Leading Order SCET Feynman Rules

1 Leading Order Lagrangian

The leading order Lagrangian, with a single set of quark and gluon collinear modes in the n direction and quark and gluon ultrasoft modes, in general covariant gauge, is,

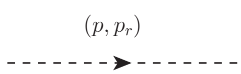
$$\mathcal{L}^{(0)} = \mathcal{L}^{(0)}_{us} + \sum_n \mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{ng},$$

$$\begin{aligned}
\mathcal{L}^{(0)}_{n\xi} &= e^{-ix\cdot\mathcal{P}}\ \bar{\xi}_n\Big(in\cdot D + i\not{D}_{n\perp}\frac{1}{i\bar{n}\cdot D_n} i\not{D}_{n\perp}\Big)\frac{\bar{\not{n}}}{2}\,\xi_n,\\
\mathcal{L}^{(0)}_{ng} &= \frac{1}{2g^2}\mathrm{Tr}\big\{[i\mathcal{D}^\mu, i\mathcal{D}^\nu][i\mathcal{D}_\mu, i\mathcal{D}_\nu]\big\} + \tau\,\mathrm{Tr}\big\{([i\mathcal{D}^\mu_{us}, A_{n\,\mu}])^2\big\} + 2\mathrm{Tr}\big\{\bar{c}_n[i\mathcal{D}^{us}_\mu, [i\mathcal{D}^\mu, c_n]]\big\},\\
\mathcal{L}^{(0)}_{us} &= \overline{\psi}_{us} i\not{D}_{us}\psi_{us} - \frac{1}{2}\mathrm{Tr}\big\{G^{\mu\nu}_{us}G^{us}_{\mu\nu}\big\} + \tau_{us}\mathrm{Tr}\big\{(i\partial_\mu A^\mu_{us})^2\big\} + 2\mathrm{Tr}\big\{\bar{c}_{us}\, i\partial_\mu iD^\mu_{us} c_{us}\big\},
\end{aligned}$$

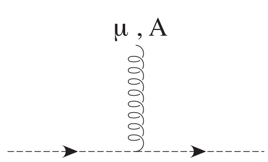
$$\begin{aligned}
iD^\mu_{n\perp} &= \mathcal{P}^\mu_\perp + gA^\mu_{n\perp},\ \ i\bar{n}\cdot D_n = \overline{\mathcal{P}} + g\bar{n}\cdot A_n,\\
i\mathcal{D}^\mu &= \frac{n^\mu}{2}(\overline{\mathcal{P}} + g\bar{n}\cdot A_n) + (\mathcal{P}^\mu_\perp + gA^\mu_{\perp,n}) + \frac{\bar{n}}{2}(in\cdot\partial + gn\cdot A_n + gn\cdot A_{us}),\\
i\mathcal{D}^\mu_{us} &\equiv \frac{n^\mu}{2}\overline{\mathcal{P}} + \mathcal{P}^\mu_\perp + \frac{\bar{n}^\mu}{2} in\cdot\partial + \frac{\bar{n}}{2} gn\cdot A_{us},\ \ iD^\mu_{us} = i\partial_\mu + A^\mu_{us}
\end{aligned}$$

2 Leading Order Feynman Rules

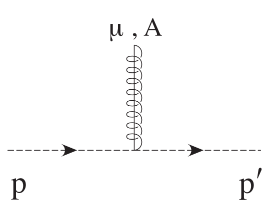
The above Lagrangian leads to the Feynman rules



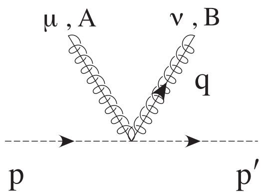
$$= \ i\,\frac{\not{n}}{2}\,\frac{\bar{n}\cdot p}{n\cdot p_r\,\bar{n}\cdot p + p_\perp^2 + i0}$$



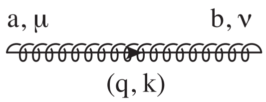
$$= ig\,T^A\, n_\mu\,\frac{\bar{\not{n}}}{2}$$



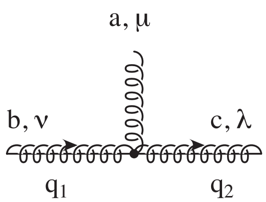
$$= ig\,T^A\left[n_\mu + \frac{\gamma^\perp_\mu \not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}'_\perp\gamma^\perp_\mu}{\bar{n}\cdot p'} - \frac{\not{p}'_\perp\not{p}_\perp}{\bar{n}\cdot p\,\bar{n}\cdot p'}\bar{n}_\mu\right]\frac{\bar{\not{n}}}{2}$$



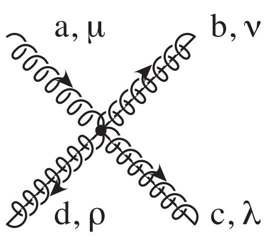
$$= \frac{ig^2\,T^A\,T^B}{\bar n\cdot(p-q)}\left[\gamma_\mu^\perp\gamma_\nu^\perp - \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\nu - \frac{\not p'_\perp \gamma_\nu^\perp}{\bar n\cdot p'}\bar n_\mu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu\right]\frac{\bar{\not n}}{2}$$
$$+ \frac{ig^2\,T^B\,T^A}{\bar n\cdot(q+p')}\left[\gamma_\nu^\perp\gamma_\mu^\perp - \frac{\gamma_\nu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\mu - \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'}\bar n_\nu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu\right]\frac{\bar{\not n}}{2}$$



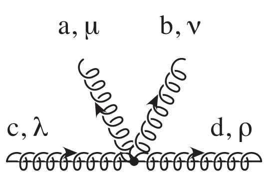
$$= \frac{-i}{\bar n\cdot q\; n\cdot k + q_\perp^2 + i0}\left(g_{\mu\nu} - (1-\tau)\frac{q_\mu q_\nu}{\bar n\cdot q\; n\cdot k + q_\perp^2}\right)\delta_{a,b}$$



$$= g f^{abc} n_\mu \left\{\bar n\cdot q_1\, g_{\nu\lambda} - \tfrac{1}{2}(1-\tfrac{1}{\tau_{us}})[\bar n_\lambda q_{1\nu} + \bar n_\nu q_{2\lambda}]\right\}$$



$$= -\tfrac{1}{2} i g^2 n_\mu \Big\{ f^{abe} f^{cde}(\bar n_\lambda g_{\nu\rho} - \bar n_\rho g_{\nu\lambda})$$
$$+ f^{ade} f^{bce}(\bar n_\nu g_{\lambda\rho} - \bar n_\lambda g_{\nu\rho}) + f^{ace} f^{bde}(\bar n_\nu g_{\lambda\rho} - \bar n_\rho g_{\nu\lambda})\Big\}$$



$$= \tfrac{1}{4} i g^2 n_\mu n_\nu \bar n_\rho \bar n_\lambda (1 - \tfrac{1}{\tau_{us}})\left\{f^{ace} f^{bde} + f^{ade} f^{bce}\right\}$$

Where collinear quarks are denoted by dashed lines, ultrasoft gluons are springs, collinear gluons are springs with a line and τ is the covariant collinear gauge fixing parameter and τ_{us} is the similar ultrasoft gauge fixing parameter.

The Feynman rules with just ultrasoft fields are the same as those in QCD, as are the Feynman rules for the interactions between n-collinear gluons. There are no Lagrangian interactions at this order between collinear fields in different sectors.