Phased Arrays 2: Fourier Analogy

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Fourier Analogy

Array factor looks like a Discrete-Space Fourier Transform

$$F(\theta) = \sum_{n=0}^{N-1} I_n e^{jnkd\cos\theta}$$

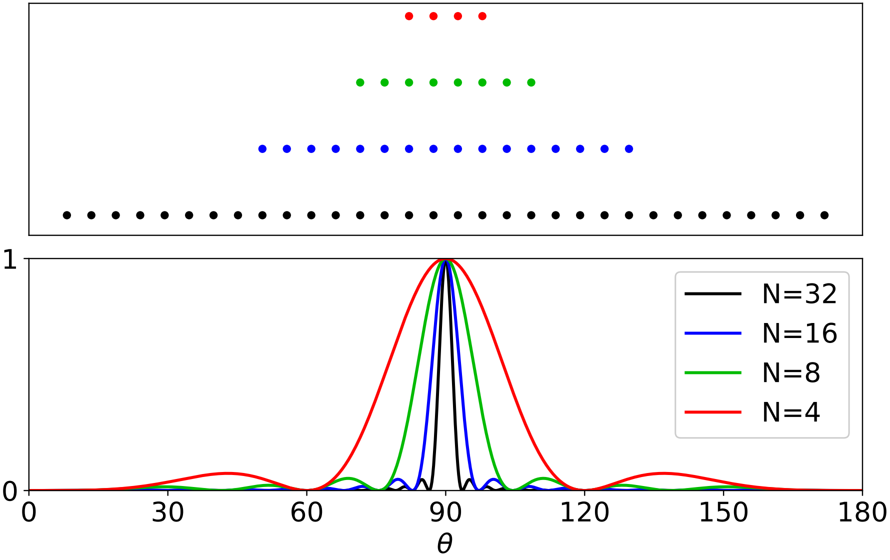
$$F(\gamma) = \sum_{n=0}^{N-1} I_n e^{jn\gamma} \qquad \gamma \equiv kd\cos(\theta)$$

Inverse Transformation

$$I_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\gamma)\, e^{-jn\gamma}\, d\gamma$$

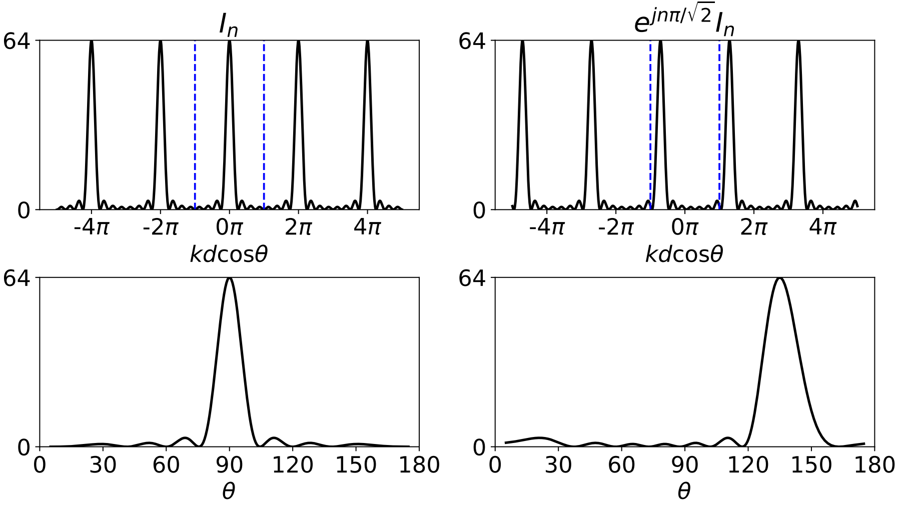
Designing I_n to approximate a desired $F(\gamma)$ is similar to designing FIR filters.

Wide in Space ↔ Narrow Beamwidth



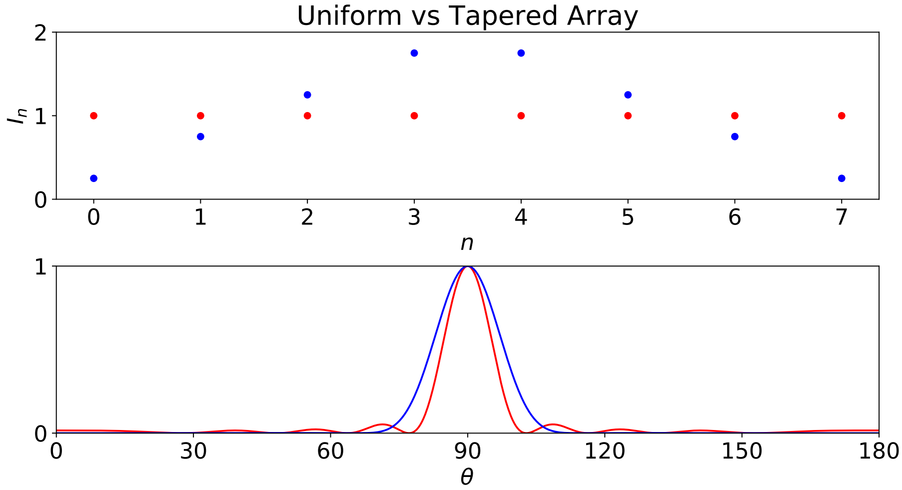
Shift Theorem

$$F(\gamma) \rightarrow F(\gamma - \gamma_0) \Leftrightarrow I_n \rightarrow e^{jn\gamma_0} I_n$$

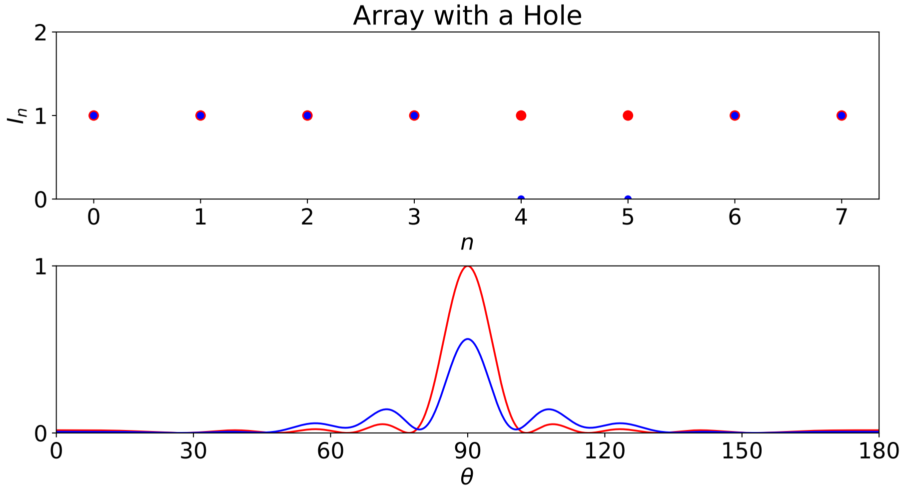


Array Tapers and Windowing

$$I_n \rightarrow w_n I_n \Leftrightarrow F(\gamma) \rightarrow W(\gamma) * F(\gamma)$$

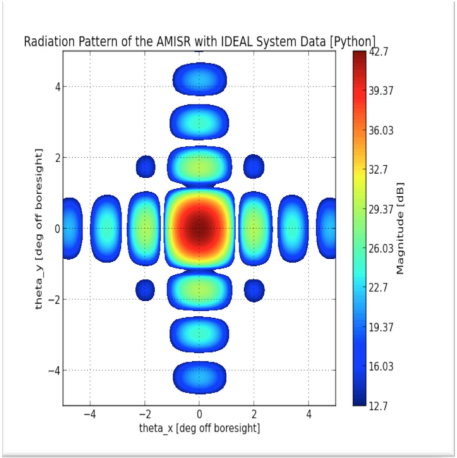


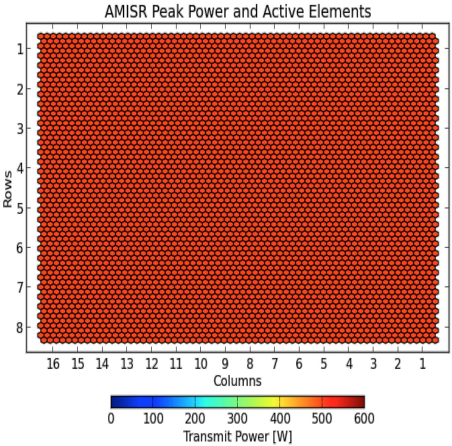
Arrays with Missing Elements



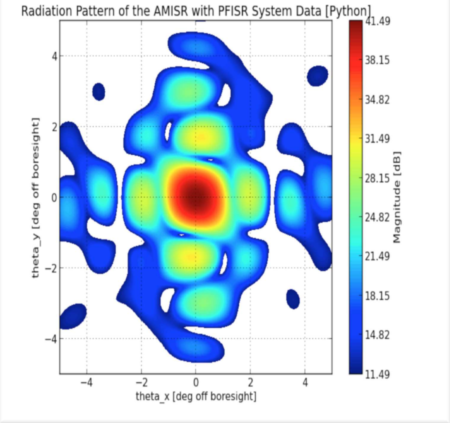
Missing elements reduces power in main beam and raises sidelobes

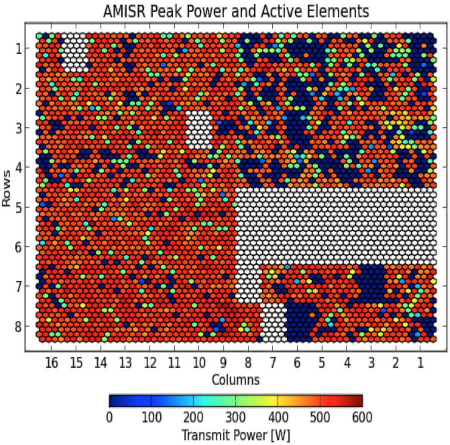
Ideal AMISR Radiation Pattern





AMISR Graceful Degradation





Fourier Analogy Summary

- In Fraunhofer diffraction the far field radiation pattern is a Fourier transform of the aperature illumination
- Phased array far field radiation patterns are a discrete-space Fourier transform of element currents
- Fourier analogy provides useful intuition for designing radiation patterns by reusing concepts from signal processing
- Shift theorem provides recipe for steering beams