Useful Integral Tricks

Feynman parameter tricks:

$$
\begin{aligned}
a^{-1}\,b^{-1} &= \int_0^1 dx\;\big[a+(b-a)x\big]^{-2} \\
a^{-n}\,b^{-m} &= \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)}\int_0^1 dx\;\frac{x^{n-1}(1-x)^{m-1}}{[a+(b-a)x]^{n+m}} \\
a^{-1}\,b^{-1}\,c^{-1} &= 2\int_0^1 dx\int_0^{1-x} dy\;\big[c+(a-c)x+(b-c)y\big]^{-3} \\
&= 2\int_0^1 dx\int_0^1 dy\;x\big[a+(c-a)x+(b-c)xy\big]^{-3} \\
a_1^{-1}\cdots a_n^{-1} &= (n-1)!\int_0^1 dx_1\cdots dx_n\;\delta\Big(\sum x_i-1\Big)\;\Big(\sum x_i a_i\Big)^{-n} \\
(a_1^{m_1}\cdots a_n^{m_n})^{-1} &= \frac{\Gamma(\sum m_i)}{\Gamma(m_1)\cdots\Gamma(m_n)}\int_0^1 dx_1\cdots dx_n\;\delta\Big(\sum x_i-1\Big)\;\Big(\sum x_i a_i\Big)^{-n}\prod x_i^{m_i-1}
\end{aligned}
\tag{0.1}
$$

To get the fourth line from the third we let $x' = 1-x$ and $y' = y/x$.

Georgi parameter tricks (when one or more propagators are linear in loop momenta):

$$
\begin{aligned}
a^{-1}\,b^{-1} &= \int_0^\infty d\lambda\;\big[a+b\lambda\big]^{-2} \\
a^{-q}\,b^{-1} &= q\int_0^\infty d\lambda\;\big[a+b\lambda\big]^{-(q+1)} = 2q\int_0^\infty d\lambda\;\big[a+2b\lambda\big]^{-(q+1)} \\
a^{-q}\,b^{-p} &= \frac{2^p\,\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\int_0^\infty d\lambda\;\lambda^{p-1}\big[a+2b\lambda\big]^{-(p+q)} \\
a^{-1}\,b^{-1}\,c^{-1} &= 2\int_0^\infty d\lambda\,d\lambda'\;\big[c+a\lambda'+b\lambda\big]^{-3} = 8\int_0^\infty d\lambda\,d\lambda'\;\big[c+2a\lambda'+2b\lambda\big]^{-3}
\end{aligned}
\tag{0.2}
$$