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Literature Review

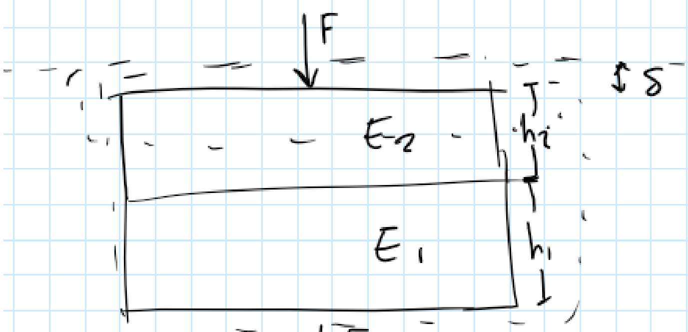
Foam properties are of interest in this project because the shoe midsole is made of this material. Foams are known as cellular solids because they are made up of many cell-like structures at the microscopic level. The article "The Mechanical Properties of Cellular Solids" by M.F. Ashby describes the structure, mechanical properties, and applications of cellular solids. Cellular solids can either be formed in nature or be man-made. Man-made cellular solids tend to be isotropic, i.e. their structure and properties do not have directionality. Cellular solids can also either be open or closed celled, describing the cells making up the foam at the microscopic level. In closed cell foams, the cells are entirely sealed, not allowing any gas to escape, and in open cell foams, the cells are not entirely sealed. Closed cell foams are of interest to us because shoe midsoles are primarily made of this type of foam. One of the most important characteristics of foam that determines its mechanical properties is the relative density of the foam, which is the density of the foam divided by the density of the solid from which it is made.

The article also states that foam has four modes of deformation, which include linear elasticity, non-linear elasticity, plastic collapse, and many different forms of fracture. There are also three sections in a stress-strain curve when a foam is compressed. These include a linear elastic section at low strains, a plateau of deformation at nearly constant stress, and a section of densification which occurs when the cell walls of the foam crush together. These three sections apply to linear, plastic, and brittle foams, but what causes the plateau is different for each type of foam. The degree of each section in the stress-strain curve depends on the relative density of the foam. Another way in which a foam can deform is by creep, which is the slow extension or compression of a material at a constant load. Creep occurs in foams when the temperature exceeds the glass temperature.

Another article "Compression properties of syntactic foams" by Nikhil Gupta describes a study in which the properties of the compression of syntactic foams were observed as the cenosphere radius ratio was varied. Cenospheres are lightweight, hollow particles that are mixed the matrix material of close cell structured polymeric foams like syntactic foams. Syntactic foams give numerous advantages over open-cell foams, including low moisture absorption and high compressive strength compared to open cell foam, high energy absorption during deformation, and a lot of design flexibility. The design flexibility can be achieved by electing the appropriate materials. The goal of this study was to investigate how the compression properties of syntactic foams changed as the cenosphere internal radius was varied, while keeping cenosphere volume fraction and the matrix resin system constant. The results of this study showed that a decrease in cenosphere internal radius corresponded to an increase in the compressive modulus and an increase in peak compressive strength. They also showed that the strain at peak compressive stress is not dependent on the cenosphere internal radius; it is a property determined by the matrix resin.

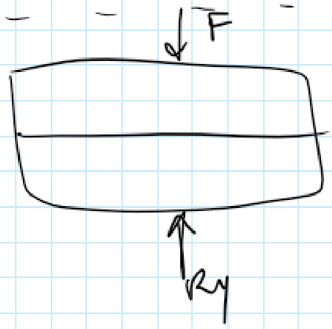
In another article by Nikhil Gupta and Eyassu Woldensenbet, the flexural behavior of syntactic foam core sandwich composites was investigated through three- and four-point bending and short beam shear strength tests. Additionally, the flexural behavior was investigated by changing the microballoon (hollow particle) radius ratio, which is the ratio of the inner to outer radius. Sandwich composites are made by attaching two thin, stiff materials around a light, thick core. The results of the study showed that the microballoon radius ratio does not affect the bending properties of syntactic foams in three- and four-point bending tests. In most of these tests, the final failure is caused by the sandwich skin fracturing on the tensile side. However, in short beam shear tests, the core shear stress and skin bending stress decrease with a decrease in the microballoon radius ratio. This means that the microballoon strength is important in determining sandwich composite mechanical properties.

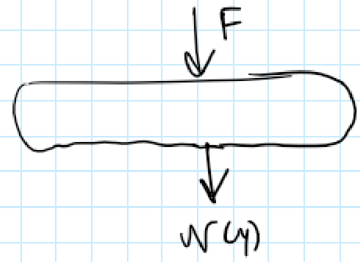
Effective Compression Modulus



undeformed

Equilibrium → Force Balance
Const. Rel → has E_1, E_2
Compatibility





$F = R_y$

$N(y) = -F$

Const. Rel

$E_1 \varepsilon_{yy} = \sigma_{yy}$
$E_2 \varepsilon_{yy} = \sigma_{yy}$

Compatibility

$\varepsilon_{1yy} = \frac{\delta_1}{h_1}$ $\quad \varepsilon_{2yy} = \frac{\delta_2}{h_2}$ $\quad \delta = \delta_1 + \delta_2$

$\sigma_{yy} = \frac{N}{A} \rightarrow N \sigma_{yy} = A$

Top Half:

$$A\sigma_{yy} = -F$$

$$AE_1 \varepsilon_{1yy} = -F$$

$$AE_1 \frac{\delta_1}{h_1} = -F$$

$$\delta_1 = \frac{-Fh_1}{AE_1} \qquad \delta_2 = -\frac{Fh_2}{AE_2} \qquad \delta = -\frac{F}{A}\left(\frac{h_1}{E_1} + \frac{h_2}{E_2}\right)$$

$$\sigma = E_{eff}\,\varepsilon \rightarrow \frac{-F}{A} = E_{eff} \cdot \frac{\delta}{h_1+h_2}$$

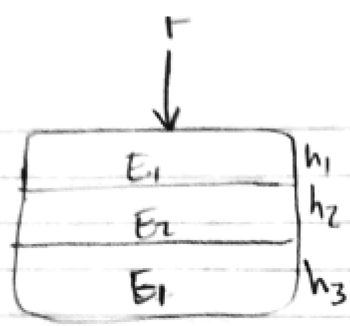
$$\rightarrow -\frac{F}{A} = \frac{\delta}{h_1+h_2} \cdot \underbrace{\frac{h_1+h_2}{\frac{h_1}{E_1} + \frac{h_2}{E_2}}}_{E_{eff}} \qquad E_{eff} = \frac{E_1E_2(h_1+h_2)}{E_2h_1 + E_1h_2}$$

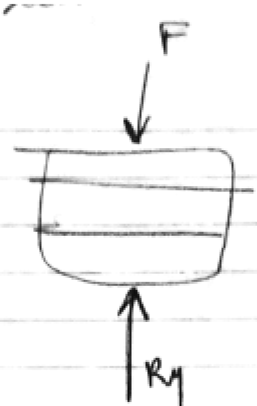
dividing top & bottom by E_1: $$E_{eff} = \frac{E_2(h_1+h_2)}{\frac{E_2}{E_1}h_1 + h_2}$$

dividing top & bottom by h_1: $$E_{eff} = \frac{E_2\left(1 + \frac{h_2}{h_1}\right)}{\frac{E_2}{E_1} + \frac{h_2}{h_1}}$$

$$\underbrace{\frac{E_{eff}}{E_2}}_{\hat{E}_{eff}} = \frac{1 + \frac{h_2}{h_1}}{\underbrace{\frac{E_2}{E_1}}_{E_R} + \underbrace{\frac{h_2}{h_1}}_{h_R}} \rightarrow \boxed{\hat{E}_{eff} = \frac{1 + h_R}{E_R + h_R}}$$

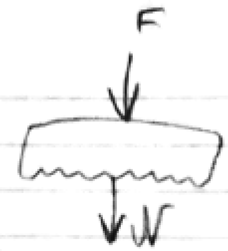
Effective Compression Modulus for N Layers





$F = R_y$

$F = -N(y)$



$$E_1 \varepsilon_1^{yy} = \sigma_{yy} \qquad \varepsilon_1^{yy} = \frac{\delta_1}{h_1} \qquad \sigma_{yy} = \frac{N}{A}$$

$$E_2 \varepsilon_2^{yy} = \sigma_{yy} \qquad \varepsilon_2^{yy} = \frac{\delta_2}{h_2} \qquad \delta = \delta_1 + \delta_2 + \delta_3$$

$$E_1 \varepsilon_3^{yy} = \sigma_{yy} \qquad \varepsilon_3^{yy} = \frac{\delta_3}{h_3}$$

$$E_1 \varepsilon_1^{yy} = N/A = -F/A$$

$$\frac{\delta_1}{h_1} E_1 = -\frac{F}{A} \rightarrow \delta_1 = \frac{-Fh_1}{E_1 A}, \quad \delta_2 = \frac{-Fh_2}{E_2 A}, \quad \delta_3 = \frac{-Fh_3}{E_1 A}$$

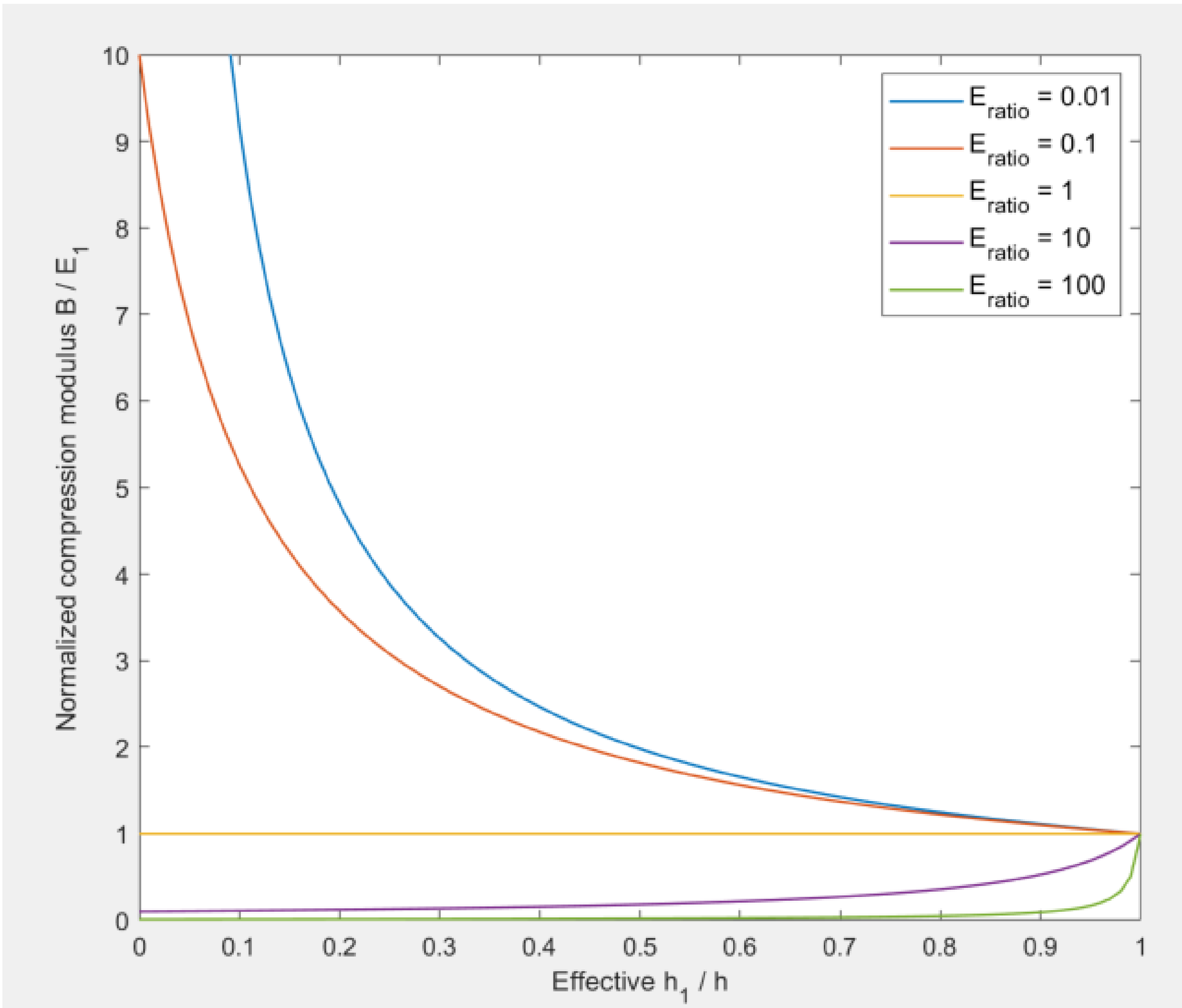
$$\delta = -\frac{F}{A}\left(\frac{h_1}{E_1} + \frac{h_2}{E_2} + \frac{h_3}{E_1}\right)$$

$$\sigma = E_{eff}\,\varepsilon \rightarrow -\frac{F}{A} = E_{eff} \cdot \frac{\delta}{h_1 + h_2 + h_3} = \frac{\delta}{h_1 + h_2 + h_3} \cdot \underbrace{\frac{h_1 + h_2 + h_3}{\frac{h_1}{E_1} + \frac{h_2}{E_2} + \frac{h_3}{E_1}}}_{E_{eff}}$$

$$E_{eff} = \frac{h_1 + h_2 + h_3}{\frac{h_1}{E_1} + \frac{h_2}{E_2} + \frac{h_3}{E_1}} \cdot \frac{E_1 E_2}{E_1 E_2} = \boxed{\frac{E_1 E_2 (h_1 + h_2 + h_3)}{h_1 E_2 + h_2 E_1 + h_3 E_2}}$$

$$\boxed{E_{eff} = \frac{E_1 E_2 \cdot \sum_{i=1}^{N} h_i}{\sum_{i=1,3,5,7}^{N} h_i E_2 + \sum_{j=2,4,6,8,\ldots}^{N} h_j E_1}}$$

Effective Modulus Plot Analysis



E_ratio = 0.01 and E_ratio = 0.1:

As the ratio of the thickness of the first material to the entire thickness of both materials increases, the normalized compression modulus of the combined materials decreases exponentially. This suggests that for a small E1 to E2 ratio, the normalized compression modulus is significantly affected by varying the thickness of material 1 by keeping the thickness of material 2 constant.

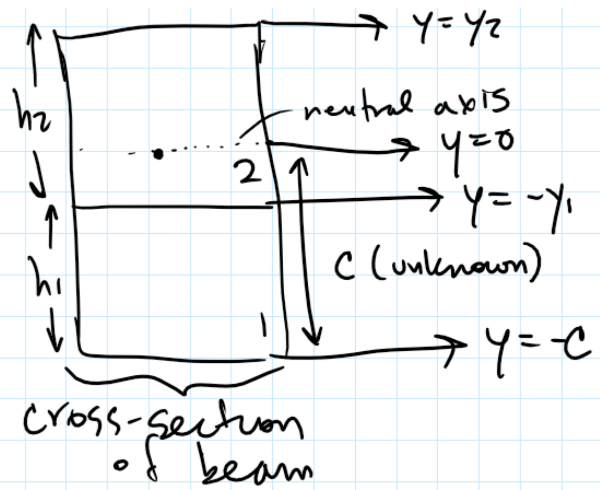
E_ratio = 1

As the h1/h ratio increases, the normalized compression modulus remains constant. This suggests that if both materials have the same Young's modulus, varying the thickness of material 1 while keeping the thickness of material 2 constant has no effect on the normalized compression modulus.

E_ratio = 10 and E_ratio = 100:

As the h1/h ratio increases, the normalized compression modulus increases very slowly from h1/h = 0 to h1/h = 0.9, then begins to rapidly increase to h1/h = 1. This suggests that for high values of E1 compared to E2, the normalized compression modulus is minimally affected by increasing the thickness of h1 while holding h2 constant.

Beam Bending



(Note: $y_2 - y_1 = h_2$
$-y_1 - (-c) = h_1$)

1st Step: find unknown c (neutral axis)

① Constitutive $\sigma_i = E_i \varepsilon_i$

② Compatibility $\varepsilon_i = -\overset{\text{curvature}}{\kappa} y$

$$\left.\begin{matrix} \sigma_i = E_i \varepsilon_i \\ \varepsilon_i = -\kappa y \end{matrix}\right\} \quad \sigma_i = -E_i \kappa y$$

③ Equilibrium

Force: $\int_{A_1} \sigma_{xx}\big|_1 \, dA + \int_{A_2} \sigma_{xx}\big|_2 \, dA = F_x = 0$

2nd Step:

Moment: $\int_{A_1} \sigma_{xx}\big|_1 \, y \, dA + \int_{A_2} \sigma_{xx}\big|_2 \, y \, dA + M = 0$

"Effective Way": $M = -E_{eff} I \kappa$

Matlab Code

```
% Step 1: defination of variables
h1_eff = 0:0.01:1;                                                 %
overall thickness of material 1
h1_eff = h1_eff';
[row, column] = size(h1_eff);
h2_eff = ones([row, column]) - h1_eff;                             %
overall thickness of material 2

E_ratio = [0.01; 0.1; 1; 10; 100];                                 %
modulus ratio of materials 1/2
B_bar = zeros(row, length(E_ratio));

% Step 2: calculation of effective compression modulus B
for i = 1:5                                                        % index
for E_ratio

    B_bar(:,i) = 1./(h1_eff + E_ratio(i)*h2_eff);                  %
dimensionless effective compression modulus B

end

% Step 3: plot results
% Plot the influence of h1_eff on B_bar
figure(1)
for i = 1:5
    plot(h1_eff, B_bar(:,i), 'LineWidth',1);
    hold on

end
hold off

% Figure setup
xlim([0,1]);
ylim([0,10]);
xlabel('Effective h_{1} / h','FontSize',11);
ylabel('Normalized compression modulus B /
E_{1}','FontSize',11);
legend({'E_{ratio} = 0.01','E_{ratio} = 0.1','E_{ratio} =
1','E_{ratio} = 10','E_{ratio} = 100'},'FontSize',11);
```