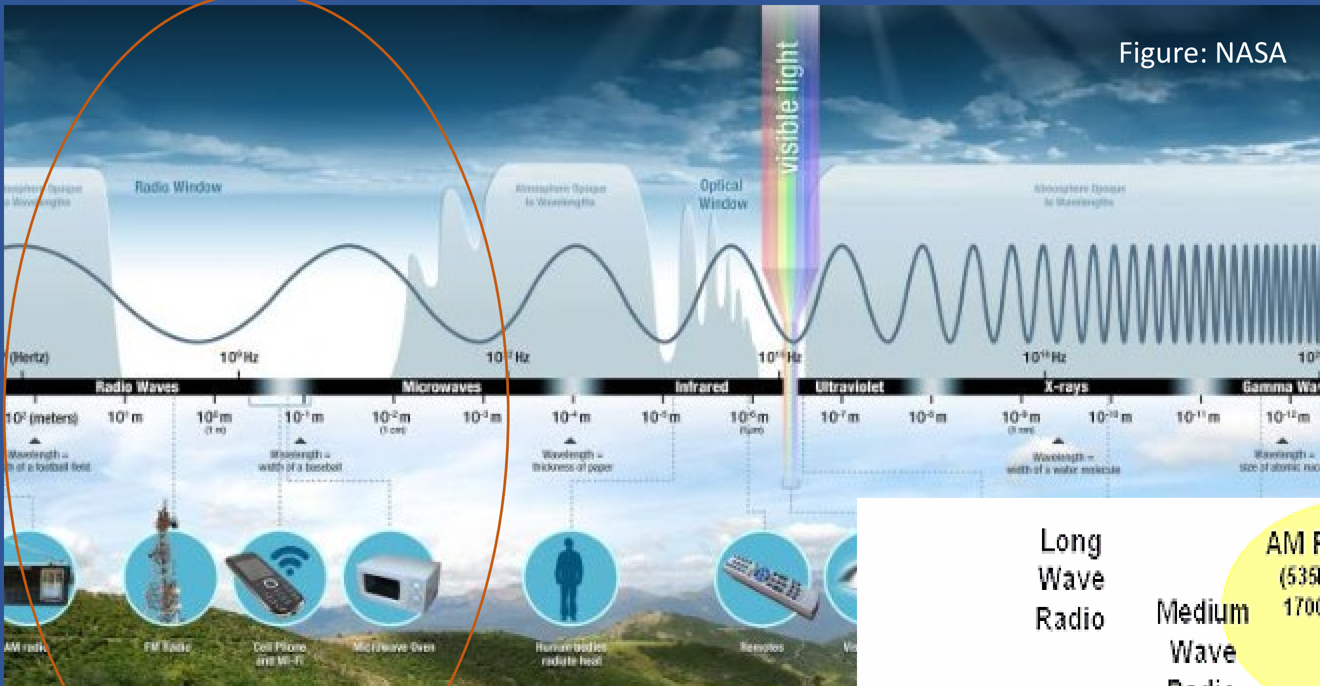
Radar Physics

Asti Bhatt, SRI

Outline - Radar Basics

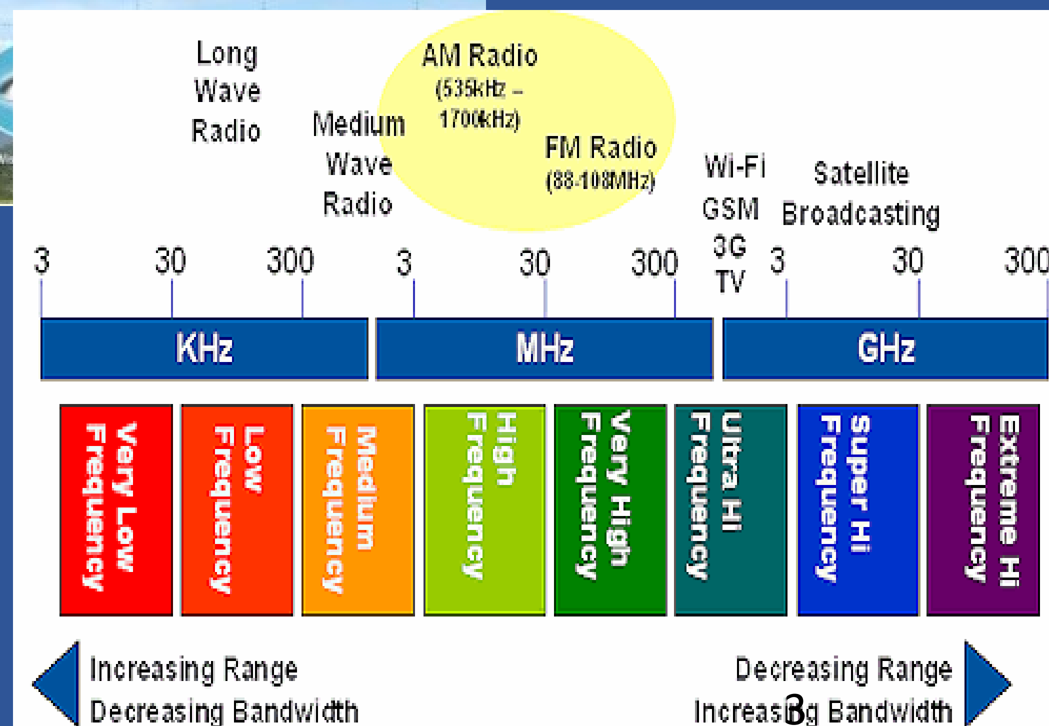
- Electromagnetic spectrum
- Radio waves and propagation
- Radar equation
- Range resolution and pulsed radars
- Doppler

The Electromagnetic Spectrum



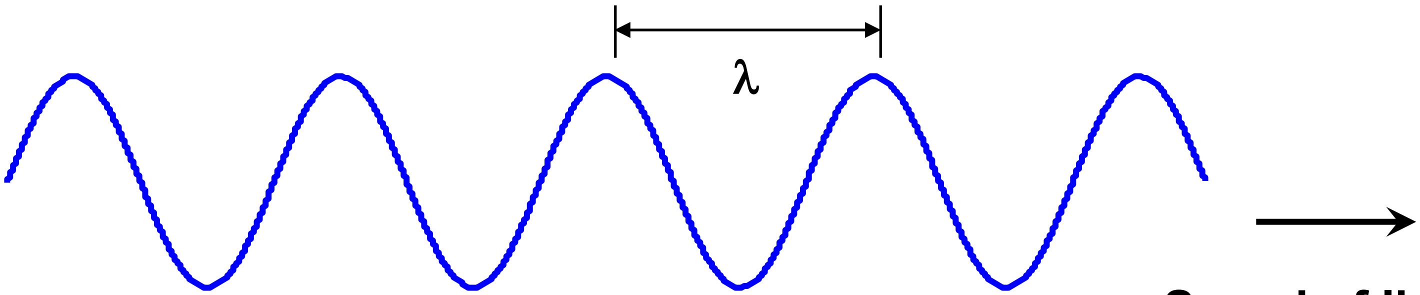
Remote sensing using radio waves:

Just light we can't see without tools.



Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c

$c = 3 \times 10^8$ m/sec

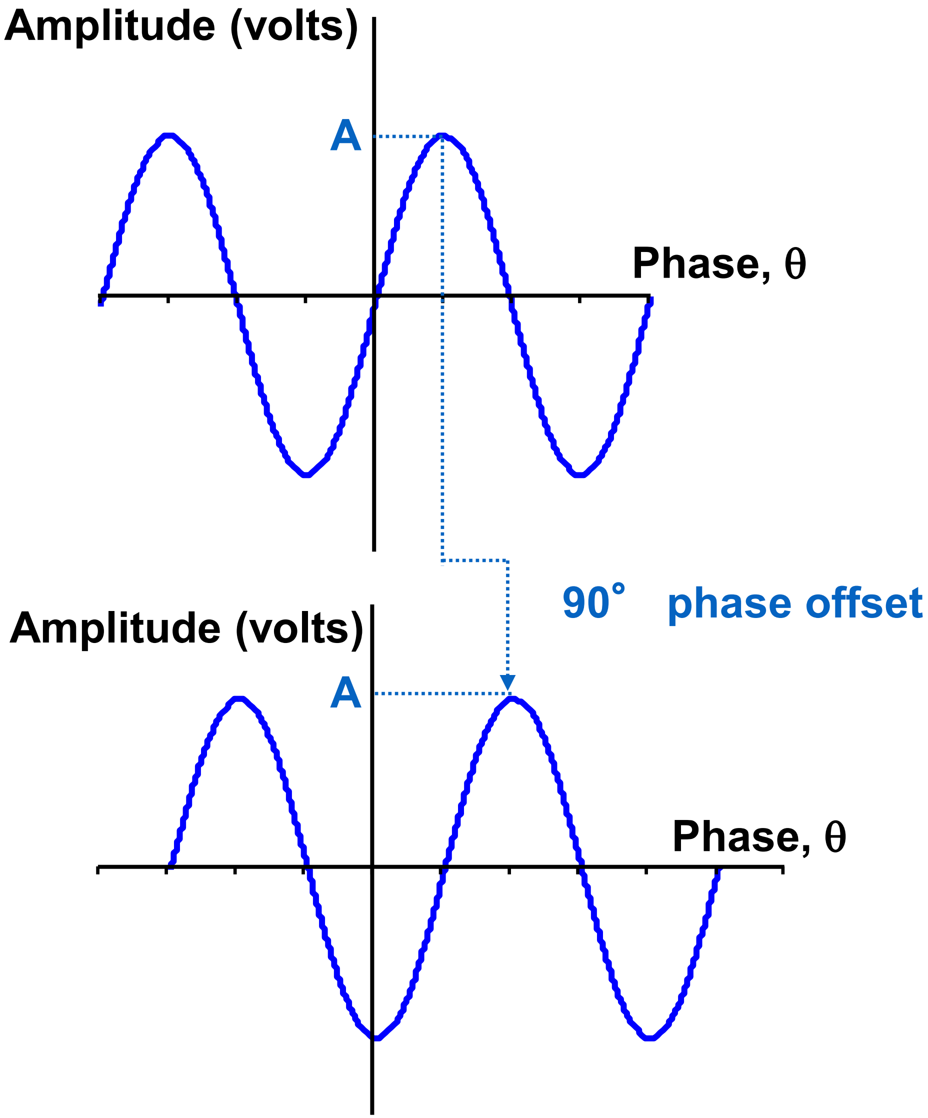
$= 300{,}000{,}000$ m/sec

$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

Frequency	Wavelength
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

Properties of Waves
Phase and Amplitude



$$A\sin(\theta)$$

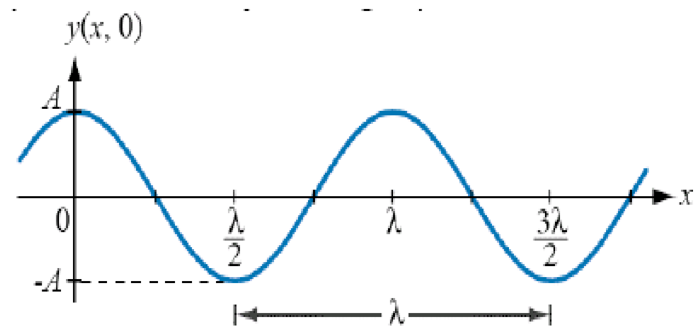
$$A\sin(\theta - 90^{o})$$

Radio Waves

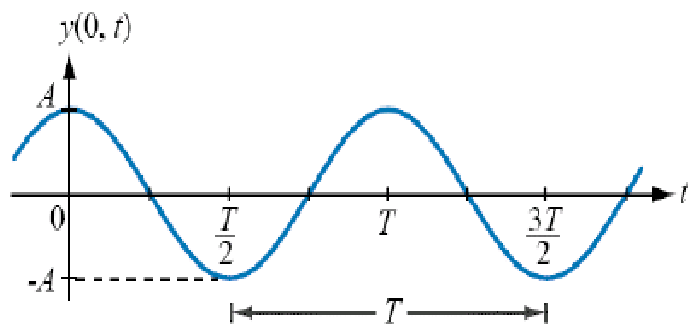
$$y(x, t) = A\cos(\omega t - kx + \phi_o)$$

Angular frequency
$\omega = 2\pi f = 2\pi/T$

Wavenumber
$k = 2\pi/\lambda$



(a) $y(x, t)$ versus x at $t = 0$

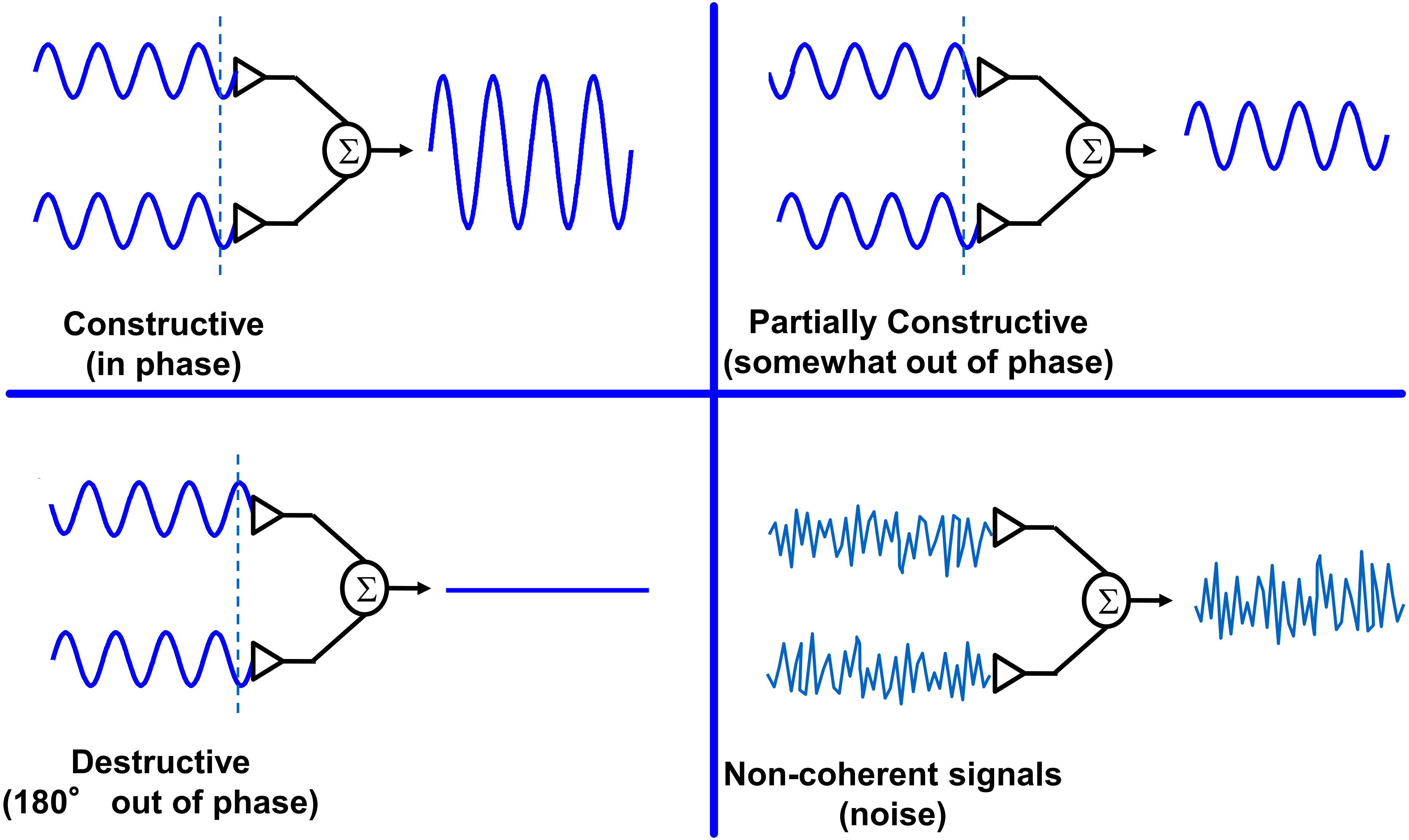


(b) $y(x, t)$ versus t at $x = 0$

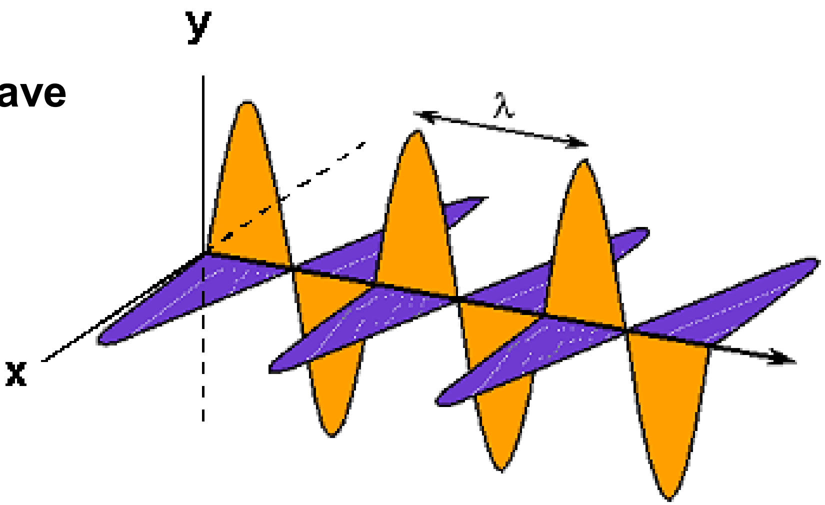
Wave phase velocity
$c = f\lambda = \omega/k = 3\text{x}10^8\ m/s$

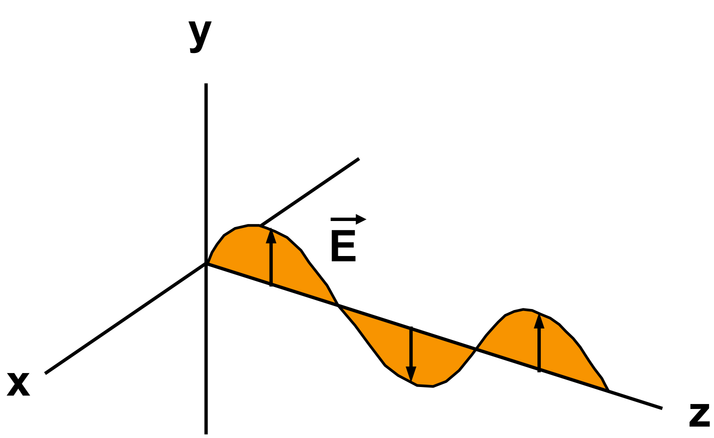
Frequency (1/s) = $\dfrac{\textbf{Speed of light (m/s)}}{\textbf{Wavelength l (m)}}$

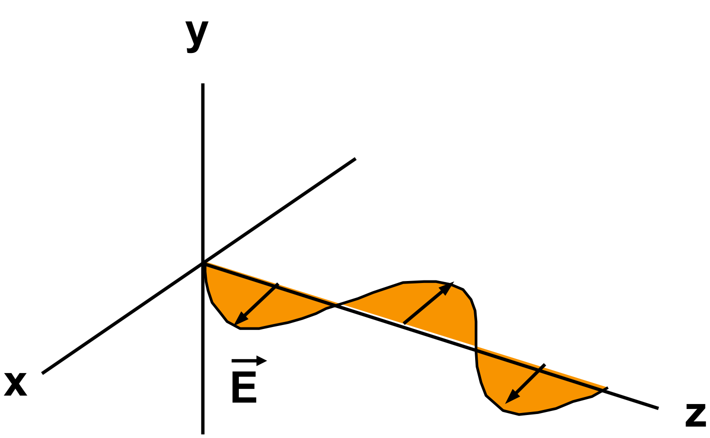
Properties of Waves
Constructive vs. Destructive Addition



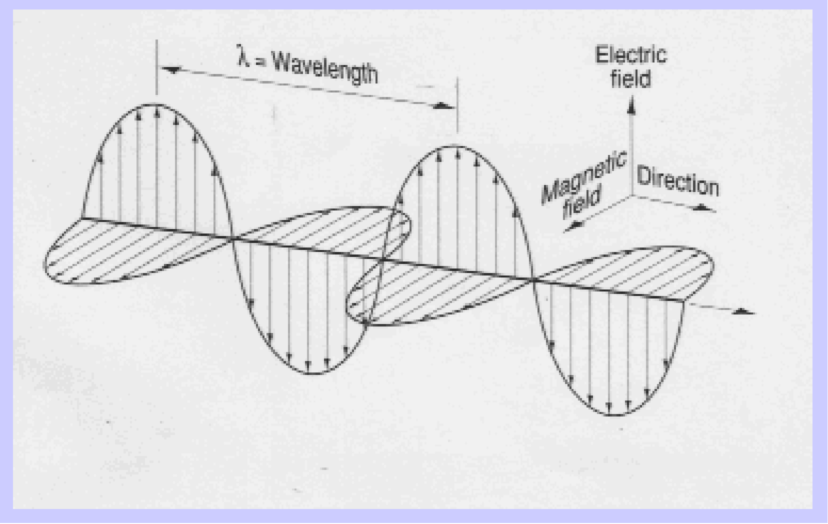
Polarization



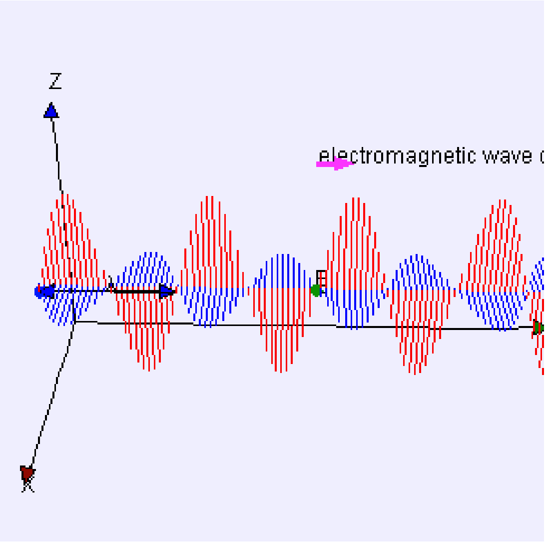


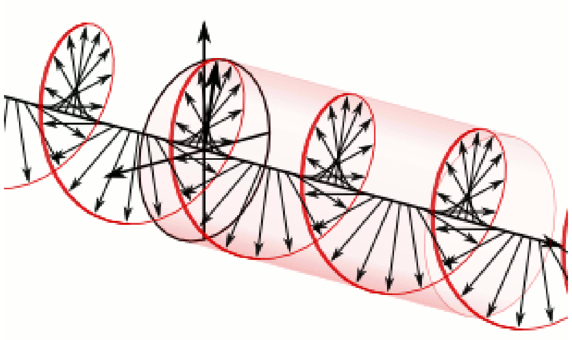


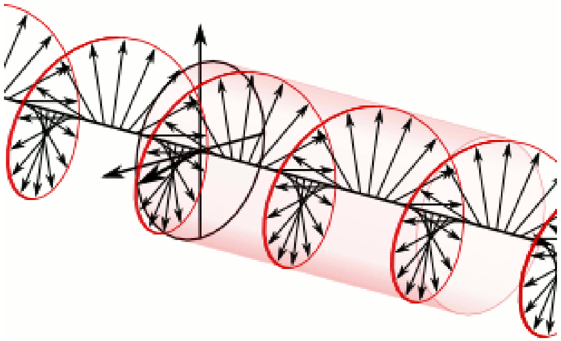
TEM Waves: ***Transverse electromagnetic*** **(*TEM*)** ***modes*** **neither electric nor magnetic field in the direction of propagation**



Electromagnetic waves in free space propagate in TEM mode







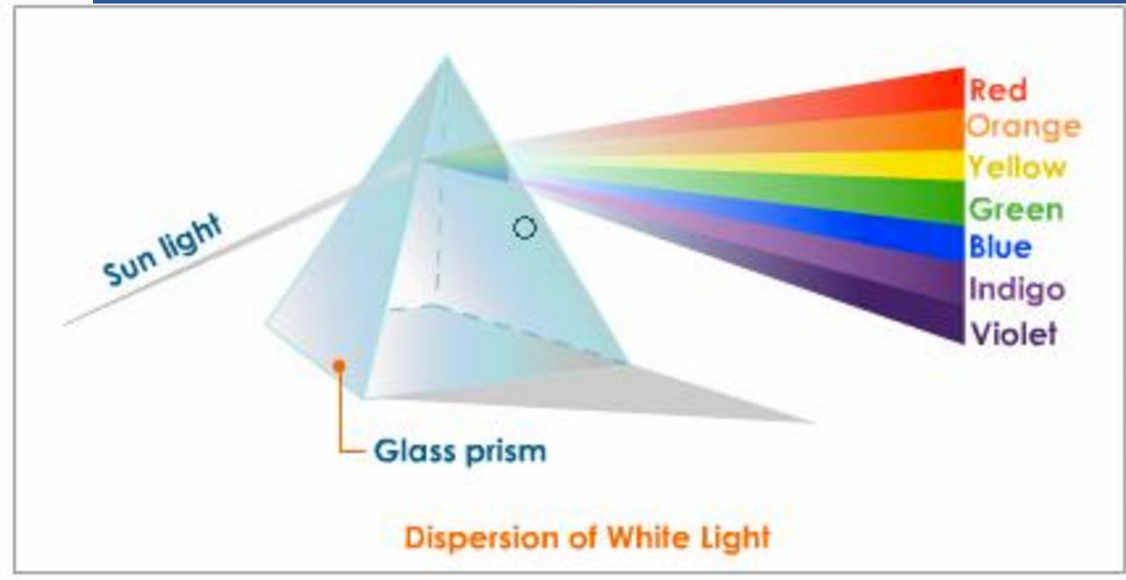
Phase Velocity, Group Velocity, Index of Refraction

$$v_\mathrm{p} = \frac{\omega}{k}$$

$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_\mathrm{p}}.$$

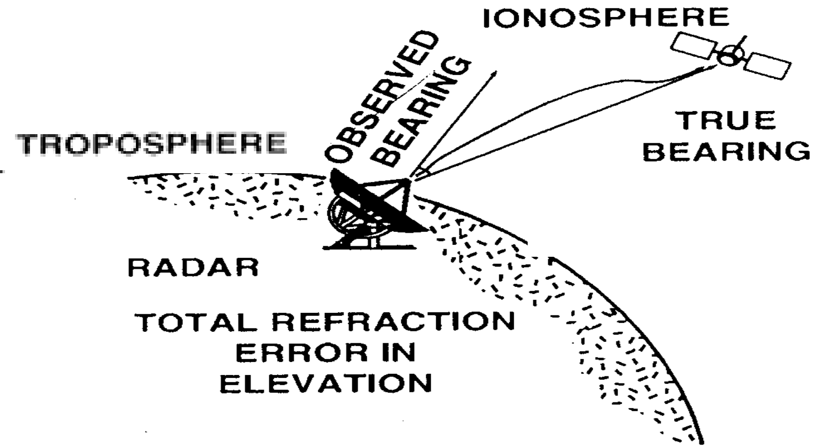
Refraction and Dispersion



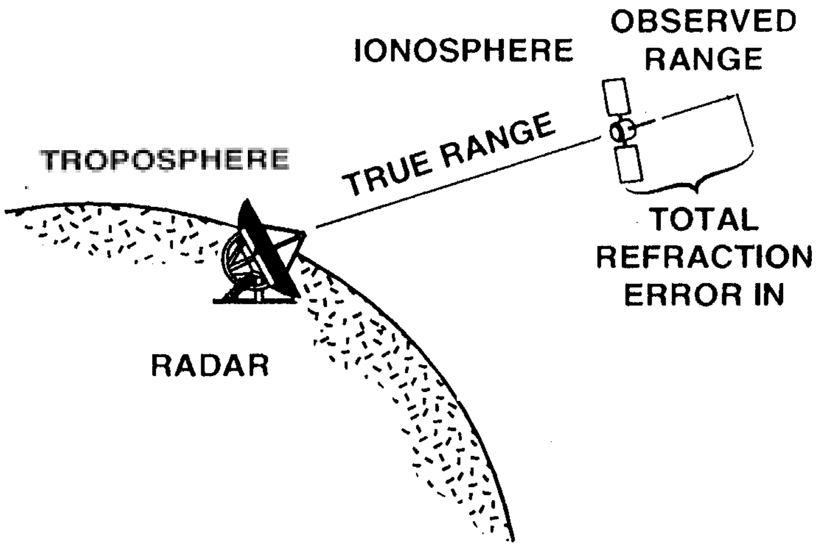
From Attila Komjathy, JPL

Illustration of Atmospheric Effects

Elevation Refraction



Range Delay



Index of Refraction $n = \frac{c}{v_p}$. in the Ionosphere is a function of frequency

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2\sin^2\theta}{1-X-iZ} \pm \frac{1}{1-X-iZ}\left(\frac{1}{4}Y^4\sin^4\theta + Y^2\cos^2\theta(1-X-iZ)^2\right)^{1/2}}$$

n is the index of refraction

$$X = \frac{\omega_N^2}{\omega^2} \quad Y = \frac{\omega_H}{\omega} \quad Z = \frac{\nu}{\omega} \quad \omega_N = \left(\frac{Ne^2}{\varepsilon_0 m_e}\right)^{1/2} \quad \omega_H = \frac{e|B|}{m_e}$$

ω = the angular frequency of the radar wave,

Y_L = Y $\cos\theta$, Y_T = Y$\sin\theta$,

θ = angle between the wave vector $\overline{k}$ and $\overline{B}$,

$\overline{k}$ = wave vector of propagating radiation,

$\overline{B}$ = geomagnetic field, N = electron density

e = electronic charge, m_e = electron mass, ν = electron collision frequency

and ε_0 = permittivity constant.

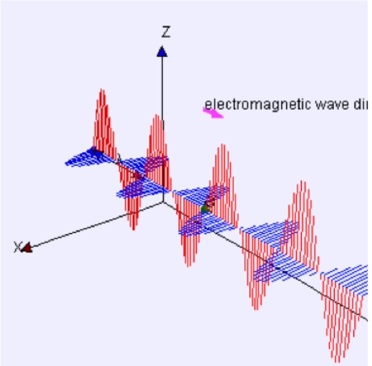
Dispersion relation: the concept

Key concept for wave behavior within a propagation medium.

Describes the relationship between SPATIAL frequency (wavelength)
and TEMPORAL frequency.

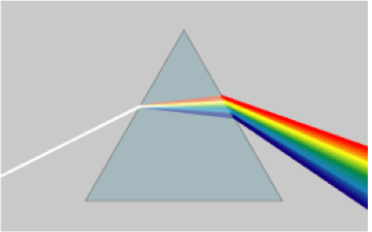
Some wave modes relate wavelength to frequency **linearly,**
but waves in most media have **nonlinear** relation between wavelength and frequency.

Linear dispersion example:
EM radiation propagation through free space
(wavelength / velocity = c)



http://weelookang.blogspot.com/2011/10/ejs-open-source-propagation-of.html

Nonlinear dispersion example:
splitting of light through a prism
(effective speed of light depends on wavelength
due to glass' non-unity index of refraction)



Wikipedia CC-3.0

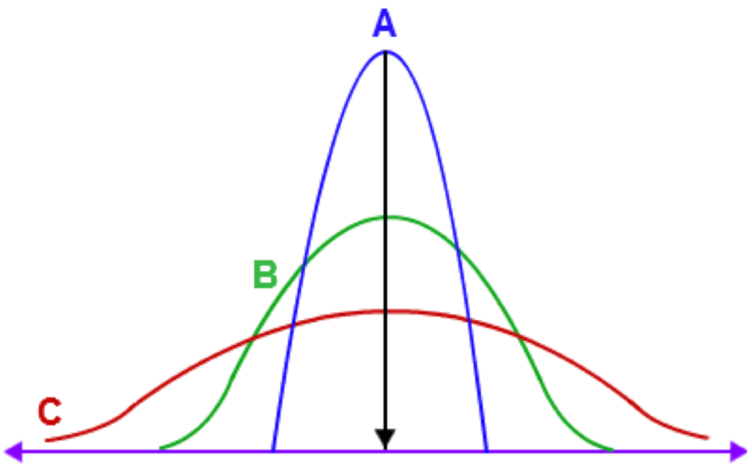
Dispersion relation: the concept

Simple linear case: uniform phase velocity

$$\omega(k) = c\ k$$

Most propagation speeds depend nonlinearly on the wavelength and/or frequency.

NB: for a **nonlinear** dispersion relation, the pulse will typically spread in either spatial frequency or temporal frequency as a function of time.



Example of pulse spreading spatially from time A to B to C.

http://www.mathcaptain.com/statistics/dispersion-statistics.html

Radio Propagation in the Ionosphere

Index of Refraction (no **B** field)

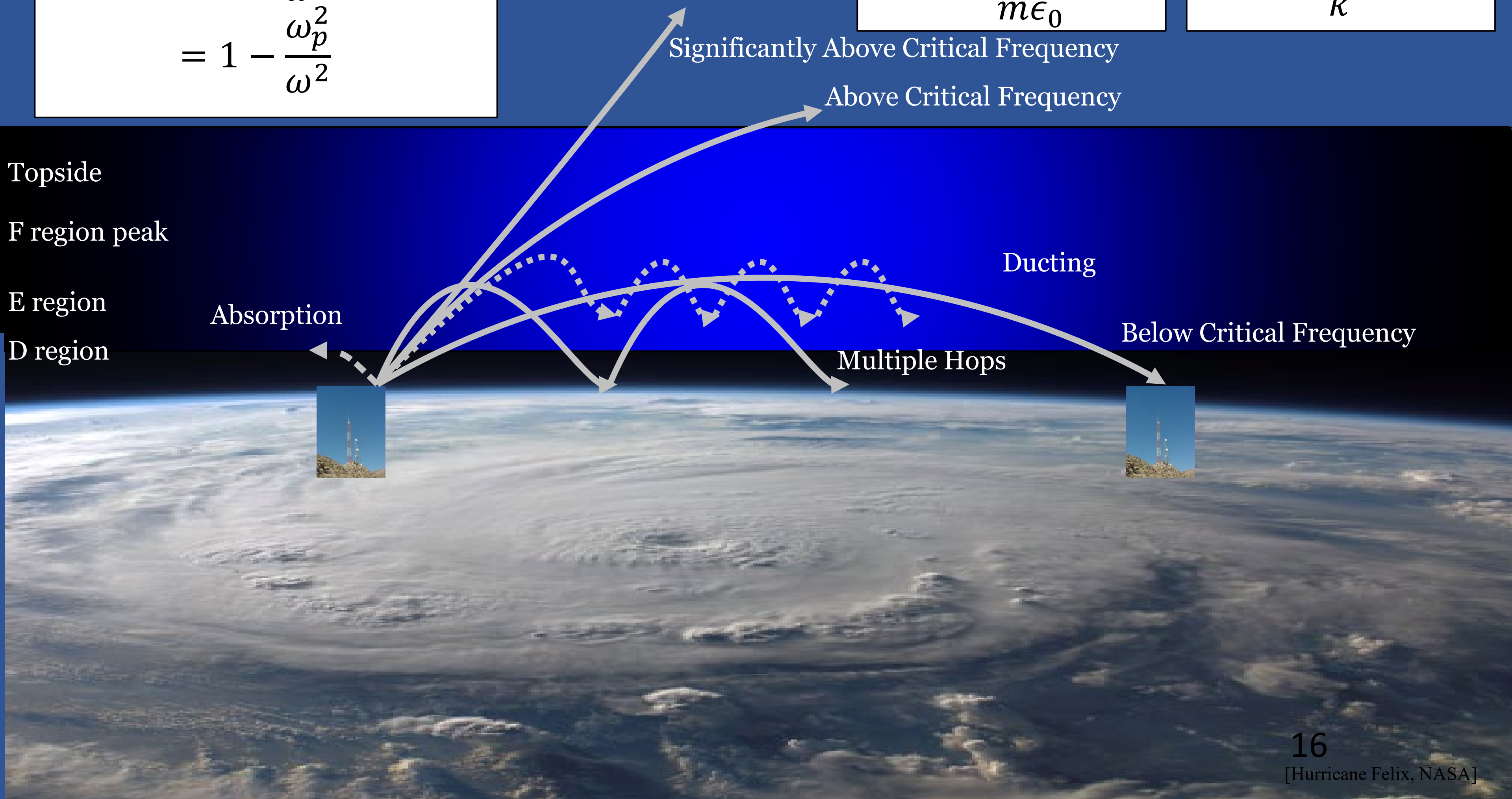
$$n^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma Frequency

$$\omega_p^2 = \frac{n_0 e^2}{m \epsilon_0}$$

Phase Velocity

$$V_{ph} = \frac{\omega}{k}$$



[Hurricane Felix, NASA]

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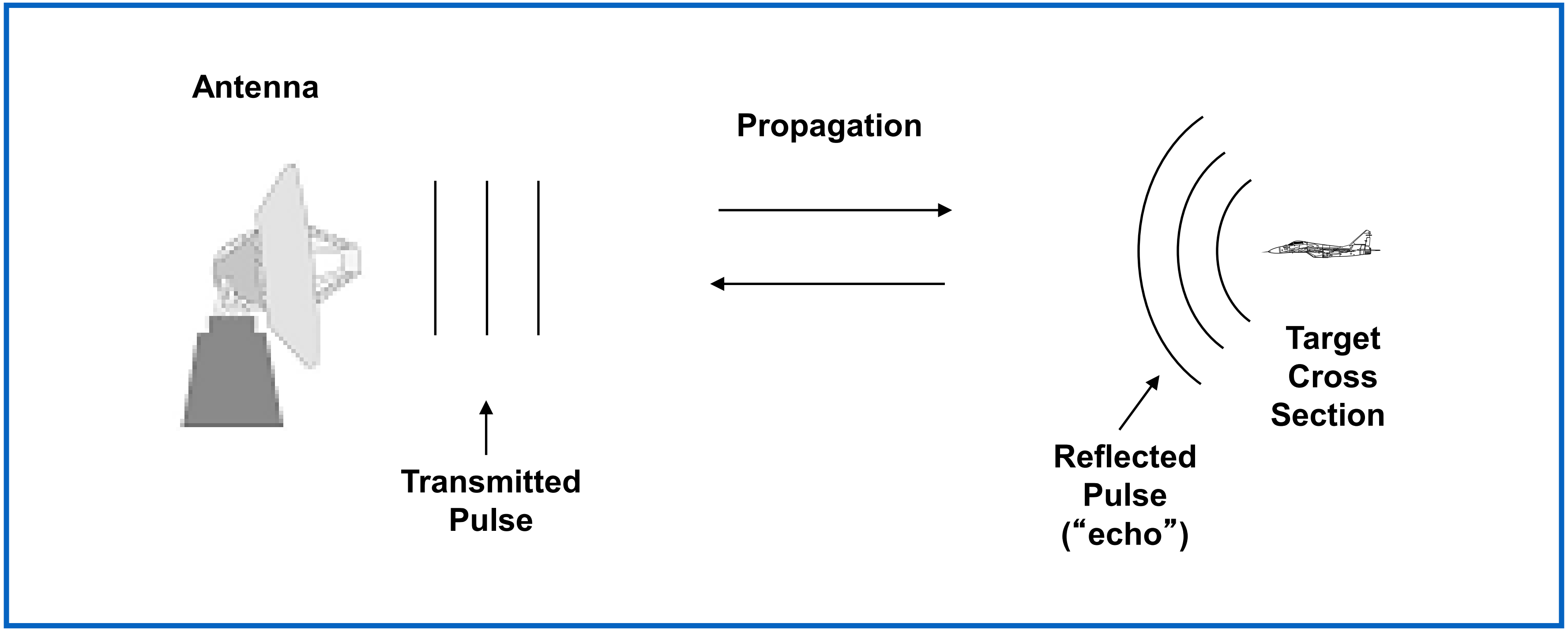
RADAR

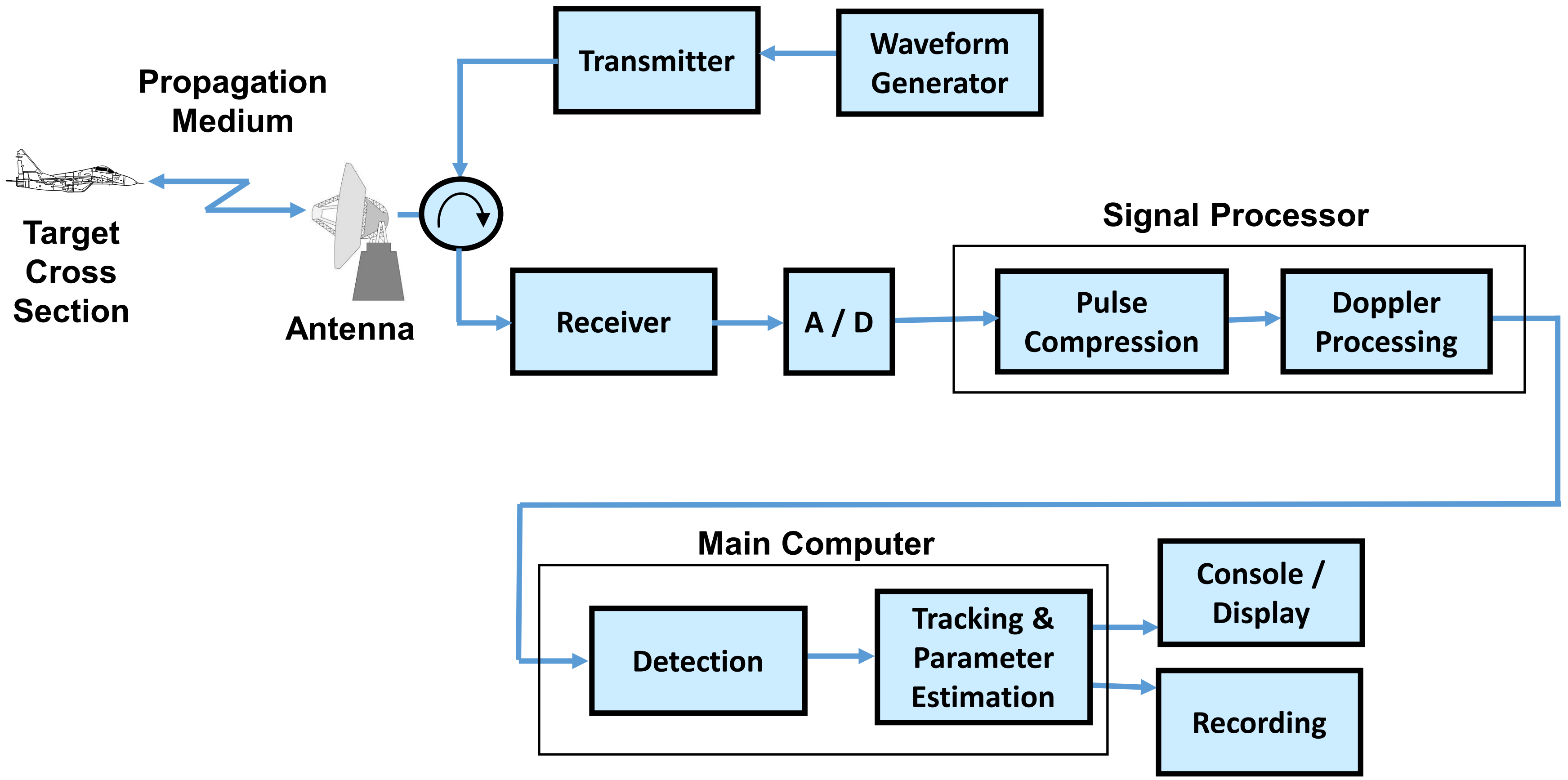
RAdio Detection And Ranging



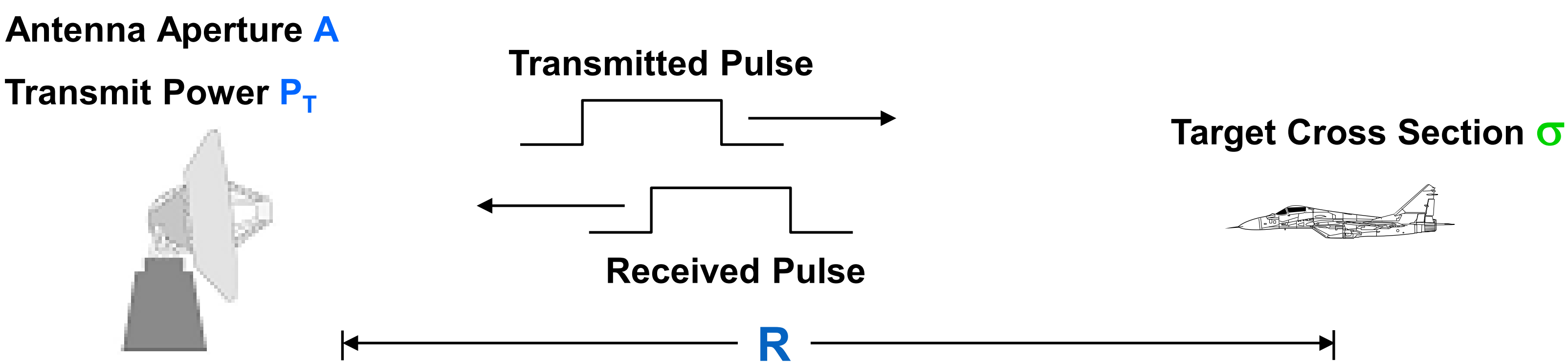
Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

Radar Block Diagram



Radar Range Equation

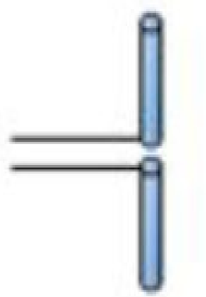


Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time

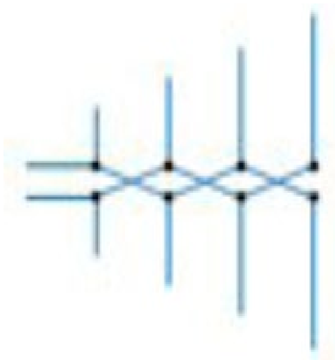
$$\text{Received Signal Energy} = \left[P_T \right] \left[\frac{4\pi A}{\lambda^2} \right] \left[\frac{1}{4\pi R^2} \right] \left[\frac{1}{L} \right] \left[\sigma \right] \left[\frac{1}{4\pi R^2} \right] \left[A \right] \left[\tau \right]$$

Antennas

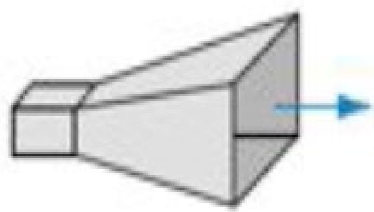
Most basic form of antennas – a wire element with a time varying current flowing in it



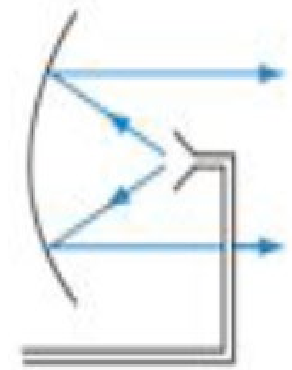
Dipole antenna



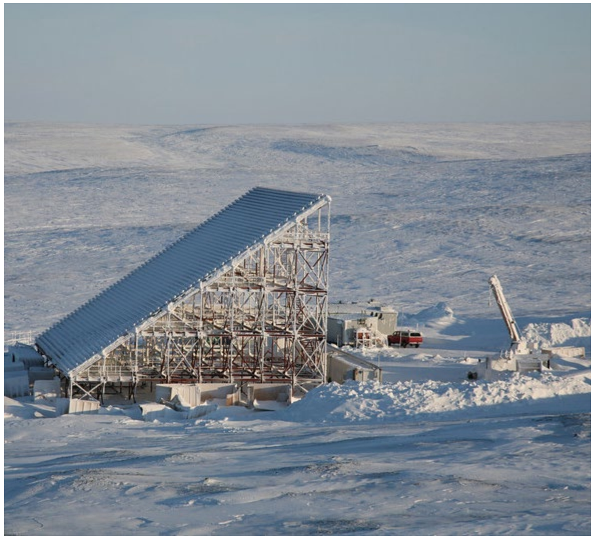
Log periodic



Horn antenna



Parabolic dish Reflector antenna

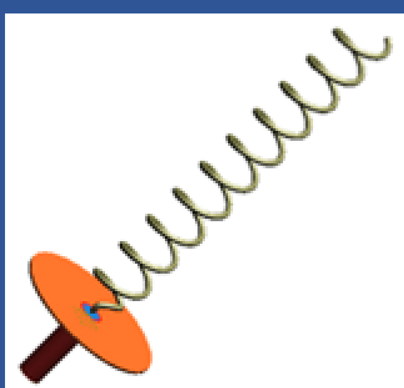


Examples of Antennas

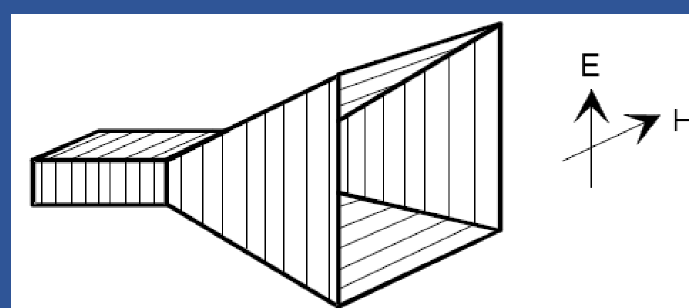
Antennas

Four primary functions of an antenna for radar applications:

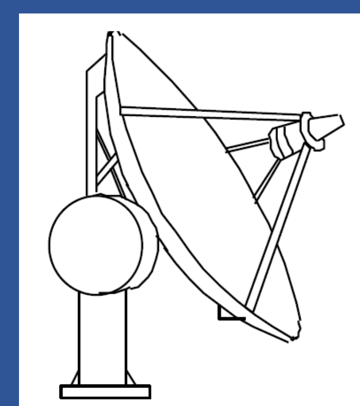
- Impedance transformation (free-space intrinsic impedance to transmission-line characteristic impedance)
- Propagation-mode adapter (free-space fields to guided waves)
- Spatial filter (radiation pattern – direction-dependent sensitivity)
- Polarization filter (polarization-dependent sensitivity)



Helical antenna



Horn antenna



Parabolic reflector antenna

Impedance transformer

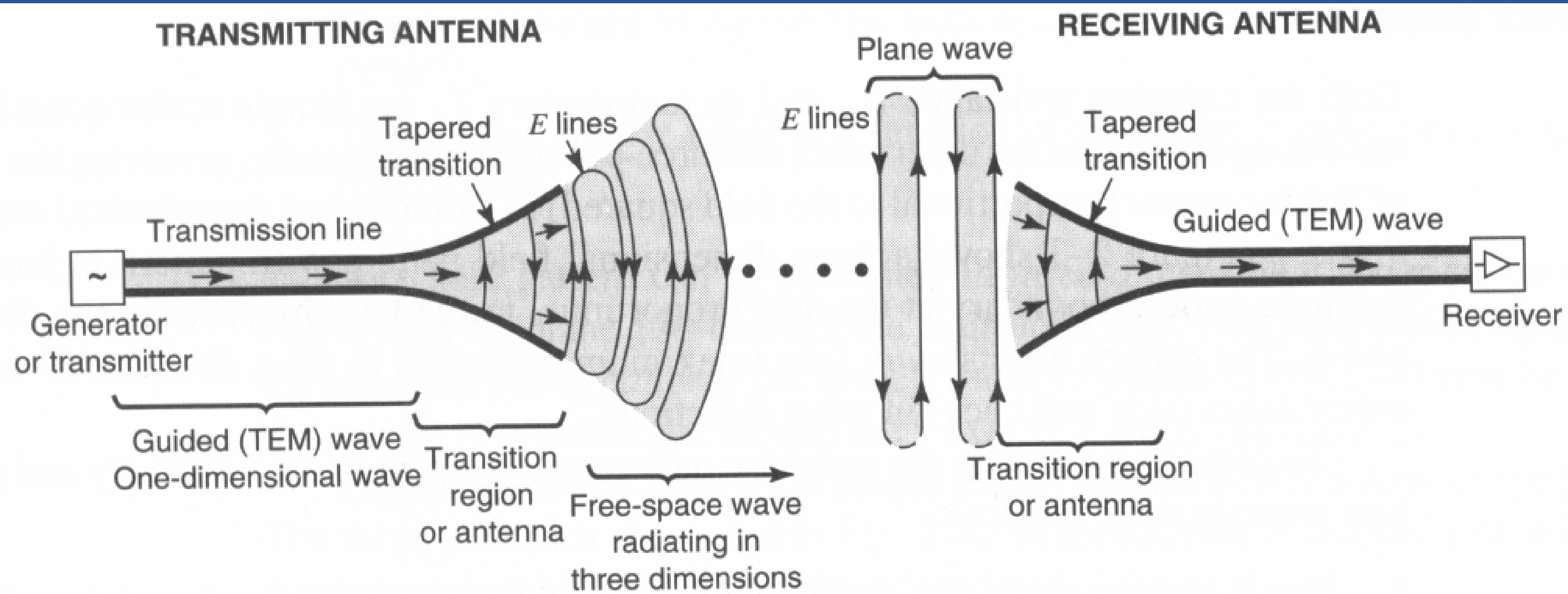
- Intrinsic impedance of free-space, $\eta_o \equiv E/H$ is

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\,\pi \cong 376.7\,\Omega$$

- Characteristic impedance of transmission line, $Z_o = V/I$
- A typical value for Z_o is 50 Ω.
- Clearly there is an impedance mismatch that must be addressed by the antenna.

Propagation-mode adapter

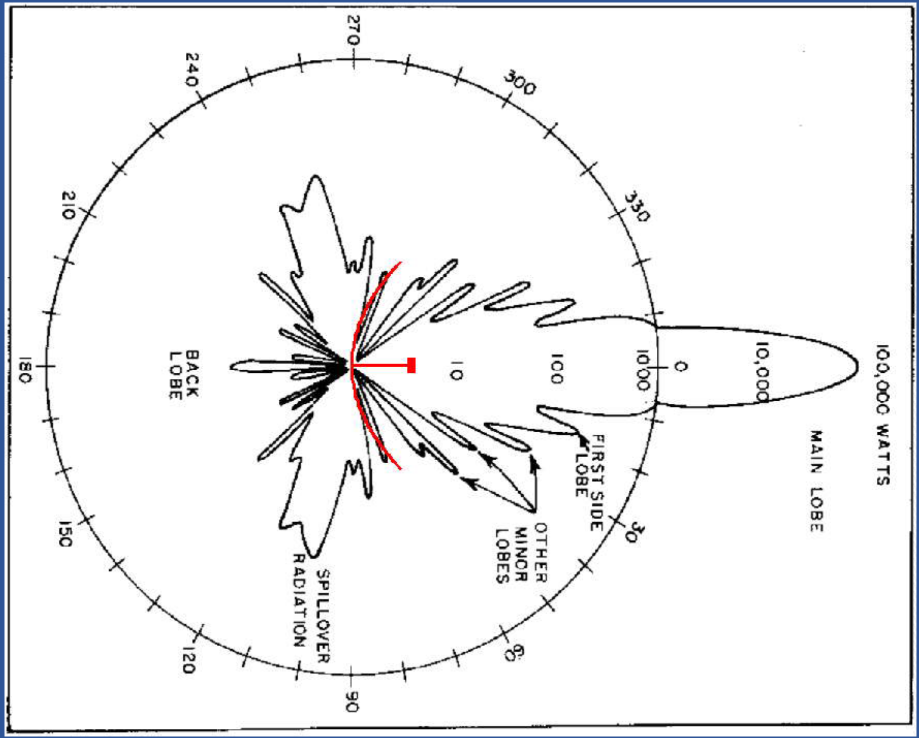
During both transmission and receive operations, the antenna must provide the transition between these two propagation modes.



Spatial filter

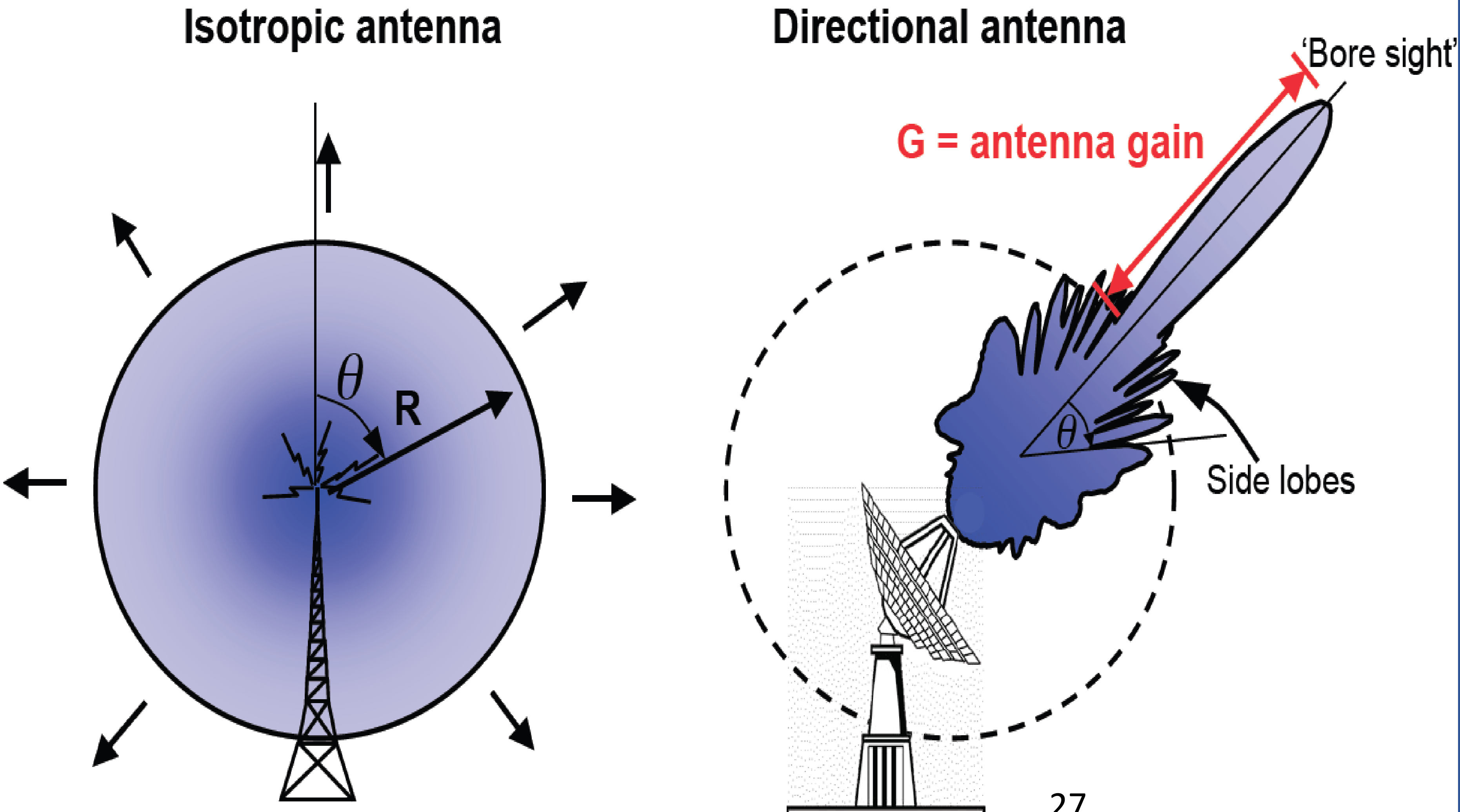
Antennas have the property of being more sensitive in one direction than in another, which provides the ability to spatially filter signals from its environment.

Directive antenna.



Radiation pattern of directive antenna.

Radiation pattern – antenna gain



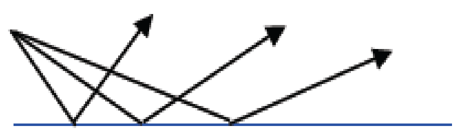
Propagation medium – losses

Radio waves are affected by the medium they propagate in. Effects dependent on the refractive index of the medium and wave frequency

Radio waves are also reflected off of the surface

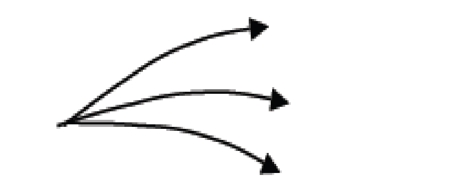
- **Atmospheric attenuation**

- **Reflection off of earth's surface**



- **Over-the-horizon diffraction**

- **Atmospheric refraction**



Attenuation usually measured in dB

$$\text{SNR dB} = 10\log_{10}\frac{\textit{signal power}}{\textit{noise power}}$$

dB value	times by
+30 dB	1000
+20 dB	100
+3 dB	2
-10 dB	0.1
-20 dB	0.01

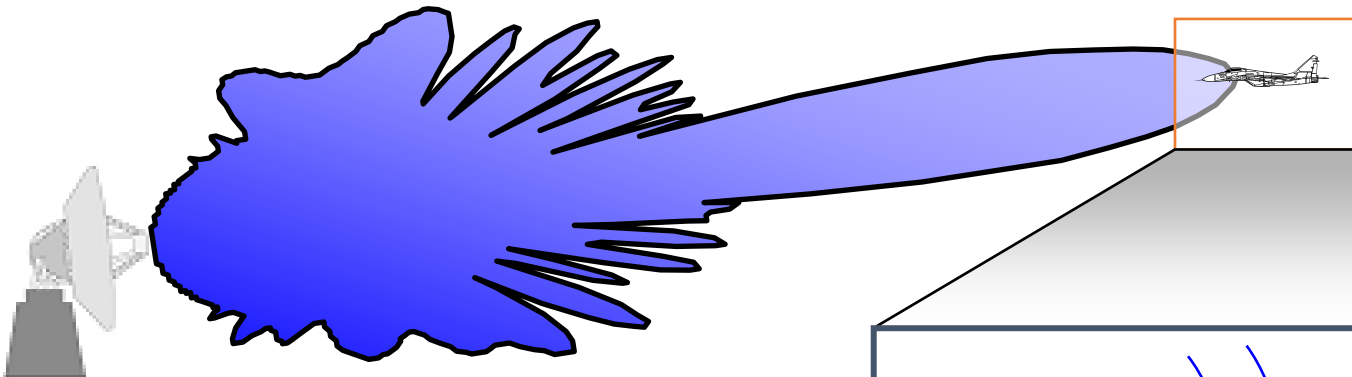
Radar equation

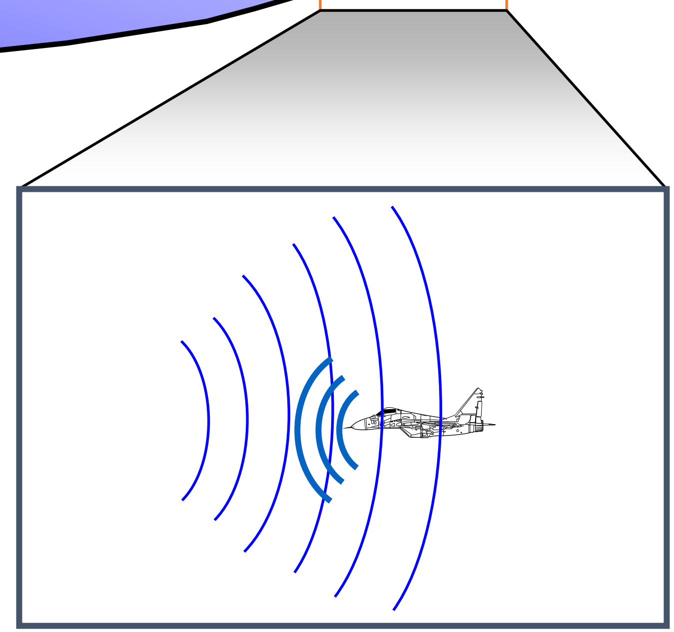
$$\text{Received Signal Energy} = \underbrace{[P_T]}_{\text{Transmit Power}} \underbrace{\left[\frac{4\pi A}{\lambda^2}\right]}_{\text{Transmit Gain}} \underbrace{\left[\frac{1}{4\pi R^2}\right]}_{\text{Spread Factor}} \underbrace{\left[\frac{1}{L}\right]}_{\text{Losses}} \underbrace{[\sigma]}_{\text{Target RCS}} \underbrace{\left[\frac{1}{4\pi R^2}\right]}_{\text{Spread Factor}} \underbrace{[A]}_{\text{Receive Aperture}} \underbrace{[\tau]}_{\text{Dwell Time}}$$

Radar cross section tells us about the target properties

It is the effective target cross section as seen by the radar

Radar Cross Section (RCS)





RCS

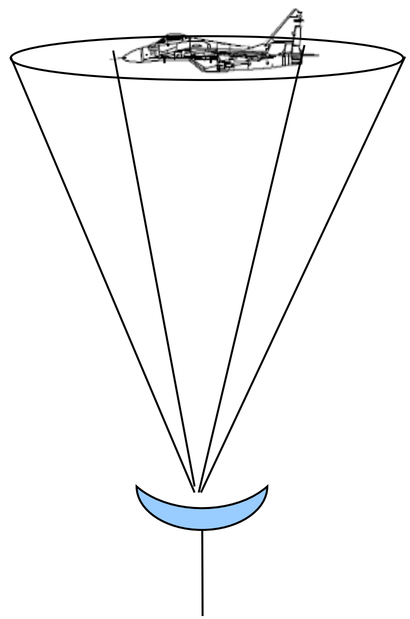
Incident Power Density $\times$ **σ** = **Reflected Power**

(Watts/m^2) **(m^2)** **(Watts)**

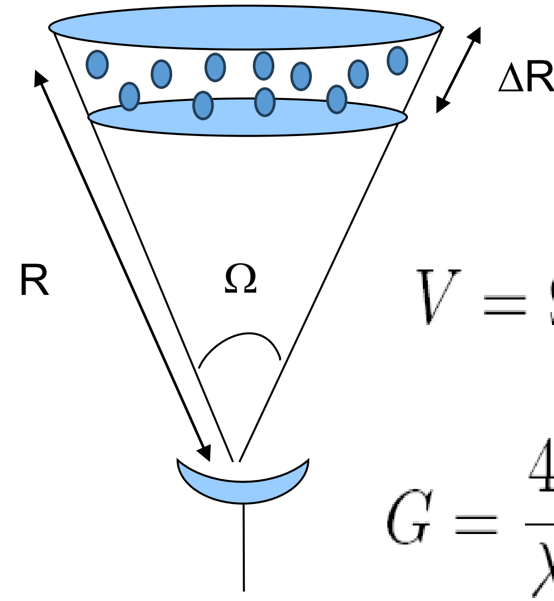
Radar Cross Section (RCS, or s) is the *effective* cross-sectional area of the target as seen by the radar

measured in m^2, or dBm^2

Hard targets vs. Soft targets



vs.



$$V = \Omega R^2 \, \Delta R$$

$$G = \frac{4\pi}{\lambda^2} A = \frac{4\pi}{\Omega}$$

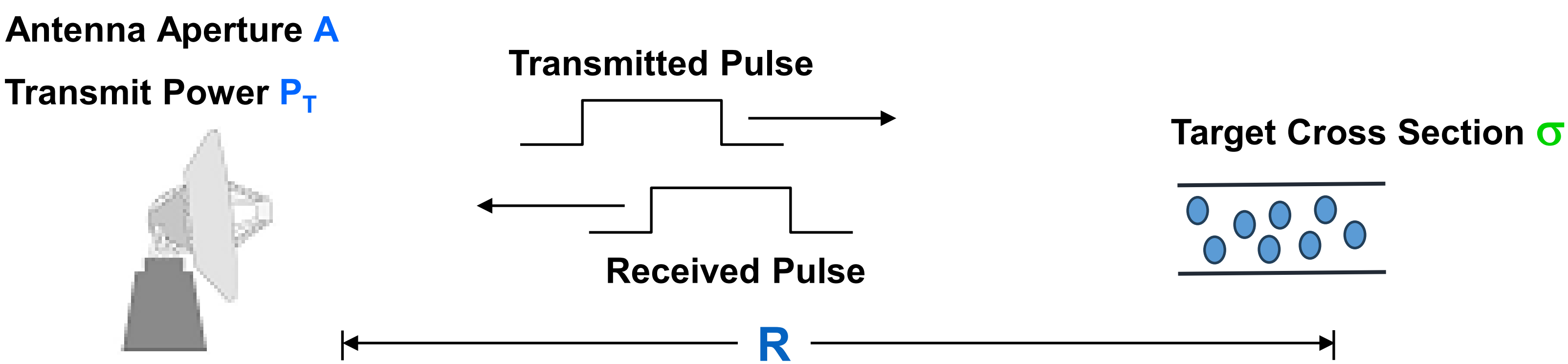
$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

$$P_r = \frac{P_t A \sigma_v \Delta R}{4\pi R^2}$$

Volume scattering - Ionosphere

- Volume scattering cross section σ_v has units of area/volume
- Signal is proportional to range resolution
- What about the ionosphere ?
  - Cross section of a single electron = 10^{-28} m^2
  - Cross section of a bunch of electrons in a 10 km^3 volume in the ionosphere assuming electron density = 10^{12} $/m^3$, is 10^{10} x 10^{12} x 10^{-28} = 10^{-6} m^2 !!)
  - **CAN be measured by an incoherent scatter radar.**

Radar Range Equation



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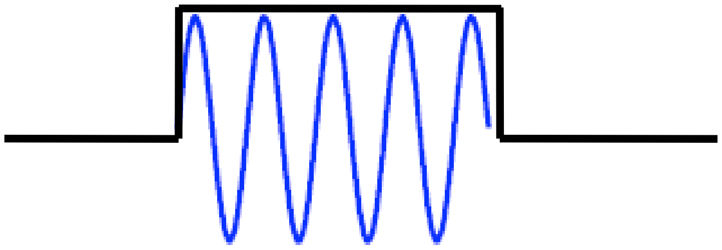
What the radar transmits: Pulses and waves



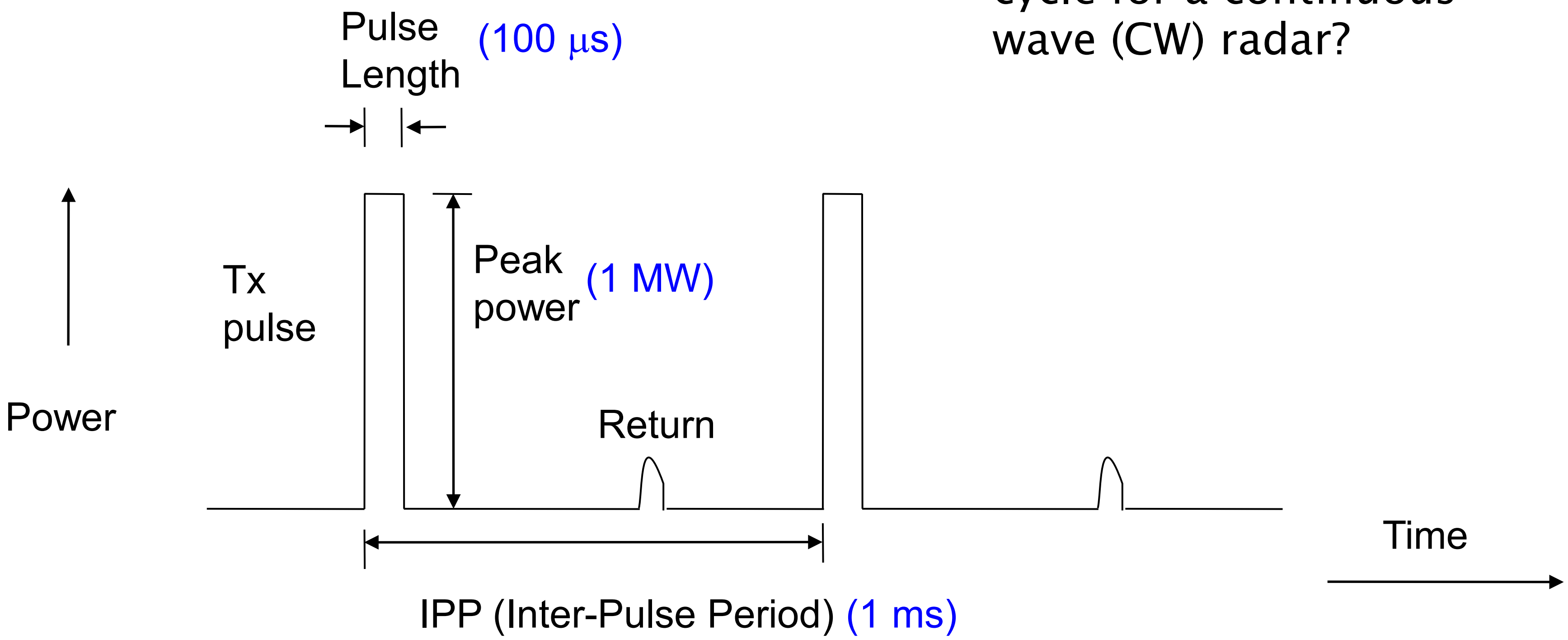
How many cycles in a pulse?

PFISR frequency = 449 MHz
Long pulse length = 330 μs
# of cycles = 148170

Radar waveforms modulate the waves with on-off sequence

Pulsed Radar

What would be the duty cycle for a continuous wave (CW) radar?



Duty cycle = Pulse Length/IPP (10%)

Average power = Peak power x Duty cycle (100 kW)

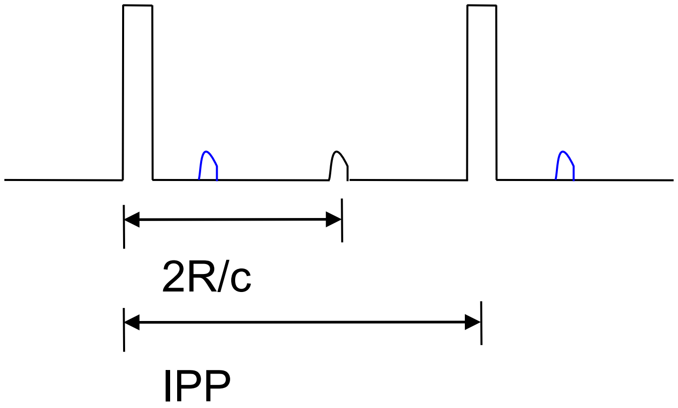
PRF (Pulse Repetition Frequency) = 1/IPP (1kHz)

Range Resolution

Range resolution is set by pulse length

Pulse length = τ_p , Range resolution = $c\tau_p/2$ for a single target.

Maximum unambiguous range



MUR = c*IPP/2

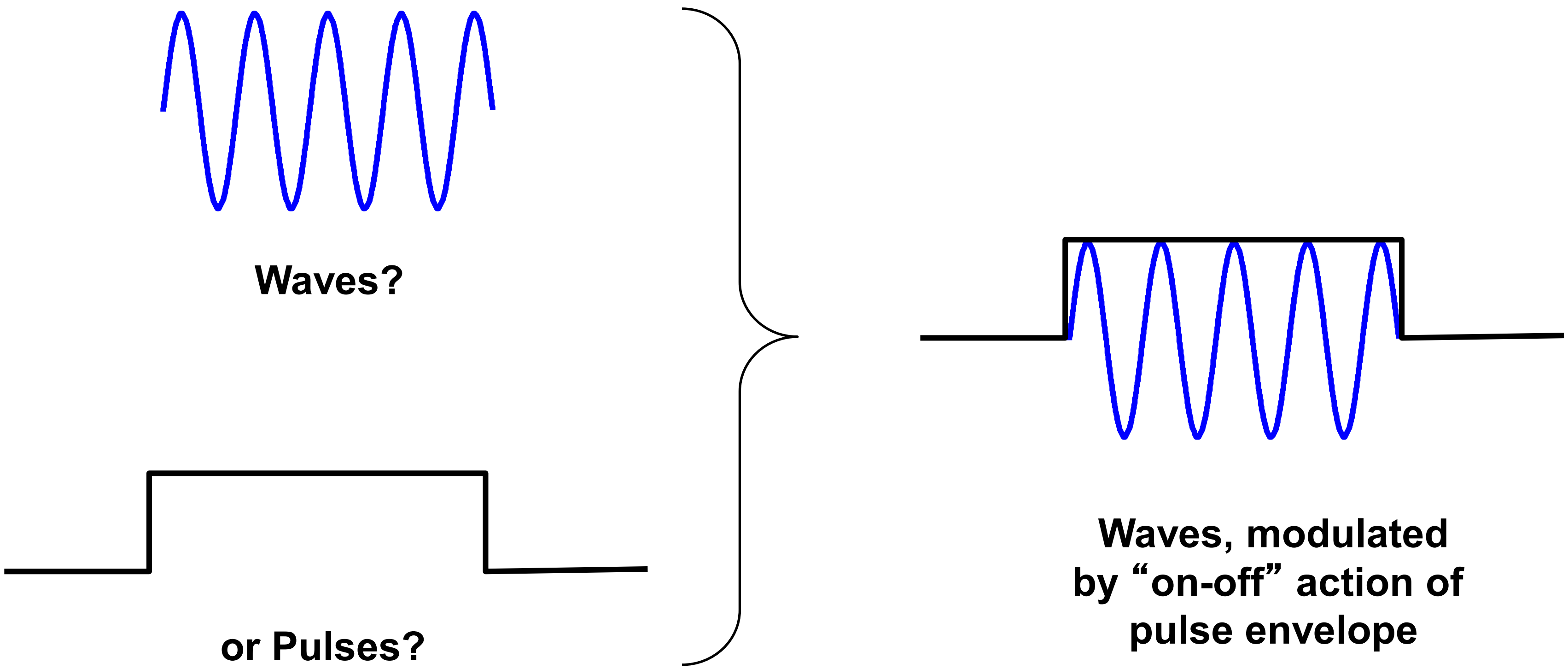
Pulse duration vs. Range resolution

Pulse Duration	Range Resolution
0.1 nsec	1.5 cm
1.0 nsec	15 cm
10 nsec	1.5 m
100 nsec	15 m
1 μsec	150 m
10 μsec	1.5 km
100 μsec	15 km
1 msec	150 km

What is a typical F region ISR pulselength?

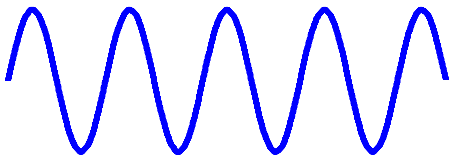
Radar Waveforms

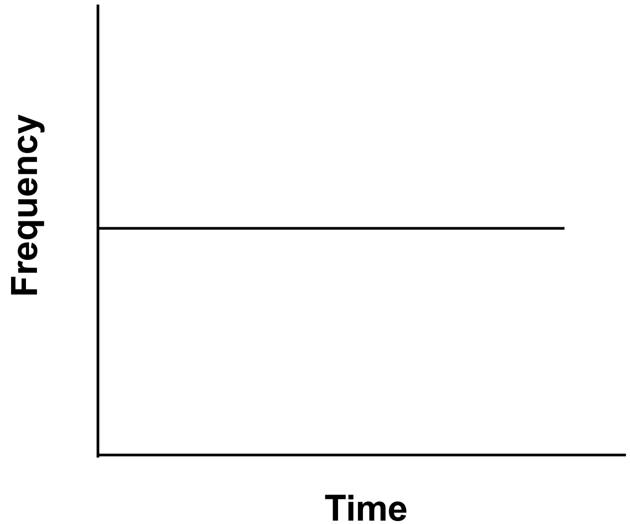
What do radars transmit?



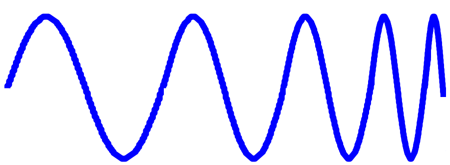
Radar Waveforms

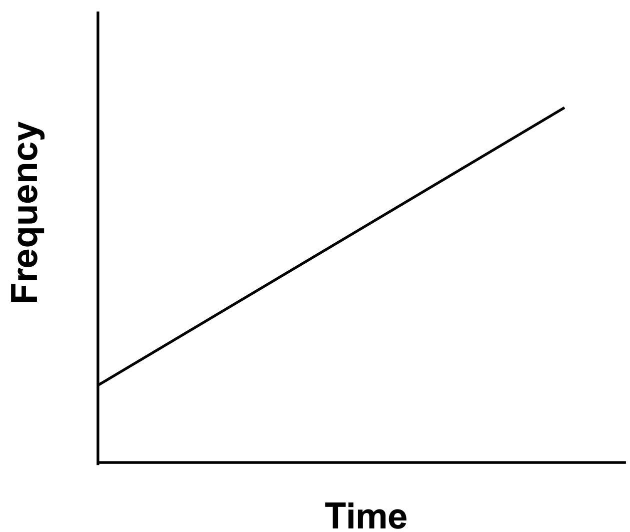
Pulse at single frequency





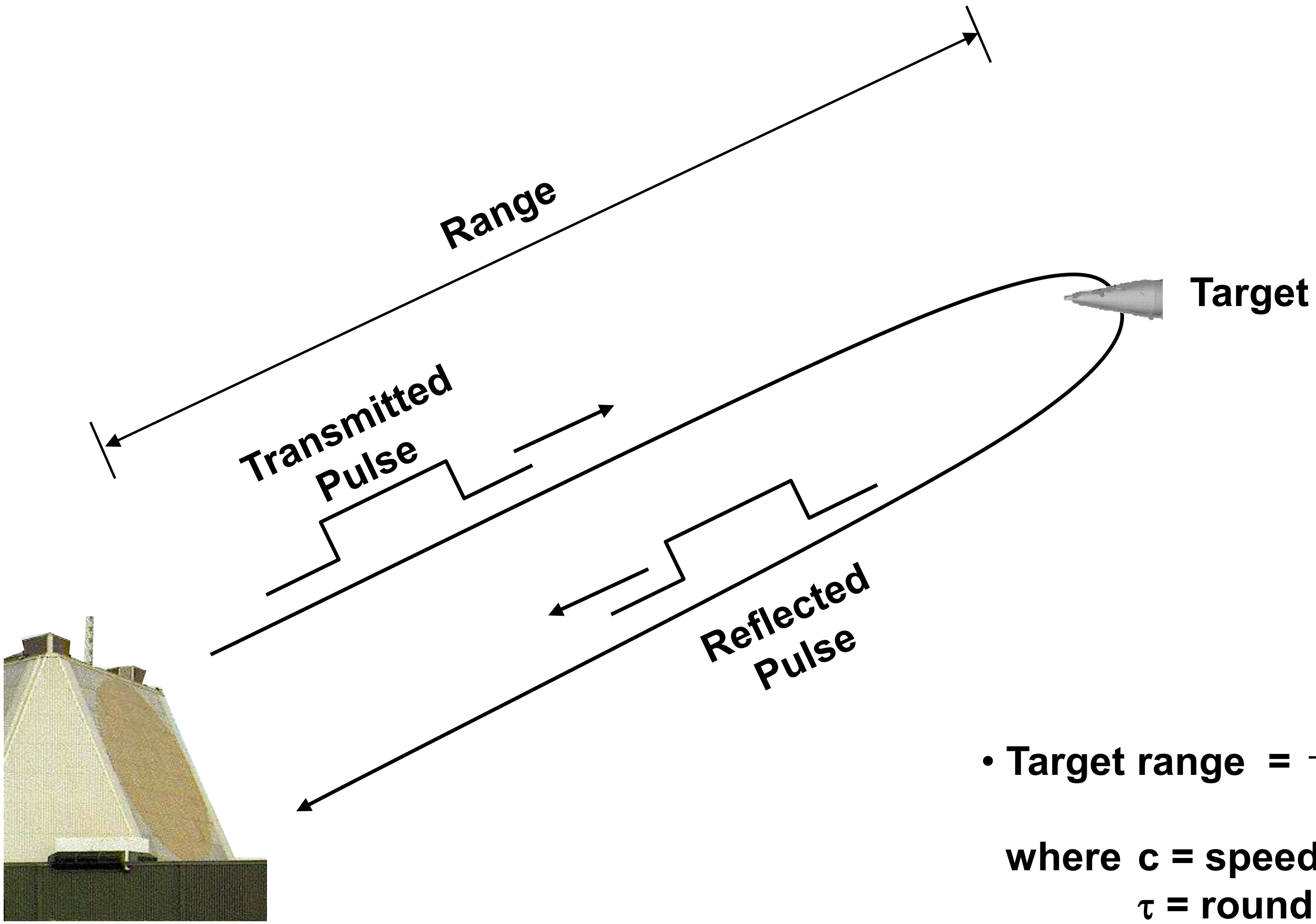
Pulse with changing frequency





Linear Frequency-Modulated (LFM) Waveform

Radar Range Measurement

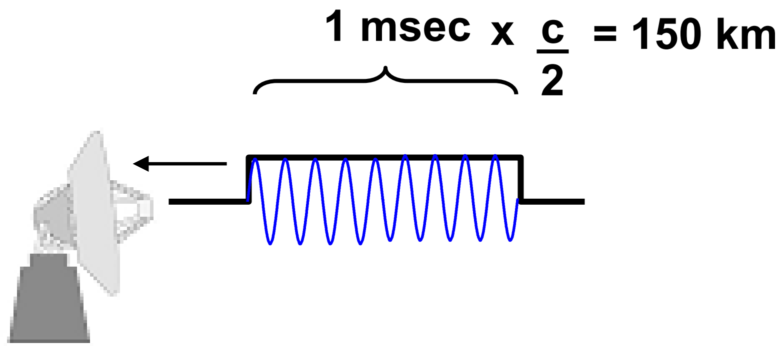


- Target range $= \frac{c\tau}{2}$

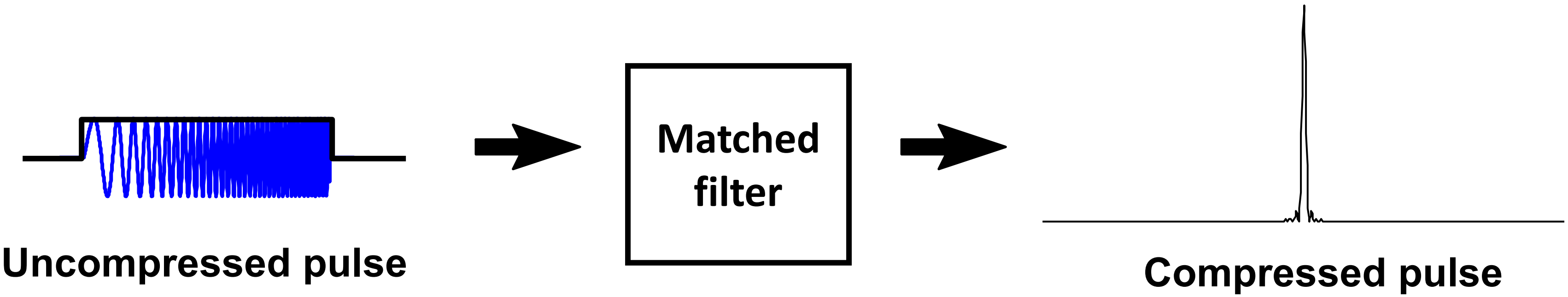
where c = speed of light
τ = round trip time

Range Resolution and pulse compression

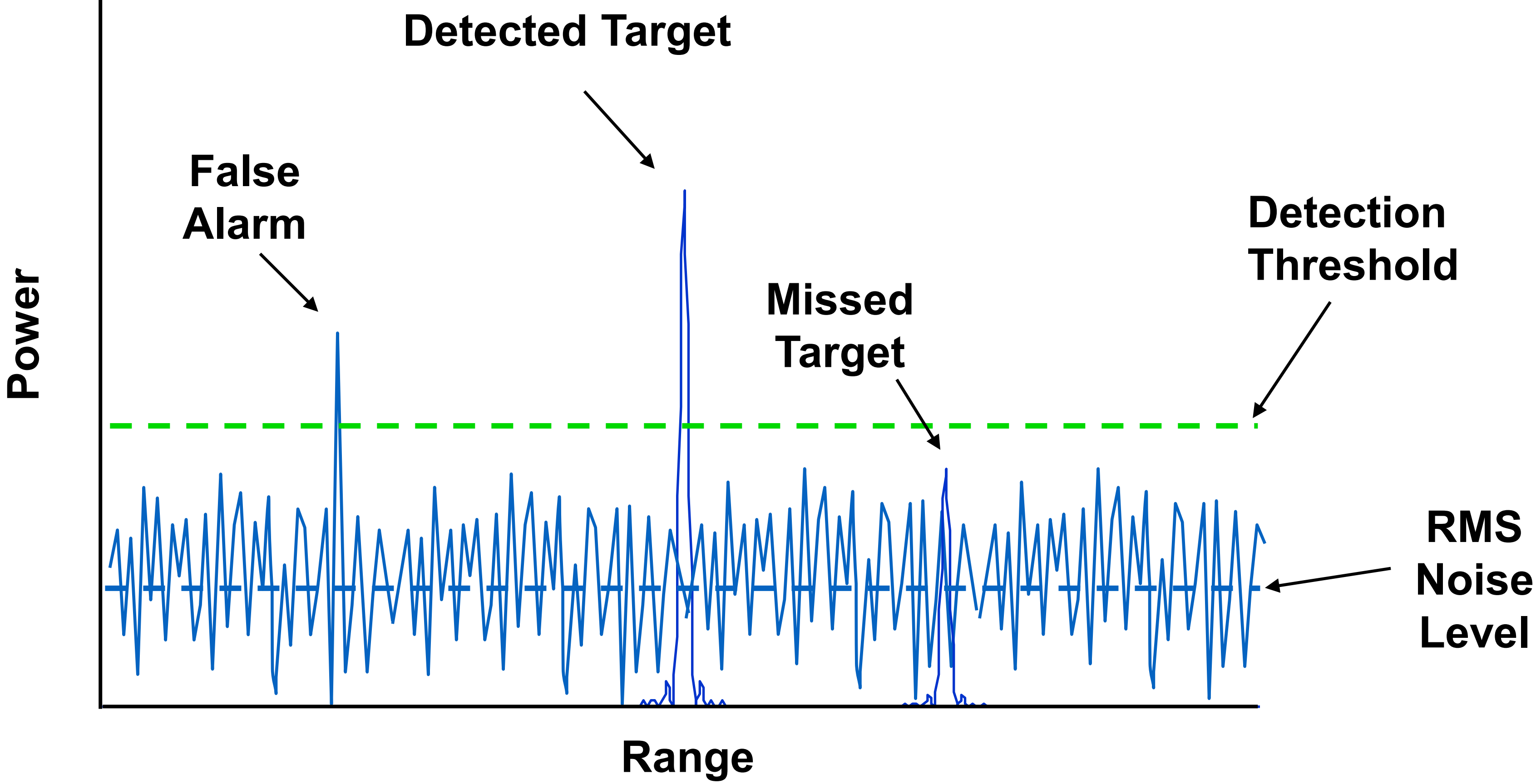
Problem: Pulse can be very long; does not allow accurate range measurement



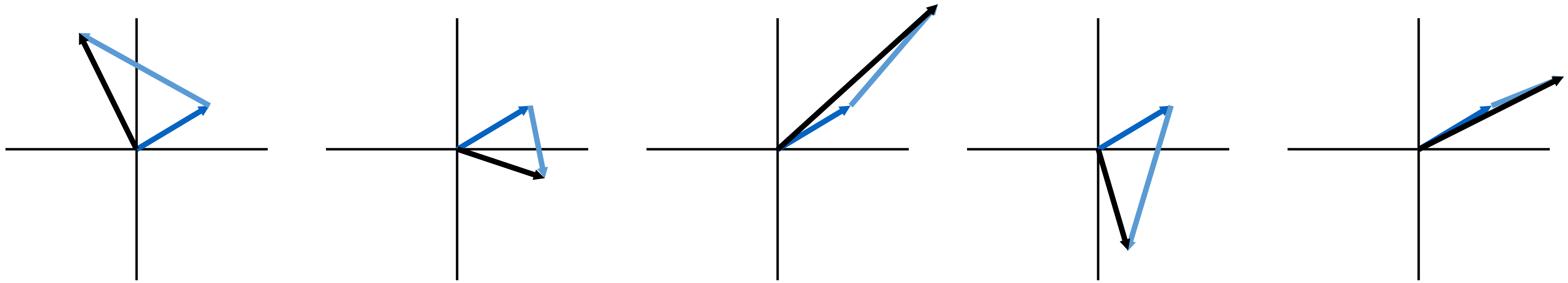
Solution: Use pulse with changing frequency and signal process using "matched filter"



Detection of Signals in Noise



Coherent Integration

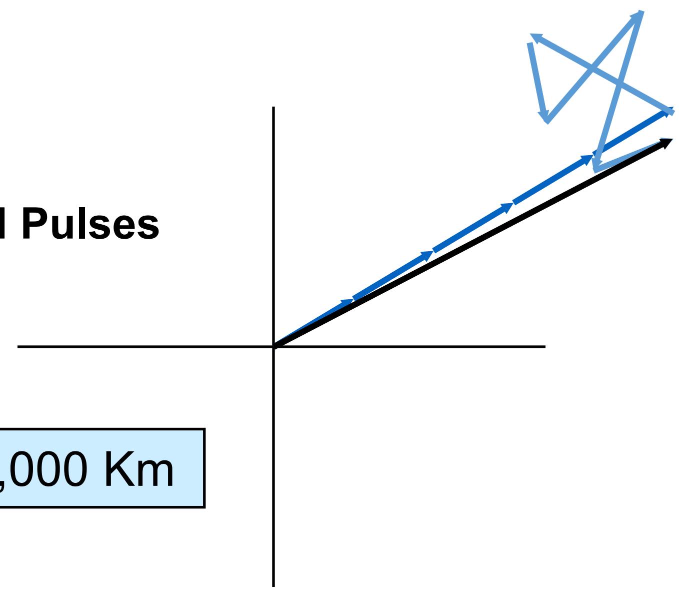


Pulse 1 **Pulse 2** **Pulse 3** **Pulse 4** **Pulse 5**

- **Coherent target returns**
- **Noise samples at low SNR**

- **Resultant signal**

Coherently Integrated Pulses

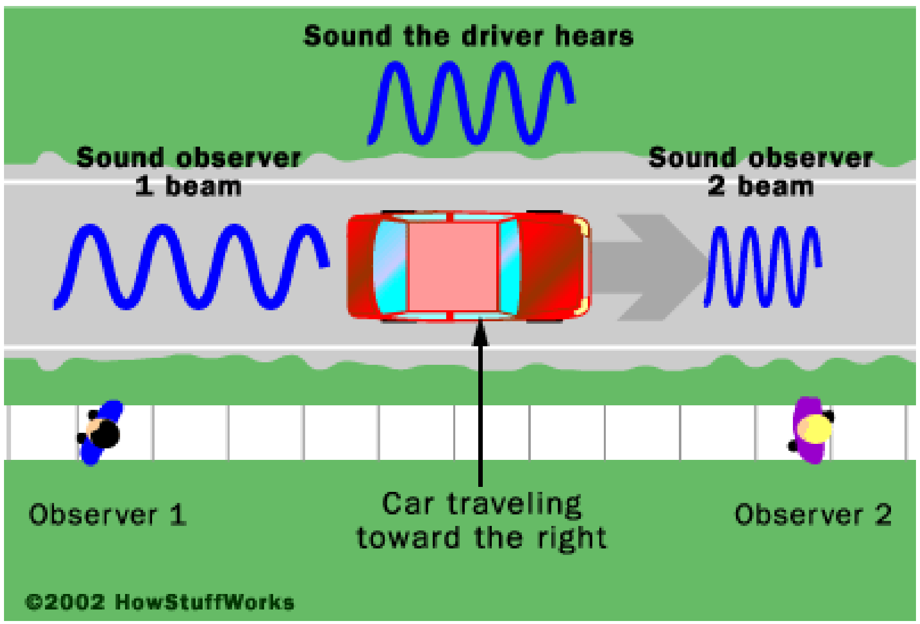


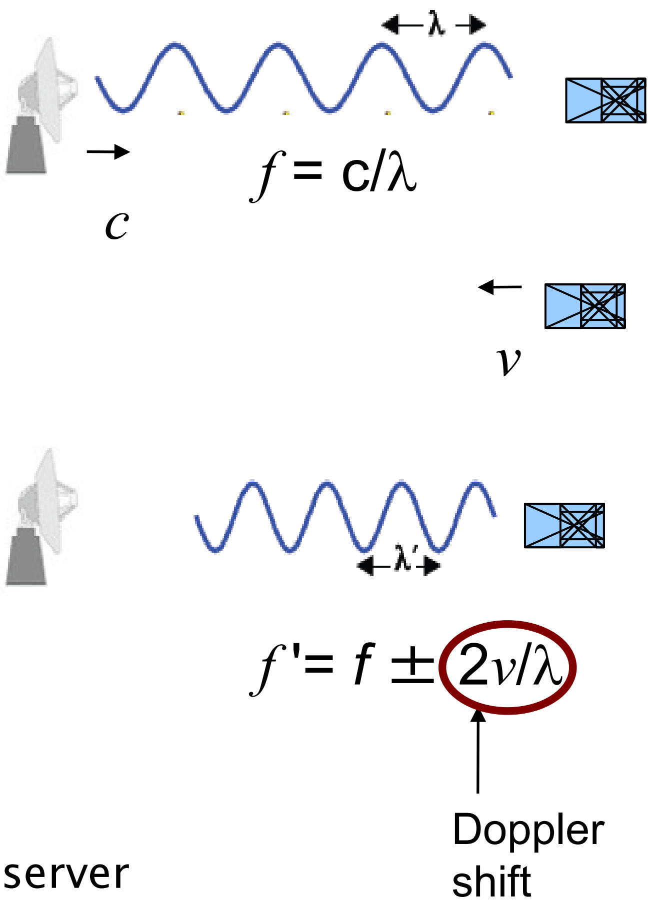
Deep space targets at 30,000 – 40,000 Km

Outline - Radar Basics

- Electromagnetic spectrum
- Radio waves and propagation
- Radar equation
- Range resolution and pulsed radars
- Doppler

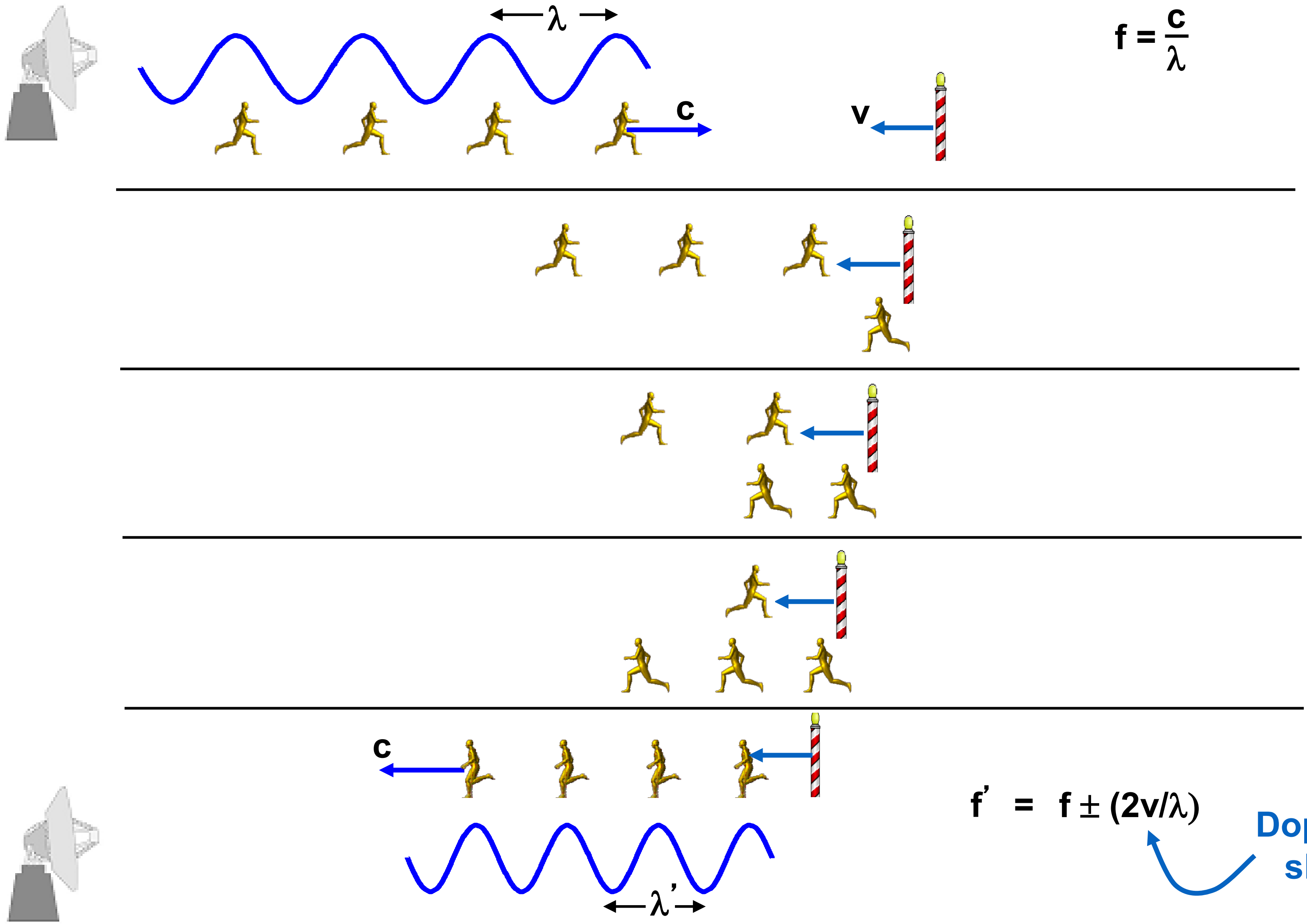
Moving target: Doppler





Positive Doppler = target moving **toward** the observer
Negative Doppler = target moving **away** from the observer

Doppler Shift Concept



$f = \frac{c}{\lambda}$

$f' = f \pm (2v/\lambda)$

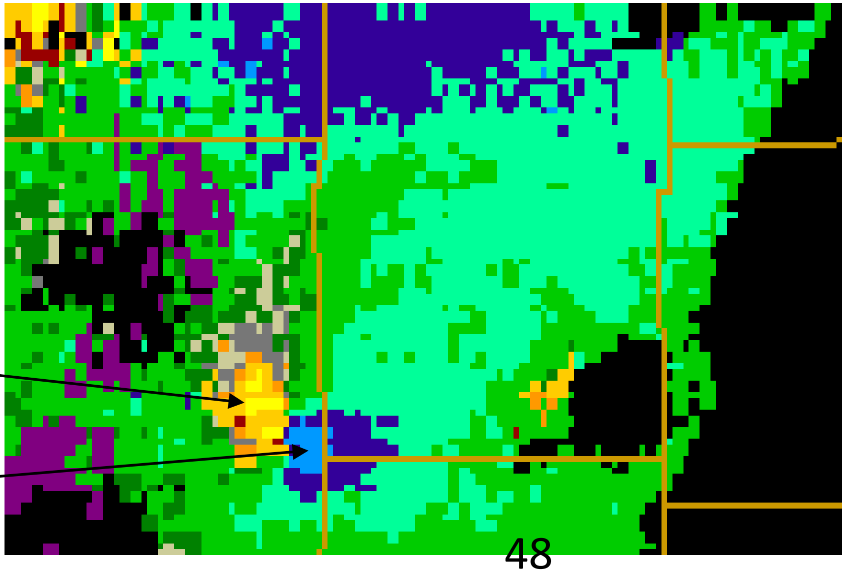
Doppler shift

Sign conventions

The Doppler frequency is negative (lower frequency, red shift) for objects moving away from the radar

The Doppler frequency is positive (higher frequency, blue shift) for objects moving toward the radar

These "color" shift conventions are typically also used on radar displays of Doppler velocity



Away from radar

Toward radar

Doppler shift frequency

Tx signal: $\cos(2\pi f_o t)$

Return from a moving target: $\cos[2\pi f_o(t + 2R/c)]$

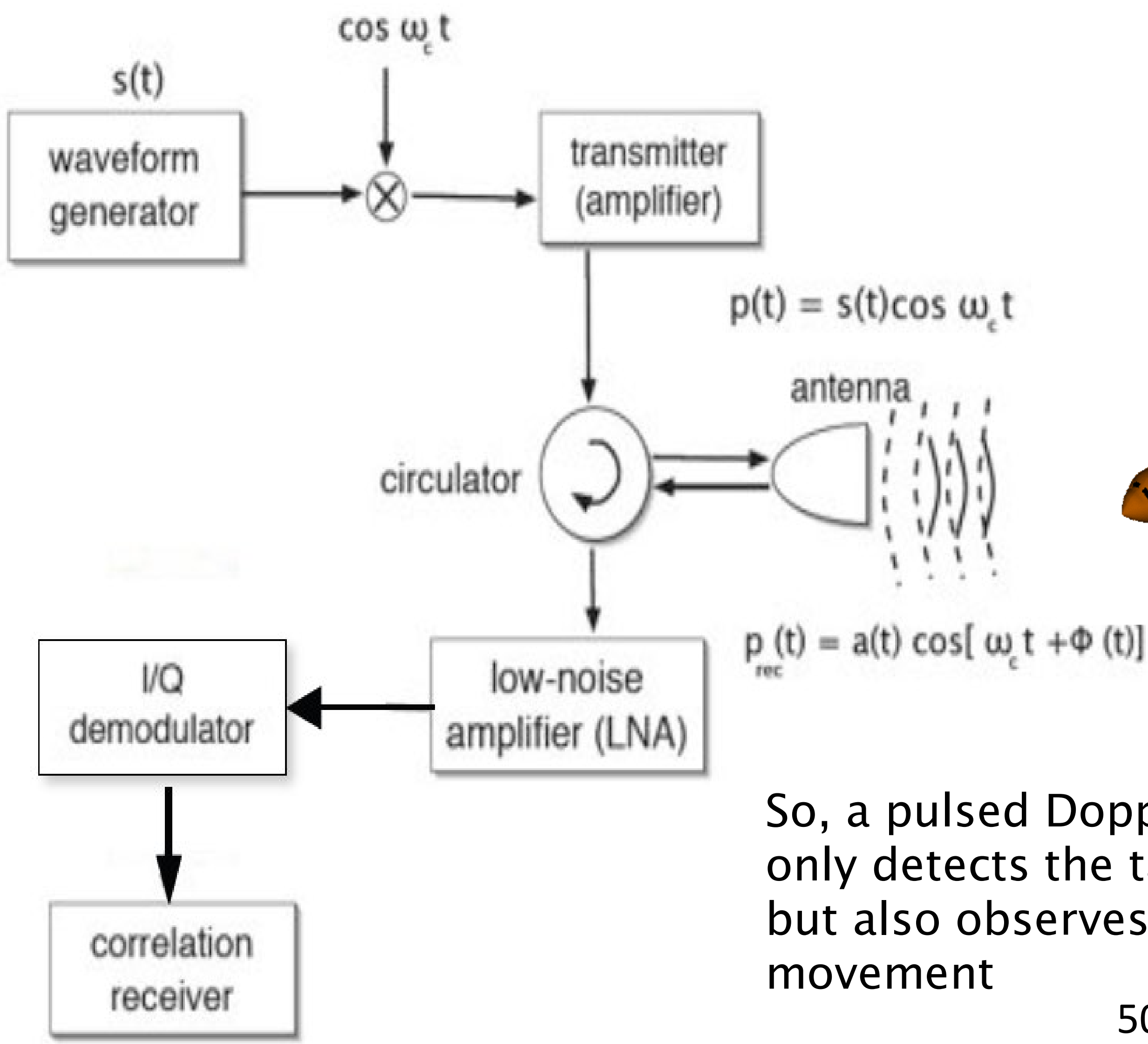
If target is moving with a constant velocity, such that $R = R_o + v_o t$

then,

Return: $\cos[2\pi(f_o + f_o 2v_o/c)t + 2\pi f_o R_o/c]$

Doppler frequency:
$-2f_o v_o/c = -2v_o/\lambda_o$

Pulsed Doppler Radar system



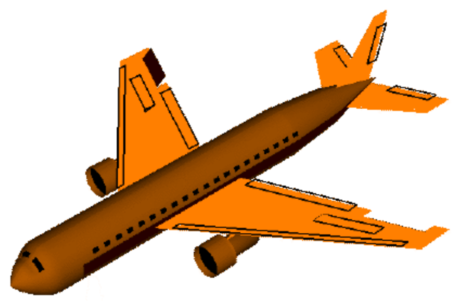


Image courtesy of NASA

So, a pulsed Doppler radar not only detects the target location, but also observes the target movement