6.012 Homework #6 Solutions

October 28, 2005

Question 1.

a)

At some point y in a channel of width W um with an inversion charge of $Q_n(y)$ C moving at $v_n(y)$ m/s the current is given by:

$$I_D(y) = WQ_n(y)v_n(y).$$

The inversion charge is given by:

$$Q_n(y) = -C_{OX}[V_{GS} - V_T - V_c(y)]$$

, where $V_c(y)$ is the vertical field from the gate to point y in the channel. The lateral velocity of the charge is due to the lateral electric field and is given by:

$$v_n(y) = -\mu\xi(y) = \mu_n \frac{d}{dy} V_c(y)$$

Hence we write the current at y as:

$$I_D(y) = WC_{OX}[V_{GS} - V_T - V_c(y)]\mu_n \frac{d}{dy} V_c(y)$$

Integrate both sides,

$$\int_0^y I_D(y)dy = WC_{OX}\mu_n \int_0^{V_c(y)} [V_{GS} - V_T - V_c(y)]dV_c(y)$$

$$\frac{yI_D}{\mu_n C_{OX} W} = (V_{GS} - V_T)V_c(y) - \frac{V_c(y)^2}{2}$$

Solve the quadratic noting that the channel voltage can't exceed Vgs-Vt.

$$V_c(y) = V_{GS} - V_T - \sqrt{(V_{GS} - V_T)^2 - \frac{2yI_D}{\mu_n C_{OX} W}}$$

Since we only want the current in saturation we substitute the formula for Idsat for Id.

$$I_D = I_{DSAT} = \frac{\mu_n C_{OX} W}{2L}(V_{GS} - V_T)^2$$

So,

$$V_c(y) = (V_{GS} - V_T)(1 - \sqrt{1 - \frac{y}{L}}) = V_{DSAT}(1 - \sqrt{1 - \frac{y}{L}})\ [V]$$

Moving along the channel the E-field will be the derivative of the potential.

$$\xi(y) = \frac{-d}{dy}V_c(y) = \frac{-V_{DSAT}}{2L}(1 - \frac{y}{L})^{-1/2}\ [V/cm]$$

The velocity will be the mobility by the field.

$$v_n(y) = -\mu_n \xi(y) = \frac{V_{DSAT}\mu_n}{2L}(1 - \frac{y}{L})^{-1/2}\ [cm/s]$$

The inversion layer charge can be calculated from the channel voltage.

$$Q_n(y) = -C_{OX}(V_{GS} - V_T - V_c(y)) = -C_{OX}V_{DSAT}\sqrt{1 - \frac{y}{L}}\ [C]$$

See the attached sketches.

b)

The transit time is the indicated integral.

$$\tau_t = \int_0^L \frac{dy}{v_n(y)} = \int_0^L \frac{2L}{V_{DSAT}\mu_n}(1 - \frac{y}{L})^{1/2}dy$$

$$\tau_t = \frac{4L^2}{3 * \mu_n(V_{GS} - V_T}\quad [s]$$

Question 2.

a)

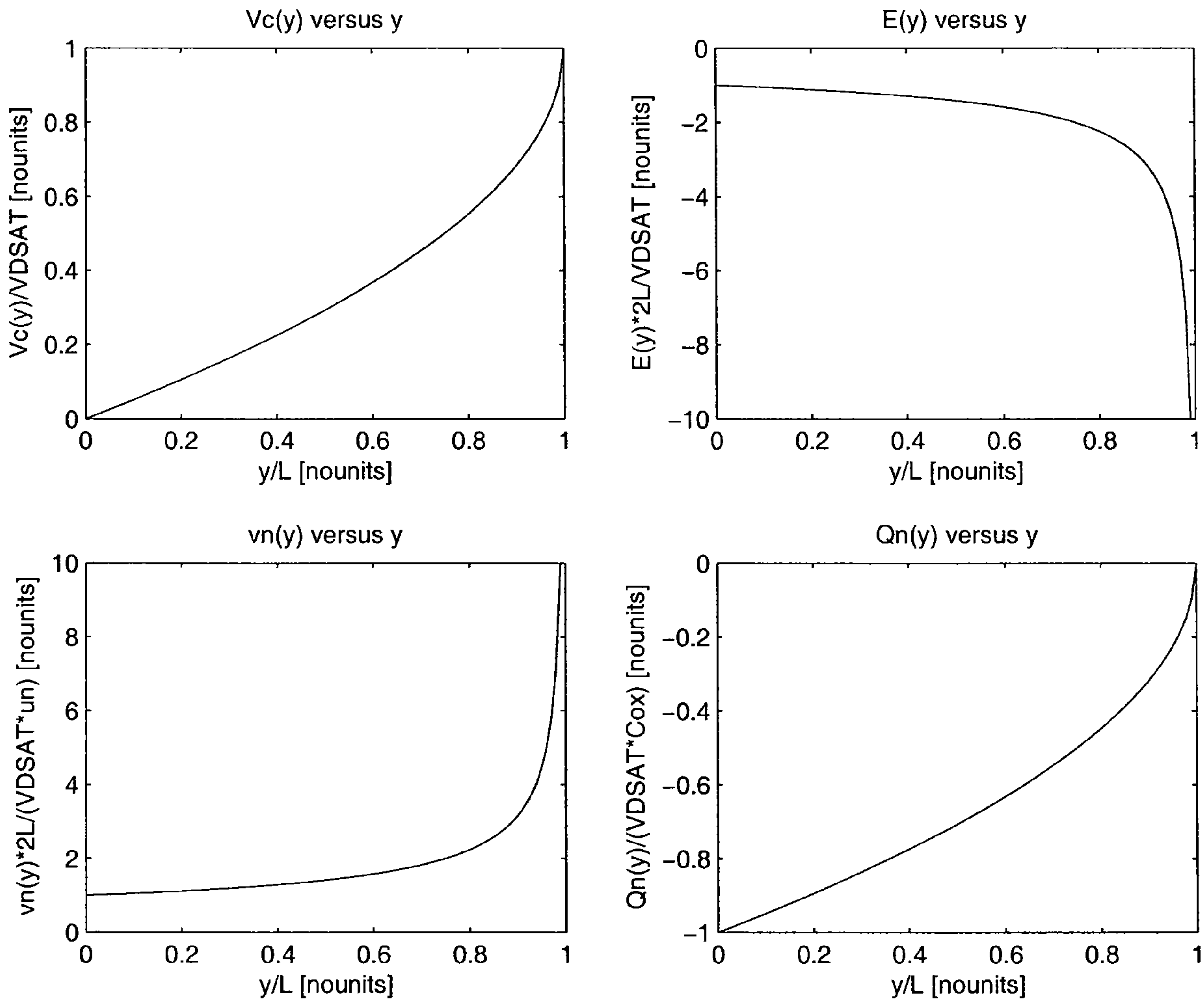
For a complete high frequency small signal model we need to find gm, gmb, ro, Cgs, Cgd, Csb, and Cdb.

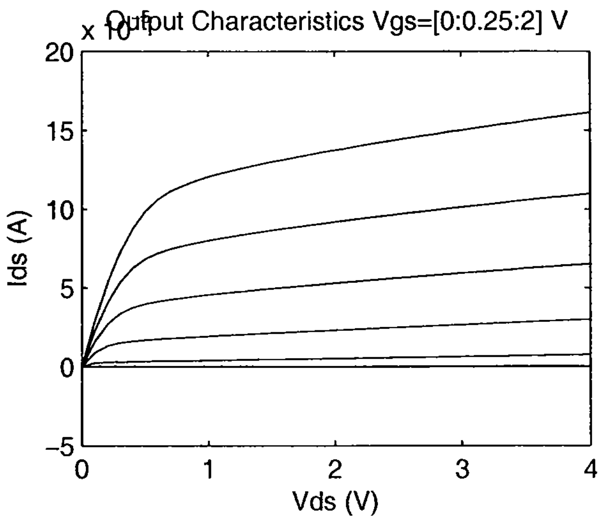
We start be determining what region of operation that we are in by finding VT(Vbs).

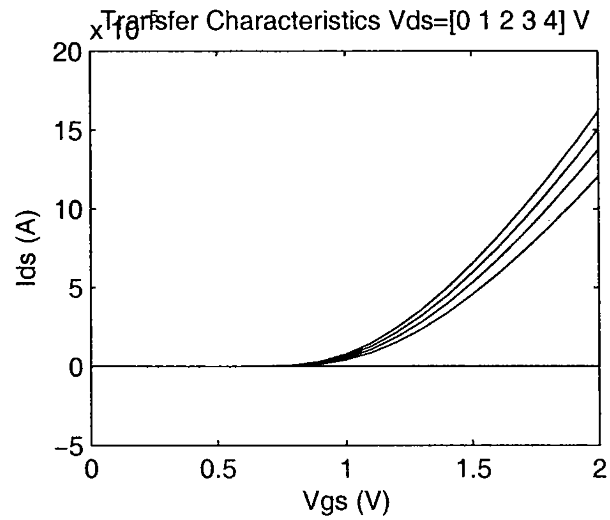
$$\phi_p = -V_t \ln N_A/n_i = -430\ [mV]$$

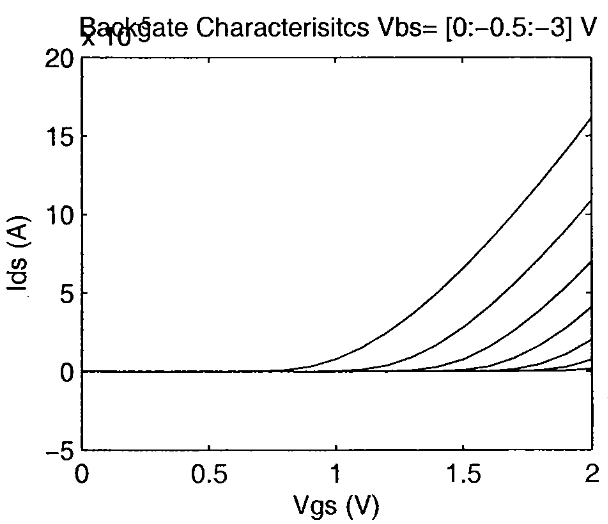
$$\gamma = \frac{\sqrt{2\epsilon_S q N_A}}{C_{OX}} = 0.502\ [\sqrt{V}]$$

2


Vc(y) versus y
Vc(y)/VDSAT [nounits]
0
0.2
0.4
0.6
0.8
1
y/L [nounits]
E(y) versus y
E(y)*2L/VDSAT [nounits]
0
−2
−4
−6
−8
−10
y/L [nounits]
vn(y) versus y
vn(y)*2L/(VDSAT*un) [nounits]
0
2
4
6
8
10
y/L [nounits]
Qn(y) versus y
Qn(y)/(VDSAT*Cox) [nounits]
0
−0.2
−0.4
−0.6
−0.8
−1
y/L [nounits]


Output Characteristics Vgs=[0:0.25:2] V
Ids (A)
Vds (V)
20
15
10
5
0
−5
0
1
2
3
4


Transfer Characteristics Vds=[0 1 2 3 4] V
Ids (A)
Vgs (V)
20
15
10
5
0
−5
0
0.5
1
1.5
2


Backgate Characterisitcs Vbs= [0:−0.5:−3] V
Ids (A)
Vgs (V)
20
15
10
5
0
−5
0
0.5
1
1.5
2

The gate potential is $\phi_{n^+} = 550$ mV.

$$V_T = -\phi_{n^+} - \phi_p + \gamma\sqrt{-2\phi_p - VBS} = 0.56\ [V]$$

The device is in saturation because Vgs-VT = 2-0.56 V < Vds = 4 V. So we continue calculating the small signal model for saturation.

$$g_m = \mu_n C_{OX}\frac{W}{L}(V_{GS} - V_T) = 2.5[mS]$$

$$g_{mb} = \frac{\gamma g_m}{2\sqrt{-2\phi_p - V_{BS}}} = 0.46[mS]$$

$$g_o = \frac{\mu_n C_{OX} W}{2L}(V_{GS} - V_T)^2\lambda = 90.9[\mu A/V]$$

$$r_o = 1/g_o = 11[k\Omega]$$

$$C_{gs} = \frac{2}{3}C_{OX}WL + WC_{ov} = 0.46[fF]$$

$$C_{gd} = WC_{ov} = 0.4[fF]$$

Note: WL_{diff} is the drain or source junction area.

$$C_{sb} = WL_{diff}\frac{C_{jo}}{\sqrt{1 - \frac{V_{BS}}{\phi_B}}} = 6.7[fF]$$

$$C_{db} = WL_{diff}\frac{C_{jo}}{\sqrt{1 - \frac{V_{BD}}{\phi_B}}} = 3.9[fF]$$

Question 3.

a)

See Graph.

b)

See Graph.

c)

See Graph.

4

d)

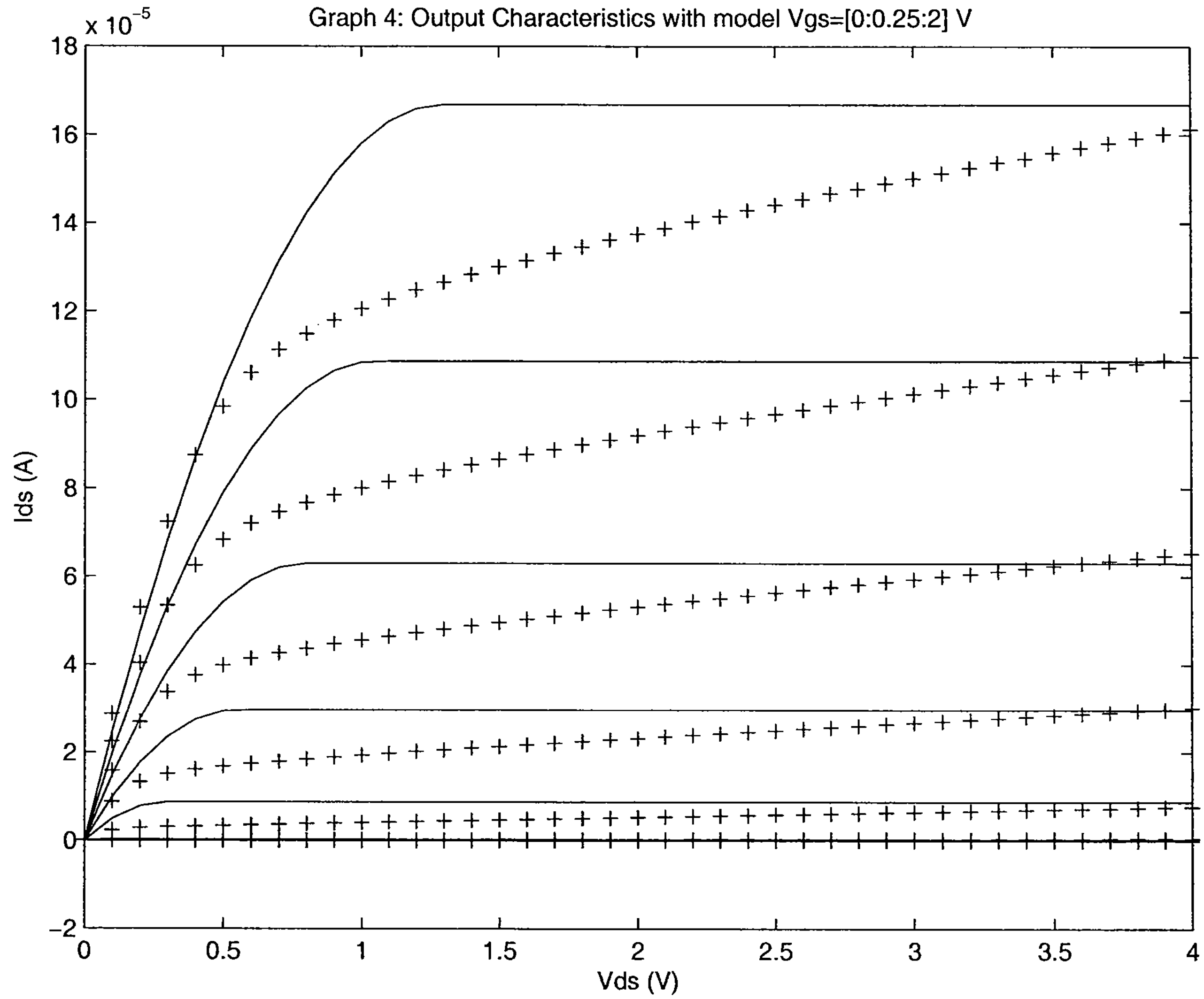
See Graphs.

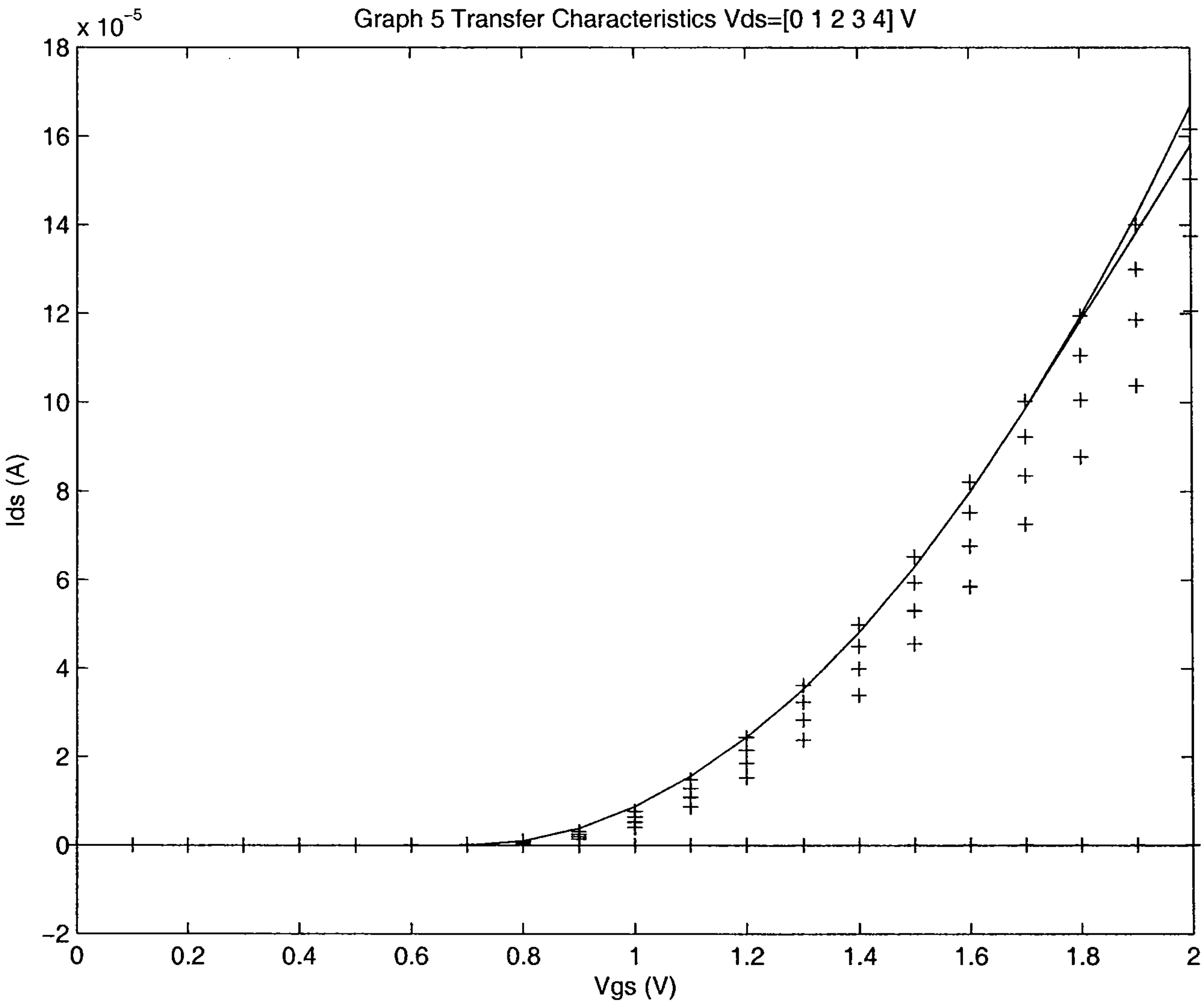
It is clear in graphs 4 and 5 that the lambda is not included in our model and that it significantly affects the performance of the device and hence degrades our model fit.

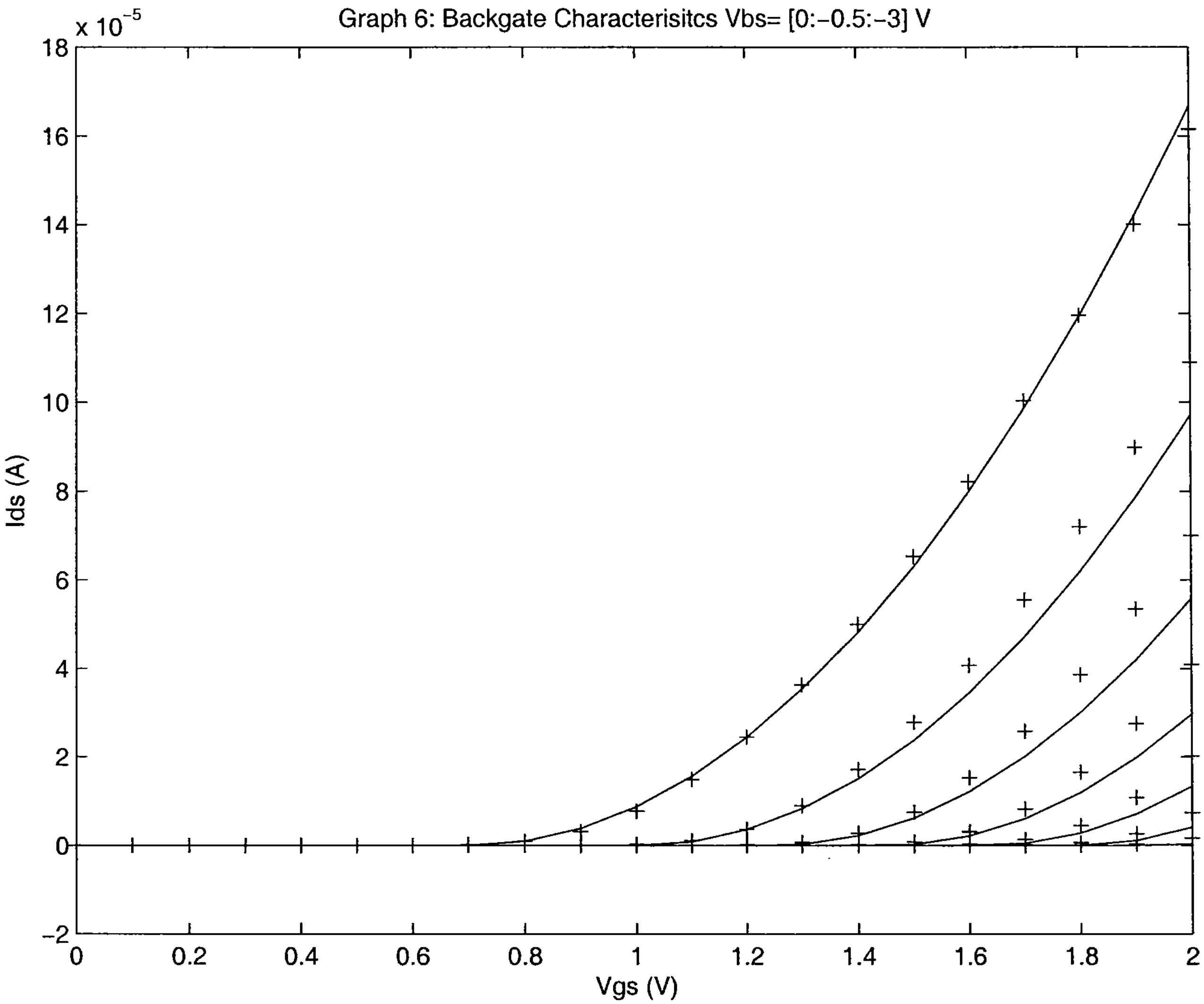
In graph 4 the fit is better at higher Vds, which is unsurprising as we fit the model to the data at Vds=4V.

In graph 5 all of the modeled curves collapse on top of each other as a consequence of an infinite output impedance (i.e. no lambda.).

In graph 6 the fit is actually pretty good. It is worth noting that the minimum of the standard deviation actually occured at phip = -0.28 , which was just outside the suggested range of values to try.


Graph 4: Output Characteristics with model Vgs=[0:0.25:2] V
x 10^-5
18
16
14
12
10
8
6
4
2
0
-2
Ids (A)
0
0.5
1
1.5
2
2.5
3
3.5
4
Vds (V)


Graph 5 Transfer Characteristics Vds=[0 1 2 3 4] V
x 10^-5
18
16
14
12
10
8
6
4
2
0
-2
Ids (A)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Vgs (V)


Graph 6: Backgate Characterisitcs Vbs= [0:-0.5:-3] V
x 10^-5
18
16
14
12
10
8
6
4
2
0
-2
Ids (A)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Vgs (V)