Data Analysis and Fitting: Review

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Data Analysis and Fitting

Questions:

- What does "fitted data" mean?
- What are the key concepts and techniques we need to fit data?
- How do we go from voltage samples to N_e, T_e, T_i, V_{los}?
- How do I work with and interpret IS Radar data products?

Topics Covered

Data Modeling:

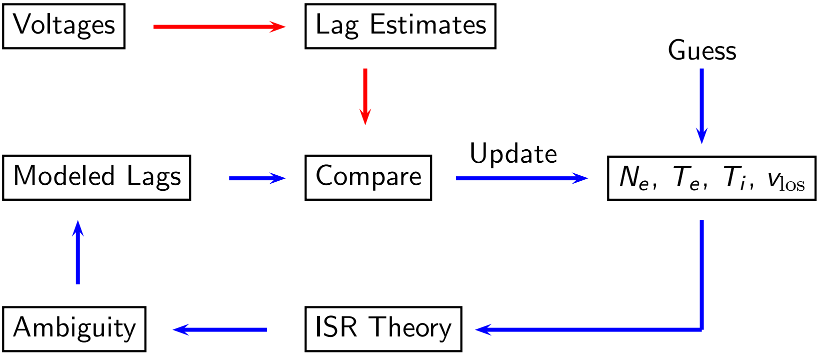
- Forward Model and Inverse Problems
  - $\langle E_s(t) E_s^*(t-\tau) \rangle = f(N_e, T_e, T_i, V_{los}, W_{t\tau}) + \mathrm{e}$
  - $f(N_e, T_e, T_i, V_{los})$ is constructed with IS Radar Theory and Measurement Ambiguity
  - Inverse problem: solving for N_e, T_e, T_i, V_{los} given the data

Fitting:

- Least-Squares
  - can be used to solve inverse problems
  - if $\mathrm{e} = \langle E_s(t) E_s^*(t-\tau) \rangle - f(N_e, T_e, T_i, V_{los})$ are Gaussian random variables
  - then, the "best-fit" of $\mathrm{p} = N_e,\ T_e,\ T_i,\ V_{los}$ minimizes the "chi-squared": $\chi^2(\mathrm{p}) = [\mathrm{y} - f(\mathrm{p})]^T\, \Sigma_{\mathrm{e}}{}^{-1}\, [\mathrm{y} - f(\mathrm{p})]$
  - this is equivalent to maximizing the likelihood function

IS Radar Inverse Problem

Compare measurements and modeled measurements



Errors and Goodness of Fit

Given:

$$\chi^2(\mathrm{p}) = [\mathrm{y} - f(\mathrm{p})]^T \, \Sigma_{\mathrm{e}}{}^{-1} \, [\mathrm{y} - f(\mathrm{p})]$$

Error Propagation:

- $\Sigma_{\hat{\mathrm{p}}_{\mathrm{LS}}} = \left[\mathrm{J}^T \Sigma_e^{-1} \mathrm{J}\right]^{-1}$, but only valid when
  - e are Gaussian and $f(\mathrm{p})$ is linear, or
  - e are Gaussian and $f(\mathrm{p})$ non-linear, but can be accurately approximated by a linear model in the region around p
- $\Sigma_{\hat{\mathrm{p}}_{\mathrm{LS}}}$ is used to construct confidence intervals (error bars)

Goodness of Fit:

- compute reduced chi-squared: $\chi^2_\nu = \chi^2/(m - n + 1)$
  - $\chi^2_\nu \approx 1$: a good fit
  - $\chi^2_\nu << 1$: an "over fit"
  - $\chi^2_\nu >> 1$: a poor fit

IS Radar Data Levels

Summary of IS Radar data products: