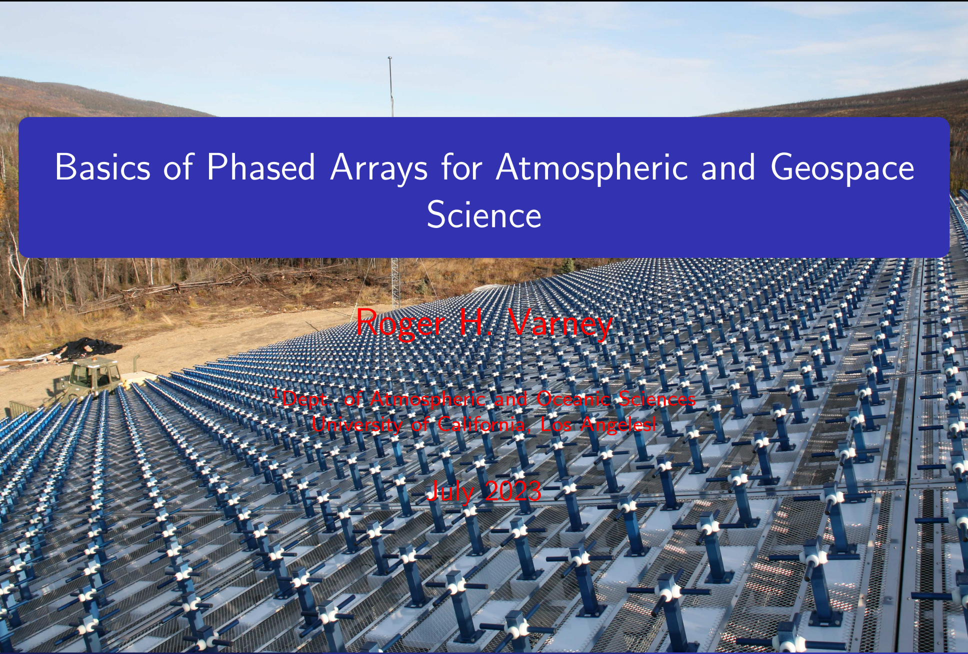

# Basics of Phased Arrays for Atmospheric and Geospace Science

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# Superposition Principle

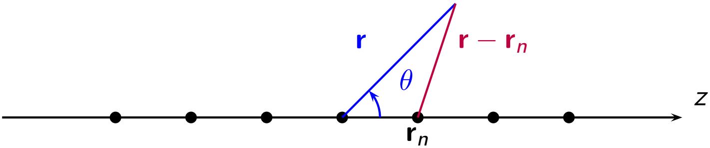
Maxwell's Equations are Linear:

$$\mathbf{J}_1 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1)$$
$$0 = \nabla \times (\mathbf{E}_1) + \frac{\partial}{\partial t}(\mathbf{B}_1)$$

$$\mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_2)$$

$$\mathbf{J}_1 + \mathbf{J}_2 = \frac{1}{\mu_0}\nabla \times (\mathbf{B}_1 + \mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t}(\mathbf{E}_1 + \mathbf{E}_2)$$
$$0 = \nabla \times (\mathbf{E}_1 + \mathbf{E}_2) + \frac{\partial}{\partial t}(\mathbf{B}_1 + \mathbf{B}_2)$$

# Superposition Applied to Antenna Arrays



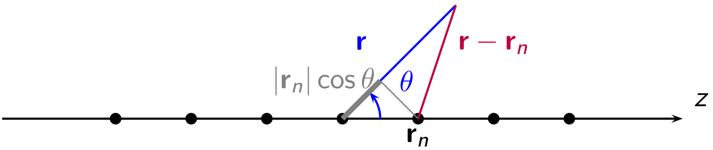

Fields radiated by single element at the origin with applied current $I_0$:

$$\mathbf{E} = \mathbf{E}_0 I_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}$$

Fields radiated by entire array:

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=0}^{N-1} I_n \frac{e^{-jk|\mathbf{r}-\mathbf{r}_n|}}{|\mathbf{r}-\mathbf{r}_n|}$$

# Far Field Approximation (Fraunhofer Zone)




If $\mathbf{r}$ and $\mathbf{r} - \mathbf{r}_n$ are almost parallel lines:

$$\mathbf{r} - \mathbf{r}_n \approx \mathbf{r} - |\mathbf{r}_n| \cos\theta \, \hat{r}$$

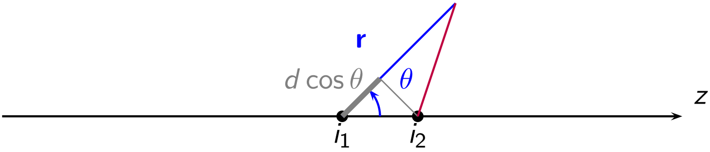
Assume $|\mathbf{r}_n| \ll |\mathbf{r}|$:

$$|\mathbf{r} - \mathbf{r}_n| \approx |\mathbf{r}| \;\text{ for demoninator terms}$$

$$-jk\,|\mathbf{r} - \mathbf{r}_n| \approx -jk\,|\mathbf{r}| + jk\,|\mathbf{r}_n| \cos\theta$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk|\mathbf{r}_n| \cos\theta}}_{\text{Array Factor}}$$

# Simple Two Element Array Example

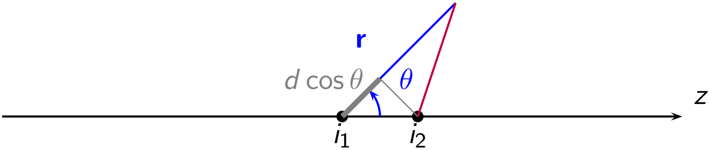



Suppose $d = \lambda/4$ and

$$i_1(t) = \cos(\omega t)$$
$$i_2(t) = \cos\left(\omega t + \frac{\pi}{2}\right)$$

How does the radiated power vary as a function of $\theta$?

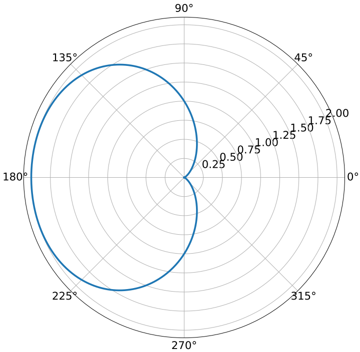
# Two Element Array Example With Phasors




$$\tilde{I}_1 = 1 \qquad \tilde{I}_2 = e^{j\pi/2}$$

$$\begin{aligned}\tilde{E} &= \frac{-j\omega}{r}\tilde{I}_1 e^{-jkr} + \frac{-j\omega}{r}\tilde{I}_2 e^{-jkr+jkd\cos\theta} \\ &= \frac{-j\omega}{r} e^{-jkr} + \frac{-j\omega}{r} e^{-jkr+jkd\cos\theta+j\pi/2} \\ &= \frac{-j\omega}{r} e^{-jkr}\left[1 + e^{jkd\cos\theta+j\pi/2}\right]\end{aligned}$$

$$\langle S\rangle \propto \frac{\omega^2}{r^2}\frac{1}{2}\left|1 + e^{jkd\cos\theta+j\pi/2}\right|^2$$



# Active Electronically Steerable Phased Arrays

## The AMISR UHF System

Antenna Element Unit (AEU)

Face

**AMISR AEU = Tx/Rx Unit**
- *500 W solid state transmitter*
- *Phasing control*
- *Status monitoring*
- *4096 AEUs/AMISR radar face*

32/panel

**AMISR Panel**
- *32 Antenna Element Units arranged in hexagonal pattern*
- *3.5 x 2 meters; 19.8 dBi / panel*
- *16 kW peak power per panel*
- *Basic system building block for AMISR*
- *Embedded linux controller*

Panel (with PCU)

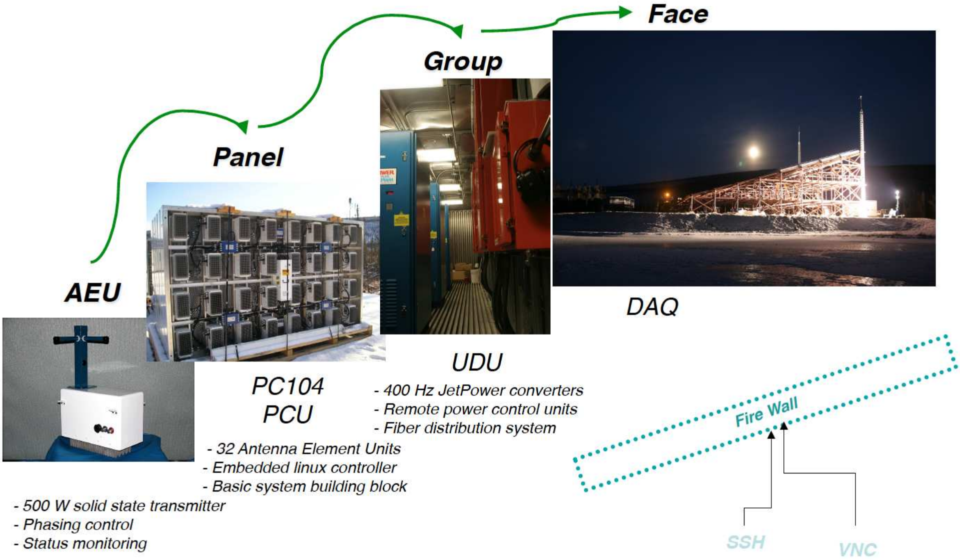
Utility Distribution Unit (UDU)

AMISR Control System (ACS)

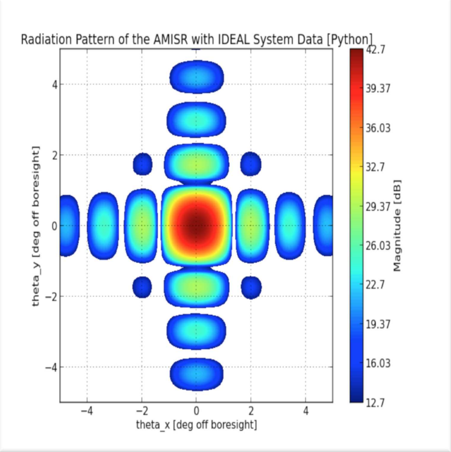
**AMISR UDU**
- *400 Hz JetPower converters*
- *Remote power control units*
- *Fiber distribution system*

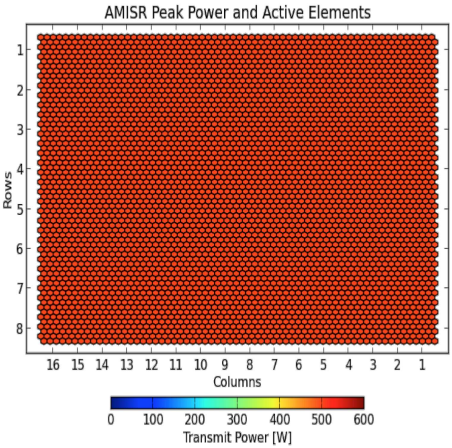
**AMISR ACS**
- *Flexible transmit and receive system*
- *Completely remotely controlled*
- *Experiments run off a scheduler*

# Advanced Modular Incoherent Scatter Radar

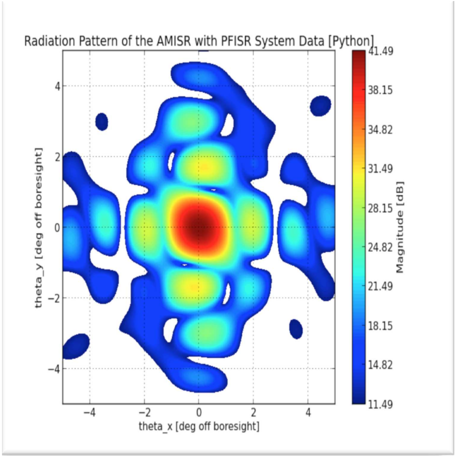


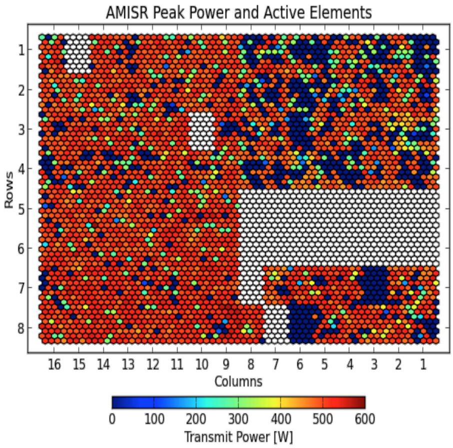
# Ideal AMISR Radiation Pattern



# AMISR Graceful Degradation

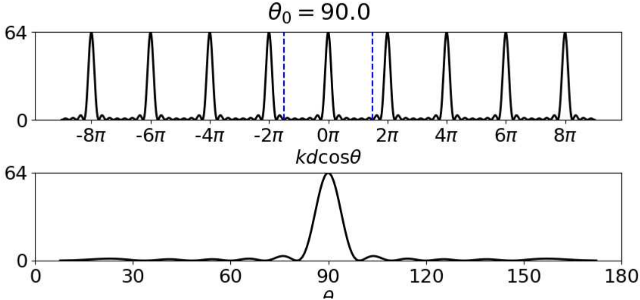


# Steering Limits and Grating Lobes

- $d < \lambda/2 \rightarrow kd < \pi$: No grating lobes will ever appear
- $\lambda/2 < kd < \lambda \rightarrow \pi < kd < 2\pi$: Grating lobes will only appear at some steering angles
- $d > \lambda \rightarrow kd > 2\pi$: Grating lobes will always appear

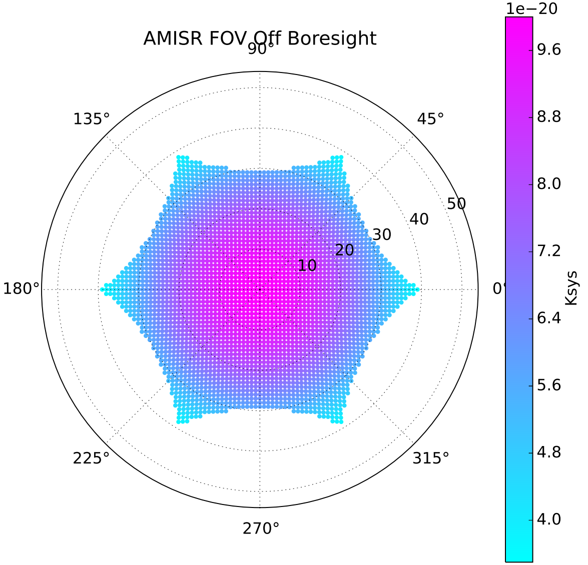
Example of linear array with $d = 3\lambda/4$ spacing




- Maximum steering angle (for a linear array): $\Delta\theta_{\max} = \left|\sin^{-1}\left(1 - \frac{2\pi}{kd}\right)\right|$

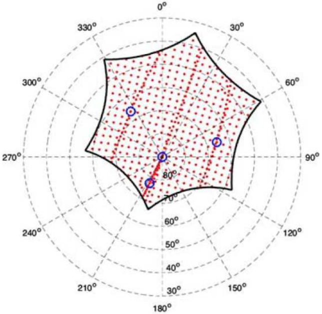
# AMISR Antenna Properties

- Hexagonal spacing with $d \approx 3\lambda/4$
- FOV limited by grating lobe limit $\sim 30^\circ - 40^\circ$
- Antenna gain decreases with steering angle off of boresight
- Antenna works best within $\sim 25^\circ$ off of boresight



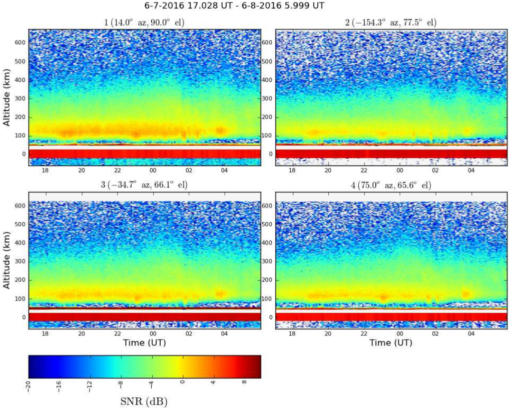
# The PFISR Up-B Compromise

## IPY Beam Pattern




The Up-B beam is close to the grating lobe limit, and therefore has reduced sensitivity.

## Reduced SNR in Up-B (Beam 2)



# Conceptual Diagram of Steering with AMISR