Radar Signal Processing: Part 3

I-Q demodulation: Time-domain perspective

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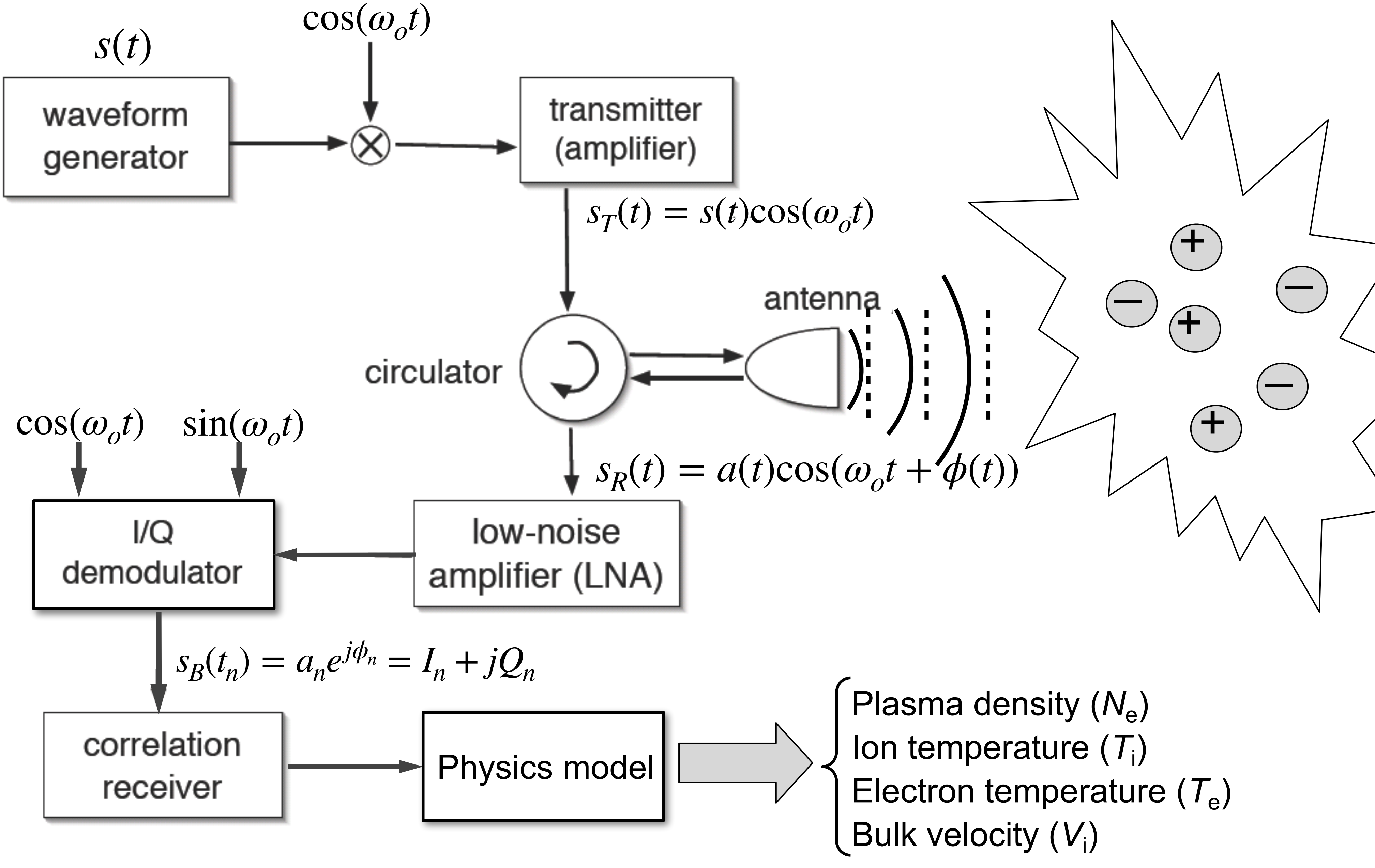
Components of a Pulsed Doppler Radar



$s(t)$

waveform generator

$\cos(\omega_o t)$

transmitter (amplifier)

$s_T(t) = s(t)\cos(\omega_o t)$

circulator

antenna

$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$

$\cos(\omega_o t)$ $\sin(\omega_o t)$

I/Q demodulator

low-noise amplifier (LNA)

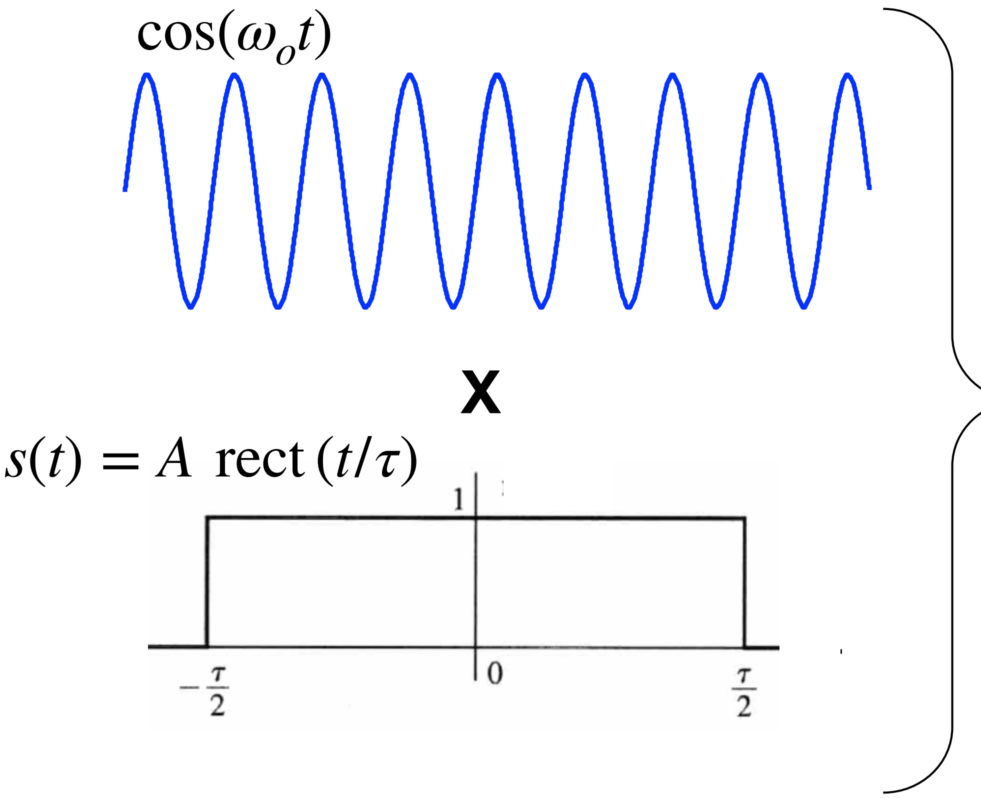
$s_B(t_n) = a_n e^{j\phi_n} = I_n + jQ_n$

correlation receiver

Physics model

- Plasma density (N_e)
- Ion temperature (T_i)
- Electron temperature (T_e)
- Bulk velocity (V_i)

A Simple Radar Pulse



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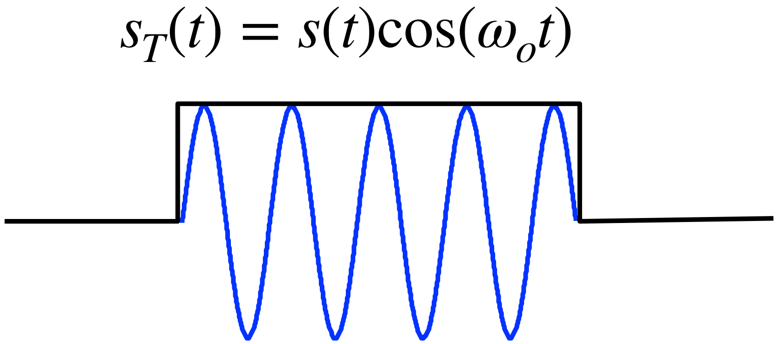
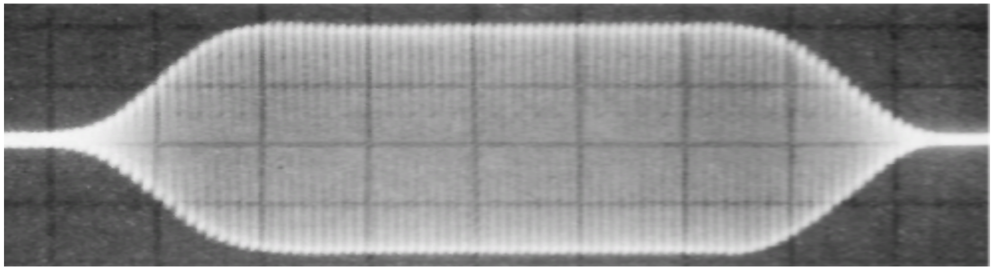
$s_T(t) = s(t)\cos(\omega_o t)$



Waves, modulated by "on-off" action of pulse envelope



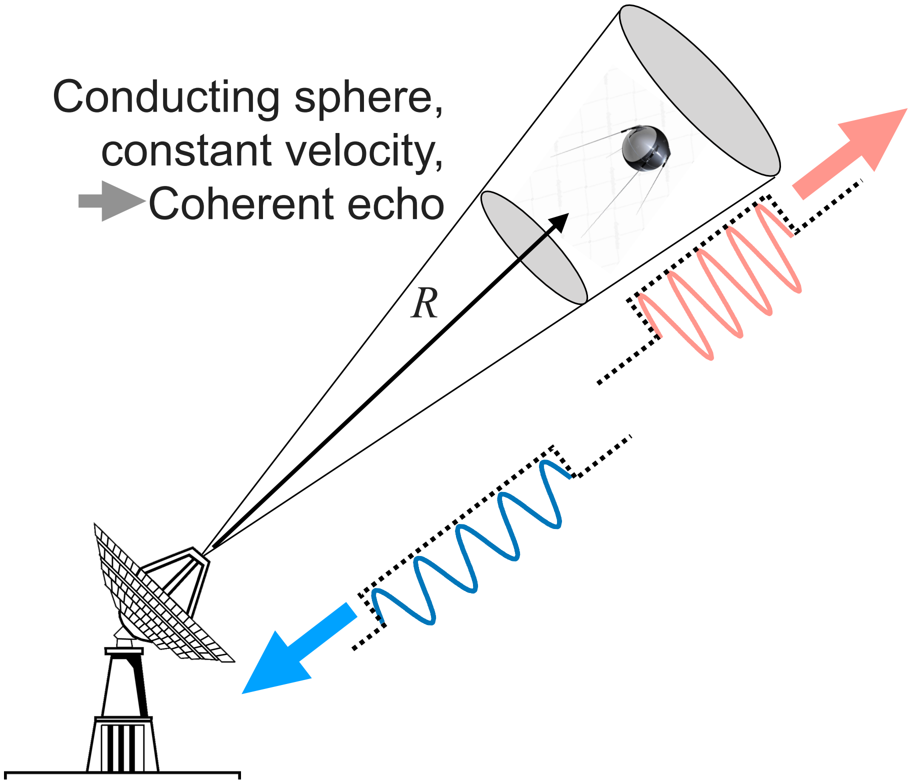
How many cycles are in a typical pulse?

PFISR frequency: 449 MHz
Typical long-pulse length: 480 μs

215,520 cycles!

Measuring Velocity

Conducting sphere,
constant velocity,
→ Coherent echo



Assume a transmitted signal: $s(t)\cos(2\pi f_o t)$

After return from target: $a(t)\cos\left[2\pi f_o\left(t + \frac{2R(t)}{c}\right)\right]$

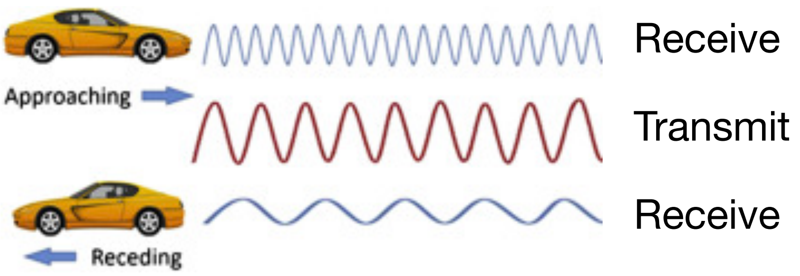
Let's assume target moves with constant velocity with respect to the radar during the measurement,

$$R = R_o + v_o t$$

Substituting we obtain:

$$a(t)\cos\left[\underbrace{2\pi f_o t}_{\omega_o t} + \underbrace{2\pi f_D t + \frac{2\pi f_o R_o}{c}}_{\phi(t)}\right] \qquad f_D = -\frac{2f_o}{c}v_o$$

$$a(t)\cos\left[\omega_o t + \phi(t)\right] \qquad \omega_D = 2\pi f_D = \frac{d\phi}{dt}$$



$f_o \sim 500$ MHz, f_D~ 50 kHz $= 0.0001 f_o$

Two issues:

1) How do we discriminate positive from negative f_D?

2) How do we remove f_o , and just sample $a(t)\cos[\phi(t)]$?

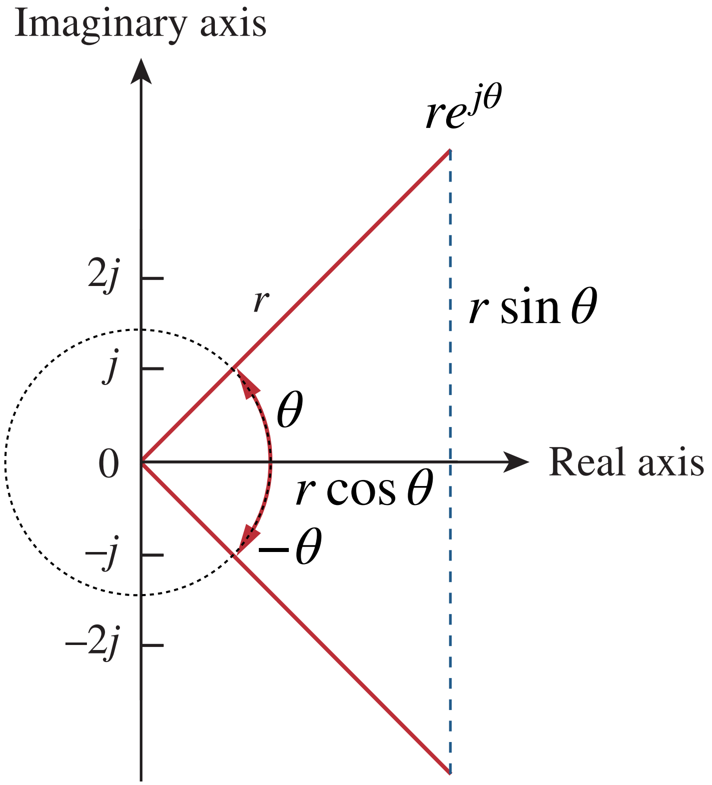
Analytic Signal Model

From Euler's identity

$$re^{j\theta} = (r\cos\theta) + j(r\sin\theta) \qquad j = \sqrt{-1}$$

$$r\cos(\theta) = \Re\{re^{j\theta}\}$$ "real part"

$$r\sin(\theta) = \Im\{re^{j\theta}\}$$ "imaginary part"



Setting $r = a(t)$ and $\theta = \omega_o t + \phi(t)$, we obtain a general complex signal model for radio and radar applications.

$$\boxed{s(t) = a(t)e^{j(\omega_o t+\phi(t))}}$$

PM

AM

Carrier

Or by letting $\omega_d = d\phi/dt \rightarrow \phi(t) = \omega_d t$

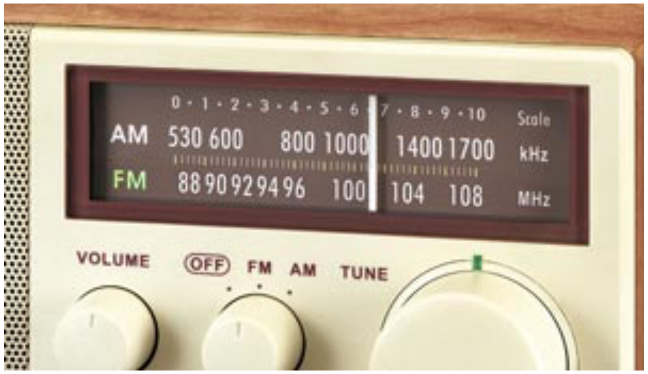
$$s(t) = a(t)e^{j(\omega_o+\omega_d)t}$$

FM

Note that:

$$\Re\{s(t)\} = a(t)\cos(\omega_o t + \phi(t))$$

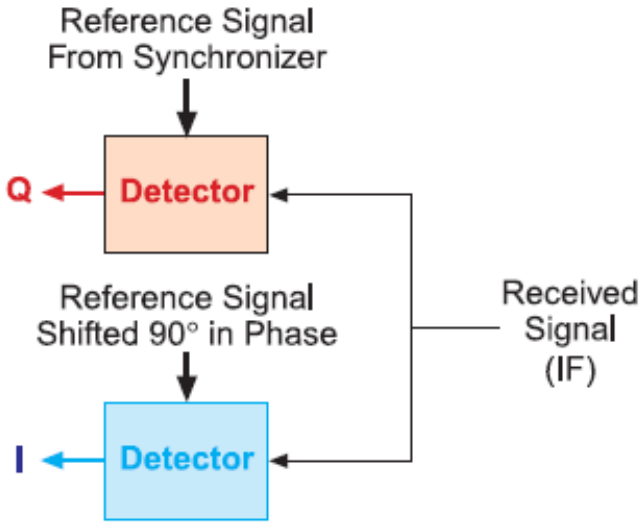
$$\Im\{s(t)\} = a(t)\sin(\omega_o t + \phi(t))$$

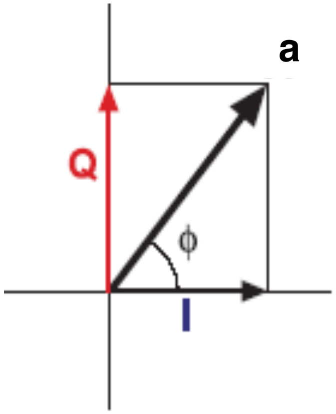


I and Q Demodulation

Consider radar transmission of a simple RF pulse. The reflected signal from the target will be the original pulse with some time varying amplitude and phase applied to it:

$$s_R(t) = a(t)\cos(\omega_o t + \phi(t))$$





We compute the analytic signal by “mixing” with cosine and sine.

Mixing with cosine give the **“in-phase” (I) channel:**

$$s_R(t)\cos(\omega_o t) = a(t)\cos(\omega_o t + \phi(t))\cos(\omega_o t)$$
$$= a(t)\frac{1}{2}\left(\underbrace{\cos[2\omega_o t + \phi(t)]}_{\text{filter out}} + \cos[\phi(t)]\right)$$

Mixing with sine give the **“quadrature” (Q) channel:**

$$s_R(t)\sin(\omega_c t) = a(t)\cos(\omega_o t + \phi(t))\sin(\omega_o t)$$
$$= a(t)\frac{1}{2}\left(\underbrace{-\sin[2\omega_o t + \phi(t)]}_{\text{filter out}} + \sin[\phi(t)]\right)$$

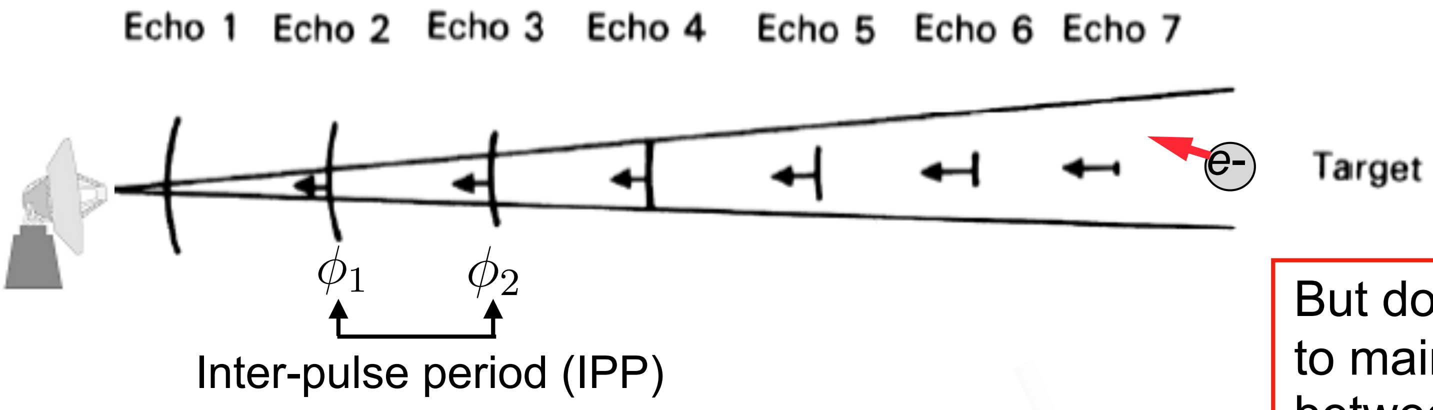
If we include a gain of 2, we retain the original signal energy. Using Euler’s identity we obtain the analytic baseband signal:

$$\boxed{s_B(t) = a(t)\cos\phi(t) + ja(t)\sin\phi(t) = I + jQ}$$

I/Q demodulation produces a time-series of complex voltage samples (I_n, Q_n) from which we can construct a discrete representation of $s_B(t)$. The Doppler frequency shift is the time rate of change of the phase,$\omega_D = d\phi/dt$.

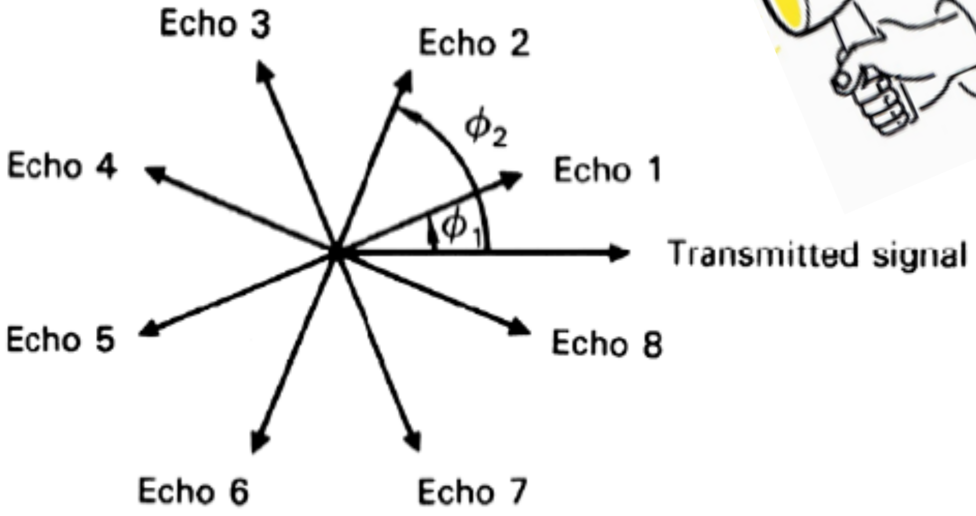
Doppler Detection: Intuitive Approach

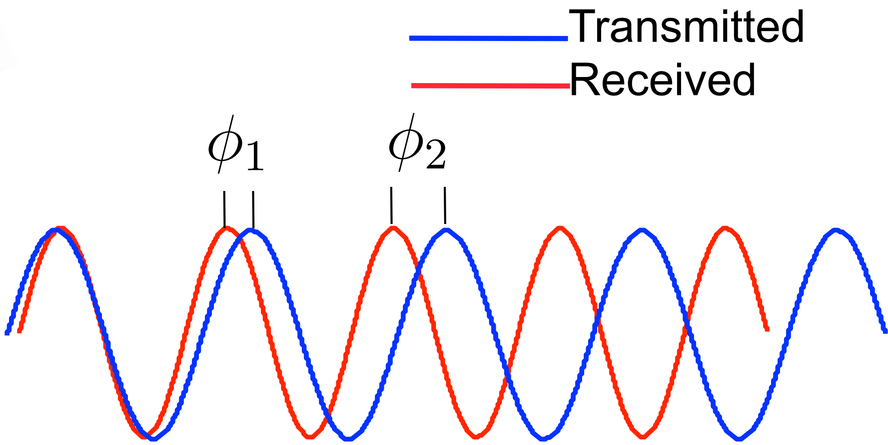
Closing on target – positive Doppler shift



But do we expect an electron to maintain a constant velocity between pulses?

Strobe light at ω_o





Target's Doppler frequency shows up as a pulse-to-pulse shift in phase.

Doppler Detection: Intuitive Approach

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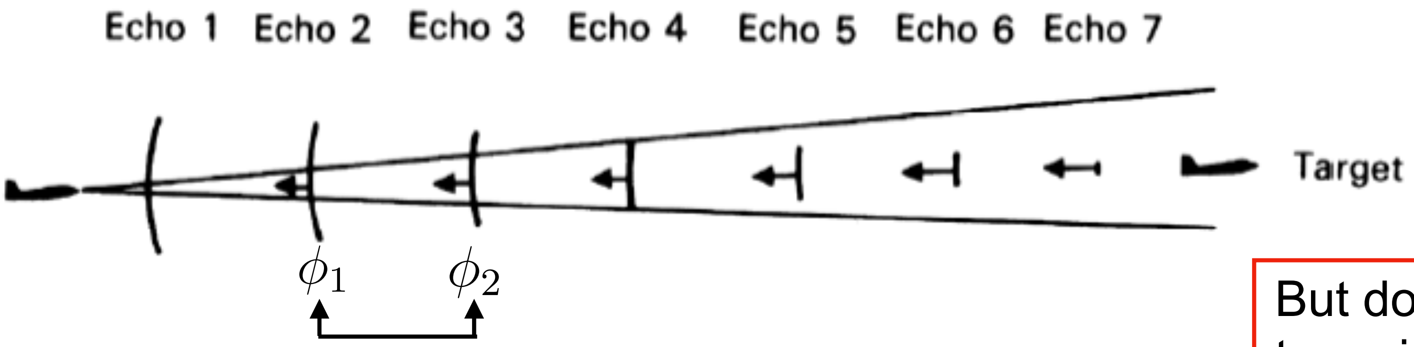
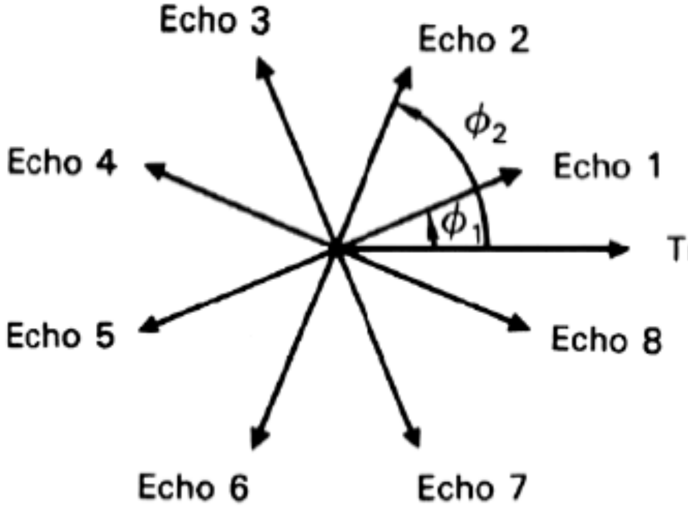
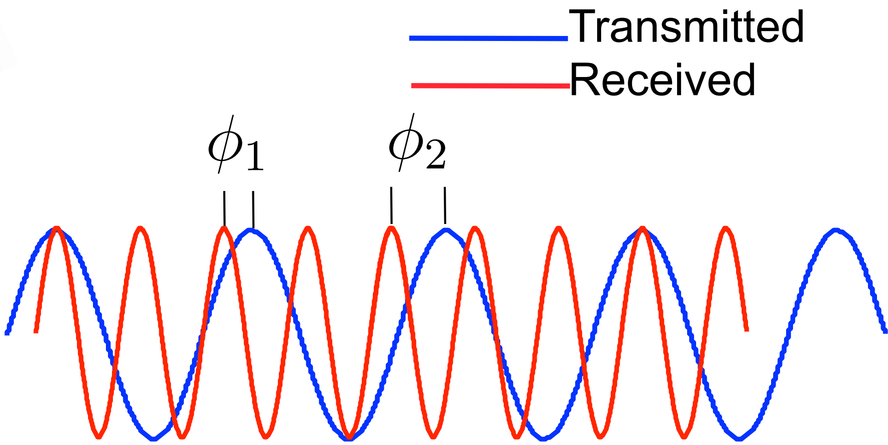


Inter-pulse period (IPP)

Strobe light at ω_o

But do we expect an electron to maintain a constant velocity between pulses?





What is the maximum Doppler shift that can be unambiguously measured?