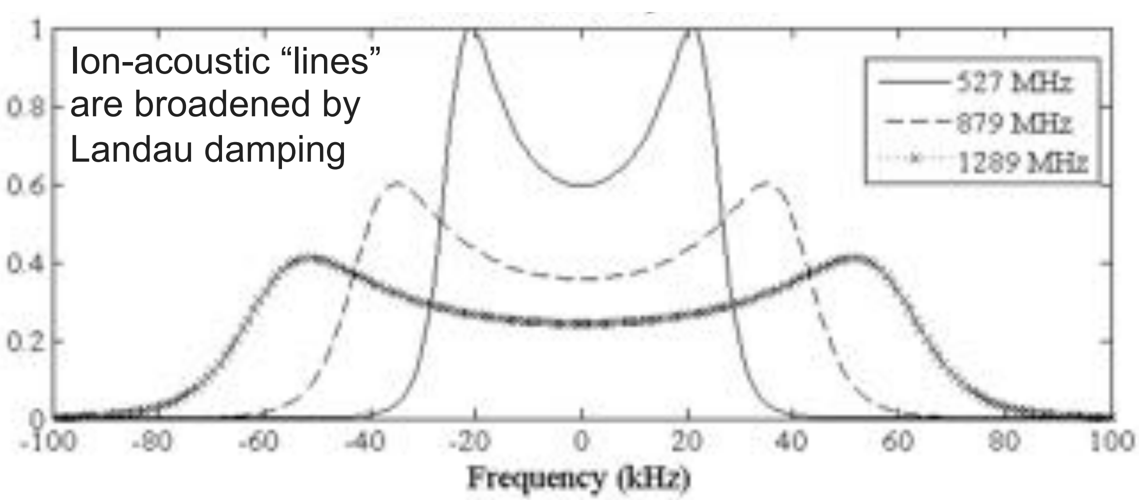


Diaz et al., RS 2008

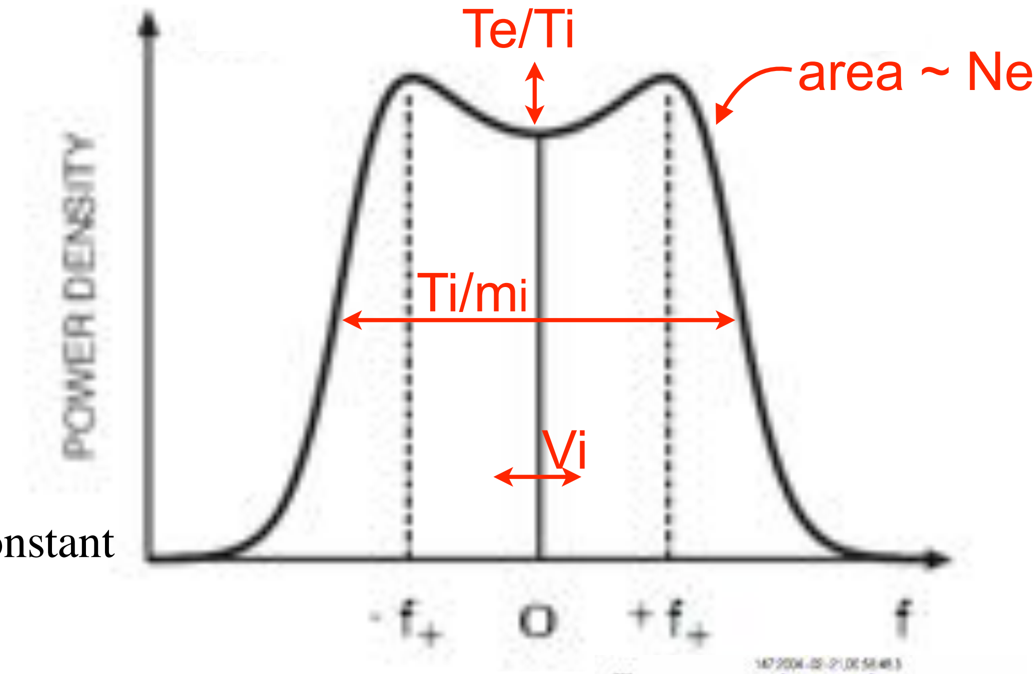
ISR Measures a Cut Through This Surface







ISR in a nutshell



Here's what we measure:

$$SNR = \frac{P_r}{P_n} = \left(\frac{P_t}{4\pi R^2}\right)\left(\frac{\sigma(\omega)}{4\pi R^2}\right)\left(\frac{GA}{KTBN_{sys}}\right)$$

P_r = Received power

P_n = Received noise power

P_t = Transmitted power

s = Radar cross section

G = Antenna gain

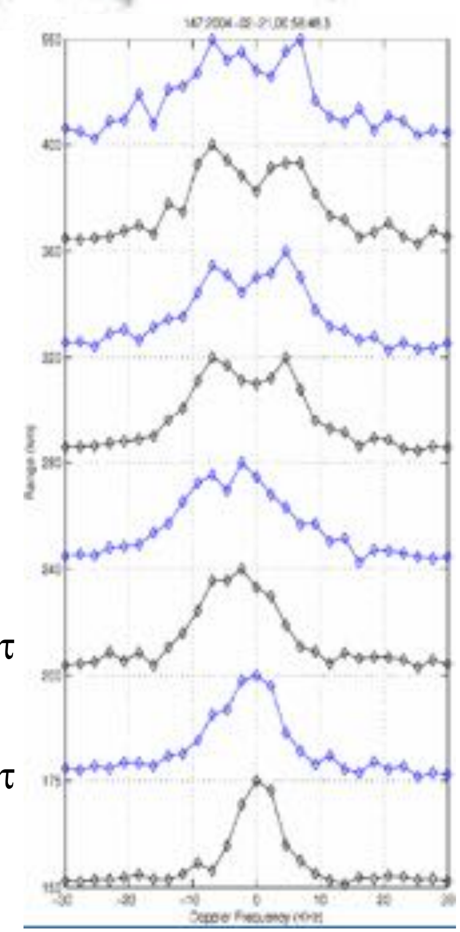
A = Antenna area

k_B = Boltzman's constant

T = Temperature

B = Bandwidth

N_{sys} = System noise temperature



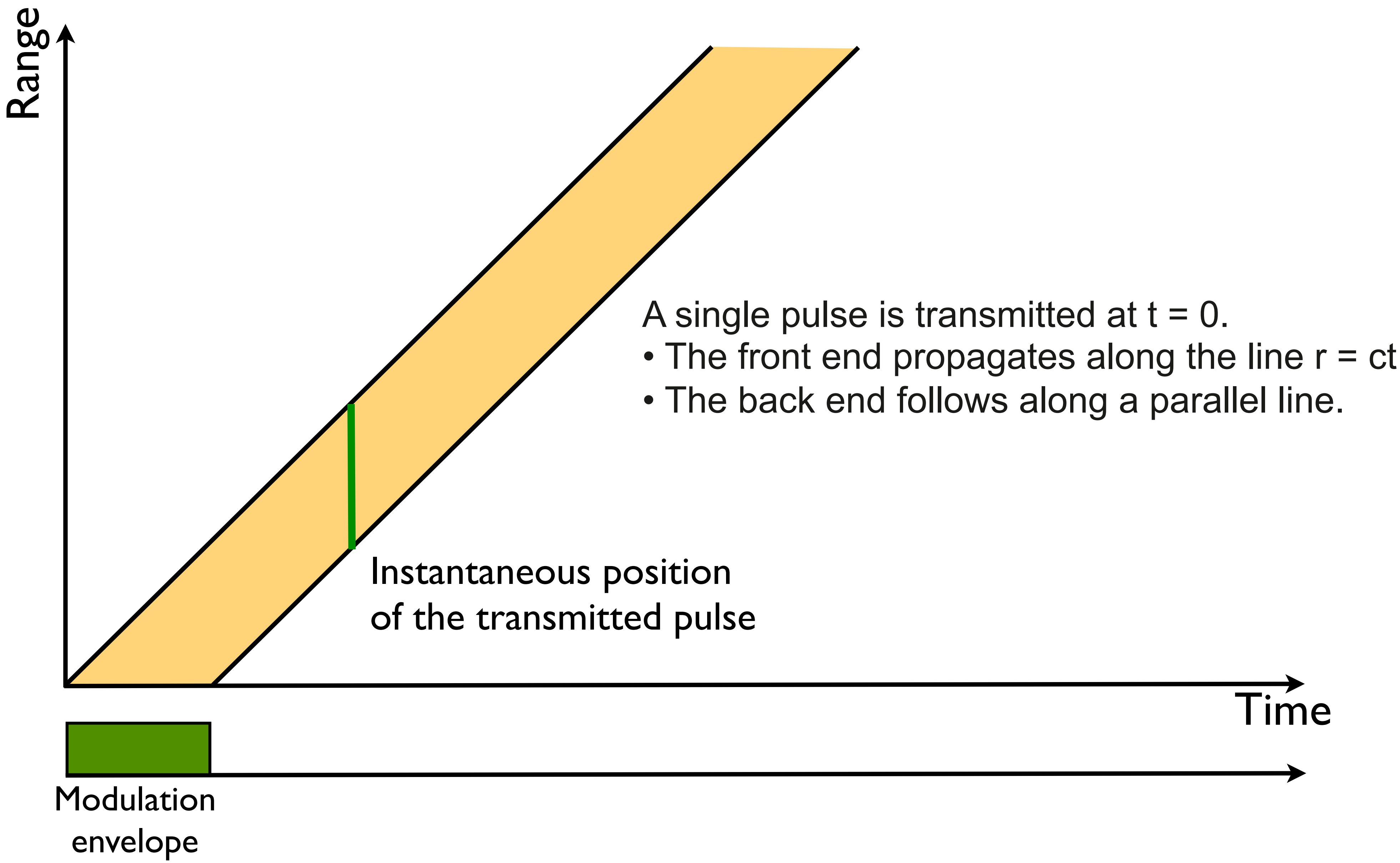
Here's the theory:

$$\sigma(\omega) = \frac{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \sum_1 \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right|^2 \overline{\left|N_e^0(\omega)\right|^2} + \left(\frac{\lambda}{4\pi D_e}\right)^4 \left|F_e(\omega)\right|^2 \sum_i \overline{\left|N_i^0(\omega)\right|^2}}{\left|1 + \left(\frac{\lambda}{4\pi}\right)^2 \left\{\left(\frac{1}{D_e}\right)^2 \times F_e(\omega) + \sum_i \left(\frac{1}{D_i}\right)^2 F_i(\omega)\right\}\right|^2}$$

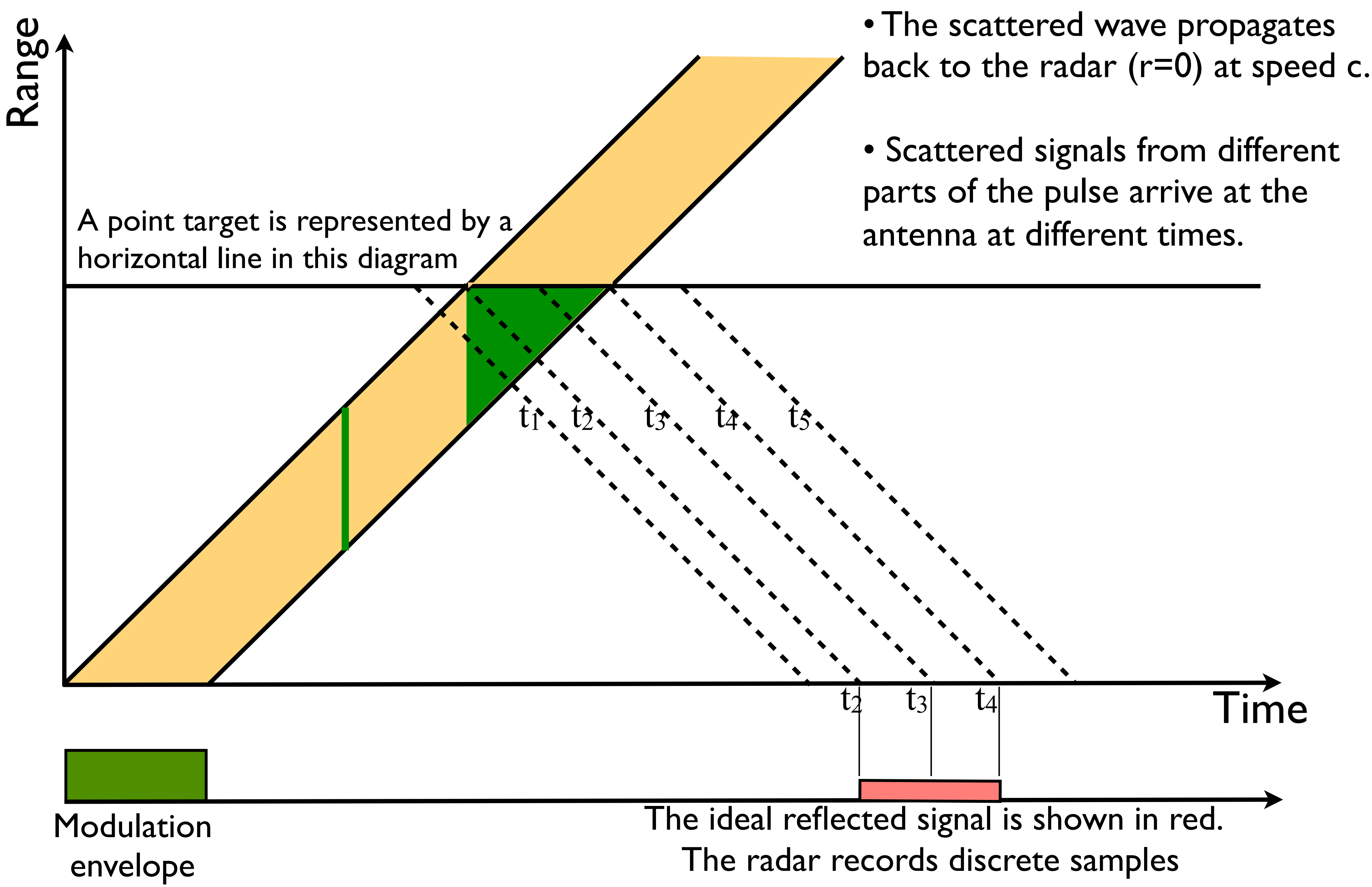
$$F_e(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_e}{\lambda^2 m_e}\tau^2\right)\cos(\omega\tau)\,d\tau$$

$$F_i(\omega) = 1 - \omega \int_o^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\sin(\omega\tau)\,d\tau - j\omega \int_0^\infty \exp\left(-\frac{16\pi^2 KT_i}{\lambda^2 m_i}\tau^2\right)\cos(\omega\tau)\,d\tau$$

Range-time analysis

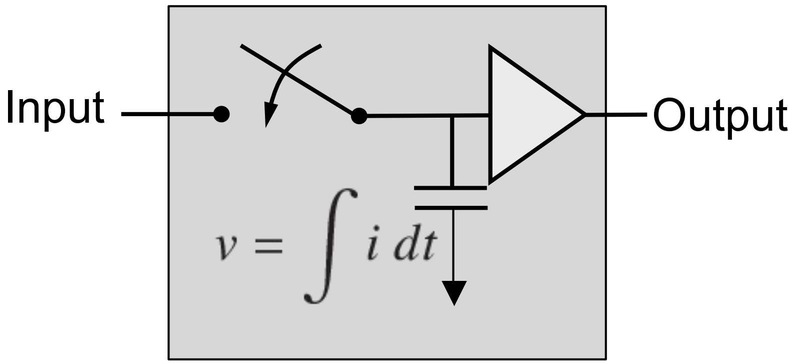


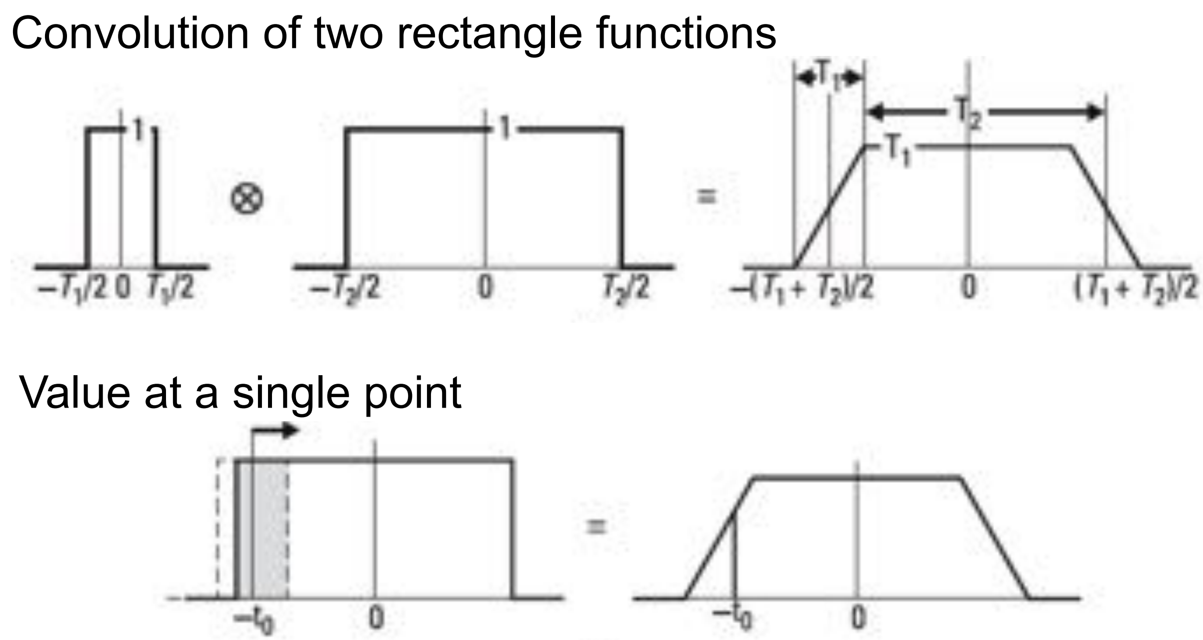
Range-time analysis



Sampling a signal require time-integration

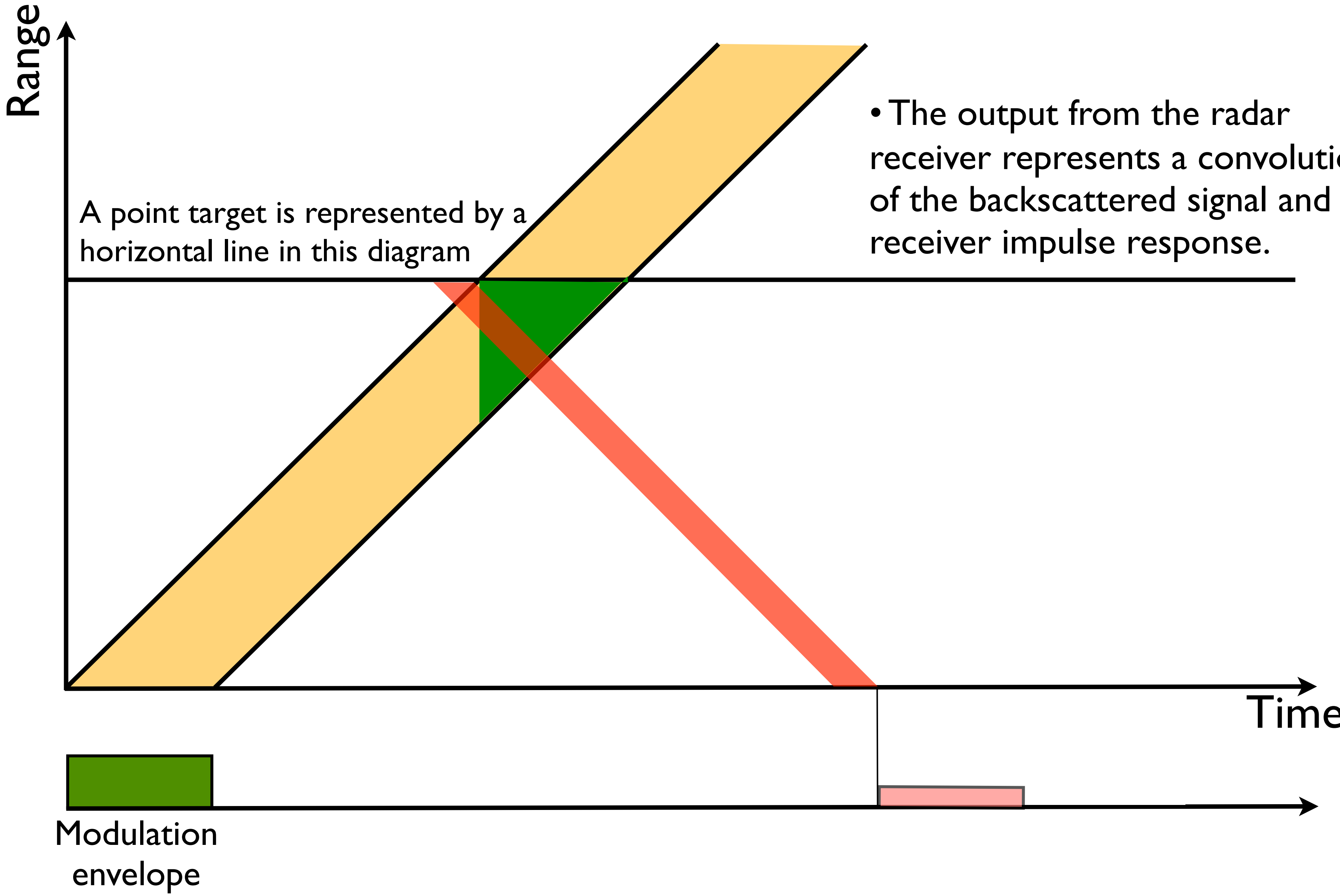
We send a pulse of duration τ. How should we listen for the echo?





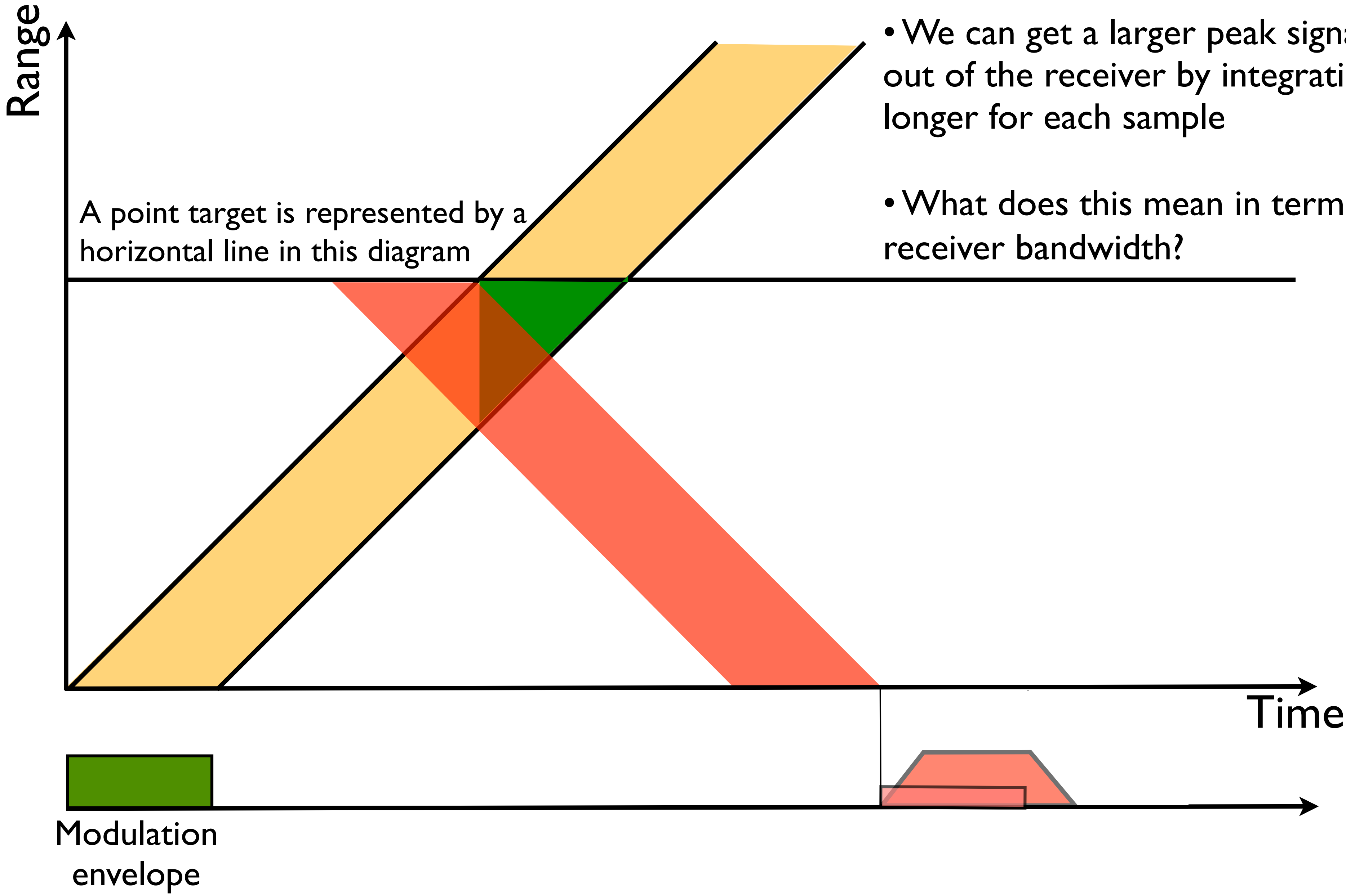
- To determine range to our target, we only need to find the rising edge of the pulse we sent. So make $T_1 << T_2$.
- But that means large receiver bandwidth, lots of noise power, poor SNR.
- Could make $T_1 >> T_2$, then we're integrating noise in time domain.
- So how long should we close the switch?

Sampling the received signal



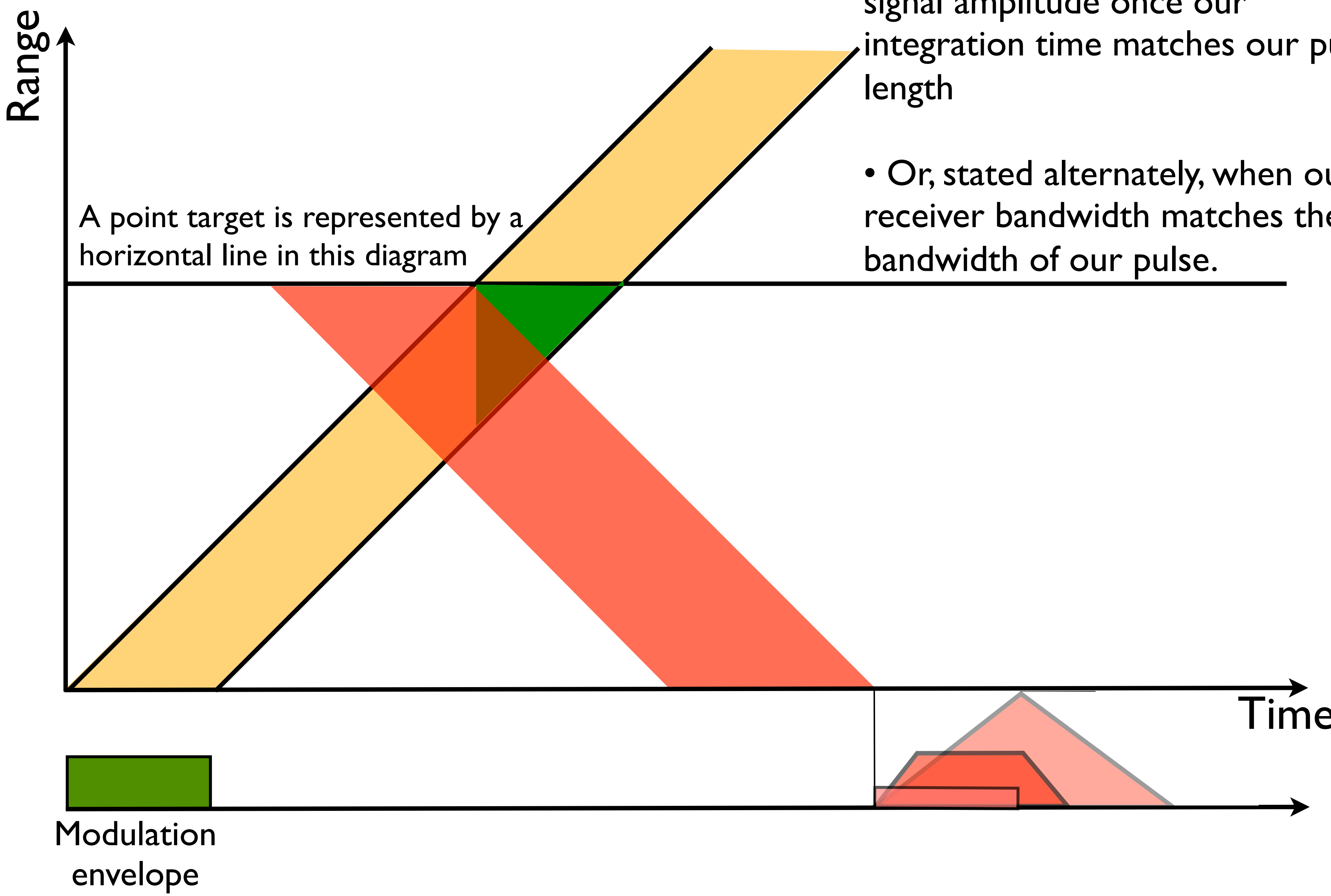
- The output from the radar receiver represents a convolution of the backscattered signal and the receiver impulse response.

Computing the ACF



- We can get a larger peak signal out of the receiver by integrating longer for each sample
- What does this mean in terms of receiver bandwidth?

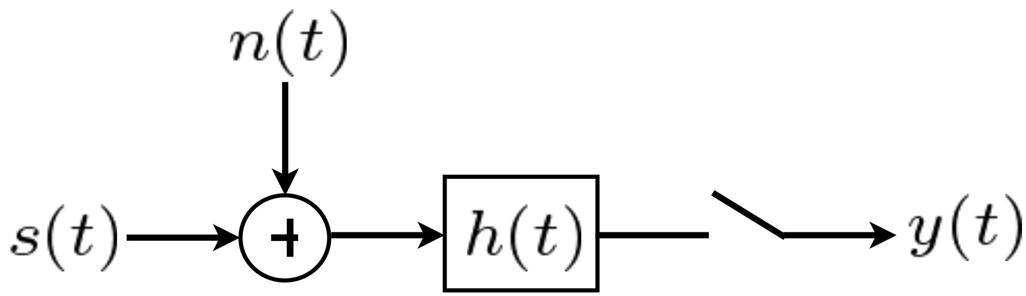
Computing the ACF



- We don't get any more gain in signal amplitude once our integration time matches our pulse length

- Or, stated alternately, when our receiver bandwidth matches the bandwidth of our pulse.

Matched Filter



$$y(t) = \int [s(\tau) + n(\tau)]\, h(t-\tau) d\tau$$
$$= \int H(f)S(f)e^{j2\pi fT} df + \int H(f)N(f)e^{j2\pi fT} df$$

How should we choose *h(t)*⇔*H(f)* such that the output SNR is maximal?

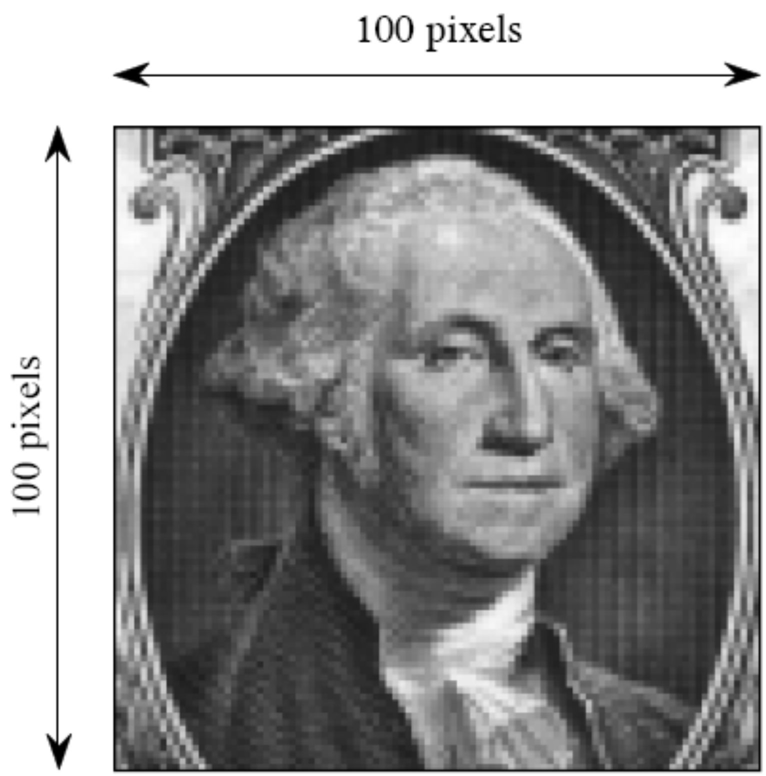
$$SNR = \frac{\left|\int H(f)S(f)e^{j2\pi fT} df\right|^2}{E\left\{\left|\int H(f)N(f)df\right|^2\right\}}$$

Assuming white Gaussian noise, it can be shown that max SNR is when

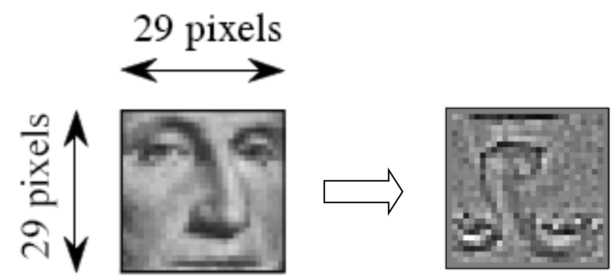
$$H(f) = S^*(f) \Longleftrightarrow h(t) = s^*(-t)$$

Pulse compression and matched filtering

"If you know what you're looking for, it's easier to find."



a. Image to be searched

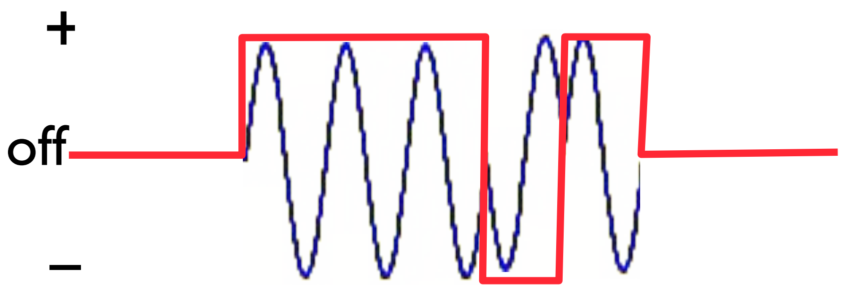


b. Target

c. Kernel

Problem: Find the precise location of the target in the image.
Solution: Correlation

Barker codes



				+	+	+	-	+	correlator output
+	+	+	-	+					1
	+	+	+	-	+				-1+1=0
		+	+	+	-	+			1-1+1=1
			+	+	+	-	+		1+1-1-1=0
				+	+	+	-	+	1+1+1+1+1=5

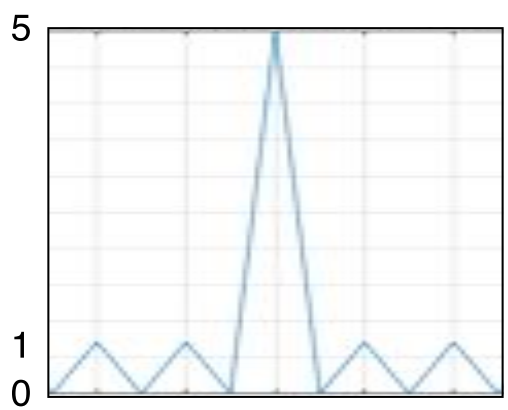
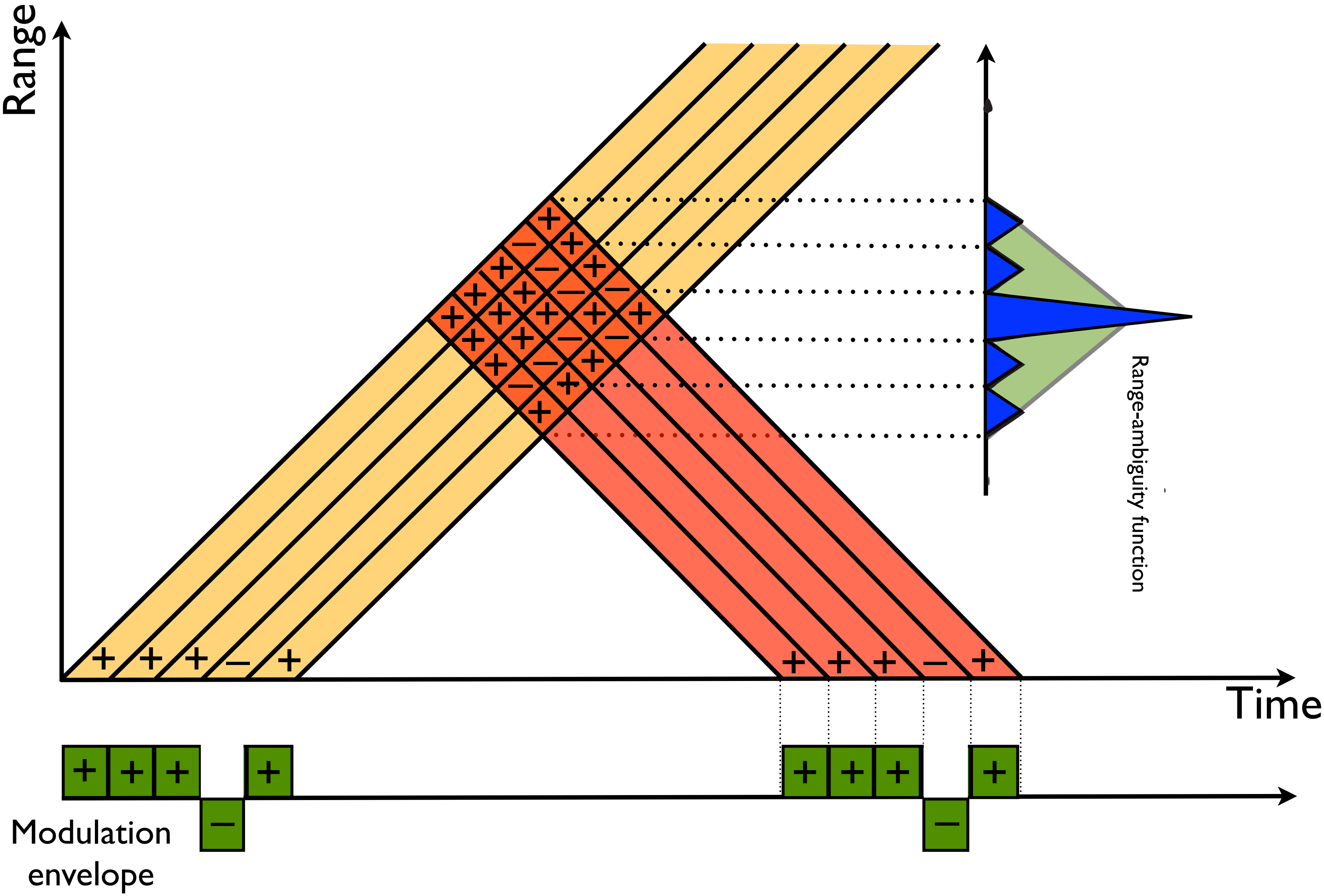


TABLE 6.2 All Known Binary Barker Codes

Code Length	Code
2	11 or 10
3	110
4	1110 or 1101
5	11101
7	1110010
11	11100010010
13	1111100110101

Volume target (e.g., the ionosphere)

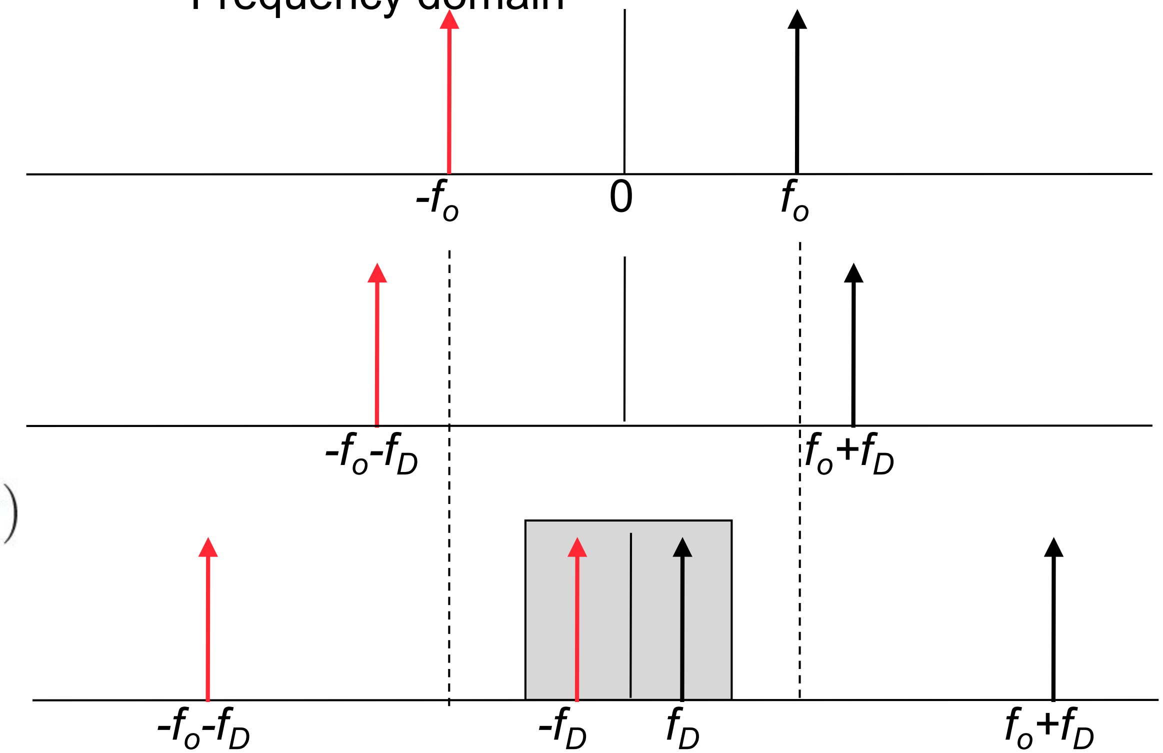


I and Q Demodulation in Frequency Domain

Transmitted signal

$$\cos(2\pi f_o t)$$

Frequency domain



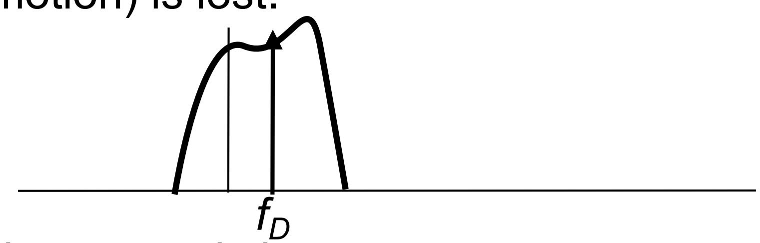
Doppler shifted

$$\cos(2\pi(f_o + f_D)t)$$

Mixed (multiplied) with carrier $\cos(2\pi f_o t)$

Cosine is even function, so sign of f_D (and, hence, direction of motion) is lost.
What we need instead is:

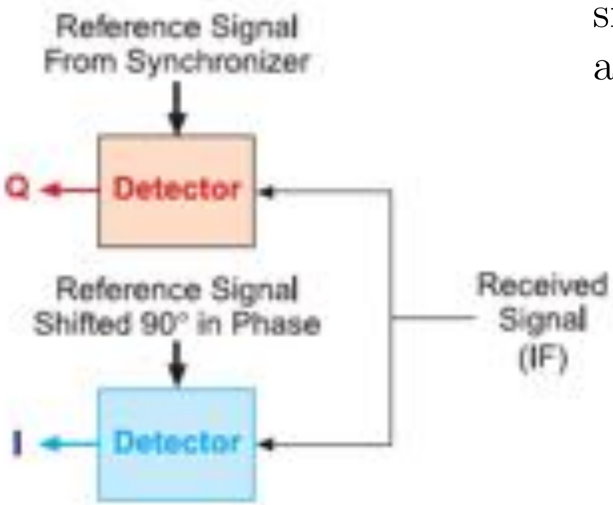
$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$



The analytic signal $e^{j2\pi f_D t}$ cannot be measured directly, but the cos and sin components via mixing with two oscillators with same frequency but orthogonal phases. The components are called "in phase" (or *I*) and "in quadrature" (or *Q*):

$$Ae^{j2\pi f_D t} = I(t) + jQ(t) \quad \xleftrightarrow{\text{FFT}} \quad A\delta(f_D)$$ (for single scatterer)

I and Q Demodulation



We transmit an amplitude-modulated cosine of frequency ω_c. The received signal will have some time varying amplitute $a(t)$ and time-varying phase $\phi(t)$ applied to this,

$$p_{rec}(t) = a(t)cos(\phi(t) + \omega_c t)$$

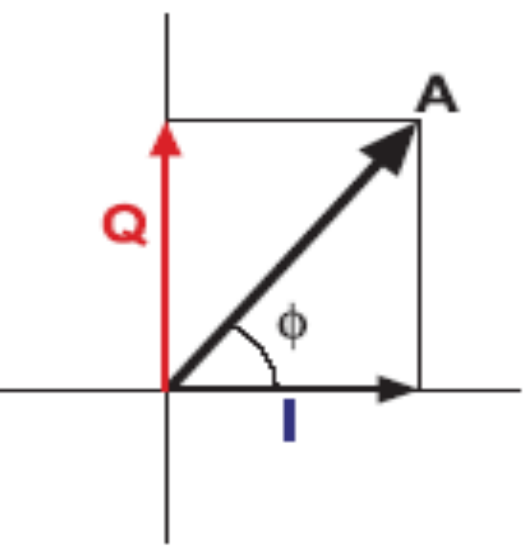
We compute the analytic signal through Euler's identity by "mixing" the signal with cosine and sine

in-phase (I) channel:

$$p_{rec}(t)\cos(\omega_c t) = a(t)\cos(\phi(t) + \omega_c t)\cos(\omega_c t)$$
$$= a(t)\tfrac{1}{2}\left(\underbrace{\cos(\phi(t) + 2\omega_c t)}_{\text{filter out}} + \cos\phi(t)\right)$$

quadrature (Q) channel (90° out of phase):

$$p_{rec}(t)\sin(\omega_c t) = a(t)\cos(\phi(t) + \omega_c t)\sin(\omega_c t)$$
$$= a(t)\tfrac{1}{2}\left(\underbrace{-\sin(\phi(t) + 2\omega_c t)}_{\text{filter out}} + \sin\phi(t)\right)$$

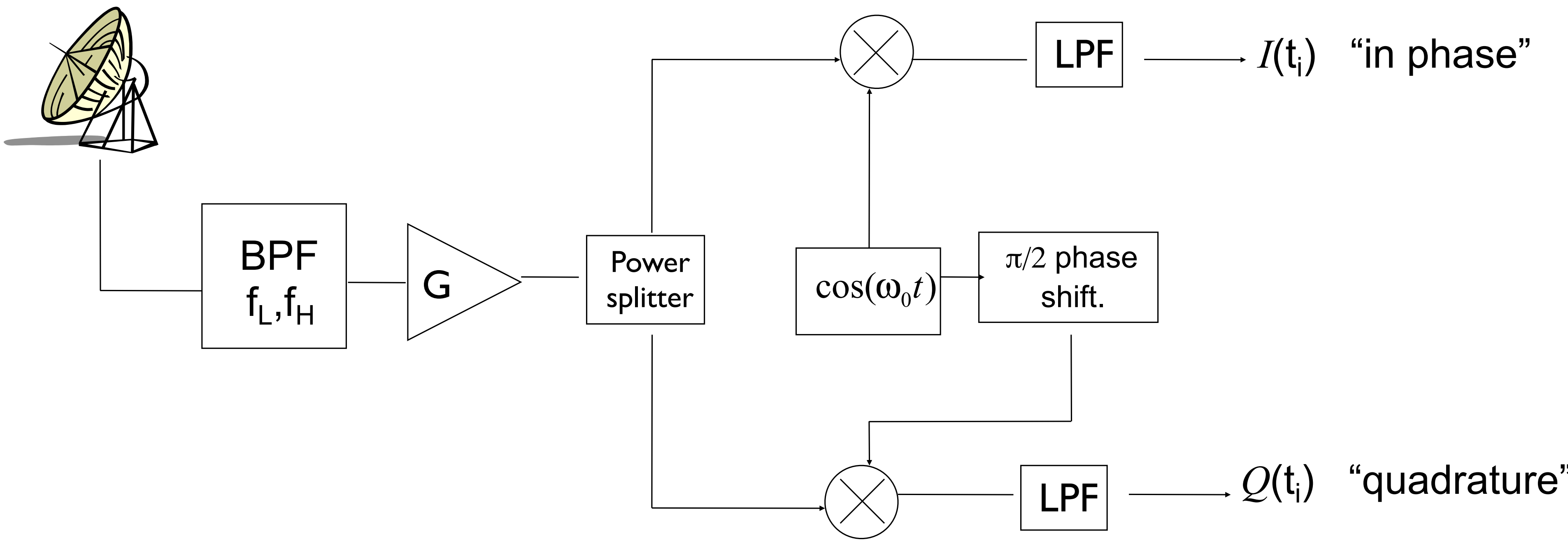


I and Q channels together give the *analytic signal*

$$\boxed{s_{rec}(t) = a(t)e^{i\phi(t)}}$$

The fundamental output of a pulsed Doppler radar is a time series of complex numbers.

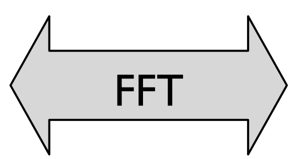
ISR Receiver: I and Q plus correlation



We have time series of *V(t) =I(t) + jQ(t)*, how do I compute the Doppler spectrum?

Estimate the autocorrelation function (ACF) by computing products of complex voltages (“lag products”)

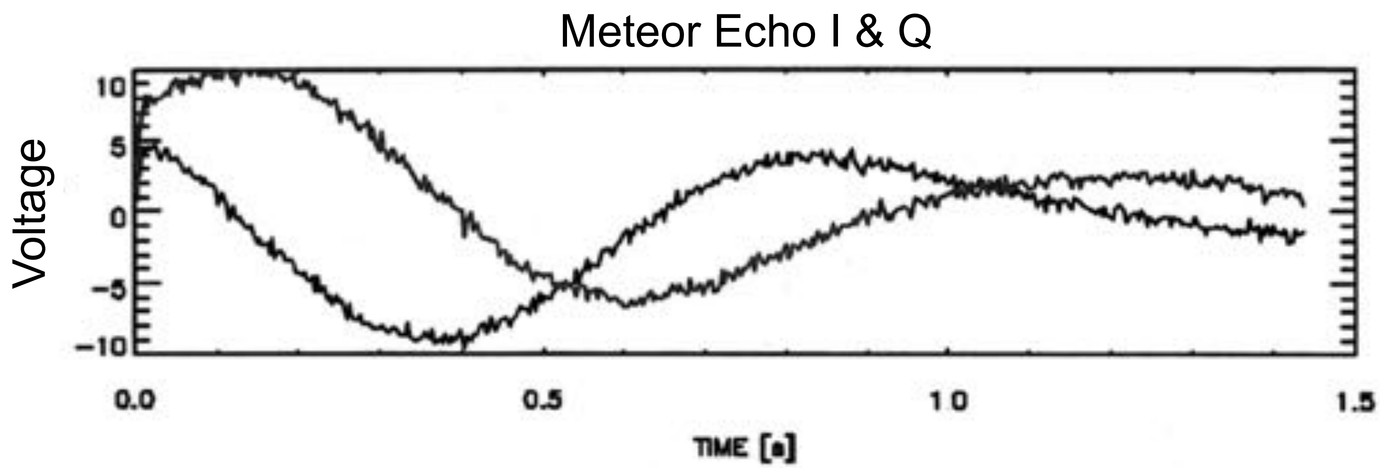
$$R_{vv}(\tau) = \frac{\langle V(t)V^*(t+\tau)\rangle}{S}$$

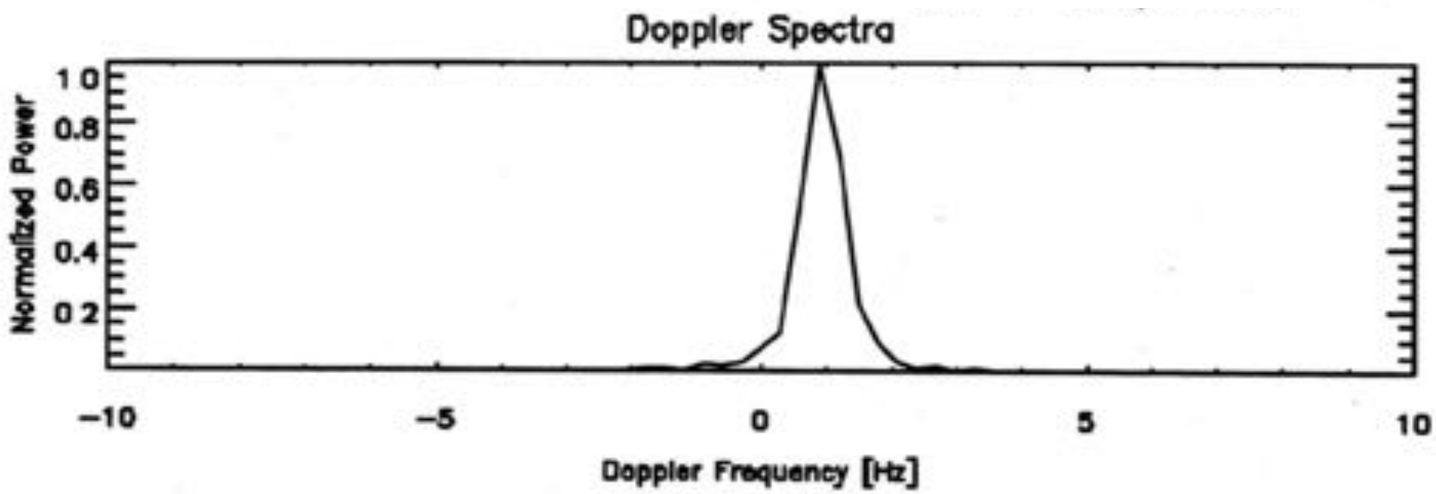


Power spectrum is Fourier Transform of the ACF

Example: Doppler Shift of a Meteor Trail

- **Collect N samples of I(t_k) and *Q*(t_k) from a target**
- **Compute the complex FFT of I(t_k)+j*Q*(t_k), and find the maximum in the frequency domain**
- **Or compute "phase slope" in time domain.**



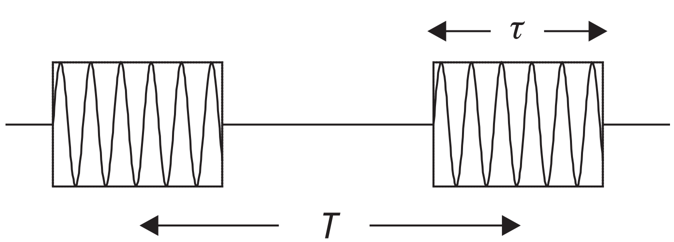


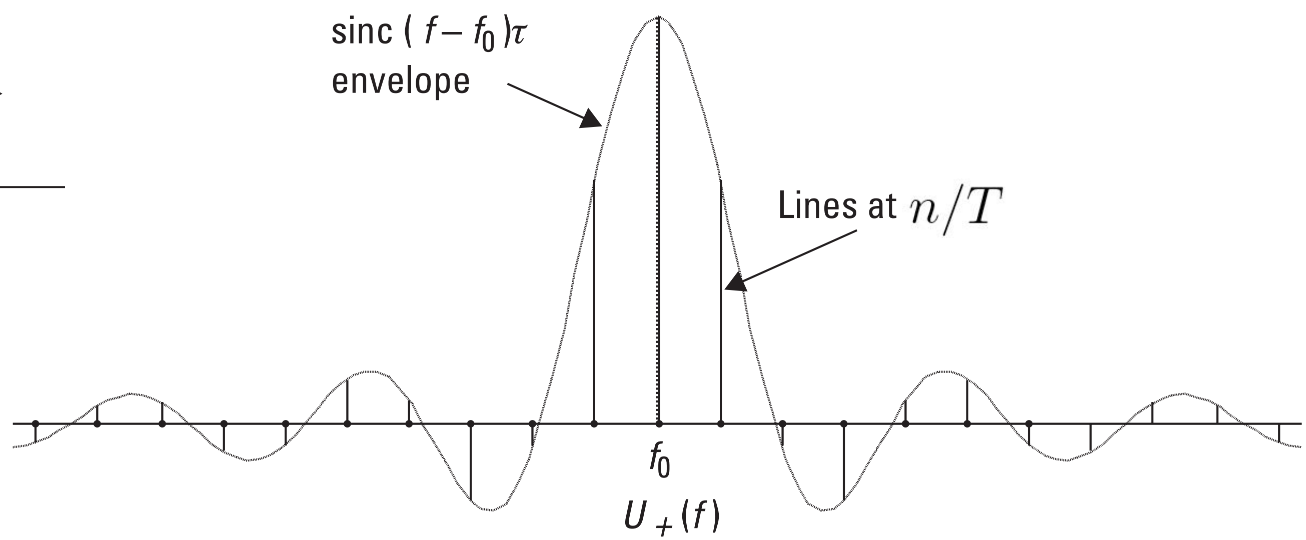
Does this strategy work for ISR?

Typical ion-acoustic velocity: 3 km/s
Doppler shift at 450 MHz: 10kHz
Correlation time: 1/10kHz = 0.1 ms
Required PRF to probe ionosphere (500km range): 300 Hz

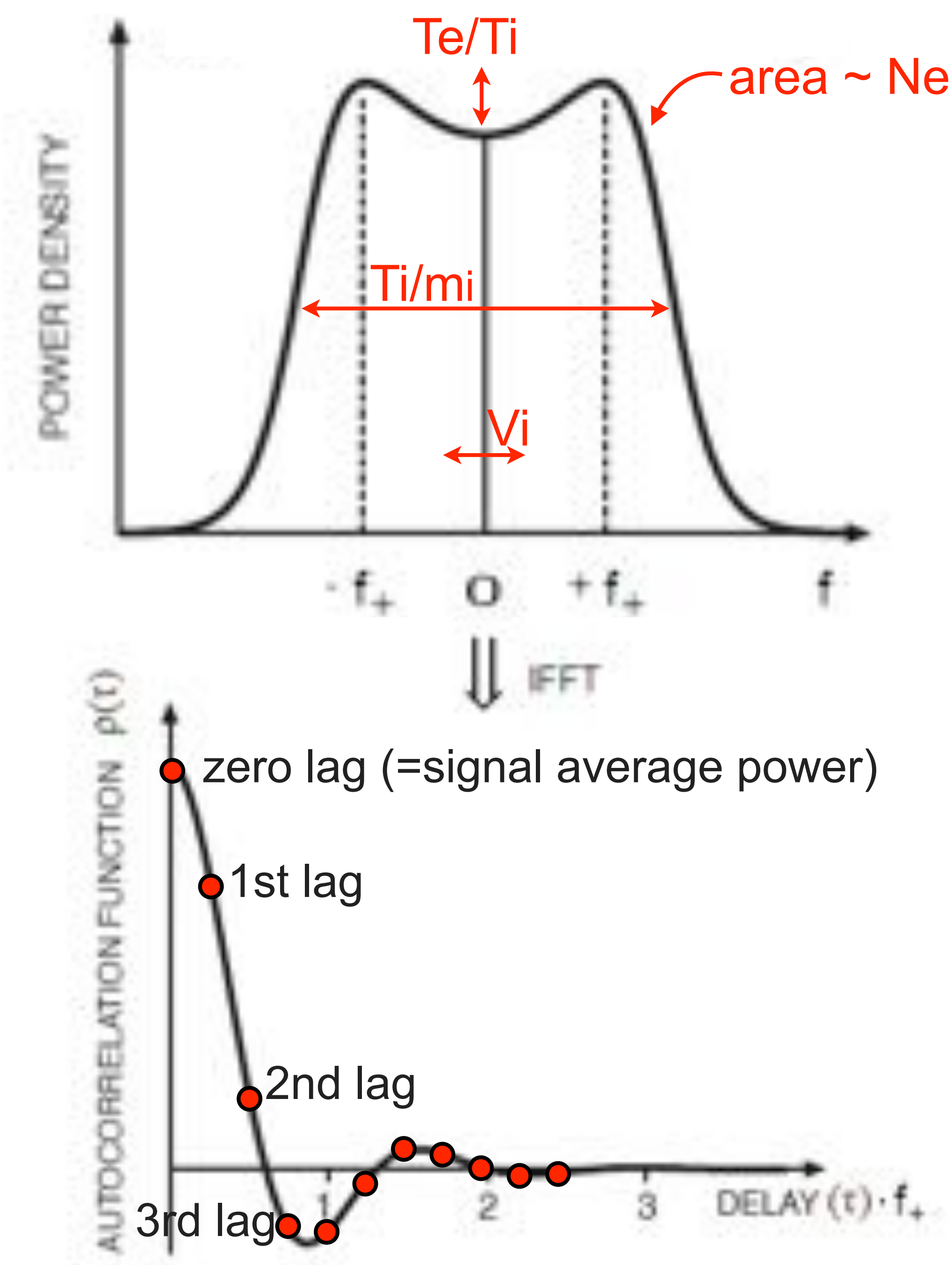
Plasma has completely decorrelated by the time we send the next pulse.

Alternately, the Doppler shift is well beyond the max unambiguous Doppler defined by the Inter-Pulse Period *T*.





Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

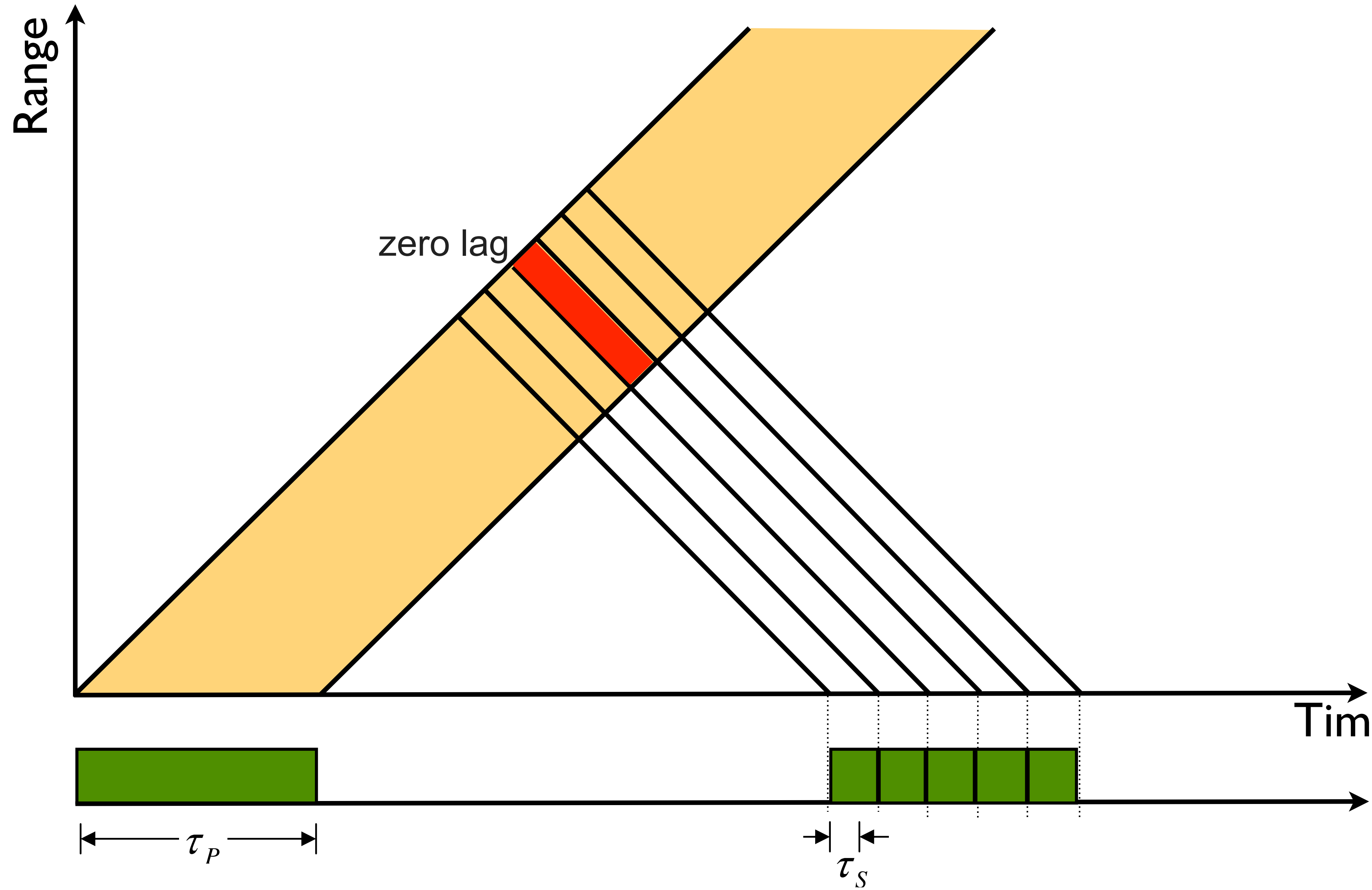
Electron to ion temperature ratio (Te/Ti) from "peak-to-valley" ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to compute lags

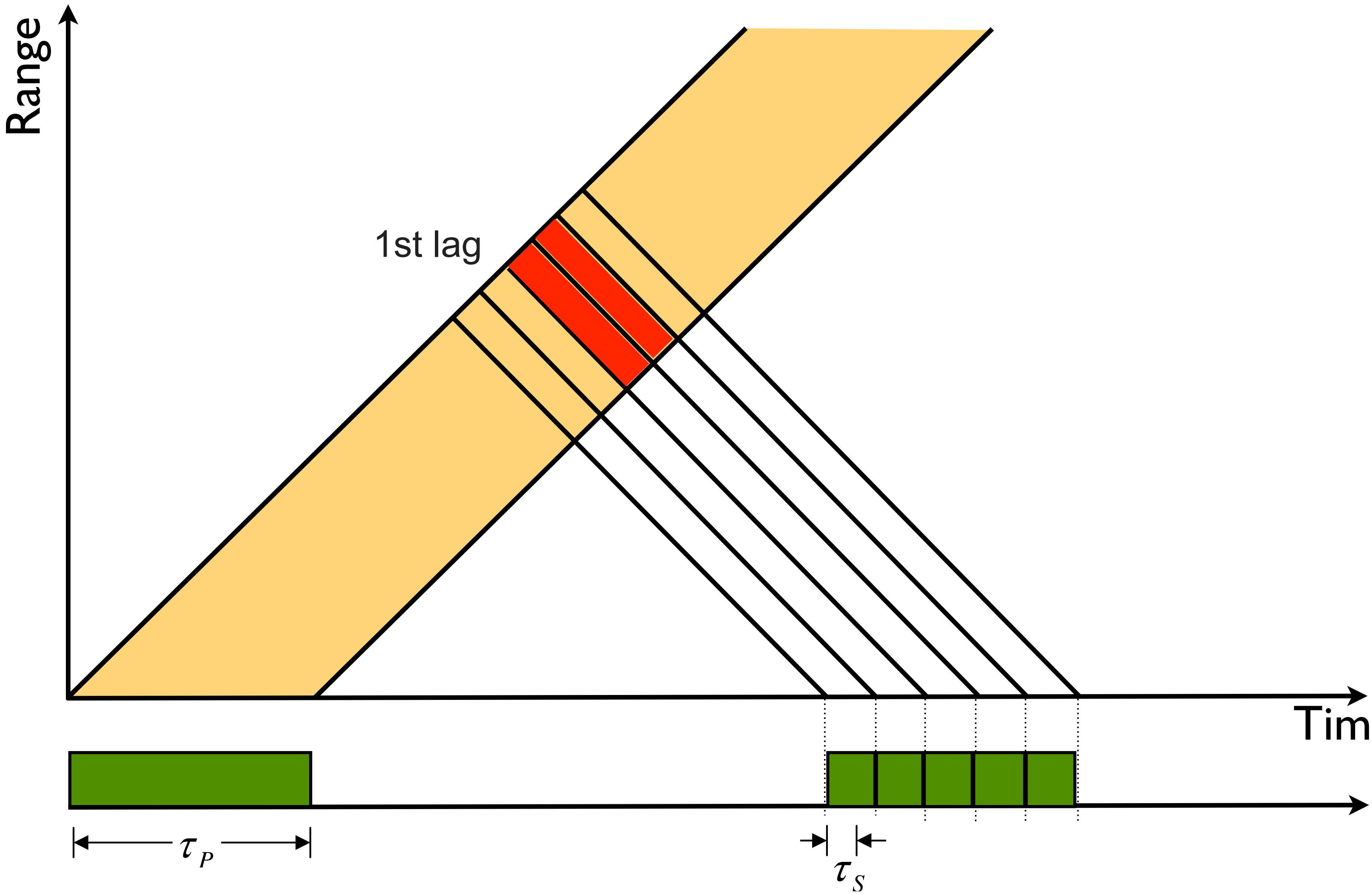
Computing the ACF (and, hence, spectrum)



τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Computing the ACF (and, hence, spectrum)



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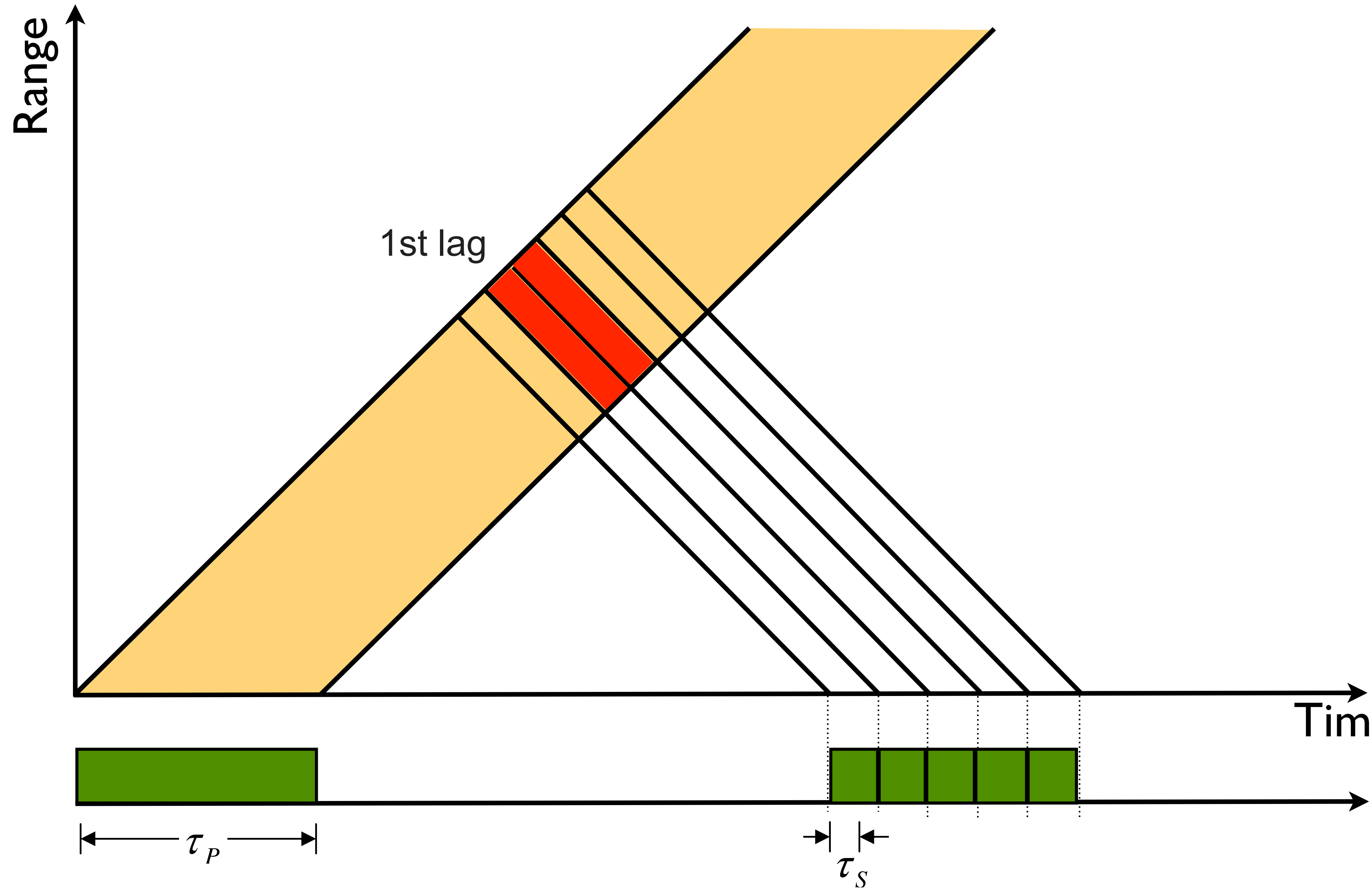

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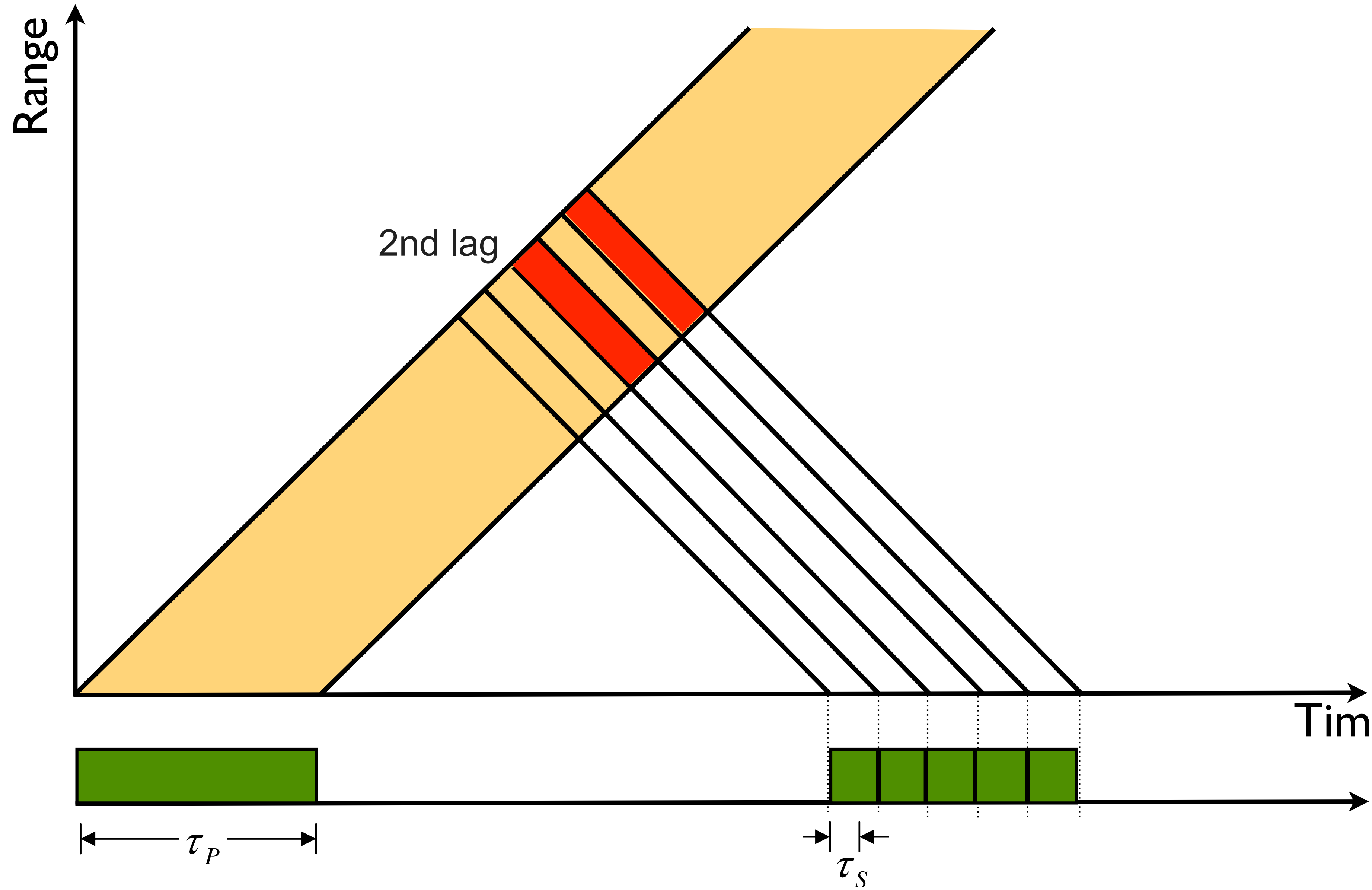

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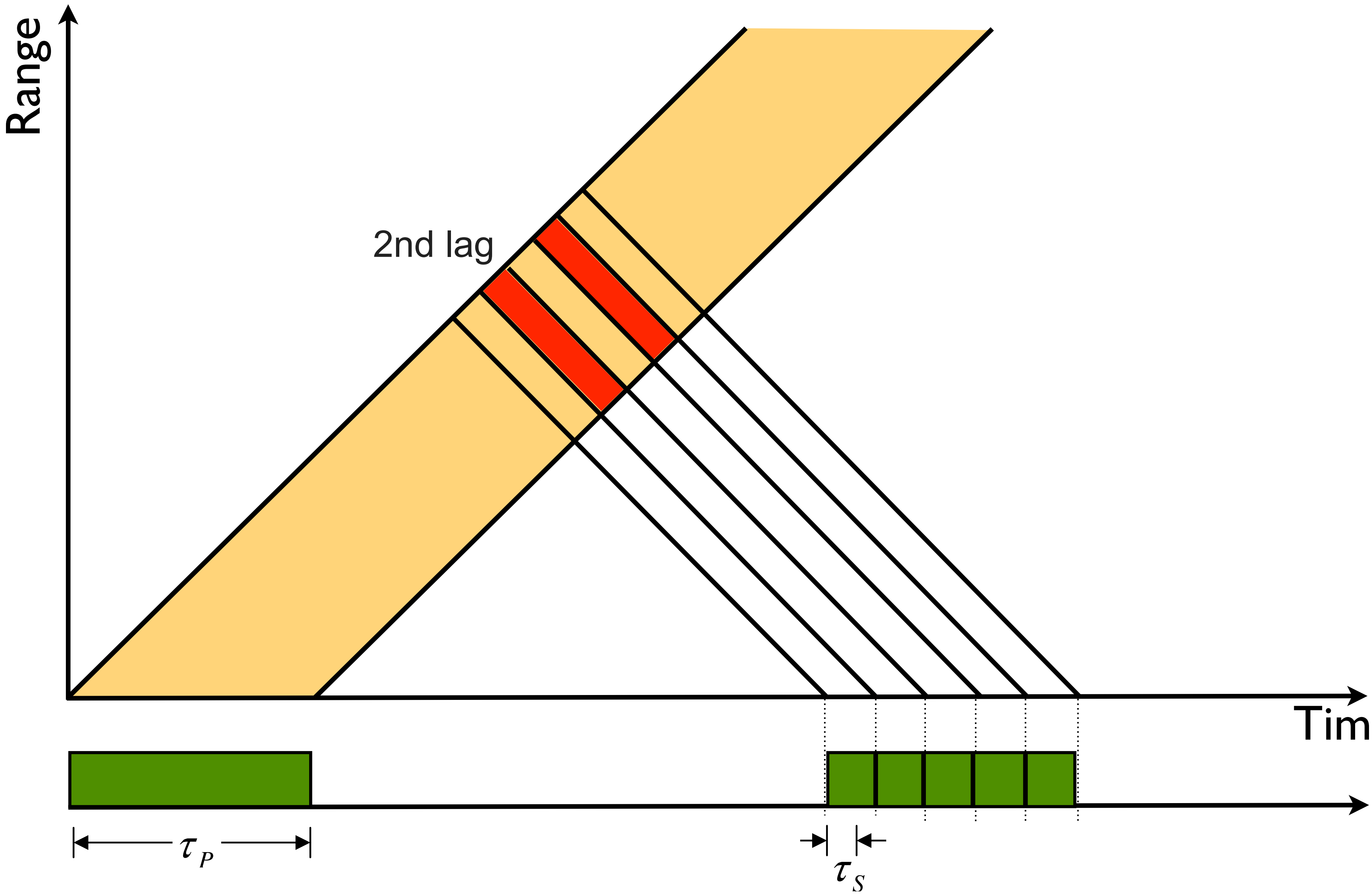

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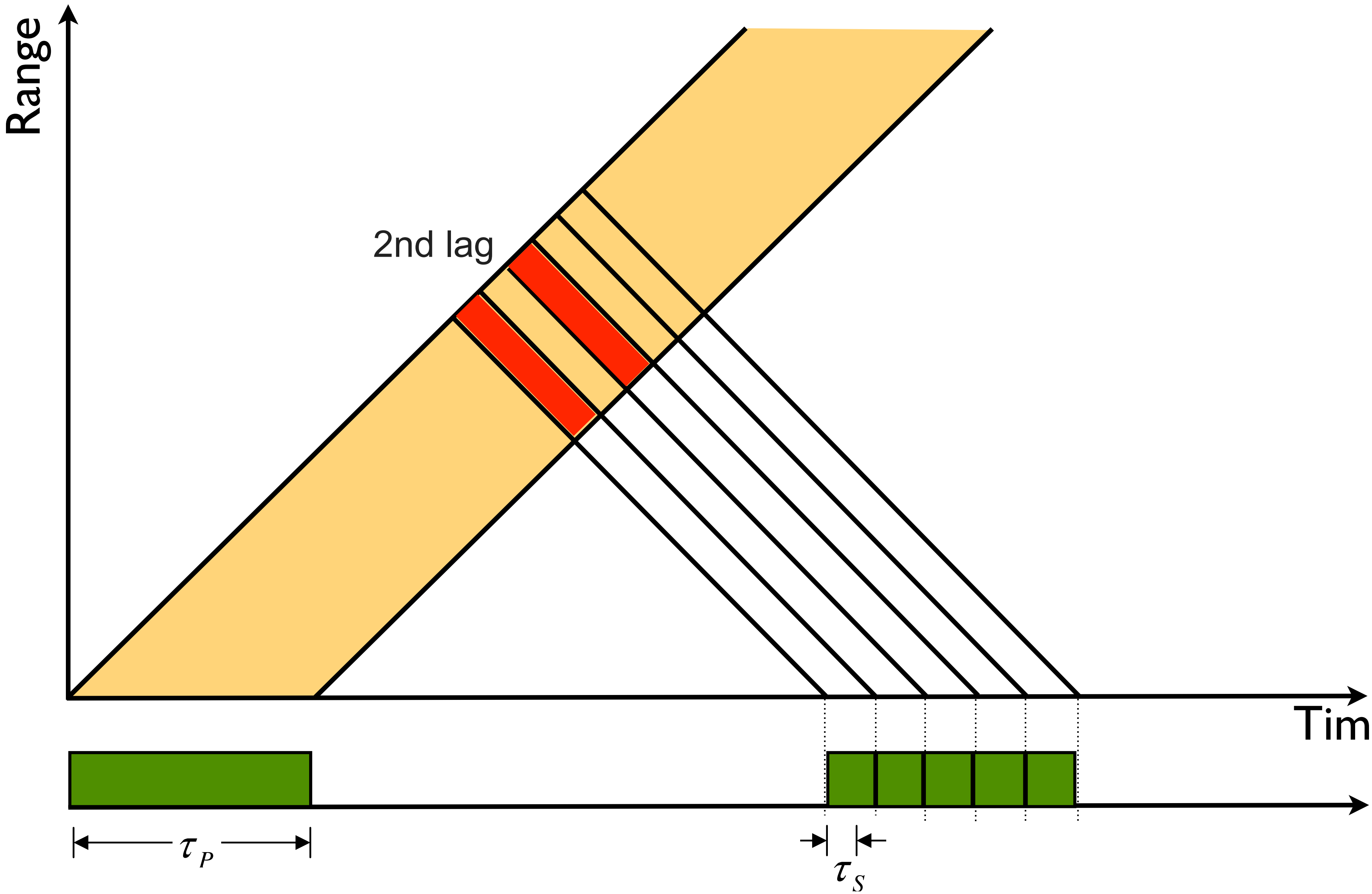

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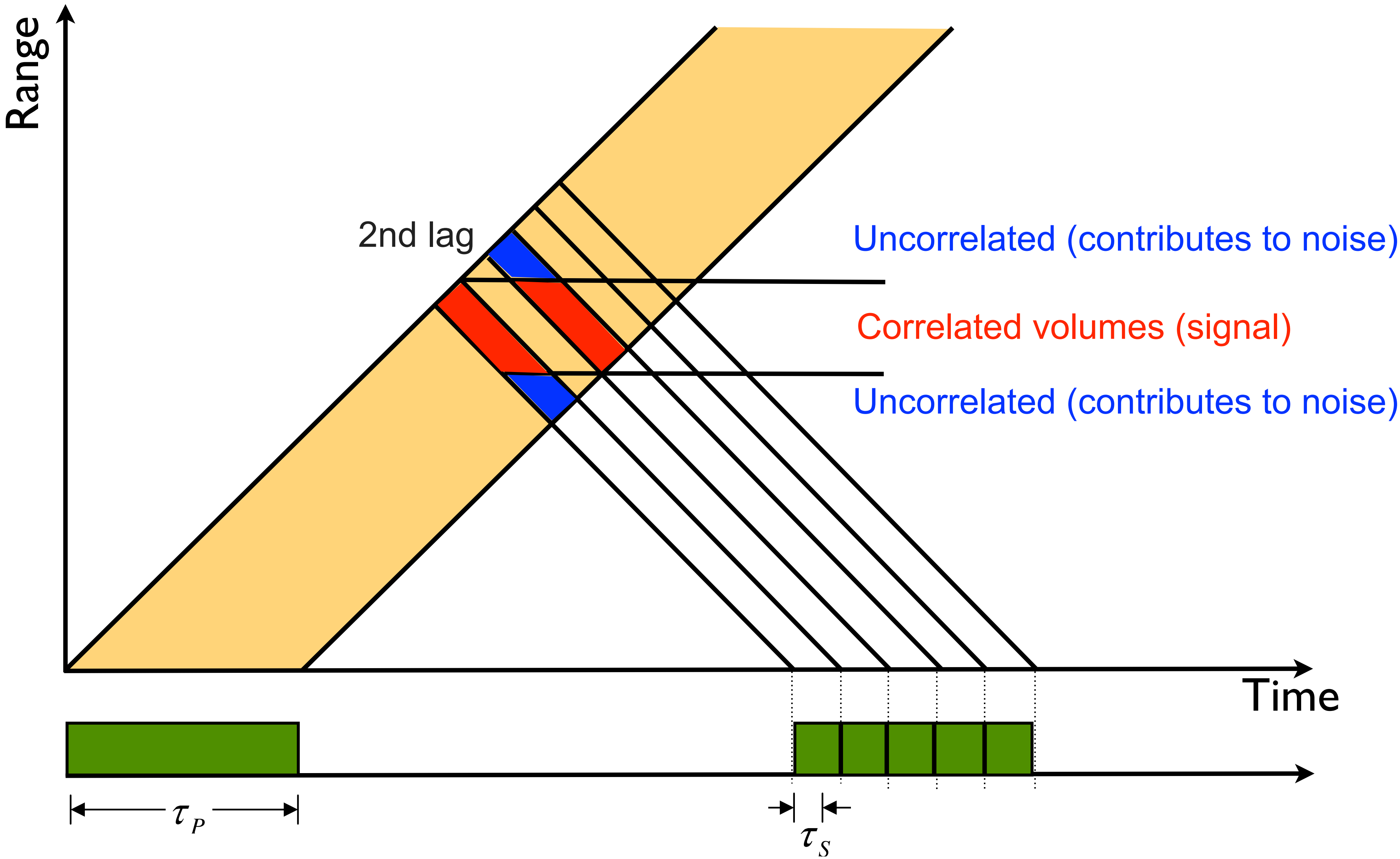

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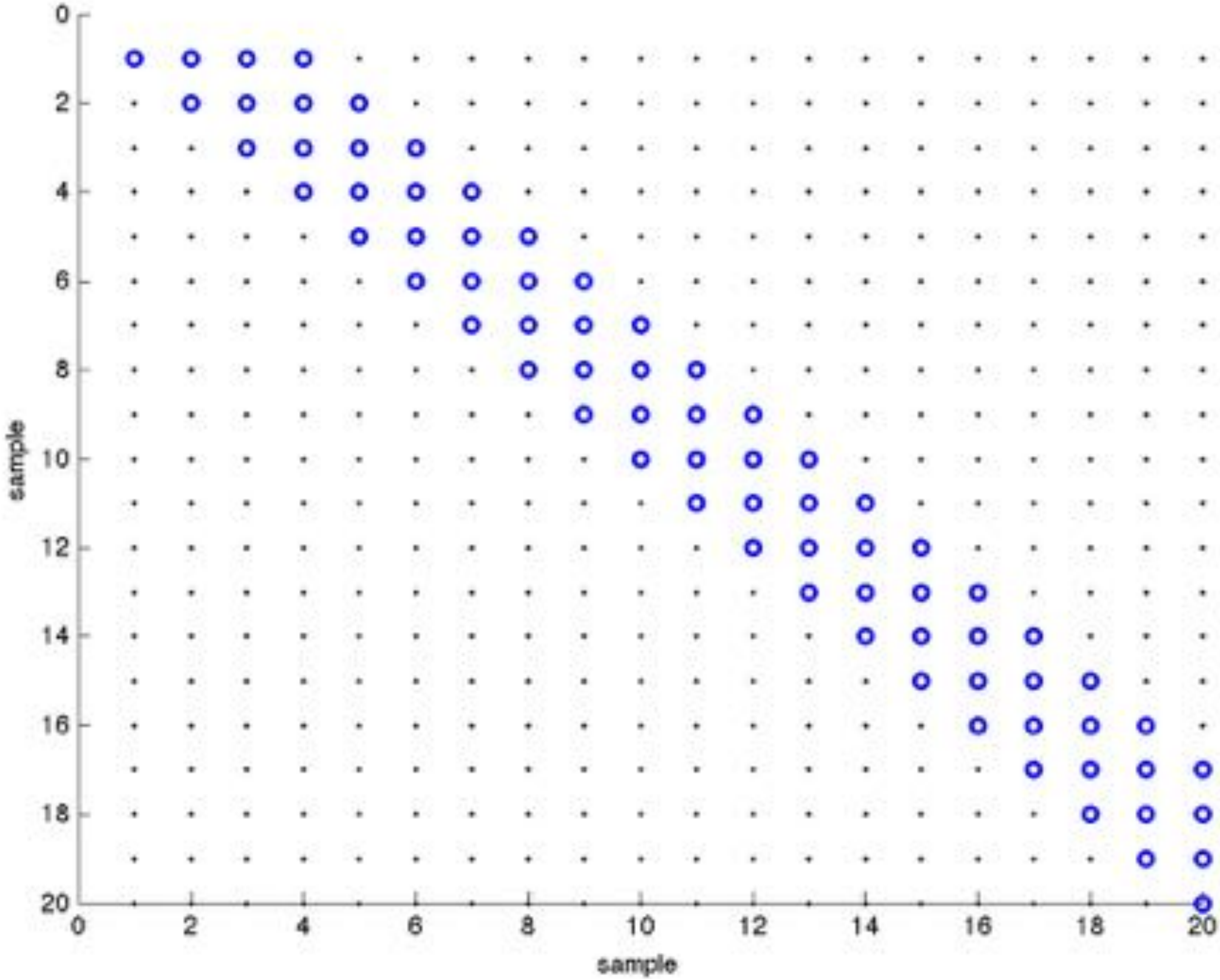
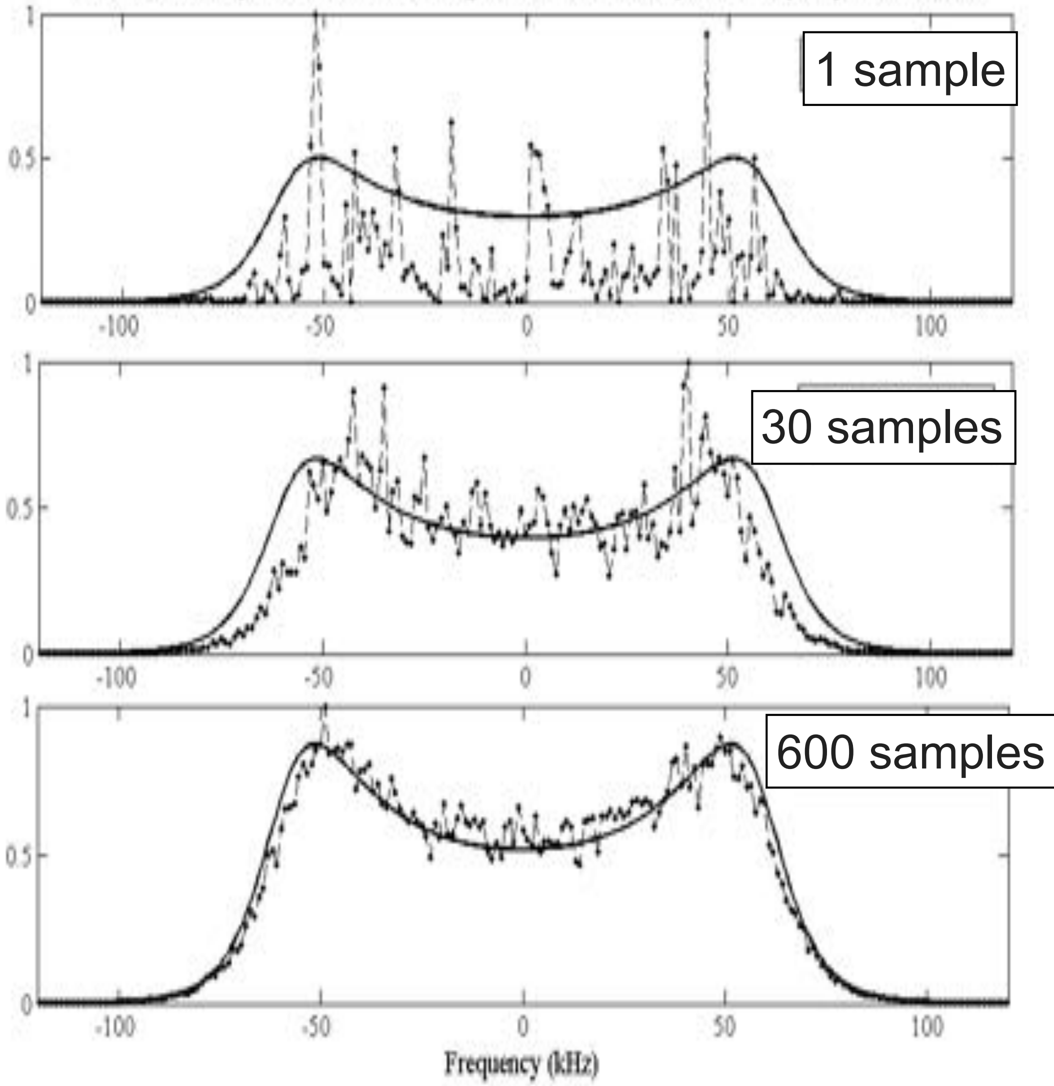
Lag-product matrix



Incoherent Averaging

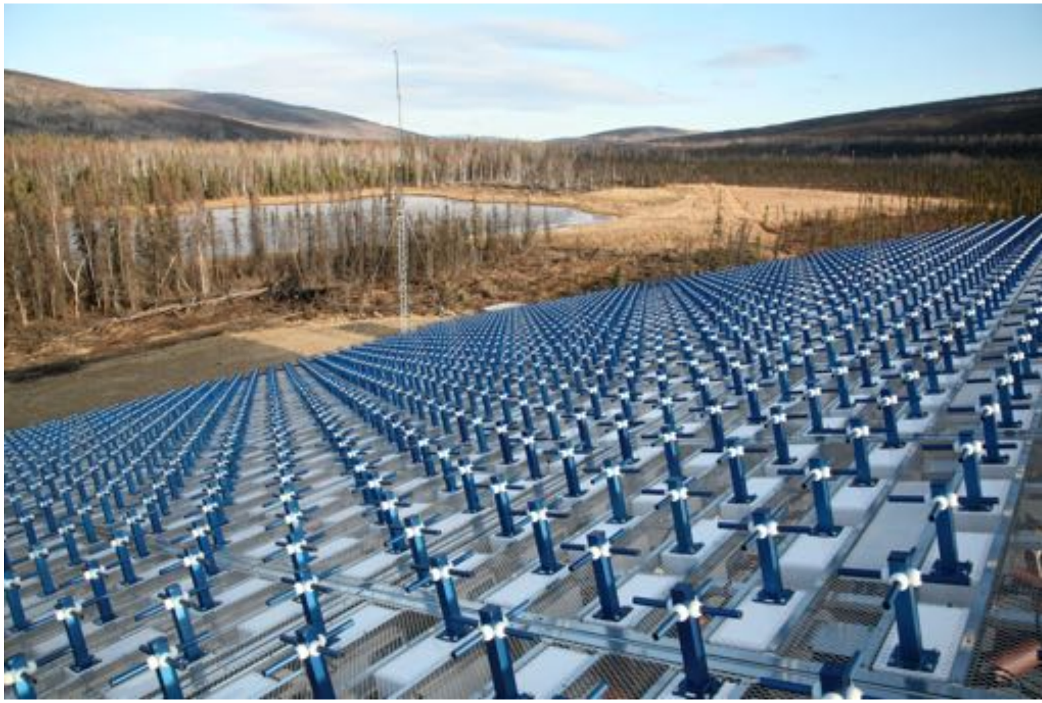
Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

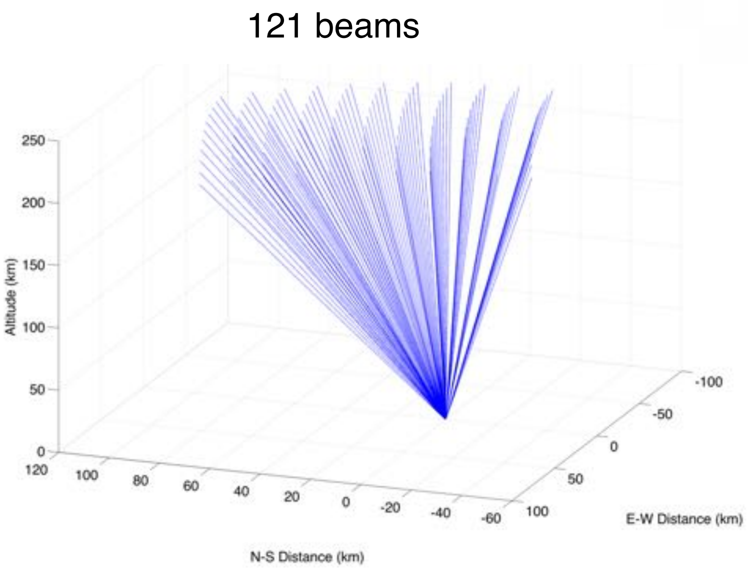
Dish Versus Phased-array

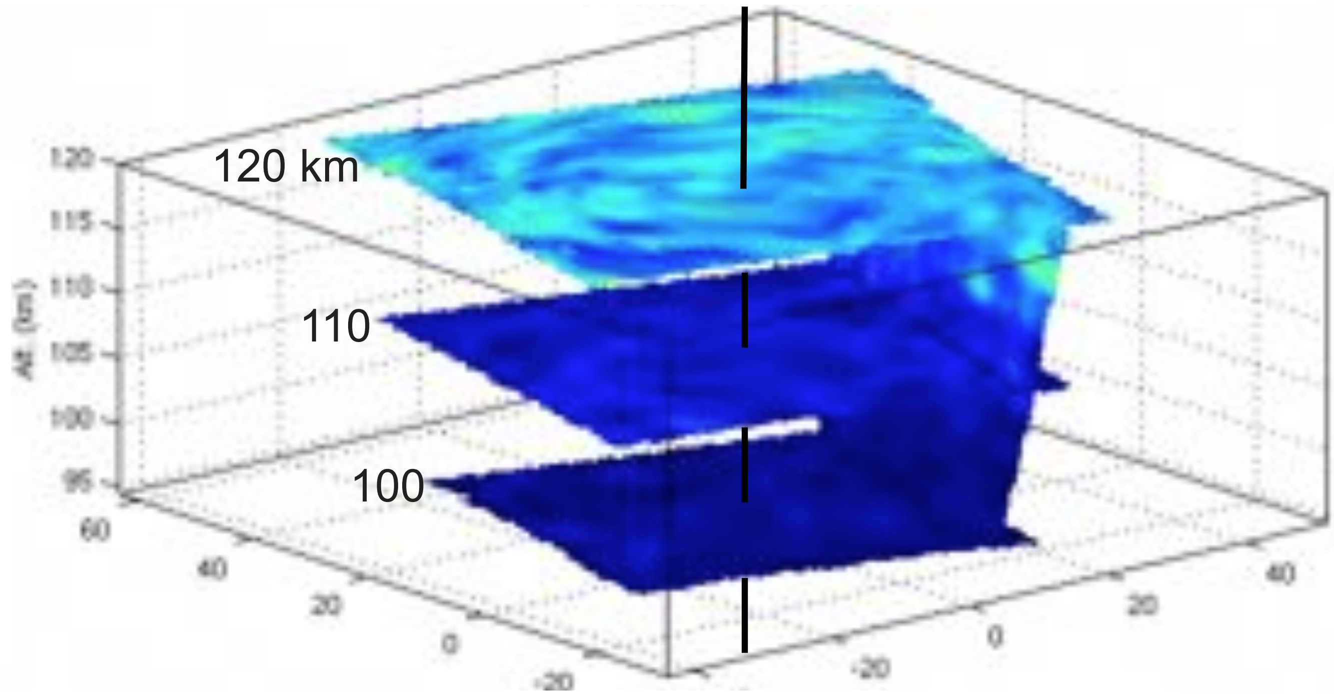


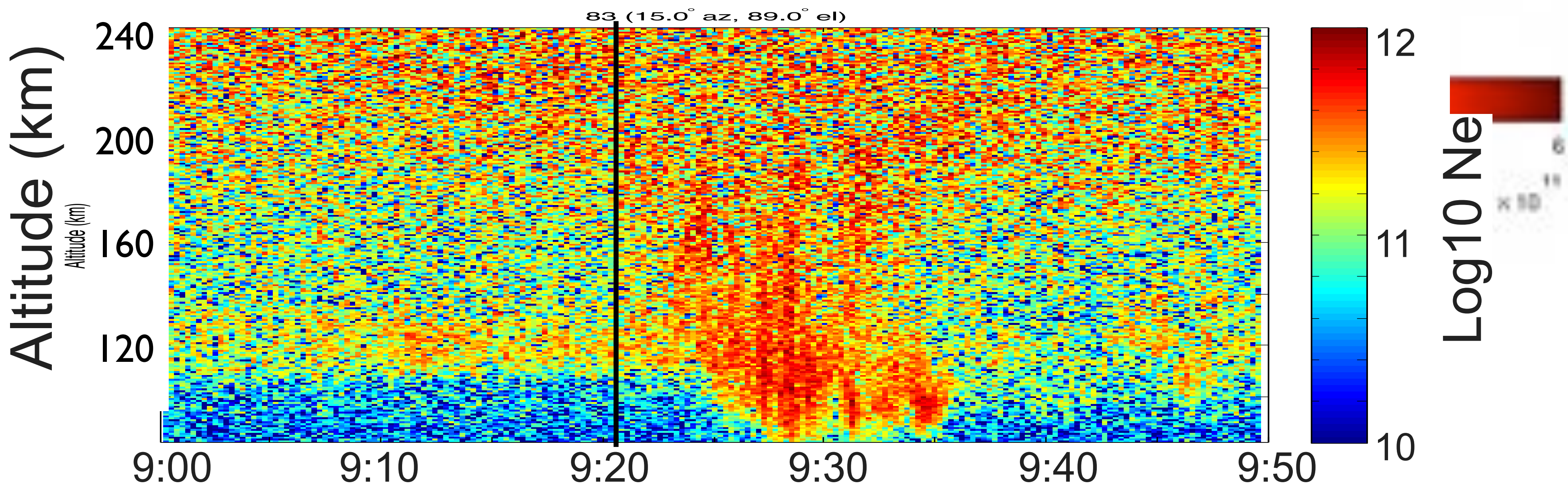
-FOV: Entire sky
-Integration at each position before moving
-Power concentrated at Klystron
-Significant mechanical complexity

-FOV: +/- 15 degrees from boresight
-Integration over all positions simultaneously
-Power distributed
-No moving parts

Three-dimensional ionospheric imaging







Semeter et al., JASTP 2009

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- Skolnik, *Introduction to Radar Systems*
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- Blahut, *Theory of Remote Image Formation*
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Background (Electromagnetics, Signal Processing:

- Ulaby, *Fundamentals of Engineering Electromagnetics*
- Cheng, *Field and Wave Electromagnetics*
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- Mitra, *Digital Signal Processing: A Computer-based Approach*

For fun:

http://mathforum.org/mbower/johnandbetty/frame.htm