Useful SCET Definitions

1 Light-Cone Coordinates

Starting with light cone basis vectors n and $\bar{n}$ which satisfy the properties

$$n^2 = 0, \qquad \bar{n}^2 = 0, \qquad n \cdot \bar{n} = 2\,, \tag{1.1}$$

it is simple to represent standard 4-vectors in the light-cone basis

$$p^\mu = \frac{n^\mu}{2} \bar{n} \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + p_\perp^\mu \tag{1.2}$$

where the $\perp$ components are orthogonal to both n and $\bar{n}$. It is customary to represent a momentum in these coordinates by

$$p^\mu = (p^+, p^-, \vec{p}_\perp) \tag{1.3}$$

where the last entry is two-dimensional, and the minkowski $p_\perp^2$ is the negative of the euclidean $\vec{p}_\perp^{\,2}$ (ie. in our notation $p_\perp^2 = -\vec{p}_\perp^{\,2}$). Here we have also defined

$$p^+ = p_+ \equiv n \cdot p\,, \qquad\qquad p^- = p_- \equiv \bar{n} \cdot p. \tag{1.4}$$

As indicated the upper or lower $\pm$ indices mean the same thing.

Using the standard $(+ - - -)$ metric, the four-momentum squared is

$$p^2 = p^+ p^- + p_\perp^2 = p^+ p^- - \vec{p}_\perp^{\,2}\,. \tag{1.5}$$

We can also decompose the metric in this basis

$$g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}\,. \tag{1.6}$$

Finally we can define an antisymmetric tensor in the $\perp$ space by $\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \bar{n}_\alpha n_\beta / 2$.

When we look at scenarios with more that one relevant direction, we set up multiple sets of light cone coordinates. For example, in the case where we think of two jets in a final state, we will have n_1 (with corresponding $\bar{n}_1$) and n_2 (with corresponding $\bar{n}_2$), which each define light cone coordinates. This can be generalized to any number of lightlike directions.