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2.980 Individual Deliverable

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Adidas: Mapping design parameters of foam composites in midsoles

1. Literature Review

1.1 Context

In 1993, Saucony was the first the first athletic shoe company to manufacture a midsole with dual-density foams. This improved stability and cushioning and also revolutionized the industry as a whole. Adidas was soon to follow with a variety of creative midsoles, including the recent Futurecraft 4D and Alphaedge 4D. Adidas has launched an effort to tailor performance products to individual physiological data and preference [1].

The midsole is of interest because significant mechanical energy is dissipated at the metatarsophalangeal (MP) joint while running or jumping. Studies have shown that increasing the midsole's longitudinal bending stiffness reduced energy dissipation and increased jump performance [2]. Specifically, insertion of carbon fiber plates into the midsole was observed to result in 1% metabolic energy savings as well as decrease in oxygen consumption rates. Study of midsole composites has potential to substantially modify shoe design.

1.2 Foam

Many types of foam are used to make shoes. Fundamentally, foam is a soft plastic with air pockets, and thus foam is lightweight, durable, and easy to compress. These properties render foam an attractive option for shoe midsoles and inserts. Generally, midsoles and inserts are both made from *closed cell foam*, in which the individual cells are closed to the environment, therefore not letting internal gas to escape or external materials to enter [3]. Ethyl vinyl acetate (EVA) is the most common type of form used for midsoles, and is available in a range of densities, stiffnesses, and formulations. For manufacturing, EVA can be hot pressed, cold presses, die cut, injected, or machined, so the material is a strong choice for a variety of needs [4]. Other common types of foam include Polyurethane (PU) and Polyethylene (PE).

Several studies have evaluated the impact of various foam properties on performance. For example, midsole hardness and thickness were both found to positively correlate with overall bending moment of the shoe during use [5]. A comparison of EVA and PU found that while EVA attained consistent values of impact damping in short-term tests, it deteriorated significantly in long-term tests while PU held up, demonstrating that PU could be the more durable and ageresistant material choice [6]. In the recent push for shoes that could provide energy return, thermoplastic polyurethane (TPU) has emerged as a popular choice, but TPU is notably heavier than EVA.

Mechanical properties of foam have also been robustly studied. Elastomeric, elastic-plastic foam, and elastic-brittle foam generally demonstrate similar trends to one another when subjected to tensile and compressive tests [7]. All foams generally display a long plateau during buckling or yielding which allows large energy absorption under a given constant load. Material, relative density and structural shape govern the behavior of the foam under load and enable us to customize foam response as desired. Elastic moduli of EVA foams increase with density, and larger shoe sizes may have higher foam densities, perhaps to provide additional support to heavier users [8].

1.3 Foam Composites

Foam composites are typically made by sandwiching a low density material between thin, stiff sheets. This effectively increases the bending stiffness of the overall structure without considerably adding weight, and improves energy absorption and shock resistance beyond that of a monolithic design [10]. Composites are used in many industries, including aerospace and biotechnology, so flexural properties of various composites have been evaluated [11]. The sheets are often manufactured from metal, glass, fiberglass, or hardened plastic.

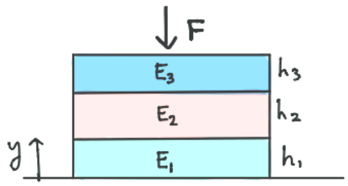
Experiments have shown that foam composite materials in midsoles exhibit strain rate-dependent response and exhibited stress accumulations at key locations on the foot. Finite element analysis has been used as an optimization method to determine ideal placement and/or structure for composites [11]. In addition, composites could prevent foam fatigue, which has been correlated to running injury due to reduced heel cushioning.

For midsoles, carbon-fiber plates have risen in popularity as a lightweight option meant to provide energy return. While foam compression itself is a prime example of energy storage with a show, the stiff plates modify the shoes' bending stiffness. They specifically reduce the flexion at the MP joint, where large amounts of energy are typically absorbed, thereby reducing energy loss during running and jumping motion [12].

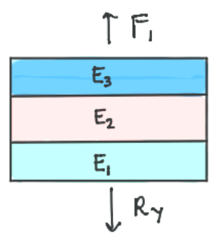
2. Compression

2.1 Stacked Composite Analysis

[Effective Compression Modulus for N-Layers]



Equilibrium (FBDs)

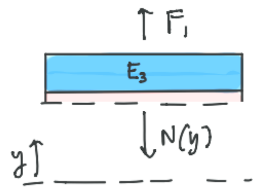


$F_1 = -F$

$\Sigma F_y = 0 = F_1 + R_y$

$F_1 = R_y$

$\Rightarrow \underline{R_y = F}$



$F_y = -F + N(y) = 0$

$\underline{F = N(y)}$

Constitutive

$\sigma_{yy} = E_1 \varepsilon_{yy,1}$

$\sigma_{yy} = E_2 \varepsilon_{yy,2}$

$\sigma_{yy} = E_3 \varepsilon_{yy,3}$

Compatibility

$$\left.\begin{aligned} \varepsilon_1 &= \delta_1 / h_1 \\ \varepsilon_2 &= \delta_2 / h_2 \\ \varepsilon_3 &= \delta_3 / h_3 \end{aligned}\right\} \quad \delta = \delta_1 + \delta_2 + \delta_3$$

$$\sigma_{yy} = N/A = F/A \Rightarrow AE_1 \varepsilon_{yy,1} = AE_2 \varepsilon_{yy,2} = AE_3 \varepsilon_{yy,3} = -F$$

$$\Rightarrow \delta_1 = -\frac{Fh_1}{AE_1} \quad (\delta_2, \delta_3 \text{ symmetric})$$

Thus, ~~total~~ deflection is:

$$\delta = \delta_1 + \delta_2 + \delta_3 \ldots + \delta_N \Rightarrow \delta = \frac{-F}{A} \sum_{i=1}^{N} \frac{h_i}{E_i}$$

Now to rewrite into E_{eff} form:

$$\underbrace{-F/A}_{=\sigma} = \underbrace{\frac{\delta}{\Sigma h_i}}_{=\varepsilon_{eff}} \underbrace{\left(\frac{\Sigma h_i}{\Sigma h_i / E_i} \right)}_{=E_{eff}}$$

$$E_{eff} = \frac{\prod_{i=1}^{N} E_i \cdot \sum_{i=1}^{N} h_i}{\sum_{j=1}^{N} \left(\left(\prod_{i=1}^{N} E_i \right) \frac{h_j}{E_j} \right)}$$

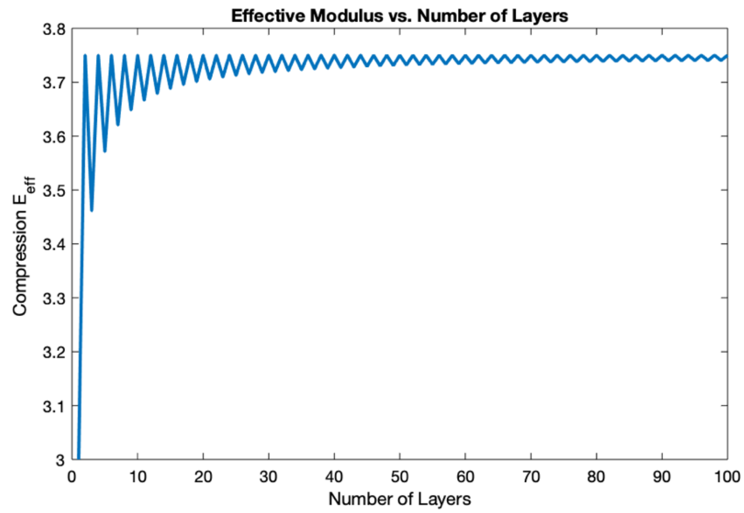
$$E_{eff} = \frac{E_1 \cdot E_2 \cdot \sum_{i=1}^{N} h_i}{\sum_{i=1,3,5\ldots}^{N} h_i \cdot E_2 + \sum_{j=2,4,6}^{N} h_j \cdot E_1}$$

Since we only have E_1 and E_2, this can further be simplified!

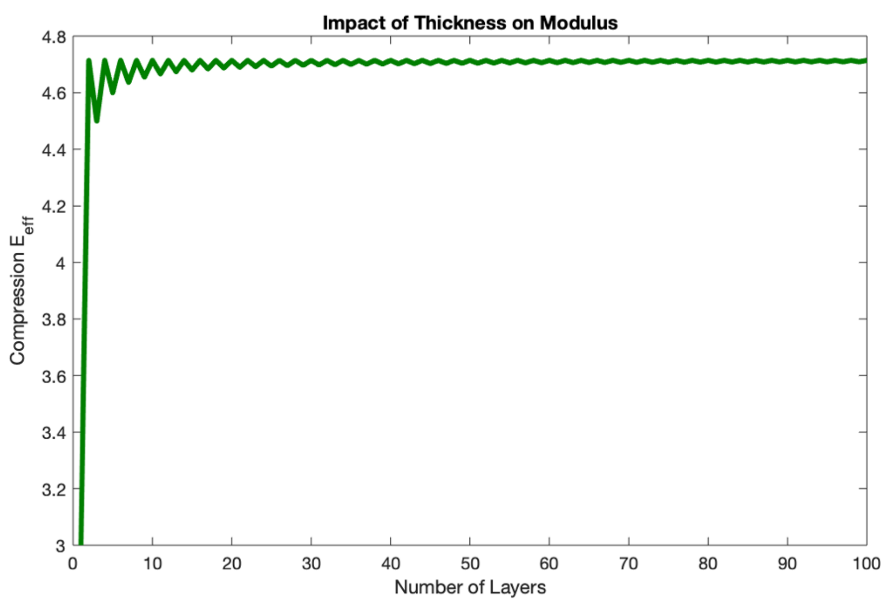
2.2 Discussion of Trends

For the following cases, an E1 value of 3 and an E2 value of 5 was used as a simple example.

Varying the **number of layers** with other parameters (ratio of E1 to E2, thickness) kept constant results in gradual convergence of the effective modulus. As the number of layers increases, E_eff oscillates around a constant value, and the amplitude of oscillations diminishes.



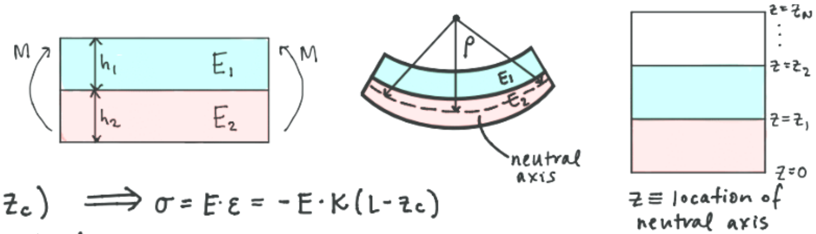
Impact of varying the **layer thickness** was also considered. If layers of both materials were changed uniformly, no notable changes were observed to have occurred from the previous case. Thus, we believe that the ratio of thicknesses is crucial, but the thicknesses alone are less important for assessing the effective modulus. As the number of layers was increased, the modulus once again oscillated around a final value. However, if the **thickness ratio** was changed — that is, if one material's thickness was changed while the other was held constant — the effective modulus value shifts drastically toward the E value of the thicker material, as expected. In general, the curve is also dampened more quickly and we observe less amplitude of oscillation at every point.



3. Bending

3.1 Stacked Composite Analysis

Effective Bending Modulus



$$\varepsilon = Ky = -K(L - z_c) \implies \sigma = E\cdot\varepsilon = -E\cdot K(L - z_c)$$

Stress along the neutral axis = 0, so assuming uniform width:

$$\int_{A_i} -E_i K (L - z_c)\, dA = 0 = \int_{L_{i-1}} E_i (L - L_c)\, dz = 0$$

$$\implies \sum_i \tfrac{1}{2} E_i \left(L_i^2 - L_{i-1}^2\right) - \sum_i E_i L_c (L_i - L_{i-1}) = 0 \implies \boxed{z_c = \frac{\sum_{i=1}^{N} E_i \left(L_i^2 - L_{i-1}^2\right)}{2\cdot\sum_{i=1}^{N} E_i \left(L_i - L_{i-1}\right)}}$$

position of neutral axis from bottom layer

For 2 layers, this simplifies to:

$$\boxed{z_c = \frac{E_1\left(L_1^2 - L_0^2\right) + E_2\left(L_2^2 - L_1^2\right)}{2\left(E_1(L_1 - L_0) + E_2(L_2 - L_1)\right)}}$$

Moment Balance (at each cross-section):

$$\int_{A_i} \sigma\cdot y\cdot A = \sum_i \int_{A_i} \sigma (L - L_c)\, dA + M = \sum_i \int_{A_i} \sigma (L - L_c)\, d(b(L - L_c)) + M = 0$$

$$= \sum_i \int -E\cdot K (L - z_c)(L - L_c)\, d(b(L - L_c)) + M = 0$$

$$M = \tfrac{1}{3}\cdot E_i\cdot (L - L_c)^3 \Big|_{L_{i-1}}^{L_i} \cdot b\cdot K \quad \text{\& since } M = E_{eff}\cdot I\cdot K,$$

$$b\cdot \sum \tfrac{1}{3} E_i \underbrace{\left[(L_i - L_c)^3 - (L_{i-1} - L_c)^3\right]}_{= I_1} = b\cdot E_{eff}\cdot \tfrac{1}{3} y^3 \Big|_{-h/2}^{h/2} = \tfrac{1}{12} h^3\cdot E_{eff}$$

To nondimensionalize, dividing both sides by $h^3 = z_n^3$

$$\implies \frac{1}{12} E_{eff} = \sum_{i=1}^{N} \frac{1}{3} E_i \left(\left(\frac{z_i}{z_n} - \frac{z_c}{z_n}\right)^3 - \left(\frac{z_{i-1}}{z_n} - \frac{z_c}{z_n}\right)^3 \right)$$

$$\implies \boxed{E_{eff} = 4 \sum_{i=1}^{N} E_i \left(\left(\frac{z_i}{z_n} - \frac{z_c}{z_n}\right)^3 - \left(\frac{z_{i-1}}{z_n} - \frac{z_c}{z_n}\right)^3 \right)}$$

To simplify:

$$z_c / z_n = \frac{\sum E_i \left(\frac{z_i}{z_n}^2 - \frac{z_{i-1}}{z_n}^2\right)}{2\cdot \sum E_i \left(z_i / z_n - z_{i-1} / z_n\right)}$$

plug back in!

For the simple case where $N = 2$:

$$z_c / z_2 = \frac{1}{2}\cdot \frac{E_1\left(z_1/z_2{}^2 - z_0/z_2{}^2\right) + E_2\left(z_2/z_2{}^2 - z_1/z_2{}^2\right)}{E_1\left(z_1/z_2 - z_0/z_2\right) + E_2\left(z_2/z_2 - z_1/z_2\right)}$$

$$\implies z_c / z_2 = \frac{E_1\left(z_1/z_2\right)^2 + E_2\left(1 - z_1/z_2{}^2\right)}{E_1\left(z_1/z_2\right) + E_2\left(1 - z_1/z_2\right)}$$

3.2 Discussion of Trends

I ran into some challenges while translating the above derivations into MATLAB! A work-in-progress script is shared on the wiki, but trends across various parameters weren't able to be evaluated yet.

Broadly speaking, based on the derivation itself, we can see that E_i is generally multiplied with its corresponding length value. This suggests that like in the compression modulus case, the influence of each material will correspond largely with its thickness. Specifically, if we non-dimensionalize the modulus, we will be able to consider one parameter — ratio of thicknesses — which should relate to how much one material will contribute to the effective modulus compared to the other.

Similarly, as N does not come into play into either equation, we should expect initially large amplitude of oscillation and eventual convergence around an asymptote as the number of layers increases. Largely, it seems as though the bending modulus follows the same general trends demonstrated by the compression modulus, although an edited MATLAB model will aid greatly in confirming this.

References

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[2] Roy et. al. "Shoe Midsole Longitudinal Bending Stiffness and Running Economy, Joint Energy, and EMG." April 2006.
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