Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

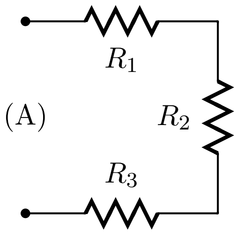
6.002 – Circuits & Electronics
Fall 2006

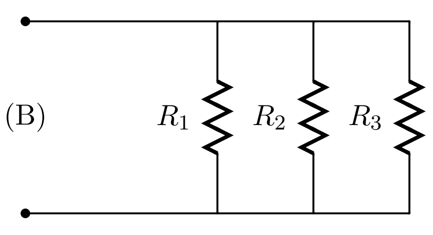
Problem Set #1

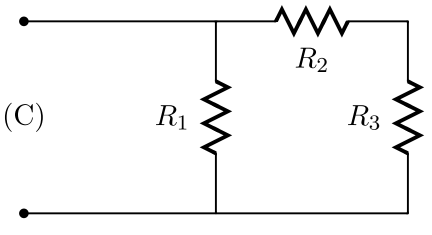
Issued 9/8/06 – Due 9/15/06

Exercise 1.0: Problem 1.4 makes use of WebLab, a physical remote-access laboratory that runs through the MIT iLab Service Broker. To obtain an iLab account, go to **http://ilab.mit.edu**, click on the *register here* link, provide the requested information, group yourself with Fall 2006 6.002 Students, and submit your request for an account. Account approval may take several hours to a day, so register early to avoid any last minute delays.

Exercise 1.1 (1 Point): Find the equivalent resistance, as viewed from its port, of each resistor network shown below.

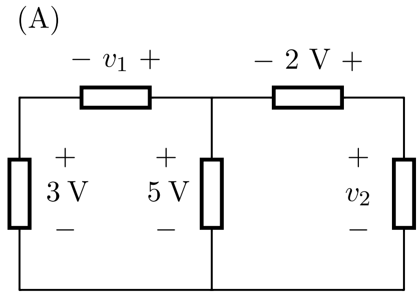


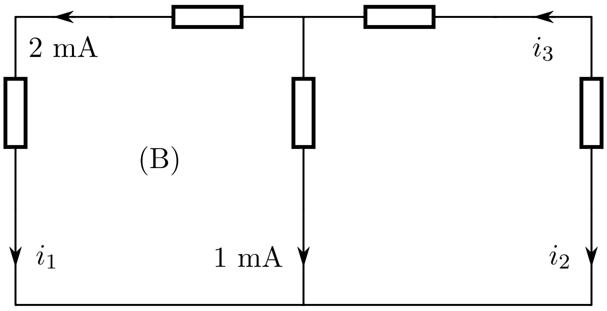


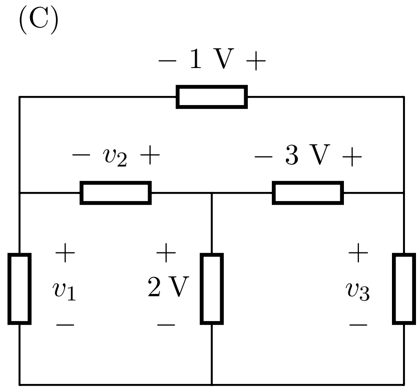


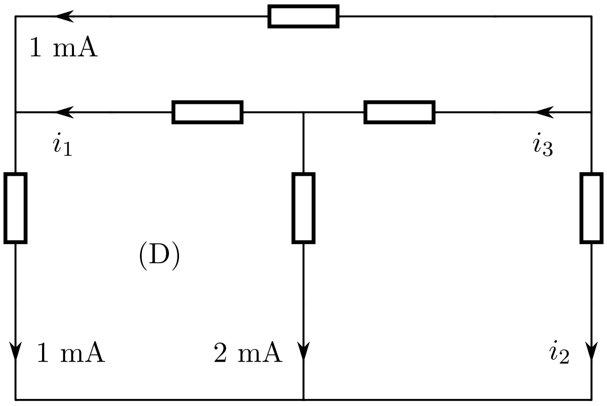
Exercise 1.2 (1 Point): What is the largest-valued resistor that can be fabricated by combining a 2-Ω resistor, another 2-Ω resistor, and a 4-Ω resistor? What is the smallest-valued resistor that can be fabricated by combining the same three resistors? How can one fabricate a 2-Ω resistor by combining the same three resistors? Suppose that each individual resistor can dissipate up to 1 W of power before burning up. How much power can the fabricated 2-Ω resistor dissipate before burning up?

Problem 1.1 (2 Points): Each network shown below has several of its branch currents or voltages specified numerically. Several other branch currents or voltages are labeled as unknowns. Find all labeled unknown branch currents and voltages. Note that the elements in the networks are drawn as non-specific boxes, indicating that they can each be a source or a resistor.









Consider the five-element network shown (twice) in the first row above. With the restriction that they must satisfy KVL, assign otherwise arbitrary non-zero voltages to the five elements using the sign conventions of the left-hand network. With the restriction that they must satisfy KCL, assign otherwise arbitrary non-zero currents to the five elements using the sign conventions of the right-hand network. (Note that the voltage and current conventions of the two networks are consistent.) The product of the voltage and current for each element represents the power absorbed by that element. Sum the five powers, and note that they sum to zero. This remarkable fact, which follows from Tellegen's theorem, illustrates that power is always conserved in an electrical independent of the elements in the network. (Note that the selection of the voltages and currents were carried out independently, and thus not predicated on the behavior of any of the network elements.)

For more on Tellegen's Theorem, see *Introductory Network Theory* by Bose and Stevens, Harper & Row, 1965, for example.

Problem 1.2 (2 Points): The network shown below has four elements: two resistors, a current source and a voltage source. The resistance of the resistors and the strengths of the sources are all given. Branch currents (i_k) and voltages (v_k) are also defined for each element.

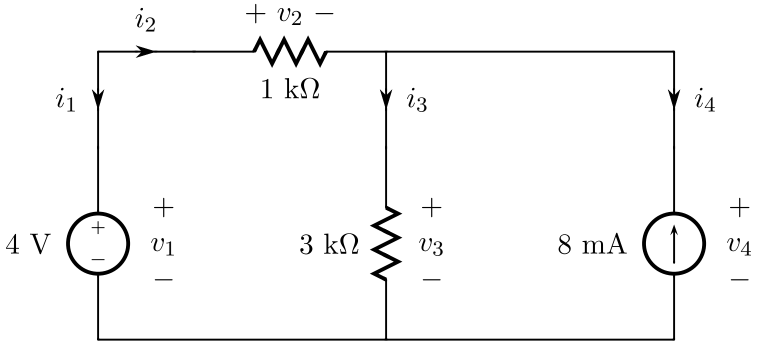
(A) How many nodes are there in the network? Write a KCL equation for each node in terms of the branch currents i_k. How many of the KCL equations are independent?

(B) How many loops are there in the network? (For the purposes of this problem, a loop is any direction-independent closed path through the network that passes through each branch no more than once.) Write a KVL equation for each loop in terms of the branch voltages v_k. How many of the KVL equations are independent?

(C) Write an expression for the v-i constitutive law for each element.

(D) By combining the independent equations from Parts (A) and (B) with the equations from Part (C), you should have a set of eight linear equations, matching in number the set of i_k plus v_k. Solve the equations to find all four i_k and all four v_k. Summarize your findings in a table.

(E) Find all four branch powers $v_k i_k$. Show that the sum of the four $v_k i_k$ is zero, and hence that power is conserved in the network. Which branch elements source power and which branch elements sink power?



Problem 1.3 (2 Points): Replace the 8-mA current source in Problem 1.2 with a 1.5-kΩ resistor. For this new network, determine all branch voltages and all branch currents. However, rather than using the eight-equations-in-eight-unknowns approach used in Problem 1.2, make use of series and parallel resistor combinations, such as explored in Exercise 1.1, and voltage and current dividers.

Problem 1.4 (2 Points): This problem makes use of WebLab, a remote-access laboratory that enables experimentation controlled through a Java-enabled web browser. The remote laboratory, actually located in Building 38 at MIT, offers access to more expensive and sophisticated equipment than is available in the 6.002 laboratory. For example, this problem uses an HP4155B Semiconductor Parameter Analyzer to measure the voltage-current relation of a real diode as follows.

- Start WebLab
  - Go to **http://ilab.mit.edu** and login to your account on the MIT iLab Service Broker. If your account does not open up under the *My Labs* section of iLab, move to that section by clicking on the *My Labs* tab on the top bar. If you have trouble running iLab on one computer, try switching to another computer, or try moving to an Athena workstation. You can request help and advice from iLab personnel, or report a bug, by clicking on the *Help* tab on the top bar.
  - Select the *Microelectronics WebLab Graphical Client* by clicking on its link. Afterwards, you can click on the *Documentation* link for detailed information about the WebLab client.
  - Start the WebLab client by clicking the *Launch Lab* link.
- Set Up Experiment

  Experiments are run through the WebLab window that will pop up following the launch of the WebLab client. In this experiment, you will measure the voltage-current relation of a diode.

  - From the drop-down *Devices* menu on the top bar, select the *p-n diode*. For this problem set, the diode is connected to the HP4155B through two Source Measurement Units (SMU). SMU #1 is connected to the positive terminal of the diode, and SMU #2 is connected to the negative terminal of the diode. Each SMU can source either the voltage across, or the current through, its terminals. It can also measure both its current and voltage.
  - Click on SMU #1. Name its current and voltage, and select the download option for both variables. Select a voltage sweep (*Mode = V & Function = VAR1*) from -1.5 V to 1.5 V with a 10-mV step. Set the compliance, or current limit, to 10 mA. Click on *Apply* and *OK* to complete the setup of SMU #1.
  - Click on SMU #2. Set this SMU to ground (*Mode = COMM*). You must also name the voltage and current, but do not select them for downloading. (The voltage will always be zero, and the current will be the opposite of that measured by SMU #1. Thus, neither variable is interesting.) Click on *Apply* and *OK* to complete the setup of SMU #2.
- Run Experiment & View Measurements
  - From the drop-down *Measurement* menu on the top bar, select *Run Measurement*. Your experiment will be queued, and run on a first-come-first-served basis. Hint: if you wait until the last minute like everyone else, you may find the queue and the wait longer than desired.
  - The measured data will be displayed on a graph at the bottom of the WebLab window. It is probably best viewed with a linear X-axis display. Both a linear and a logarithmic

Y-axis display are useful for answering the questions below. Note that the logarithmic display first takes the absolute value of the data.

  - Note the temperature of the diode. It is shown just above the measurement graph.
  - If for any reason you wish to clear your data and run the experiment again, you can first clear the data by selecting *Clear Data* from the *Results* drop-down menu on the top bar.

- Download Measurements

  To download the measured voltage and current data to your local computer, select *Download Data* from the *Results* drop-down menu on the top bar.

Use your voltage and current data measured for the diode via WebLab to answer the following questions. As you view the data, remember that SMU #1 was set up to limit the diode current to 10 mA, even if the applied voltage attempts to drive a larger current.

(A) In theory, the voltage-current (v_{D}-i_{D}) relation for a diode is

$$i_{\mathrm{D}} = I_{\mathrm{S}}(e^{v_{\mathrm{D}}/V_{\mathrm{T}}} - 1) \ .$$

The thermal voltage V_{T} is given by $V_{\mathrm{T}} = kT/q$, where $k = 1.38 * 10^{-23}$ J/K is Boltzmann's constant, T is the diode temperature in degrees Kelvin, and $q = 1.60 * 10^{-19}$ C is the electron charge. Given the measured diode temperature, compute V_{T}.

(B) Over what range of voltage does the measured diode data exhibit $i_{\mathrm{D}} \approx -I_{\mathrm{S}}$? Is this as expected from the theoretical relation given above? Using the measured data, estimate I_{S}.

(C) Over what range of voltage does the measured diode data exhibit the exponential relation $i_{\mathrm{D}} \approx I_{\mathrm{S}} e^{V_{\mathrm{D}}/V_{\mathrm{T}}}$? Is this as expected from the theoretical relation given above?

The following hints may help in answering Parts (B) and (C) above.

- Parts (B) and (C) can be answered either graphically or numerically. If you choose a graphical method, then you should plot the downloaded data using MatLab, Excel or another program, or print a screen-shot of the graph produced by WebLab. Capturing screen-shots is a platform specific activity; consult the WebLab documentation for Windows- and Athena-specific advice. If you choose a numerical method, then you could process the downloaded data using MatLab, Excel or another program.
- Before answering Parts (B) and (C), you may find it instructive to examine the measured data visually using WebLab. In this case, a logarithmic y-axis display is probably best used for Parts (B) and (C).