ISR Theory 4: Total Scattered Power

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Total Scattered Power

$$\left\langle |n_e(\mathrm{k},\omega)|^2 \right\rangle = \left|1 - \frac{\chi_e}{\epsilon}\right|^2 \left\langle |n_{te}(\mathrm{k},\omega)|^2 \right\rangle \qquad \text{Electron Line}$$

$$+ \left|\frac{\chi_e}{\epsilon}\right|^2 \left\langle |n_{ti}(\mathrm{k},\omega)|^2 \right\rangle \qquad \text{Ion Line}$$

Total radar cross section

$$\sigma = \sigma_e V \int \left\langle |n_e(\mathrm{k},\omega)|^2 \right\rangle \frac{d\omega}{2\pi}$$

$$= \sigma_e V \left[\frac{k^2\lambda_{De}^2 N_e}{1 + k^2\lambda_{De}^2} + \frac{N_e}{\left(1 + k^2\lambda_{De}^2\right)\left(1 + k^2\lambda_{De}^2 + \frac{T_e}{T_i}\right)} \right]$$

Where

$$\sigma_e = 10^{-28}\ \mathrm{m}^2 \qquad k = \frac{4\pi}{\lambda} \qquad \lambda_{De} = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 N_e}}$$

Collective and Non-Collective Limits

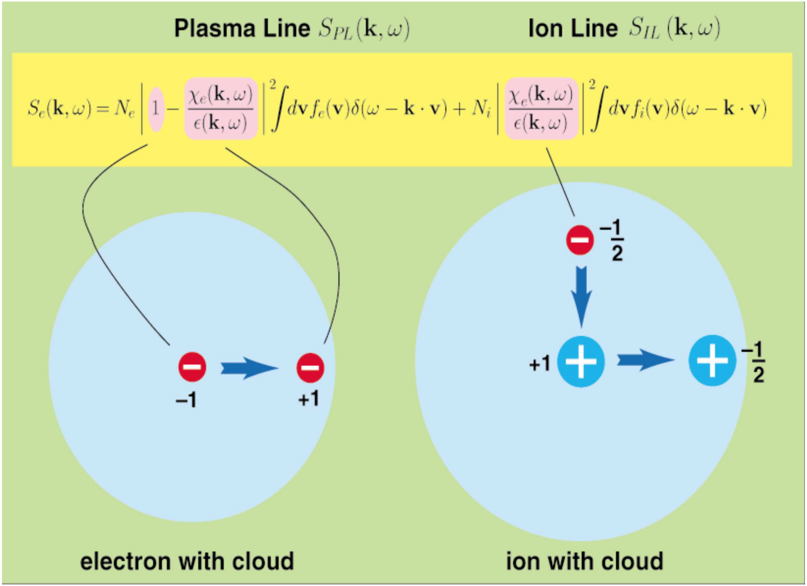
- **Non-Collective Limit:** $k^2\lambda_{De}^2 \gg 1$
  Electron line dominates (wide bandwidth)

$$\sigma = \sigma_e V N_e$$

- **Collective Limit:** $k^2\lambda_{De}^2 \ll 1$
  Ion line dominates (narrow bandwidth)

$$\sigma = \sigma_e V \frac{N_e}{1 + \frac{T_e}{T_i}}$$

Dressed Particle Concepts



Reporting Electron Density from Ion Line Power

Ion Line Cross Section:

$$\sigma = \sigma_e V \frac{N_e}{2} \zeta$$

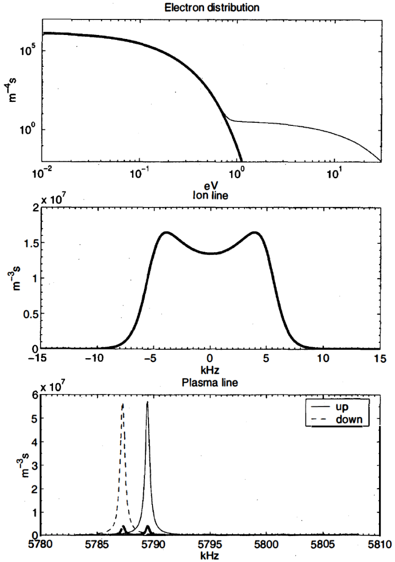
Temperature Correction: $\zeta = \dfrac{2}{\left(1 + k^2\lambda_{De}^2\right)\left(1 + k^2\lambda_{De}^2 + T_e/T_i\right)}$

- Uncorrected N_e: Assume $\zeta = 1$.
  - $T_e/T_i = 1$
  - $k^2\lambda_{De}^2 \ll 1$.
- N_e with model: Compute ζ using an empirical model of T_e/T_i as a function of altitude.
- N_e with fits: Compute ζ with T_e and T_i estimated from fitted ACF.

Enhanced Plasma Lines

- Circumstances where
  $\sigma_{PL} \gg \sigma_e V N_e \frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}$
  - Daytime (Photoelectrons)
  - Conjugate Sunrise (Conjugate Photoelectrons)
  - Auroral Upwards FAC (Precipitating Electrons)
- Fast electrons resonantly enhance Langmuir waves when they match the wave phase velocity
- Enhanced plasma lines are scatter from "dressed" fast electrons

Häggström et al. (2000) Adv. Polar Upper Atmos. Res., **14**, 103-121



Total Scattered Power Summary

- In the collective regime $\sigma \neq \sigma_e V N_e$
- Correction terms can be understood using dressed particle theory concepts
- Corrections depend on temperature ratio (T_e/T_i) and Debye length
- ISR typically report both uncorrected N_e (from power) and corrected N_e (from fitted ACFs)
- Dressed particle theory concepts also explain enhanced plasma line observations