Radar Signal Processing: Part 2

Doppler Spectrum, Bragg Scatter, and the ISR Power Spectrum

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Doppler Radar Summary: "Coherent" hard targets

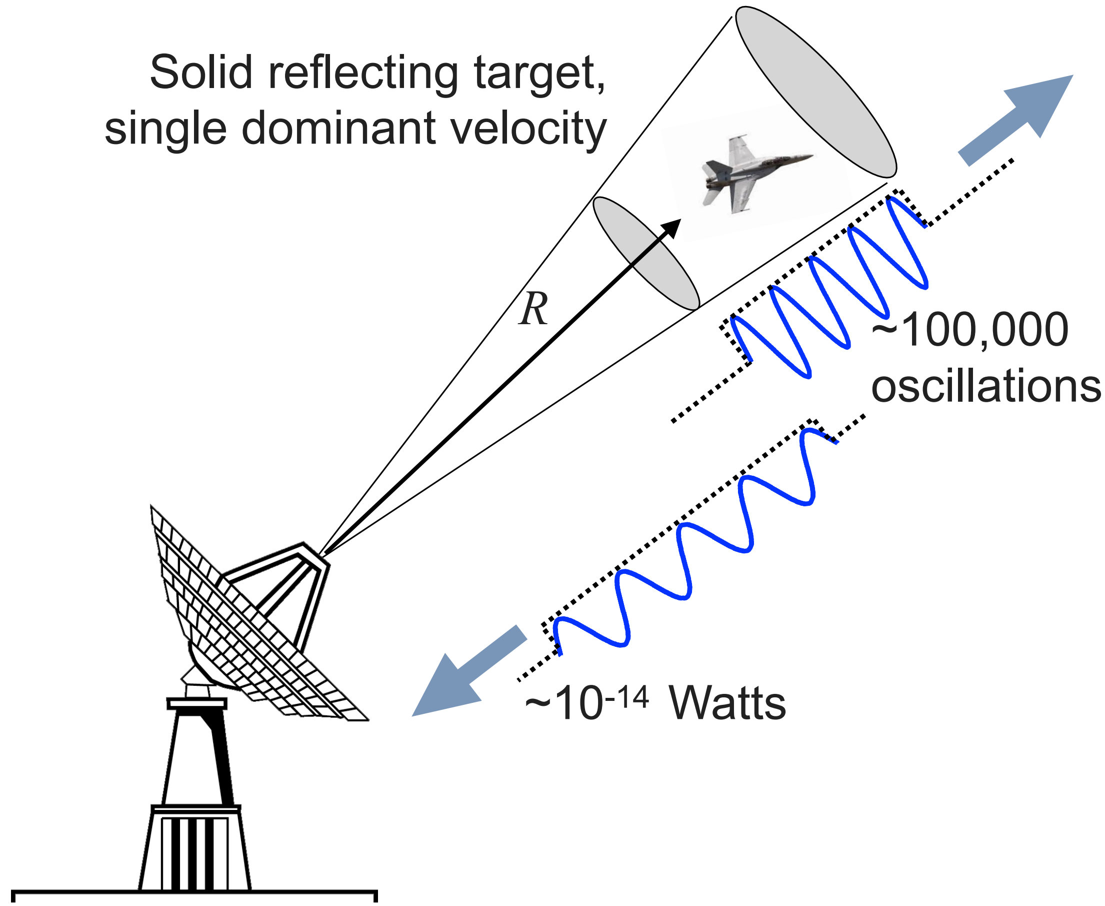
Two key concepts:

Time ⇔ Distance

$$R = -\frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$



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Velocity is manifested as a Doppler frequency shift in the received signal.

What happens when we have multiple targets in the radar volume, moving at different velocities?

Concept of a "Doppler Spectrum"

Superposition of targets moving with different velocities within the radar volume

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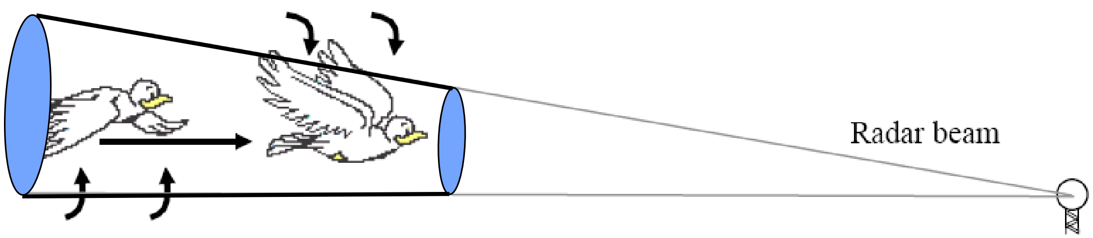

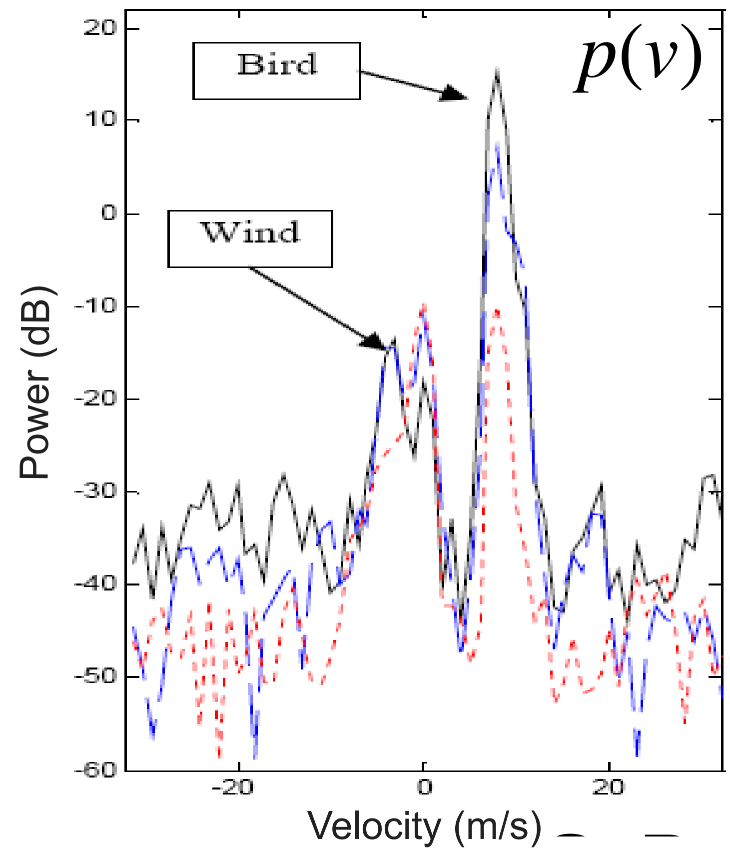

Processing: $p(R, f_D) \rightarrow p(v)$

If there is a distribution of targets withdifferent velocities (e.g., bird, flapping wings,wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed "beam filling" Target

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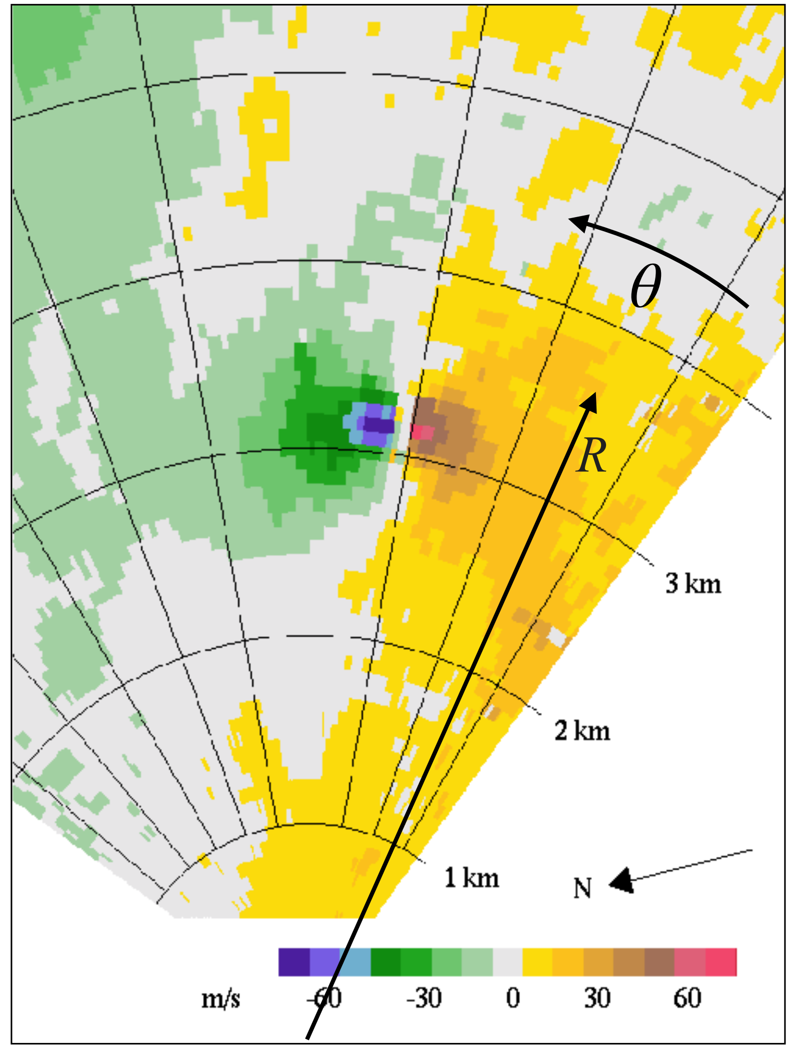


Processing:

$$p(R, f_D, t) \rightarrow f_D(R, t) \rightarrow v(R, \theta)$$

For a beam-filling target (like water droplets in a tornado), the radar can be used to construct insightful images of velocity relative to the radar.

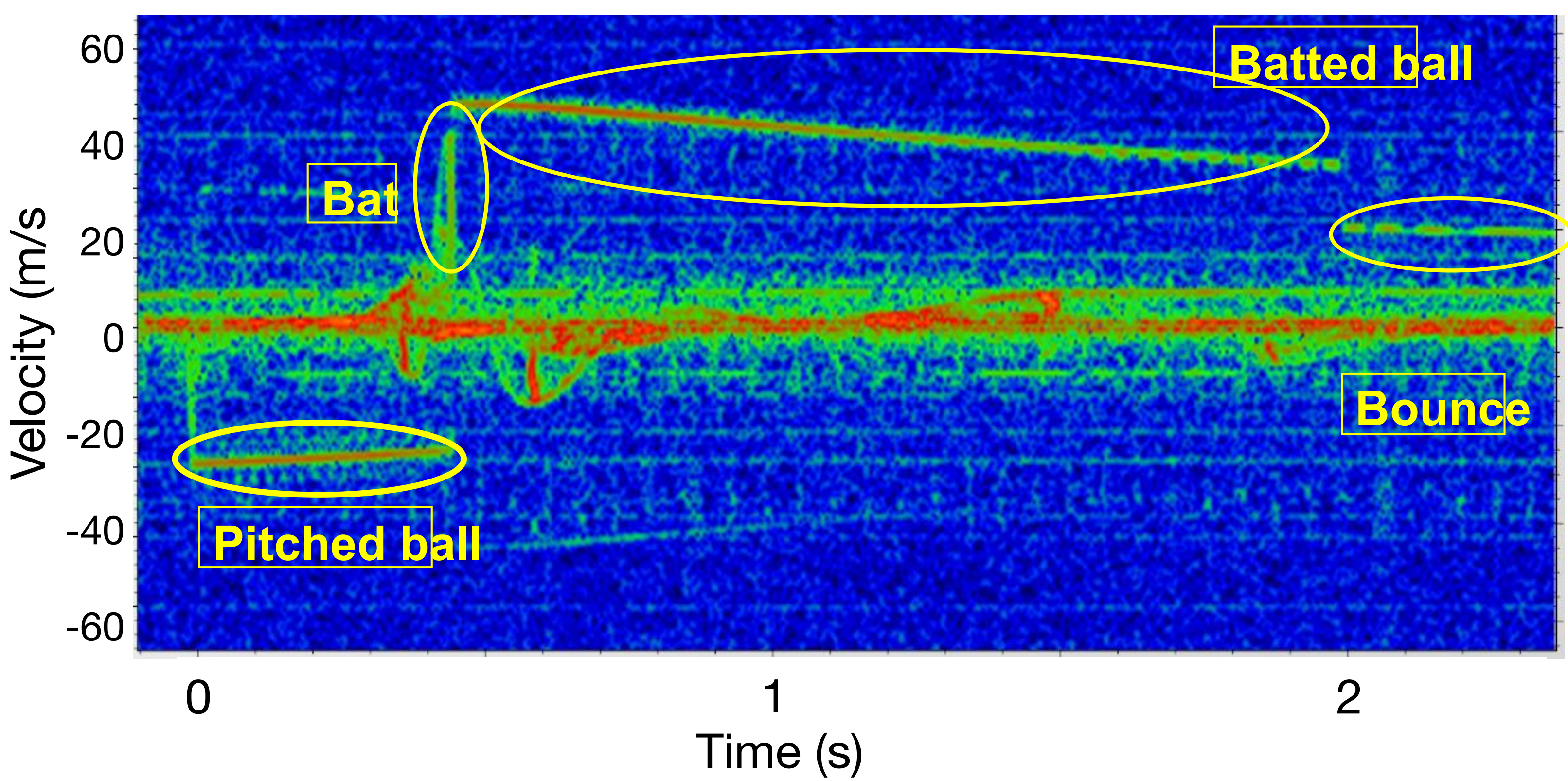
Micro-Doppler Analysis

Trackman radar: "continuous wave" (CW) radar: precise Doppler but no range information.

Can identify targets and actions based on Doppler signatures!

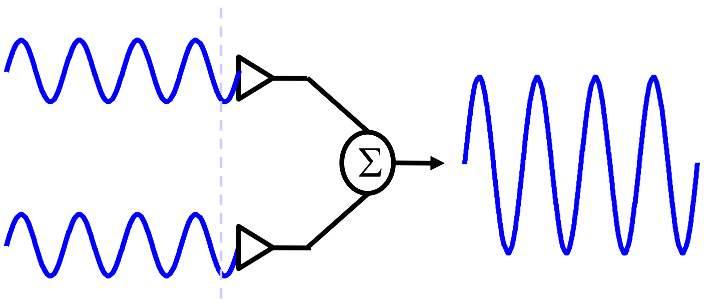
Processing:

$$p(f_D, t) \rightarrow p(v, t)$$

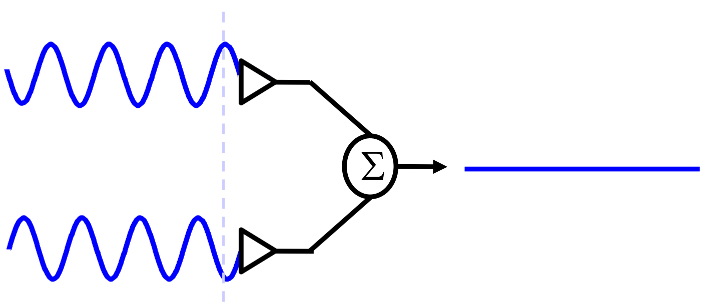


Wave Interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

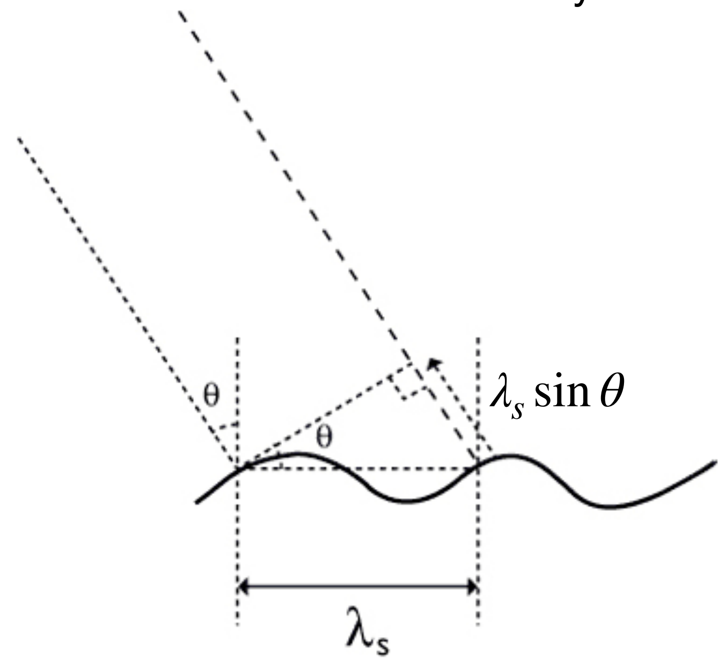


Constructive
(in phase)



Destructive
(180° out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

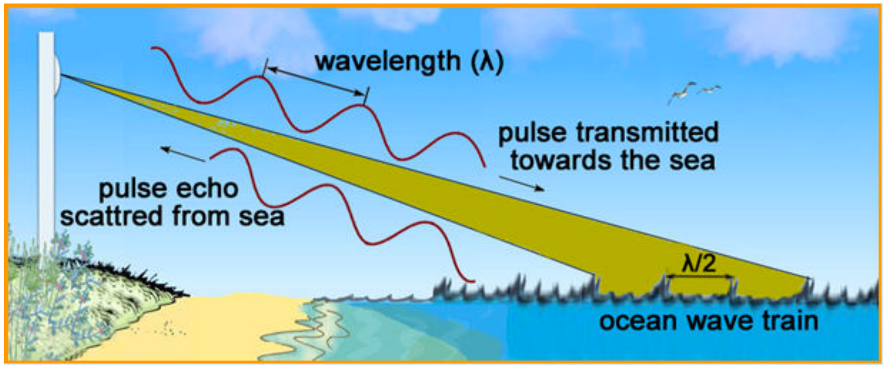


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

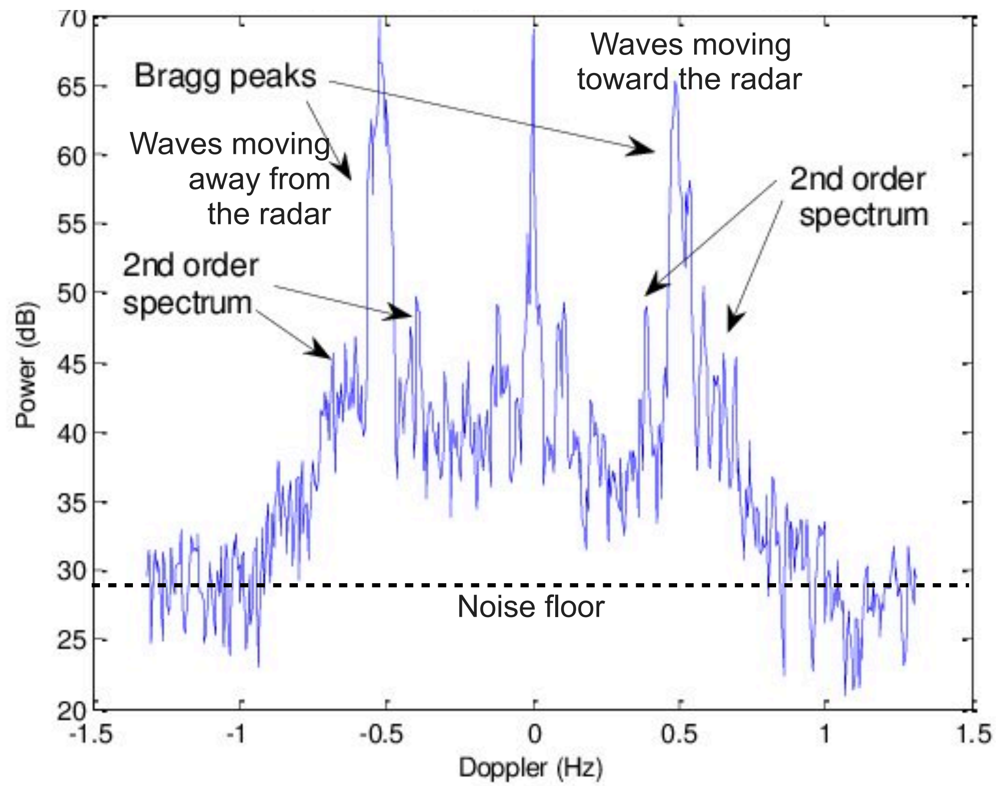
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90°$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing $\simeq$ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.
The scattering is from free electrons in the conducting sea water.
The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.
The frequency of the peaks tells us the velocity and direction of the waves.
The height of the peaks tells us something about the amplitude and density of the waves.
The width of the peaks tells us something about the spread in velocity of the waves

Doppler spectrum of the ionosphere

Let's put this all together for the ionosphere. The two predominant longitudinal modes in a thermal plasma:

Ion-acoustic mode:

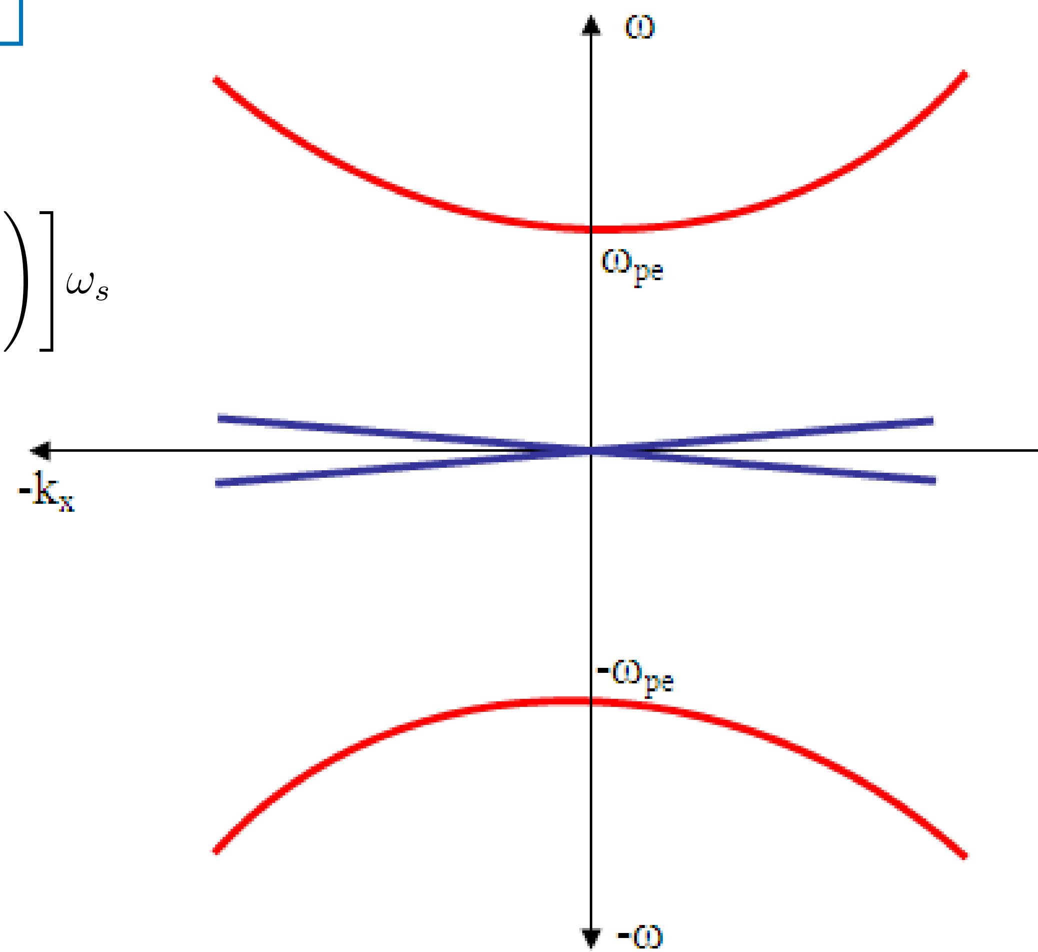
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

Langmuir mode:

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the}\lambda_{De}k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$



Computer simulation of the ionosphere

Simple rules yield complex behavior

Particle-in-cell (PIC) simulation:

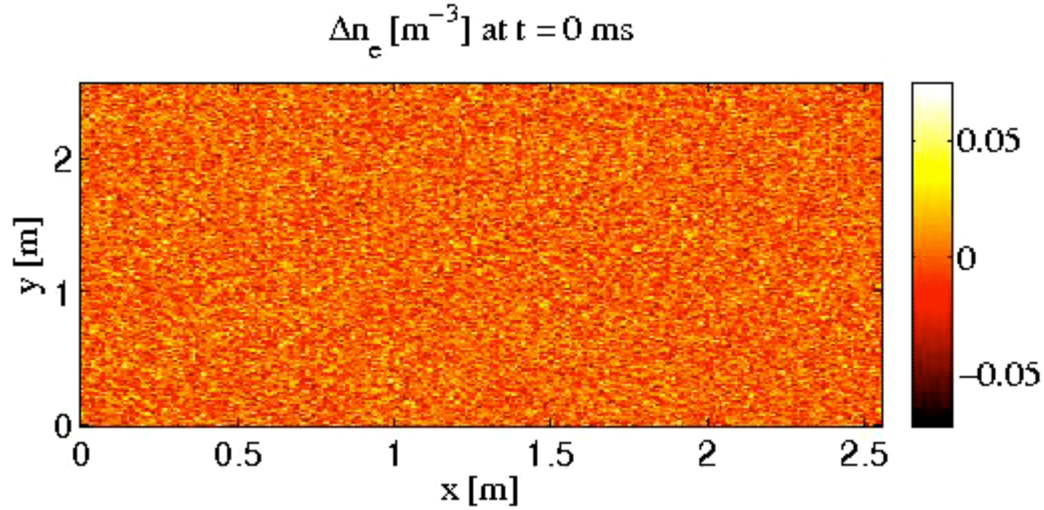
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

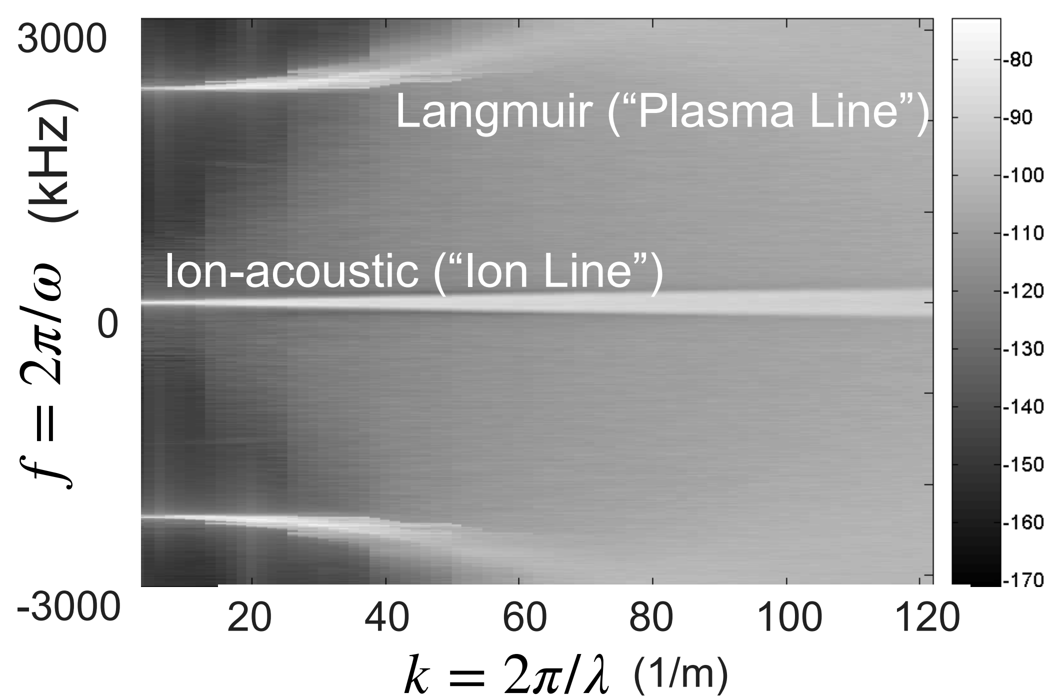
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

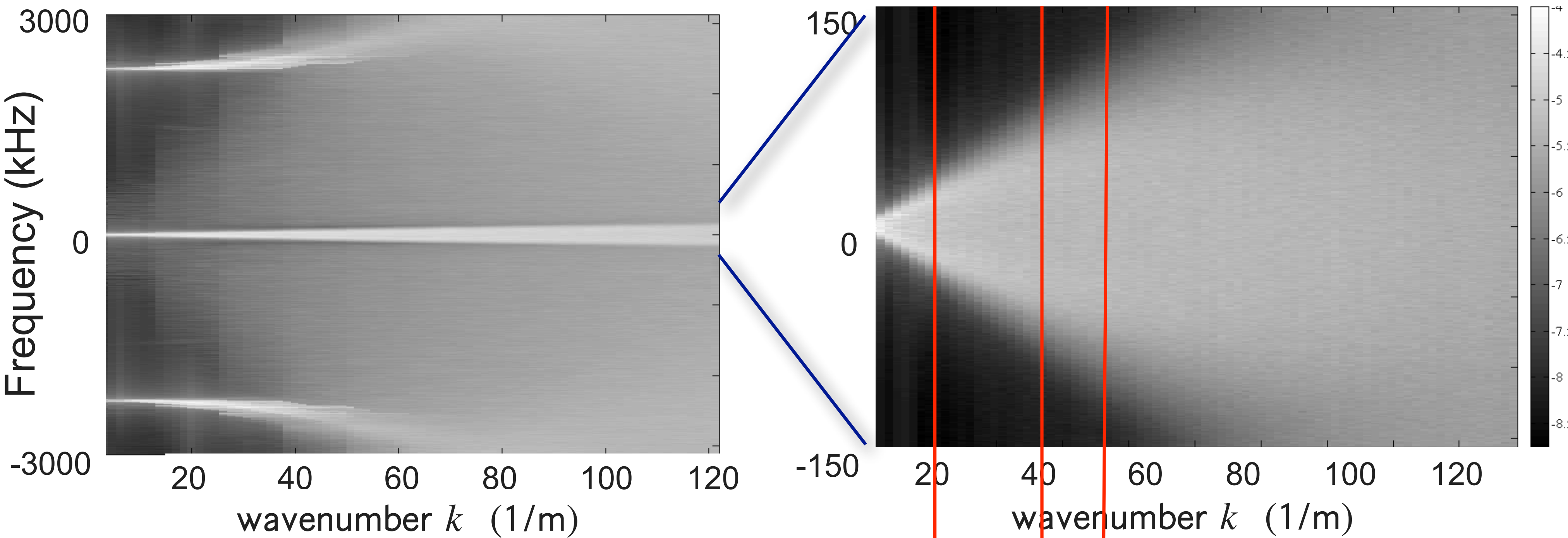
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$





ISR measures a cut through this surface

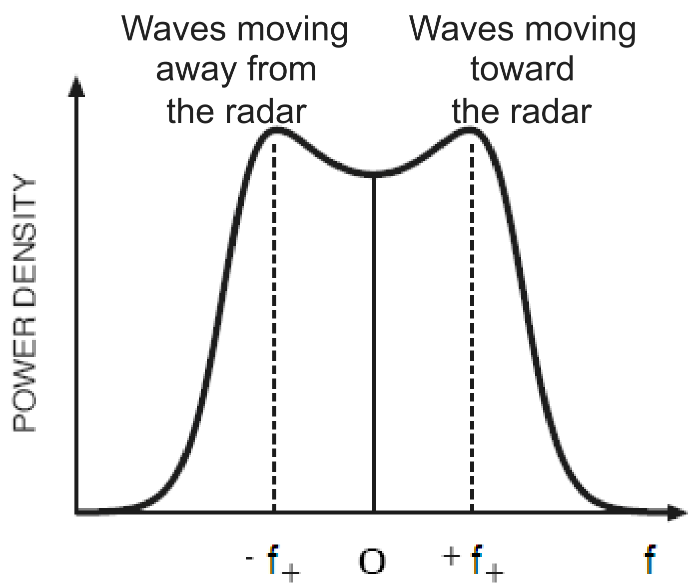


Sondrestrom (1.2 GHz)

EISCAT UHF (930 MHz)

AMISR (450 MHz),
Millstone (440 MHz)

Ion-acoustic "lines" are broadened by Landau damping



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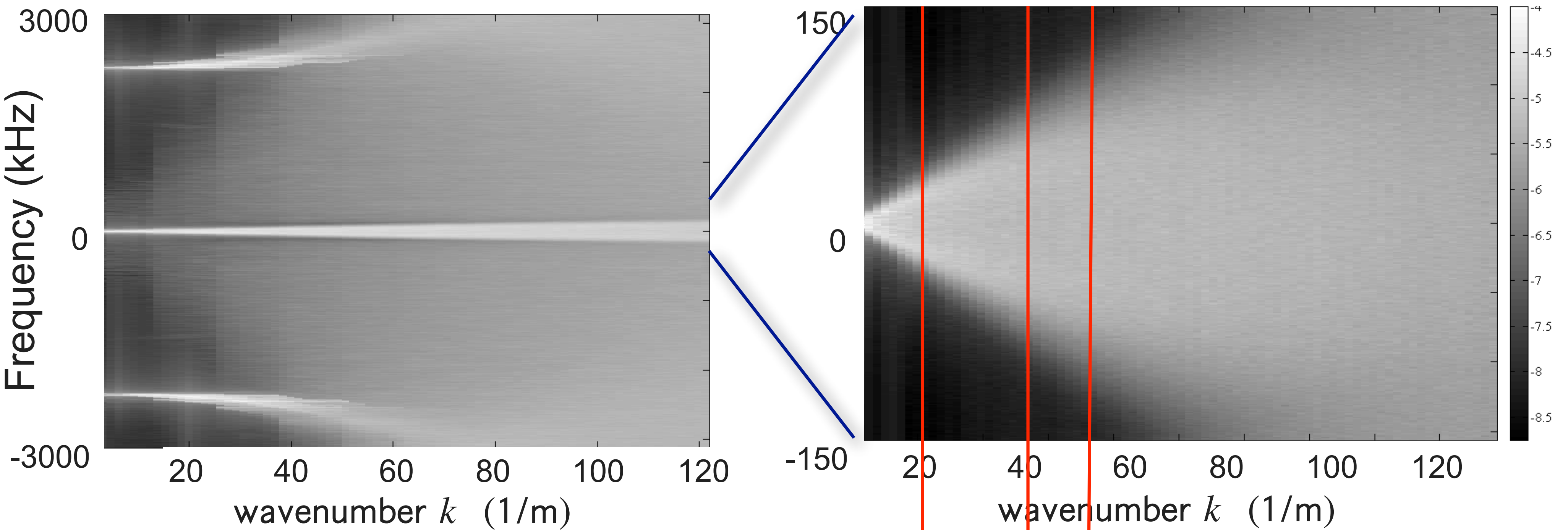

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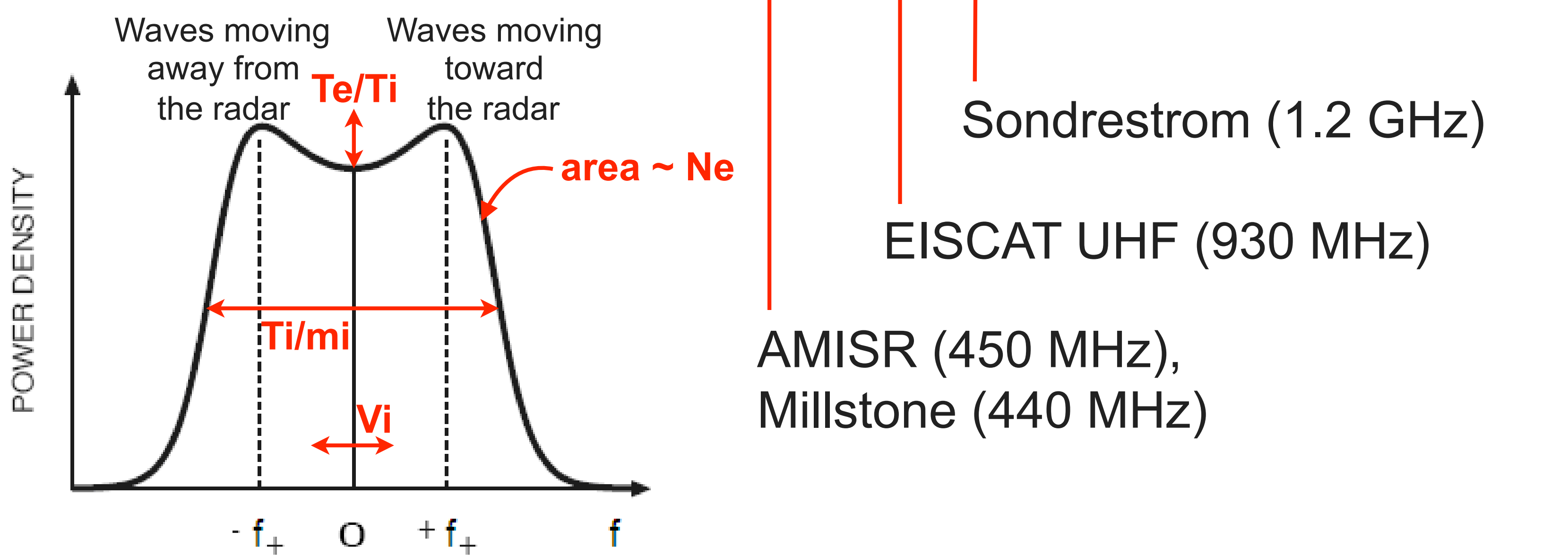


Doppler Radar: "Incoherent" Distributed Target

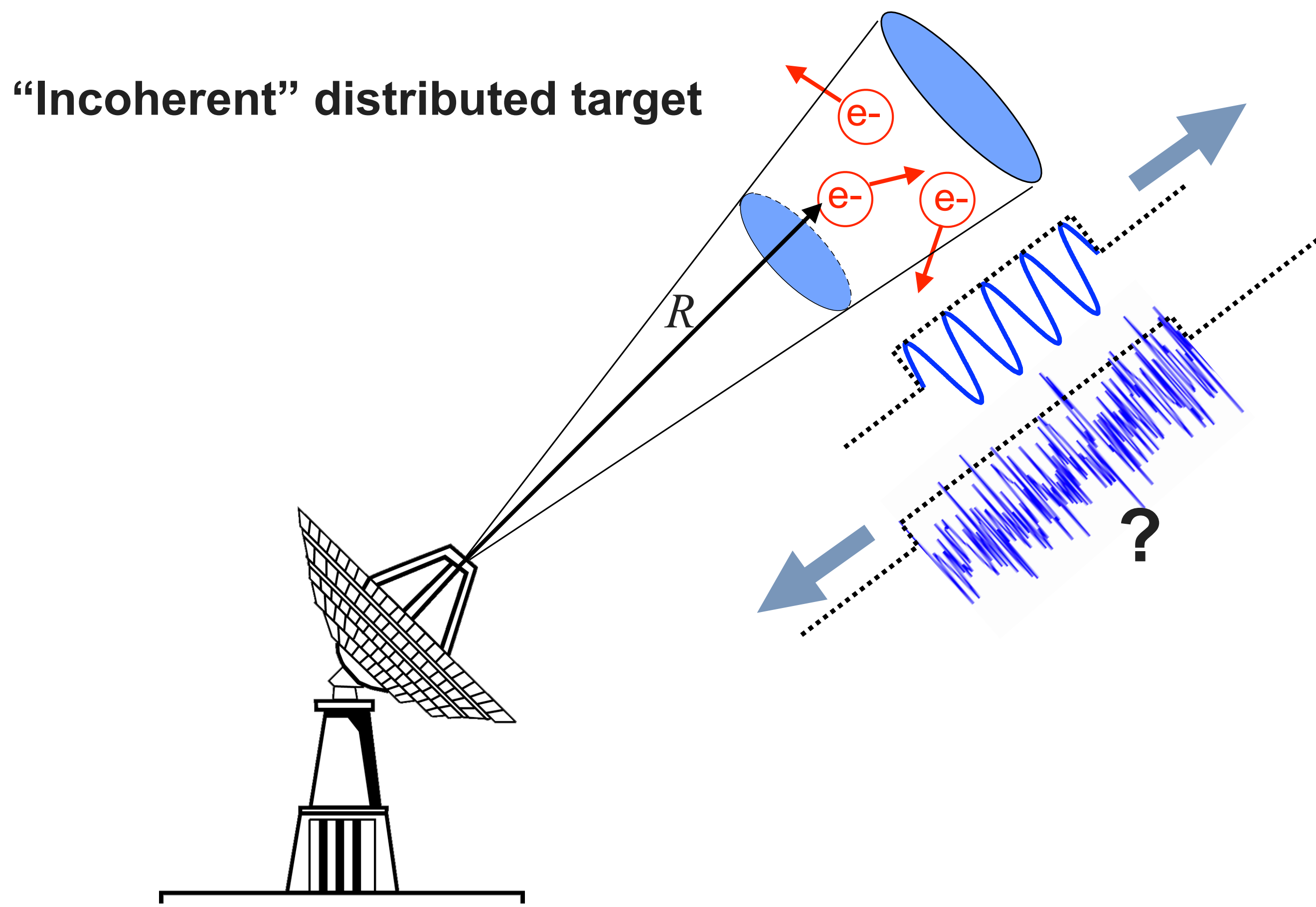
Two key concepts:

Distant ⟺ Time

$$R = -\frac{c\Delta t}{2}$$

Velocity ⟺ Frequency

$$f_D = -\frac{2f_o}{c}v_o$$



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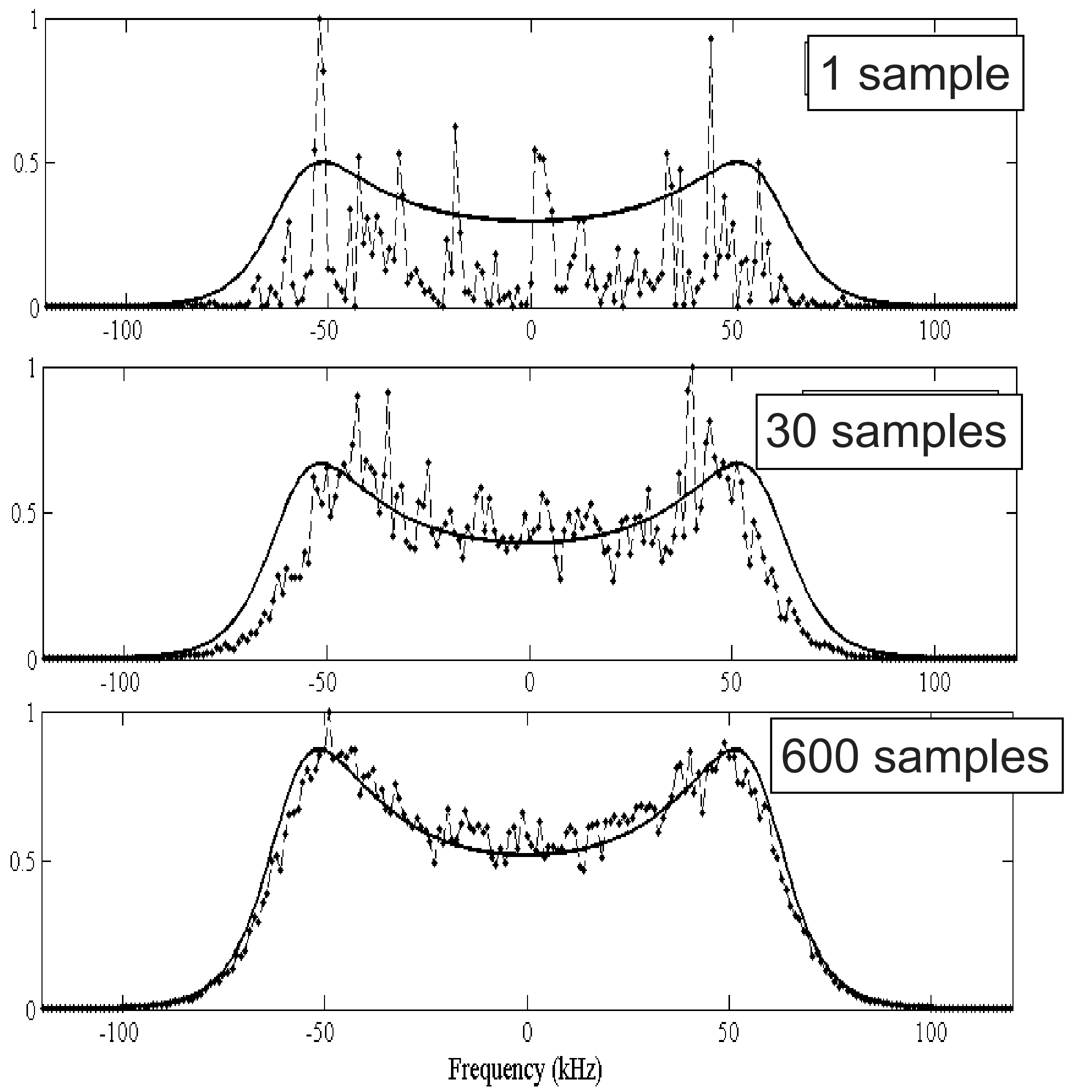
Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz



We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\rho_e \sim \frac{1}{K}\left(1 + \frac{1}{SNR}\right)$$

$\rho_e =$ Mean Square Error
$K =$ number of samples
$SNR =$ per-pulse Signal-to-Noise Ratio