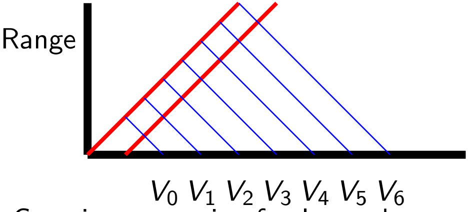
Radar Statistics: Lag-Product Estimation

Roger H. Varney

[1]Dept. of Atmospheric and Oceanic Sciences
University of California, Los Angeles

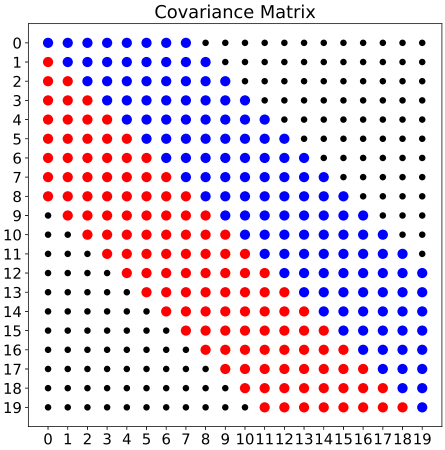
July, 2023

Covariance Matrix of Voltages



Covariance matrix of voltages has a particular form

- Zero for samples more than a pulse length apart.
- conjugate symmetric.
- Lag-product array is the subset of interest.



AMISR Index Convention for LPA

Lag-Product Array:

$$L_\ell^i \equiv \left\langle V^*_{i-\left\lfloor \frac{\ell}{2} \right\rfloor} V_{i+\left\lfloor \frac{\ell}{2} \right\rfloor + (\ell \bmod 2)} \right\rangle$$

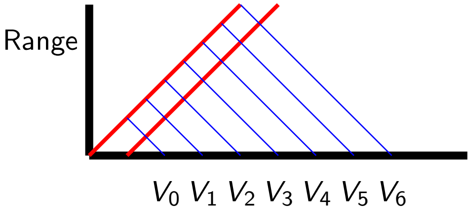
$$L_0^i = \langle V_i^* V_i \rangle$$

$$L_1^i = \langle V_i^* V_{i+1} \rangle$$

$$L_2^i = \left\langle V_{i-1}^* V_{i+1} \right\rangle$$

$$L_3^i = \left\langle V_{i-1}^* V_{i+2} \right\rangle$$

etc.



Interpretation

- i is a "range" index
- ℓ is the lag index
- Sometimes called the "radar ACF" at range i and lag ℓ

LPA Estimators

This definition

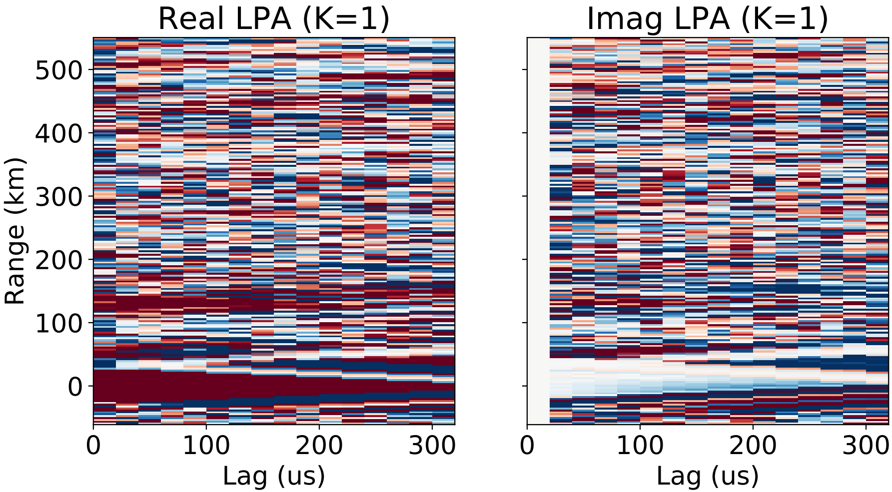
$$L_\ell^i \equiv \left\langle V^*_{i-\lfloor\frac{\ell}{2}\rfloor} V_{i+\lfloor\frac{\ell}{2}\rfloor+(\ell \bmod 2)} \right\rangle$$

refers to the *expected value* of the products of samples.
Can be estimated by taking products samples and averaging over many pulses

$$\hat{L}_\ell^i = \frac{1}{K}\sum_{k=0}^{K-1} V^*_{i-\lfloor\frac{\ell}{2}\rfloor} V_{i+\lfloor\frac{\ell}{2}\rfloor+(\ell \bmod 2)}$$

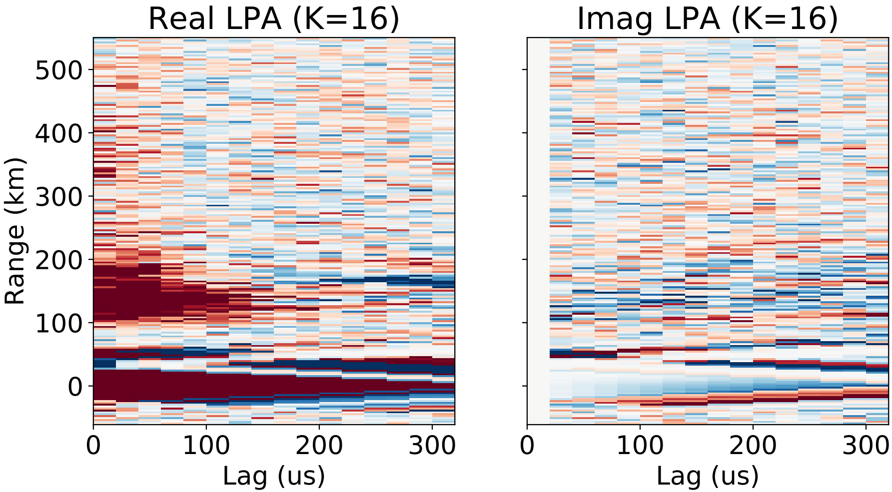
By virtue of the central limit theorem, these estimators will be Gaussian random variables with variances that decrease as $1/K$ (error decreases as $1/\sqrt{K}$).

LPA Estimator from 1 Pulse



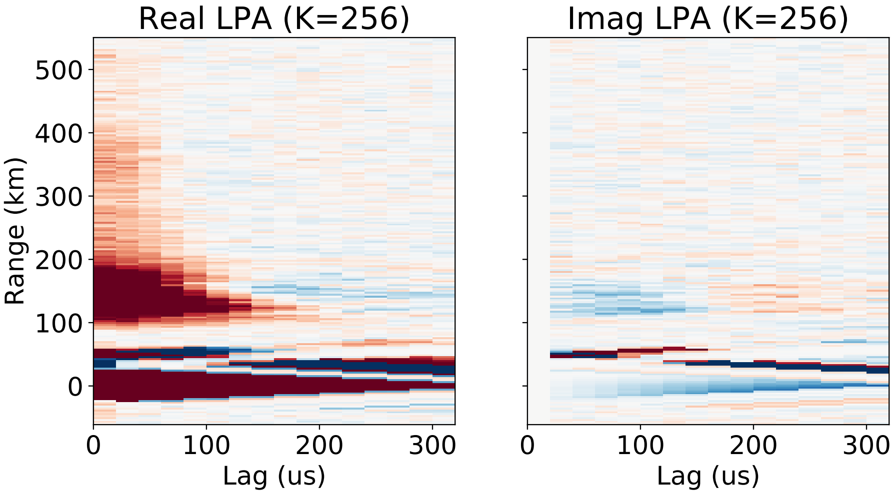
Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 16 Pulses



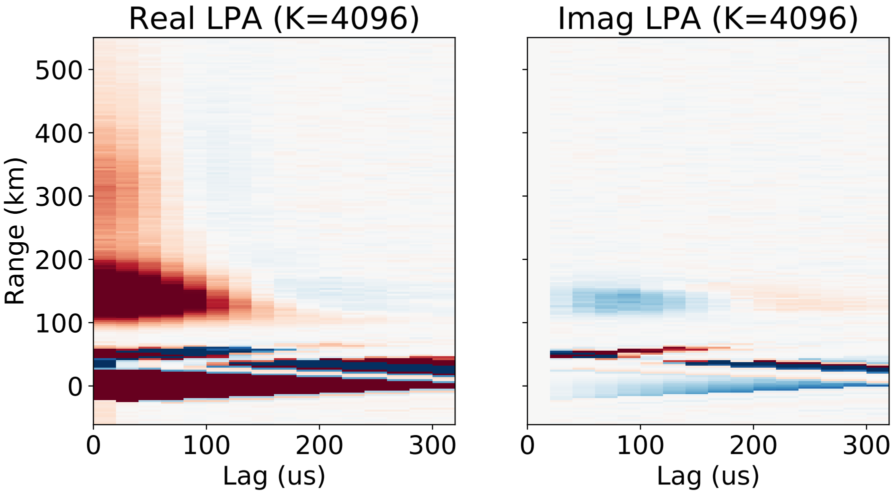
Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 256 Pulses



Note the Noise ACF has been subtracted before plotting.

LPA Estimator from 4096 Pulses



Note the Noise ACF has been subtracted before plotting.

Forward Model for ISR LPA

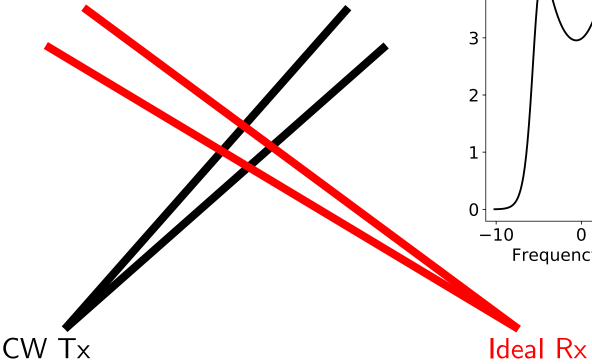
Goal: Map ionospheric physical parameters to the LPA the radar is expected to measure

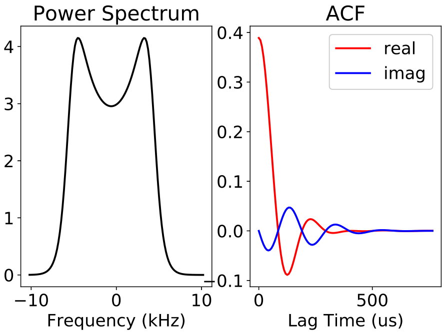
$$N_e(r),\, T_e(r),\, T_i(r),\, u_{\mathrm{los}}(r) \xrightarrow{\text{Forward Model}} L_\ell^i$$

Components of forward model:

1. Physics portion (ISR theory)
2. Engineering portion (Ambiguity function theory)
3. Noise background

Physics Portion

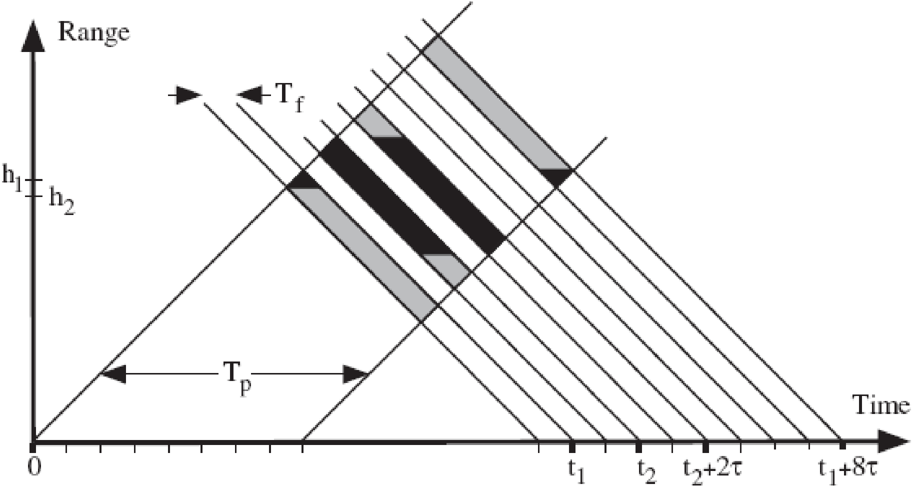




ISR theory gives the PSD and ACF of the received voltages as a function of N_e, T_e, T_i, and u_{los} in the overlap volume.

Engineering Portion

LPA Mixes Range and Time Information



Information smeared in range over the dark overlap regions.

Ambiguity Functions

Measured lag-products:

$$\langle V^* (t_{s1})\, V(t_{s2}) \rangle = \int d\tau dr\, A(r,\tau)\, W_{t_{s1},t_{s2}}(\tau, r)$$

The measured lag-product is the ACF from the physics portion $A(r,\tau)$ blurred by the **range-lag ambiguity function** $W_{t_{s1},t_{s2}}(\tau, r)$

The ambiguity function is a function of the transmit waveform $s(t)$ and the receiver impulse response $h(t)$

$$W_{t_s}(t, r) \equiv s\left(t - \frac{2r}{c}\right) h^* (t_s - t)$$

$$W_{t_{s1},t_{s2}}(\tau, r) = \int dt\, W_{ts2}(t+\tau, r)\, W^*_{ts1}(t, r)$$

Full Forward Model Including External Noise

$$\langle V^*(t_{s1})\, V(t_{s2})\rangle = \int d\tau dr\, A(r,\tau) W_{t_{s1},t_{s2}}(\tau, r) + R_N(t_{s2} - t_{s1})$$

Measured elements of the LPA are determined by

- Physics of the ionosphere (unknown)
- Engineering details of radar experiment (known)
- ACF of external noise (estimate from long ranges)

Goal of fitting is to go backwards from measurements to parameters determining the physics of the ionosphere.