Radar Signal Processing

Principles of Pulse-Doppler Radar

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Essential mathematical operations

Fourier Transform: Expresses a function as a weighted sum of complex exponentials.

analysis equation: $$F(\omega) = \mathscr{F}\left[f(t)\right] = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

synthesis equation: $$f(t) = \mathscr{F}^{-1}\left[F(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}d\omega$$

$$f(t) \quad \Longleftrightarrow \quad F(\omega)$$

Convolution: Expresses the action of a linear, time-invariant system on a function.

$$f(t) * g(t) = \int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$

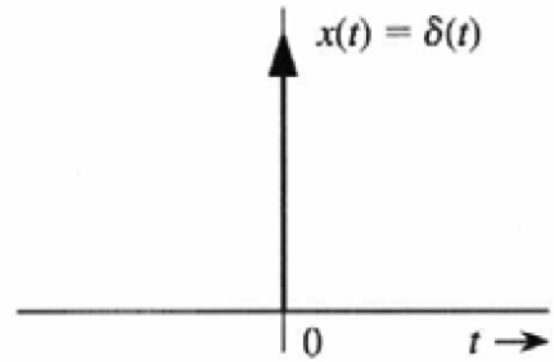
$$f(t) * g(t) \quad \Longleftrightarrow \quad F(\omega)G(\omega)$$

$$f(t)g(t) \quad \Longleftrightarrow \quad F(\omega) * G(\omega)$$

Dirac Delta Function $\delta(x)$

A generalized function, or distribution, with the properties

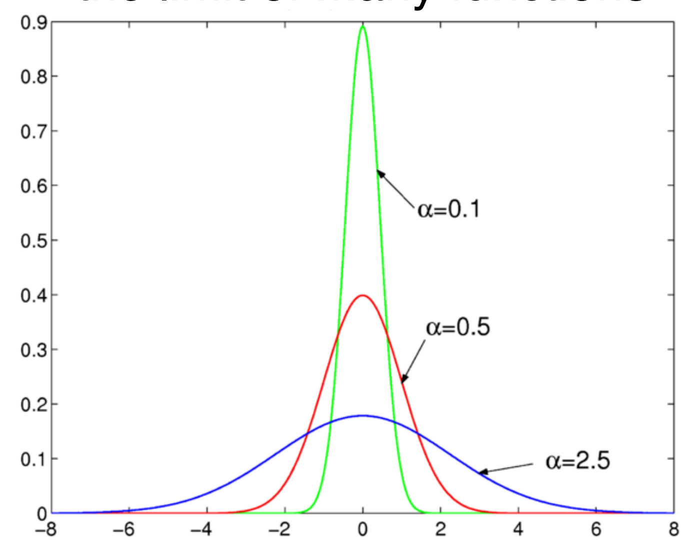
$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \delta(x)dx = 1$$



Sampling property: From the above it follows that

$$f(t_o) = \int_{-\infty}^{+\infty} f(t)\delta(\underbrace{t - t_o}_{\text{argument is zero at } t = t_0})dt$$

$\delta(t)$ may be expressed as the limit of many functions



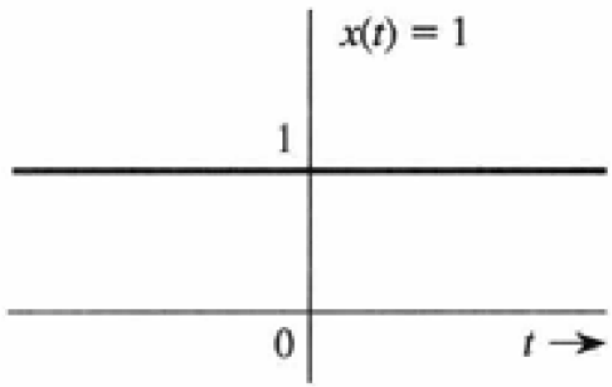
$$\delta(t) = \lim_{\alpha \to 0} \frac{1}{\sqrt{4\pi\alpha}} e^{-t^2/(4\alpha)}$$

Shift property: Convolution of a function $F(x)$ with $\delta(x - x_o)$ shifts the entire function by x_o. We will use this property to understand mixing. Specifically:

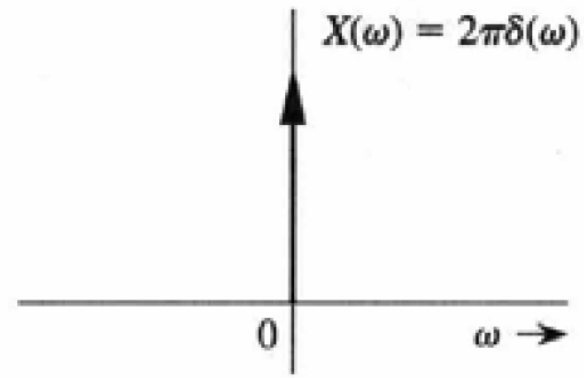
$$F(\omega) * \delta(\omega - \omega_0) = \int_{-\infty}^{+\infty} F(\Omega)\delta(\underbrace{\omega - \omega_0} - \Omega)d\Omega$$

$$= F(\omega - \omega_0)$$

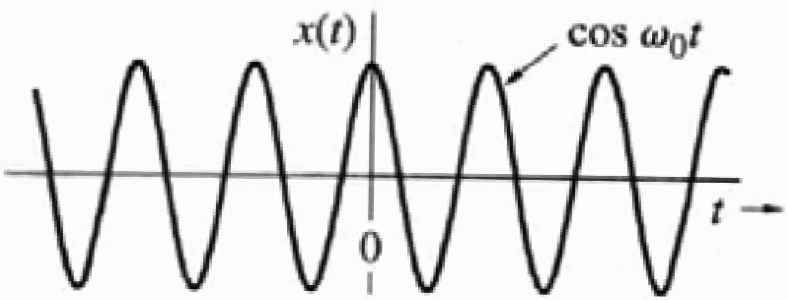
Fourier analysis of harmonic functions



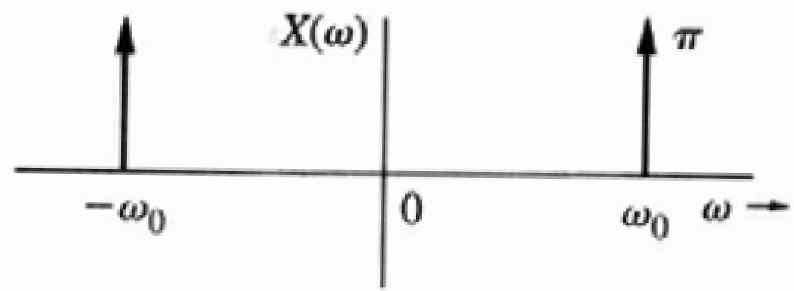
$\Longleftrightarrow$



$$\cos \omega_0 t \Longleftrightarrow \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$



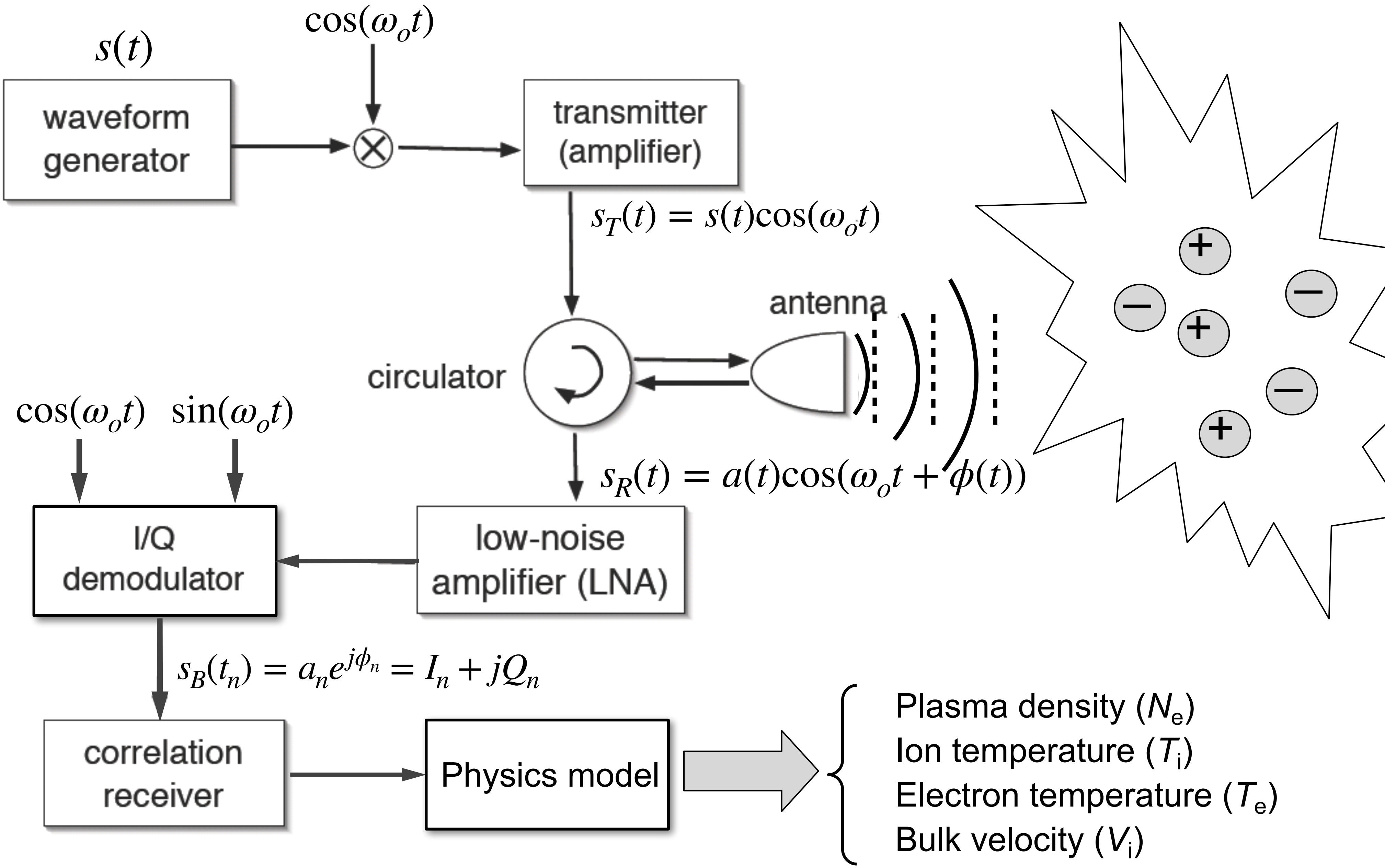
$\Longleftrightarrow$



$$\sin \omega_0 t \quad \Longleftrightarrow \quad j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$$

$$e^{j\omega_o t} = \cos(\omega_o t) + j\sin(\omega_o t) \quad \Longleftrightarrow \quad 2\pi\delta(\omega-\omega_0)$$

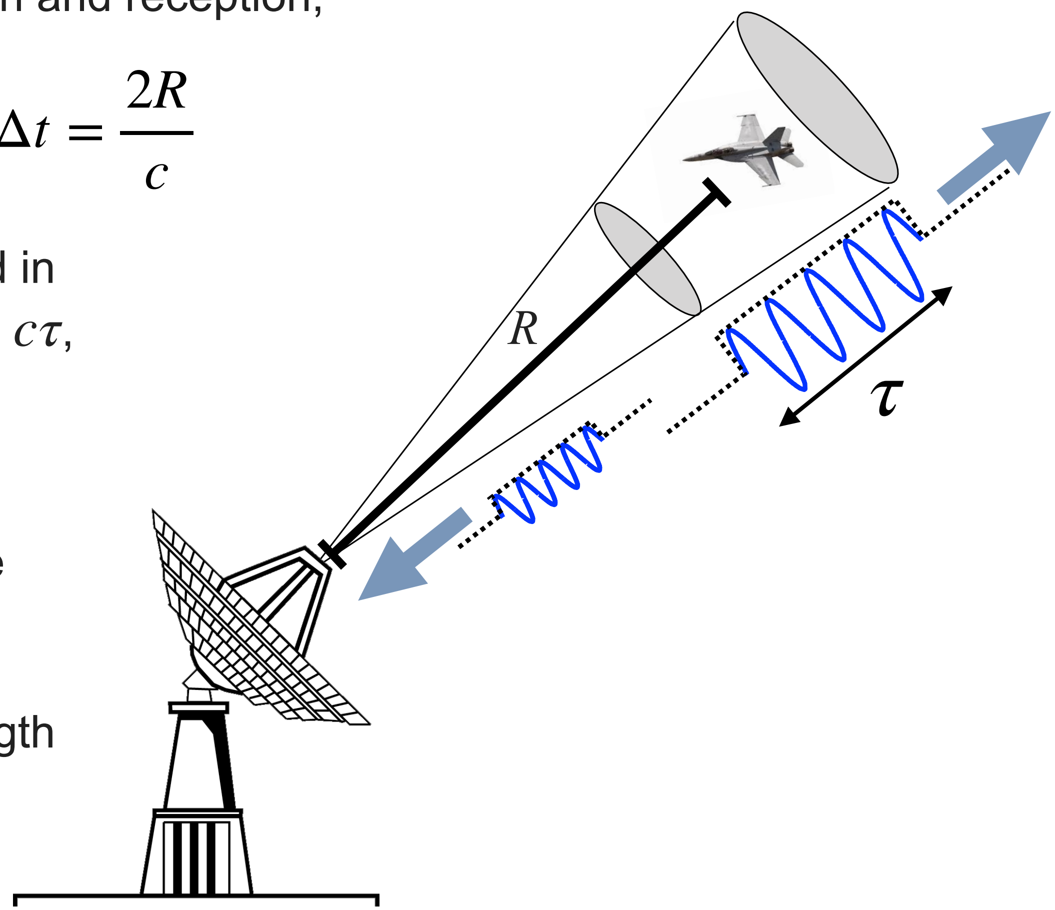
Components of a Pulsed Doppler Radar



Measuring Range

Range R to the target is measured by transmitting a pulse of electromagnetic waves, and measuring the time Δt between transmission and reception,

$$R = \frac{c\Delta t}{2} \qquad \text{or} \qquad \Delta t = \frac{2R}{c}$$

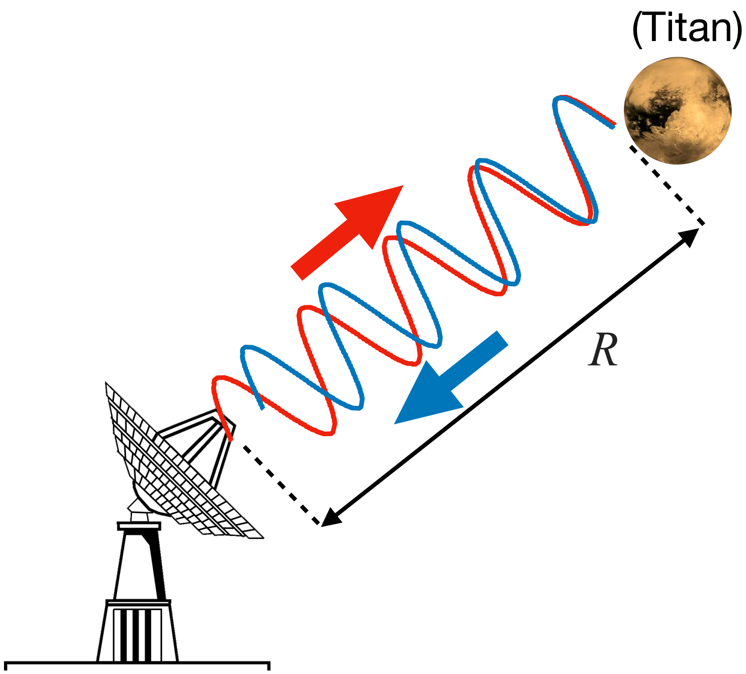


The ***pulse length*** τ is most often expressed in units of time, and corresponds to a distance $c\tau$, where $c = 3 \times 10^8$ m/s.

Range resolution depends on how well we can resolve Δt. For the case of a simple on-off pulse, the optimal approach is to match the sampling period to the pulse length (the so-called "matched filter" approach).

Range resolution for a simple on-off pulse ("uncoded pulse") is controlled by τ. Shorter τ yields higher range resolution. But a shorter pulse also carry less total energy, and so the reflected signal is more difficult to discriminate from background noise.

Measuring Velocity



Assume a transmitted signal: $\cos(2\pi f_o t)$

After return from target: $\cos\left[2\pi f_o \left(t - \frac{2R}{c}\right)\right]$

Now let us allow range *R* to vary with time. Let's assume the target moves at a constant velocity, with positive away from the radar and negative toward the radar:

$$R = R_o + v_o t$$

Substituting we obtain:

$$\cos\left[2\pi \Big(\underbrace{f_o - f_o\frac{2v_o}{c}}_{f_D}\Big) t - \underbrace{\frac{4\pi f_o R_o}{c}}_{\text{constant}}\right]$$

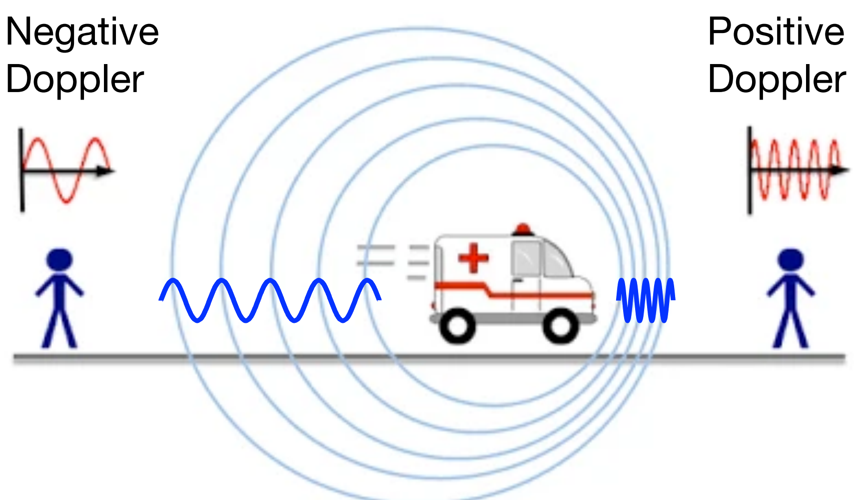
The change in frequency caused by a moving target is proportional to the *component* of the velocity vector along the radar line of sight:

$$\boxed{f_D = -\frac{2f_o}{c}v_o = -\frac{2v_o}{\lambda}}$$

For ISR:

$f_o \sim 500$ MHz,

$f_D \sim$ 50 kHz $= 0.0001 f_o$

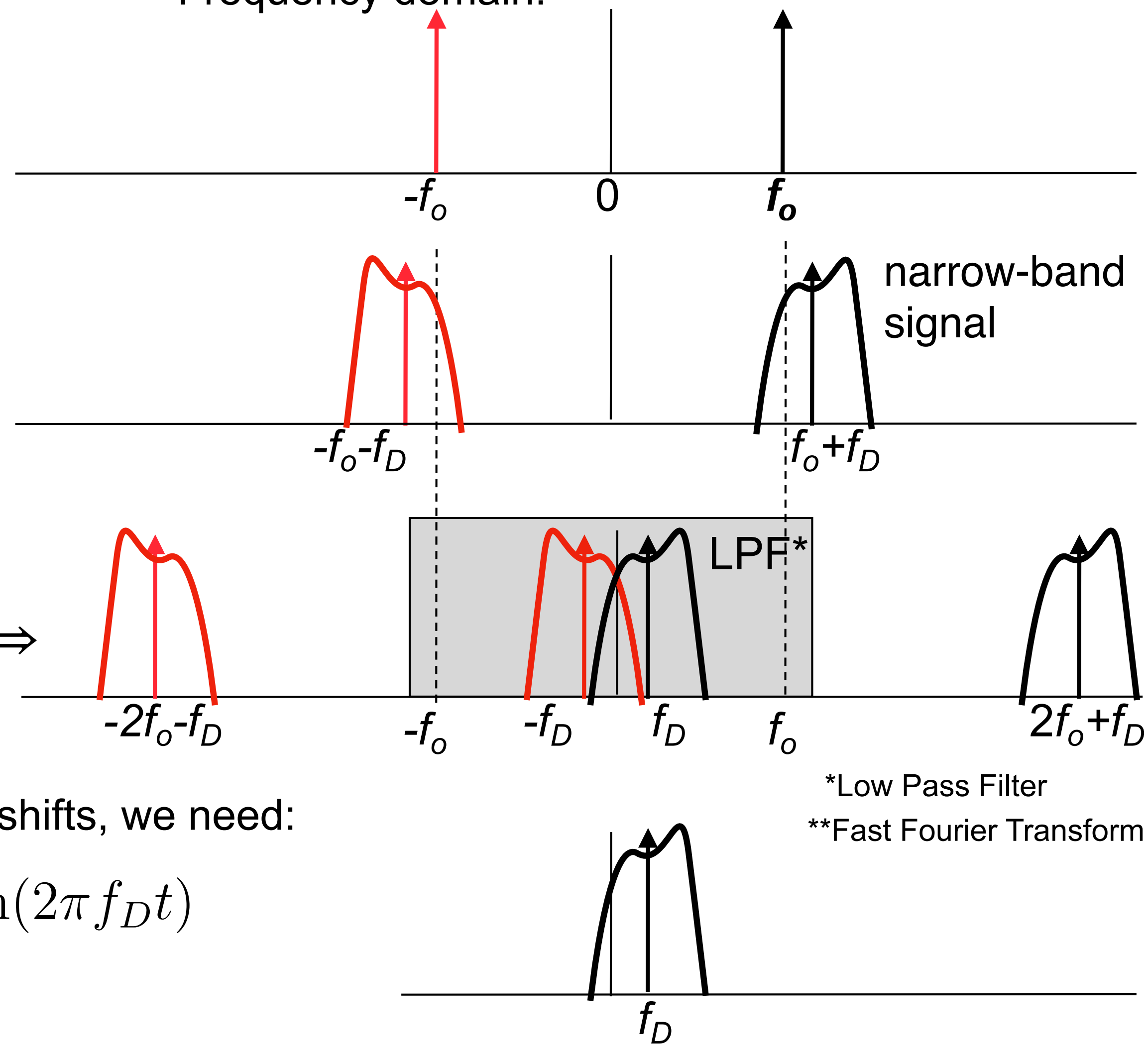


I/Q Demodulation: Frequency Domain

Transmitted signal:

$$\cos(2\pi f_o t) \quad \Longleftrightarrow$$

Frequency domain:



Reflected signal from moving target

$$\cos(2\pi(f_o + f_D)t) \quad \Longleftrightarrow$$

Mix (multiply) with oscillator $\cos(2\pi f_o t)$

$$\frac{1}{2}\cos\left[2\pi(2f_o + f_D)t\right] + \frac{1}{2}\cos[2\pi f_D t] \quad \Longleftrightarrow$$

*Low Pass Filter

**Fast Fourier Transform

To resole both positive and negative Doppler shifts, we need:

$$e^{j2\pi f_D t} = \cos(2\pi f_D t) + j\sin(2\pi f_D t)$$

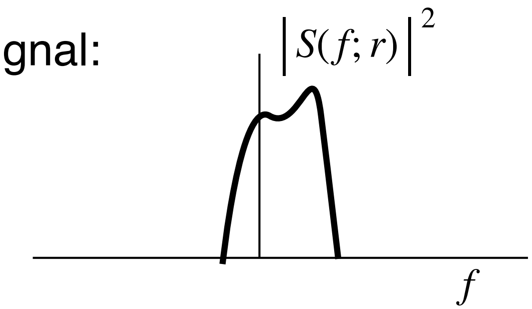
We thus need to mix with a second oscillator at same frequency but $90°$ out of phase.
For a cosine reference, the quadrature function is sine. The two components are called "in phase" (*I*) and "in quadrature" (Q). Together *I* and *Q* represent discrete samples of the baseband analytic signal,

$$s_B(t) = Ae^{2\pi f_D t} = I(t) + jQ(t) \quad \xleftrightarrow{\text{FFT**}} \quad A\delta(f - f_D)$$ (for a single scatterer)

Correlation and the ISR Spectrum

How do we compute the power spectrum from our complex voltages ?

One approach is to compute Fourier transform of the range-resolved signal:



$$s(r,t) = I(r,t) + Q(r,t) \quad \Longleftrightarrow \quad S(r,f)$$

from which the power spectrum may be represent as $\left|S(r,f)\right|^2$

For both practical and theoretical reasons, we usually compute the auto-correlation function (ACF) first. The discrete representation of $R_s(r,\tau)$ is constructed through appropriate scaling and multiplication of the complex voltage samples $s(r_k, t_n)$.

$$R_s(r,\tau) = \frac{\left\langle s(r,t)\overline{s(r,t+\tau)}\right\rangle}{\left\langle \left|s(r,t)\right|^2 \right\rangle}$$

where the angle brackets denote the ensemble average, or the expected value.
Rhe power spectral density is then given by the Fourier transform of the R_s

$$R_s(r,\tau) \Longleftrightarrow \left|S(r,f)\right|^2 \qquad \text{(Wiener-Khinchin theorem)}$$

Concept of a “Doppler Spectrum”

Superposition of targets moving with different velocities within the radar volume

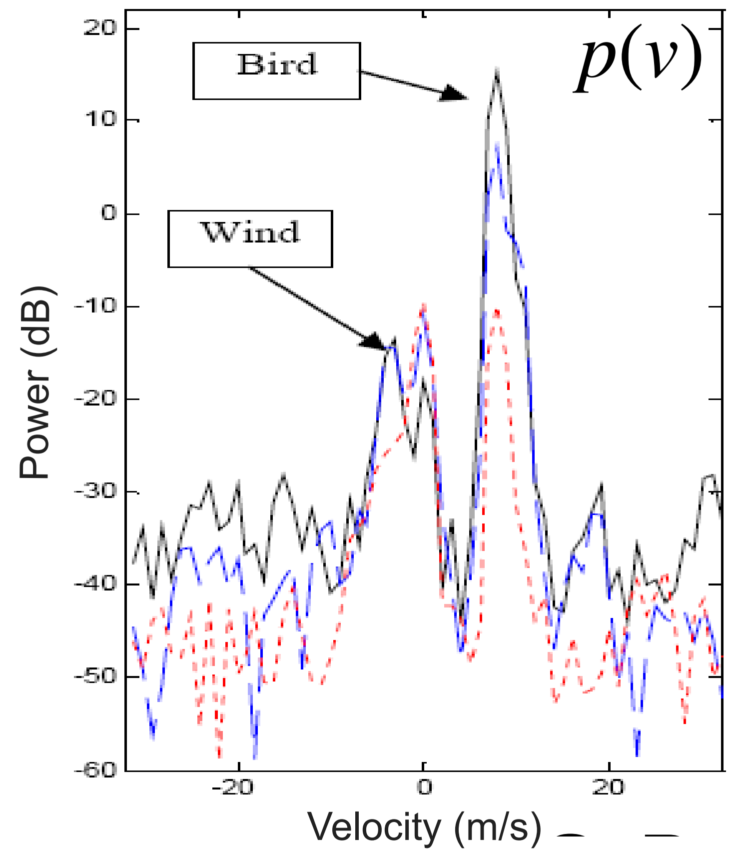
Two key concepts:

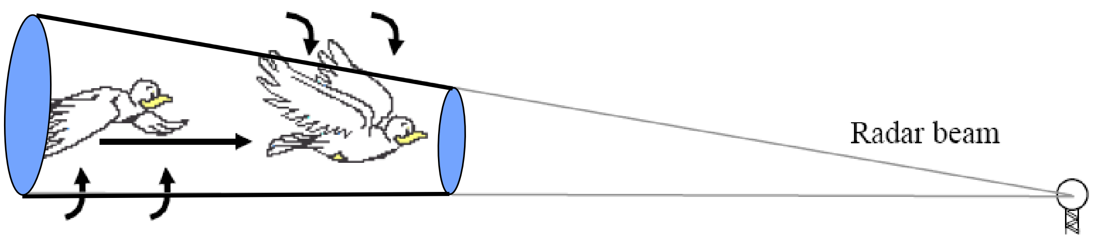
Time ⟺ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$





Processing: $p(R, f_D) \rightarrow p(R, v)$

If there is a distribution of targets withdifferent velocities (e.g., bird, flapping wings,wind) then there is no single Doppler shift but, rather, a Doppler spectrum.

Distributed "beam filling" Target

A pulse-Doppler radar measures backscattered power as a function range, velocity, and time.

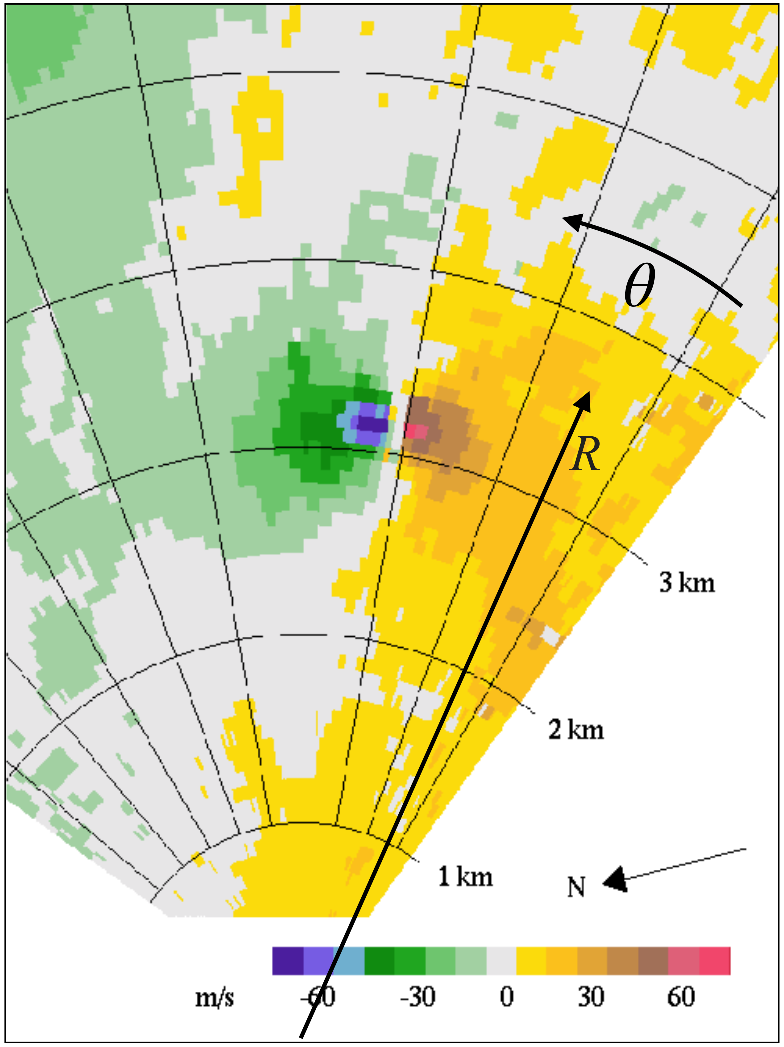
Two key concepts:

Time ⇔ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⇔ Velocity

$$f_D = -\frac{2f_o}{c} v_o$$



Processing:

$$p(R, f_D, t) \rightarrow f_D(R, t) \rightarrow v(R, \theta)$$

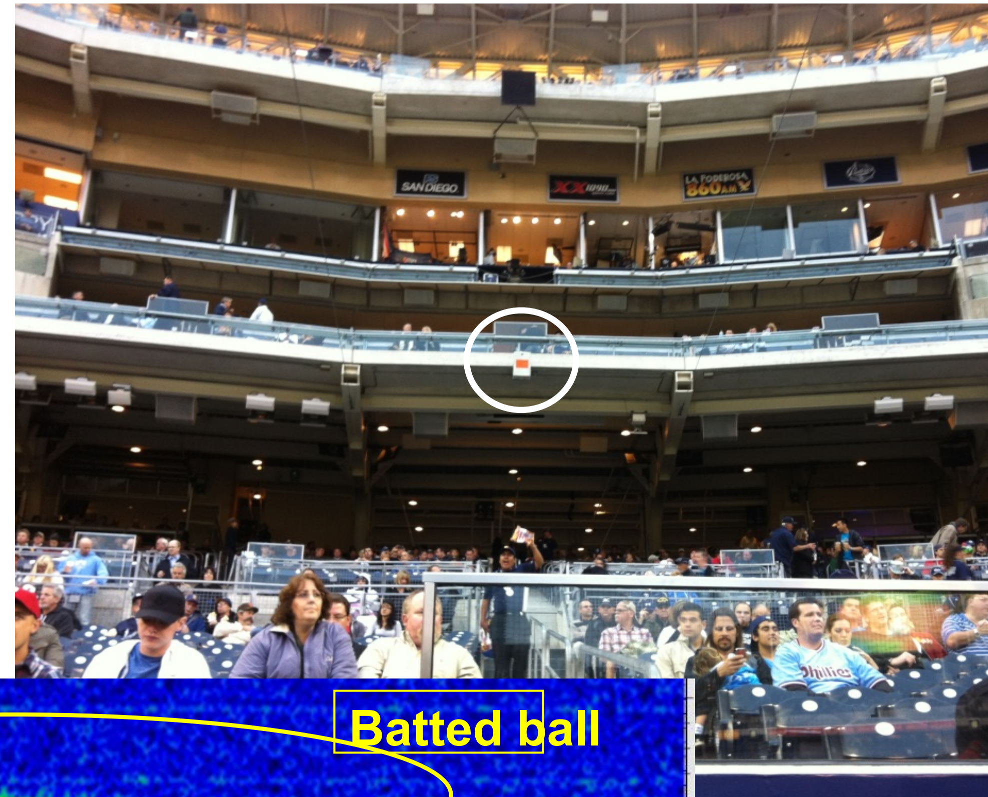
Micro-Doppler Analysis

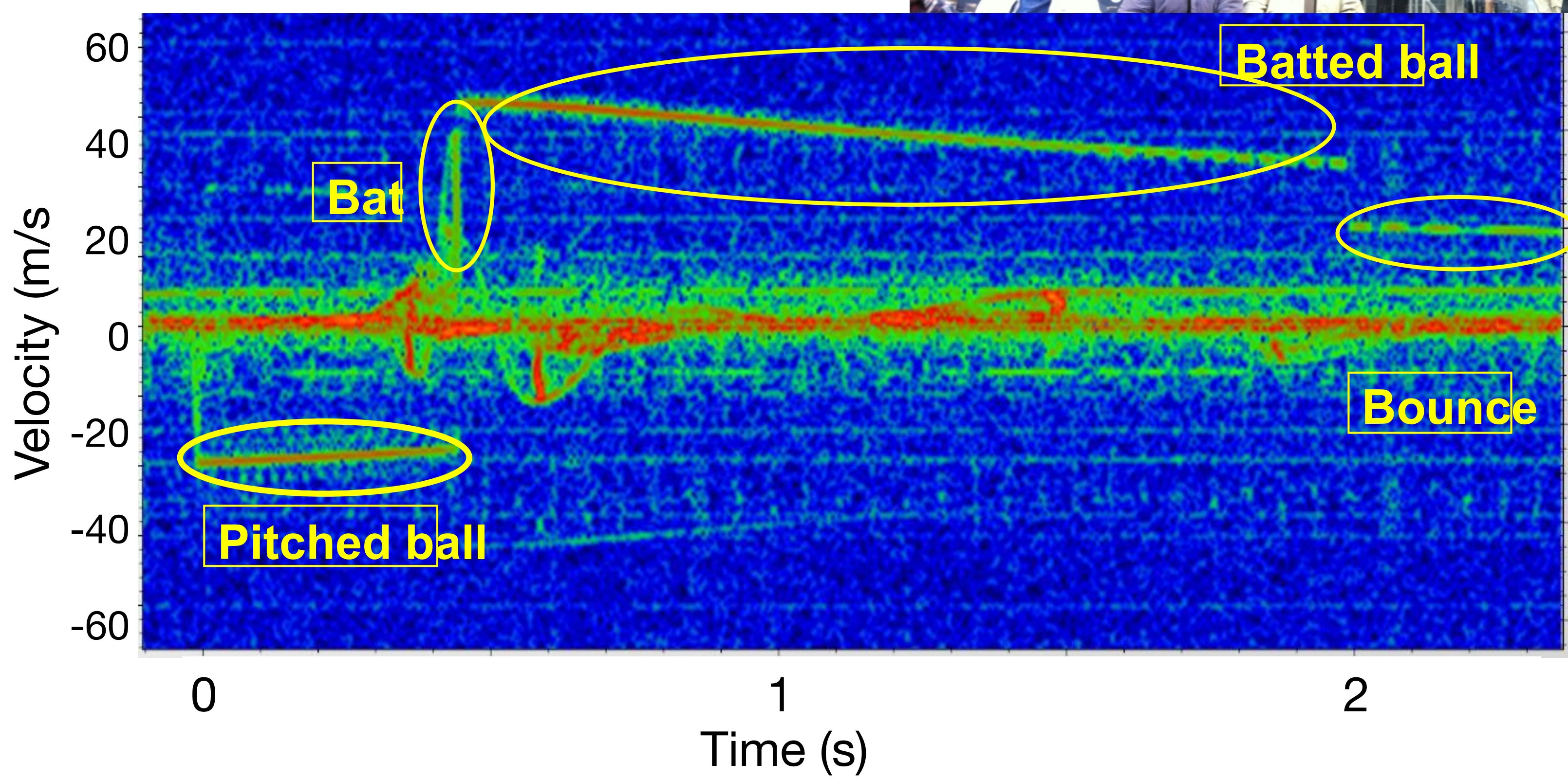
Trackman radar: "continuous wave" (CW) radar: precise Doppler but no range information.

Can identify targets and actions based on Doppler signatures!

Processing:

$$p(f_D, t) \rightarrow p(v, t)$$





Doppler Radar Summary: Distributed "Incoherent" Targets

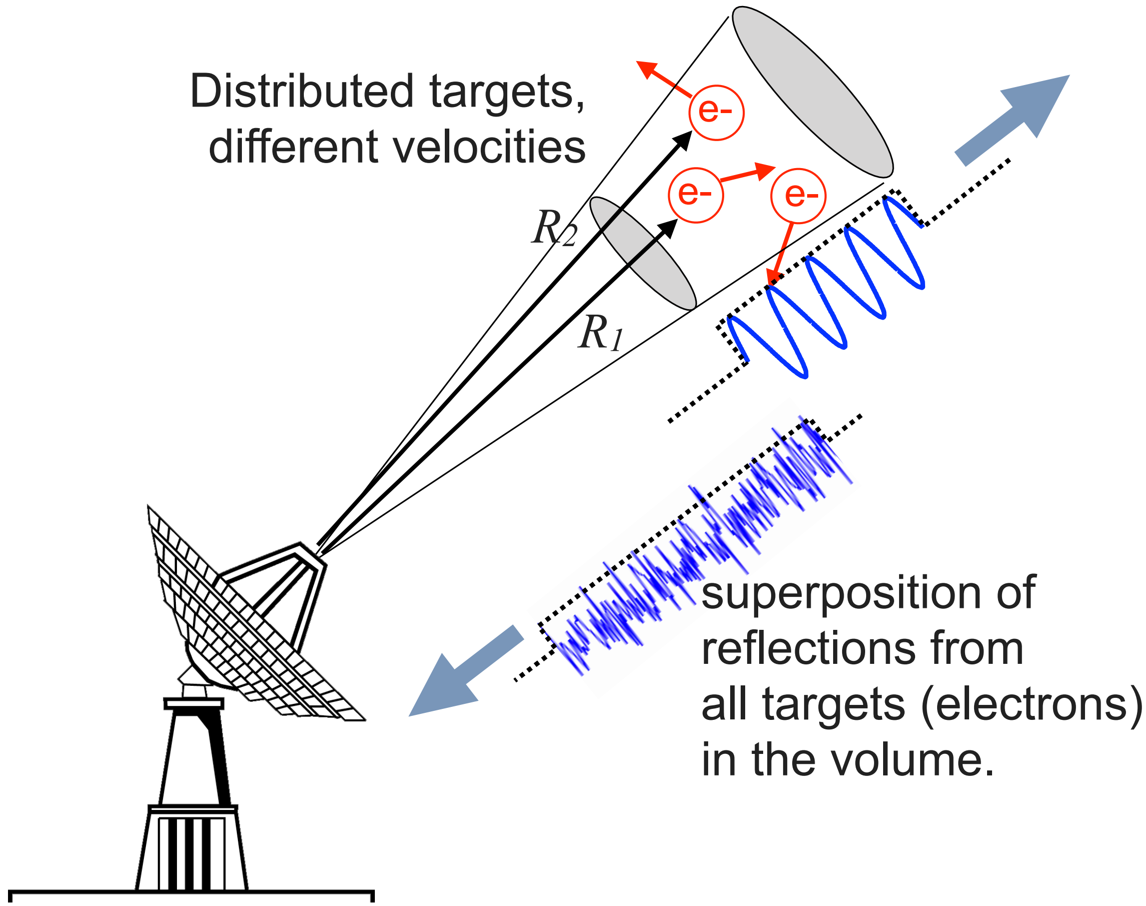
Two key concepts:

Time $\Leftrightarrow$ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency $\Leftrightarrow$ Velocity

$$f_D = -\frac{2f_o}{c}v_o$$

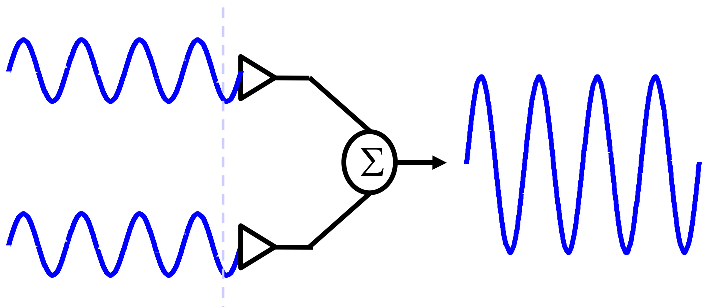


A Doppler radar measures backscattered power as a function range and velocity.
Velocity is manifested as a Doppler frequency shift in the received signal.

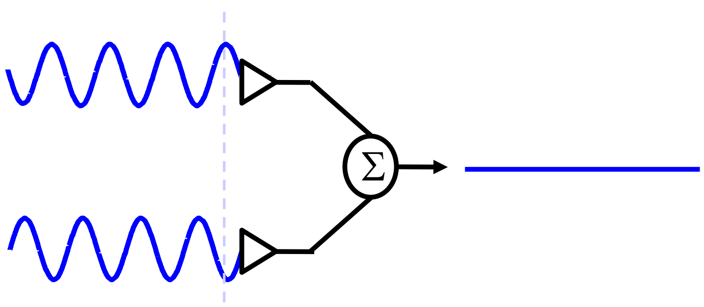
What happens when we have multiple targets in the radar volume, moving at different velocities?

Wave Interference and Bragg Scatter

Consider two waves with the same frequency but different phase.

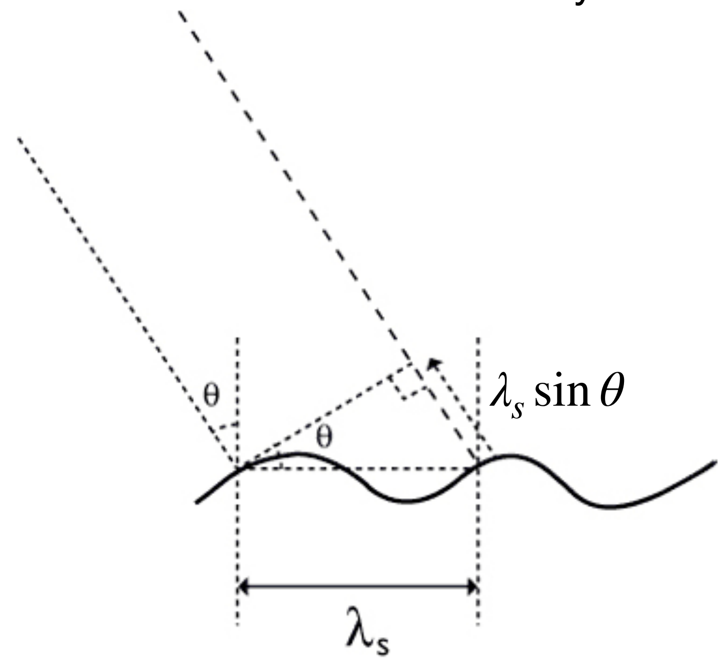


Constructive (in phase)



Destructive (180° out of phase)

Consider a wave along the interface between a dielectric and a conducting (reflective) medium, as depicted below. This is representative of an air-ocean boundary.

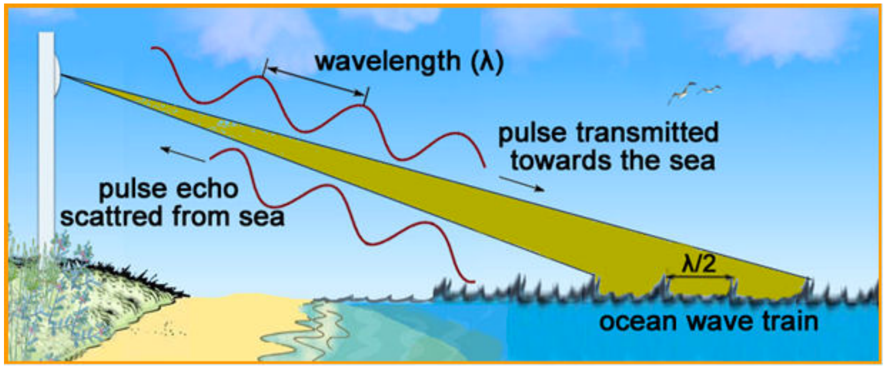


Suppose waves are observed at angle θ using a radar with wavelength λ_o. The condition for maximum constructive interference is

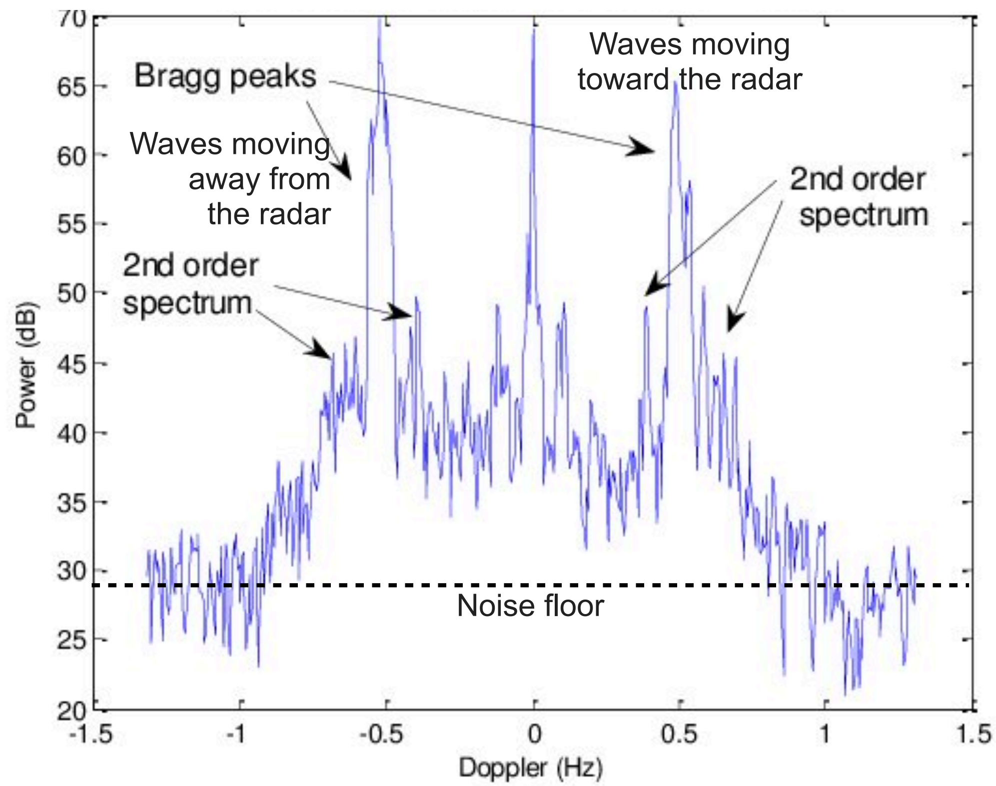
$$n\lambda_0 = 2\lambda_s \sin\theta$$

If $\theta = 90^\circ$ (or if these waves are propagating isotropically), then the Bragg condition is met for $n\lambda_0 = 2\lambda_s$

Doppler spectrum of ocean waves



Backscatter from the ocean at low aspect angle shows peaks in the Doppler spectrum from the subset of waves matching the Bragg condition for the radar (spacing ≃ half the radar wavelength)



Important points:

The target is distributed over the entire radar beam width.
The scattering is from free electrons in the conducting sea water.
The Doppler spectrum has peaks due to Bragg scatter from waves in the medium.
The frequency of the peaks tells us the velocity and direction of the waves.
The height of the peaks tells us something about the amplitude and density of the waves.
The width of the peaks tells us something about the spread in velocity of the waves

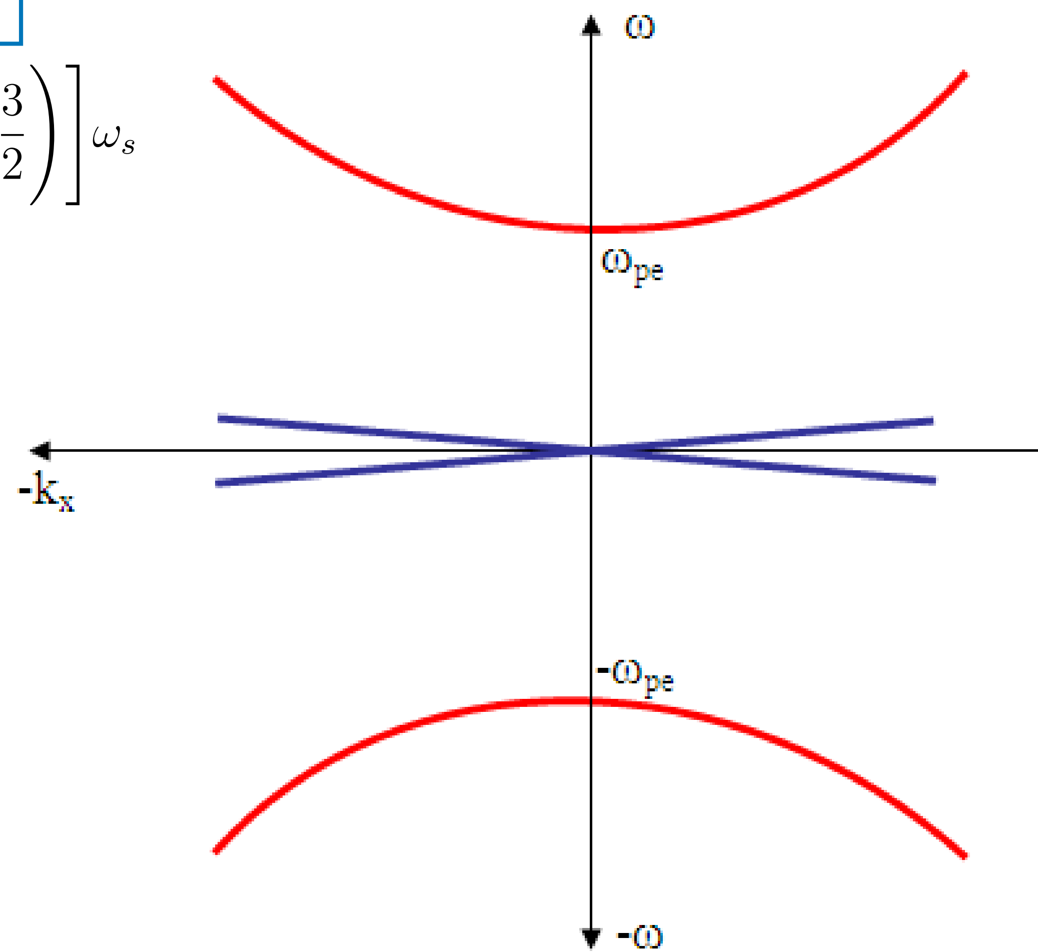
Doppler spectrum of the ionosphere

Let's put this all together for the ionosphere. The two predominant longitudinal modes in a thermal plasma:

Ion-acoustic mode:

$$\boxed{\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$



Langmuir mode:

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$

Computer simulation of the ionosphere

Simple rules yield complex behavior

Particle-in-cell (PIC) simulation:

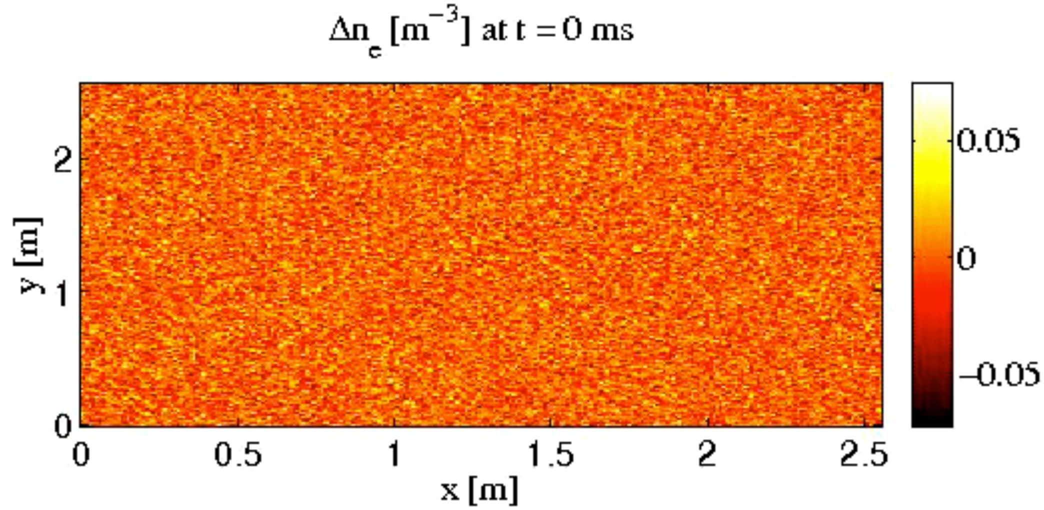
$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

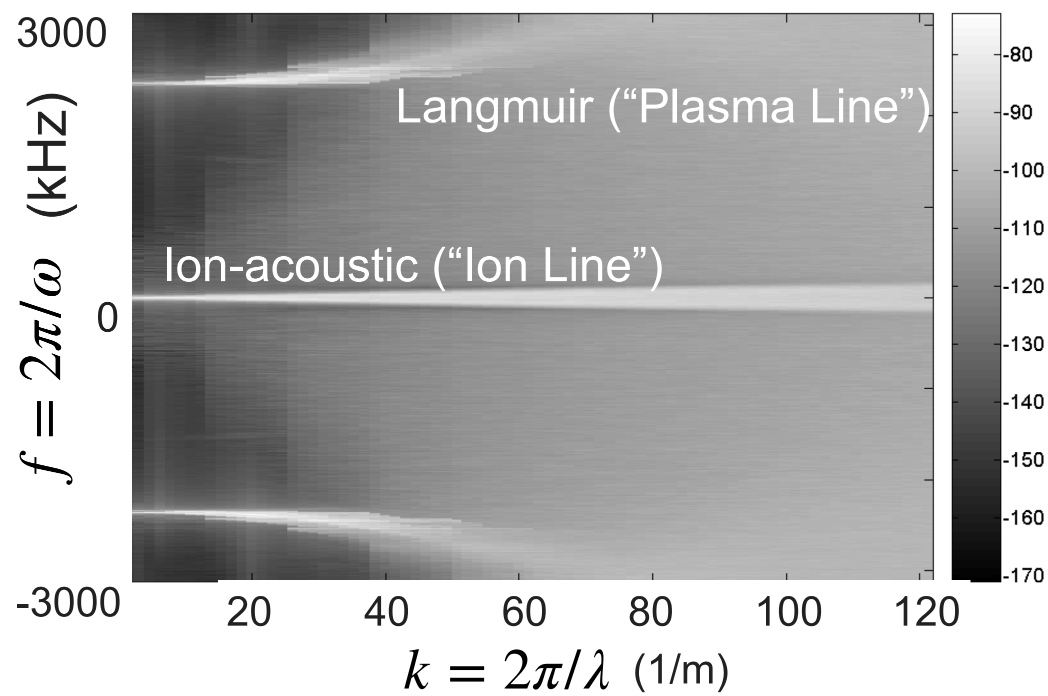
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

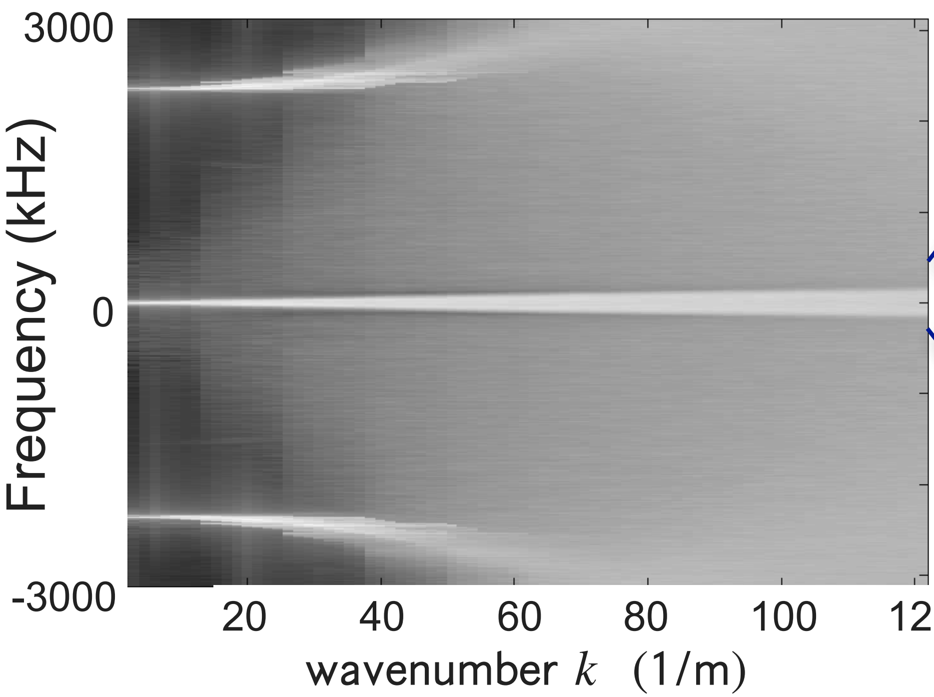
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

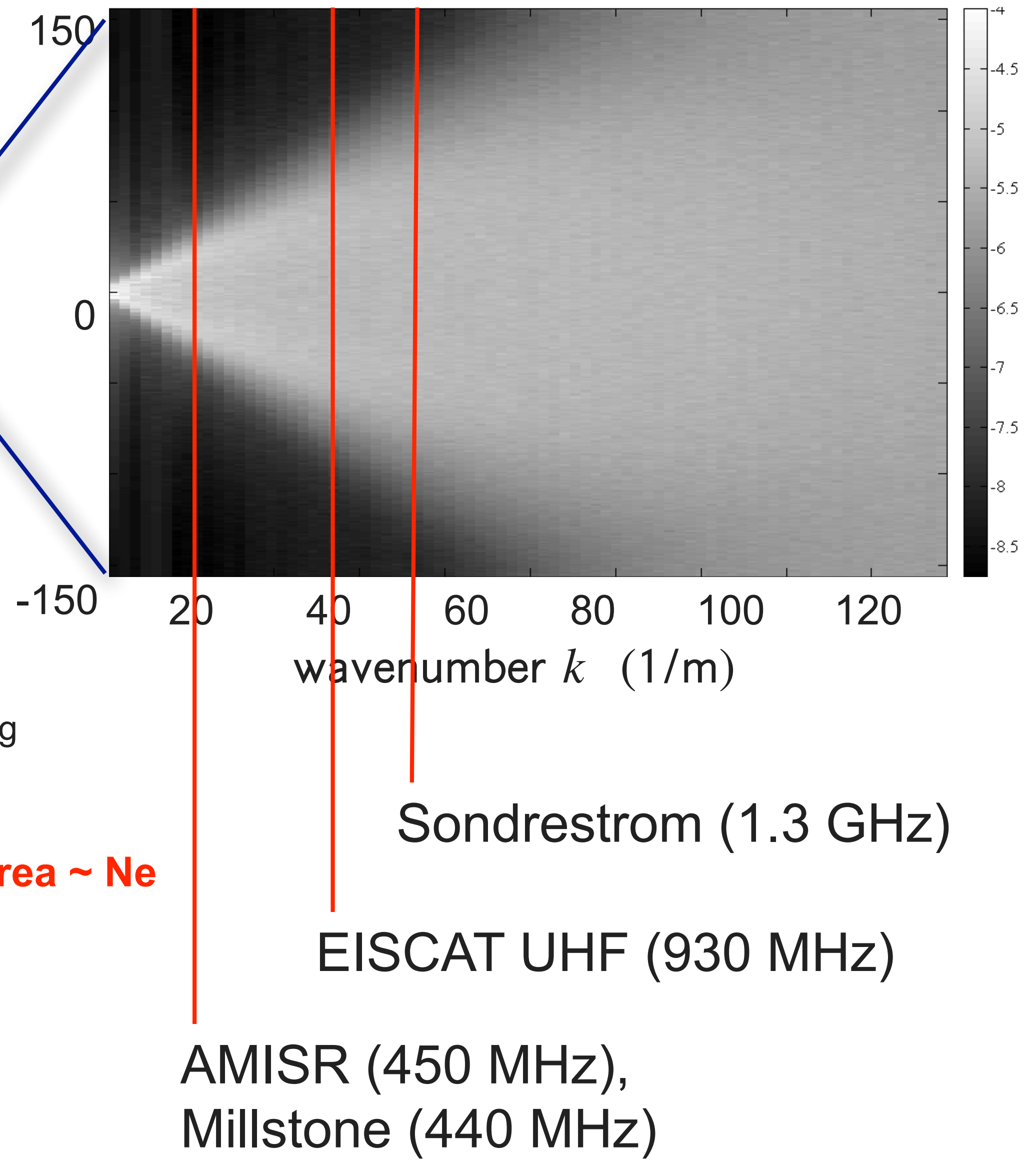
$$\nabla \cdot \mathbf{B} = 0$$



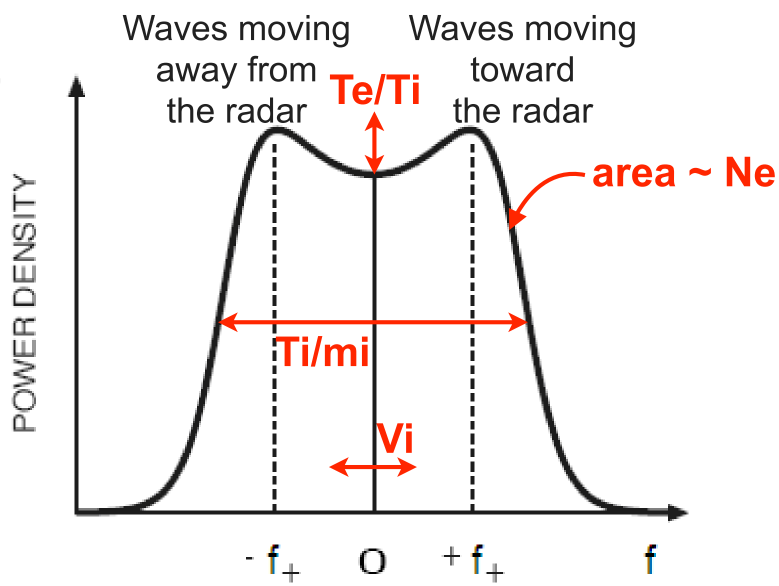


ISR measures a cut through this surface





Ion-acoustic “lines” are broadened by Landau damping



Doppler Radar Summary: "Coherent" hard targets

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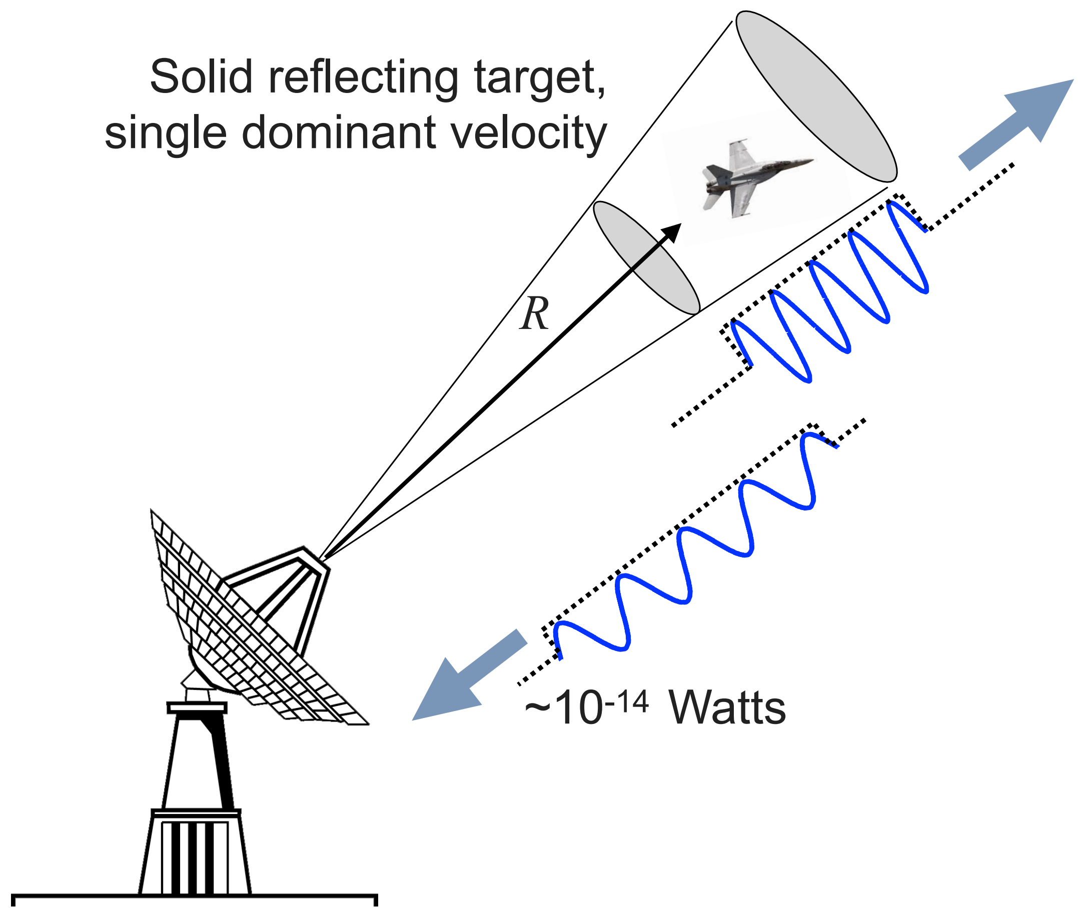

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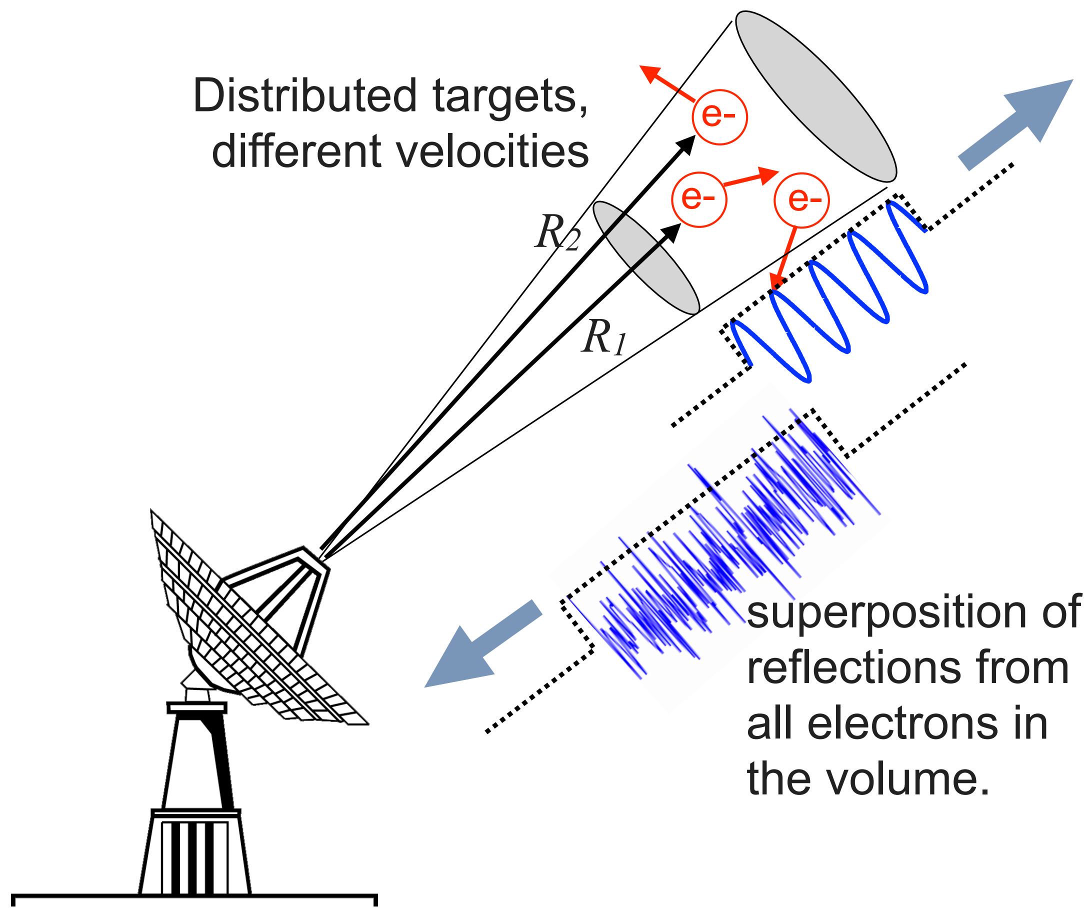

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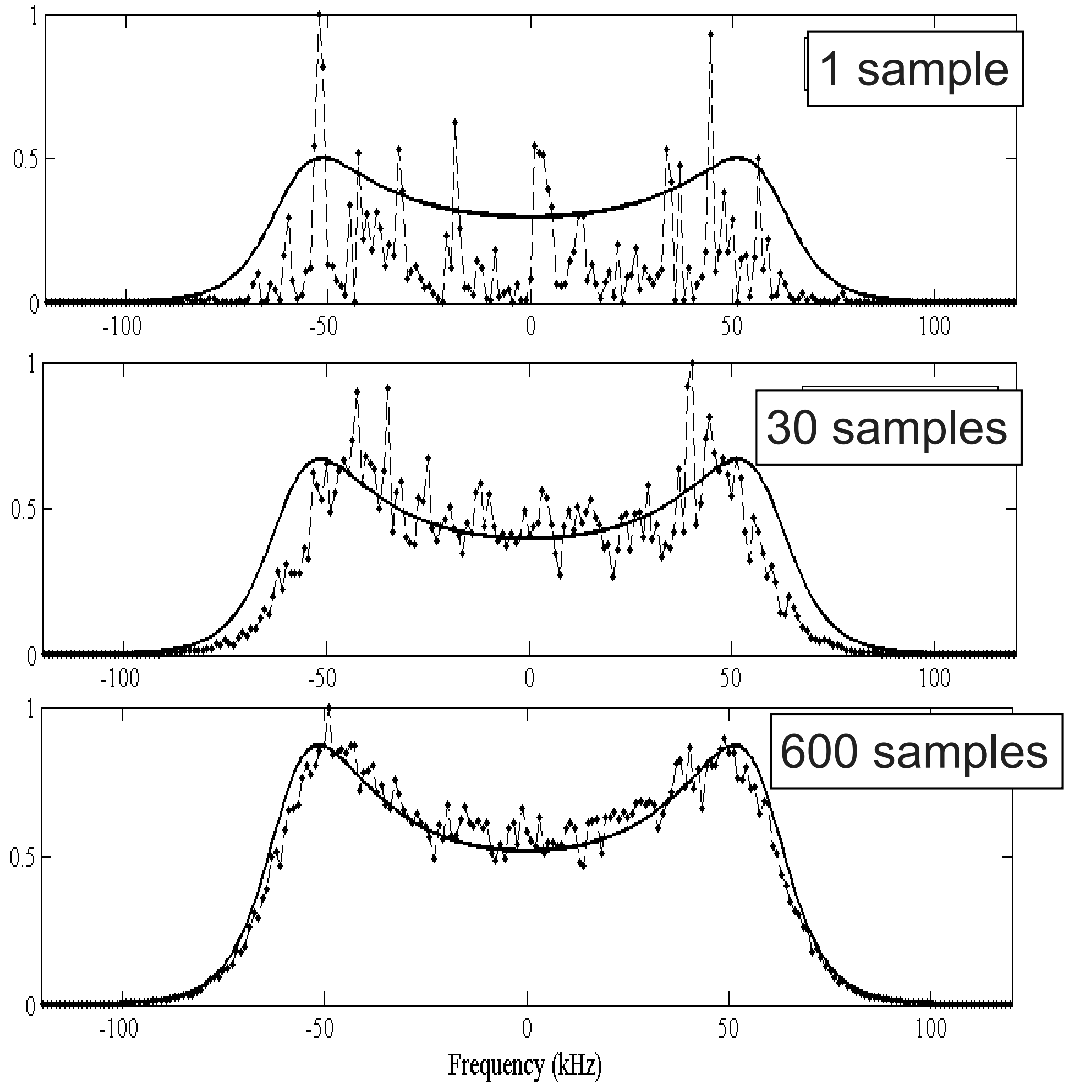
Incoherent Averaging

Normalized ISR spectrum for different integration times at 1290 MHz

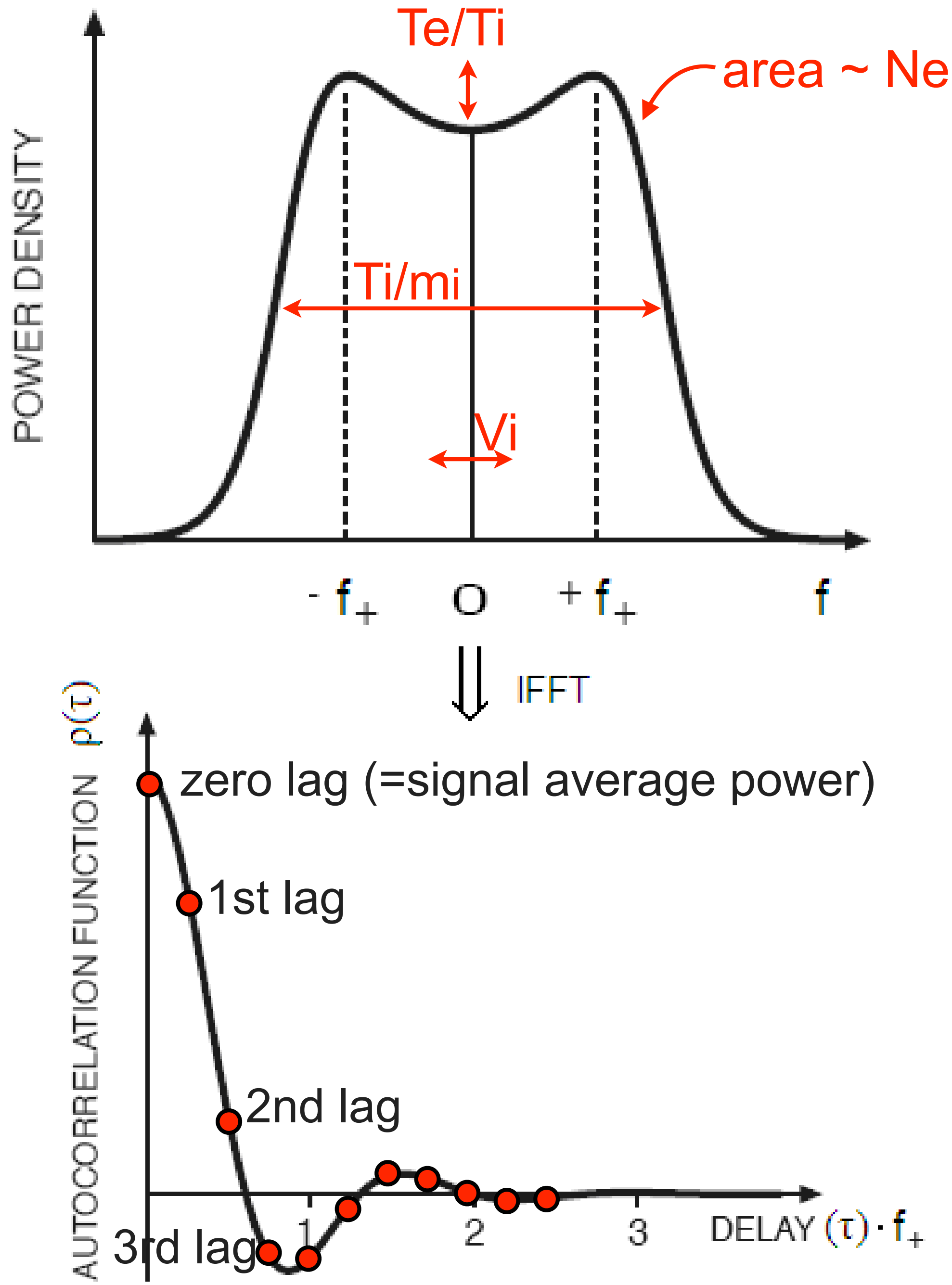


We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\rho_e \sim \frac{1}{K}\left(1 + \frac{1}{S/N}\right)^2$$

$\rho_e =$ Mean Square Error
$K =$ number of samples
$SNR =$ per-pulse Signal-to-Noise Ratio

Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

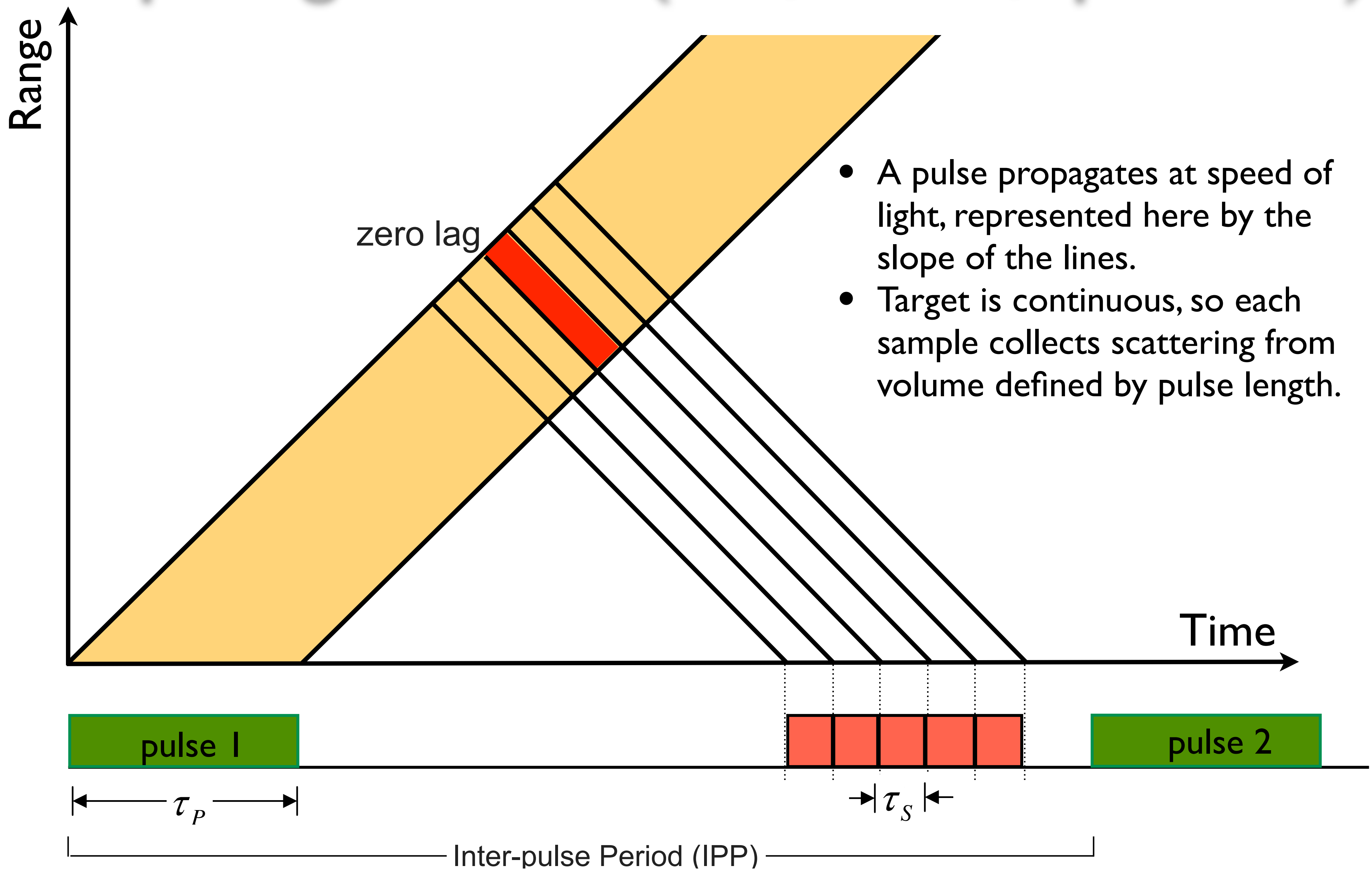
Electron to ion temperature ratio (Te/Ti) from “peak-to-valley” ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to sample lags with sufficient fidelity to provide meaningful estimates of plasma parameters

Computing the ACF (and, hence, spectrum)

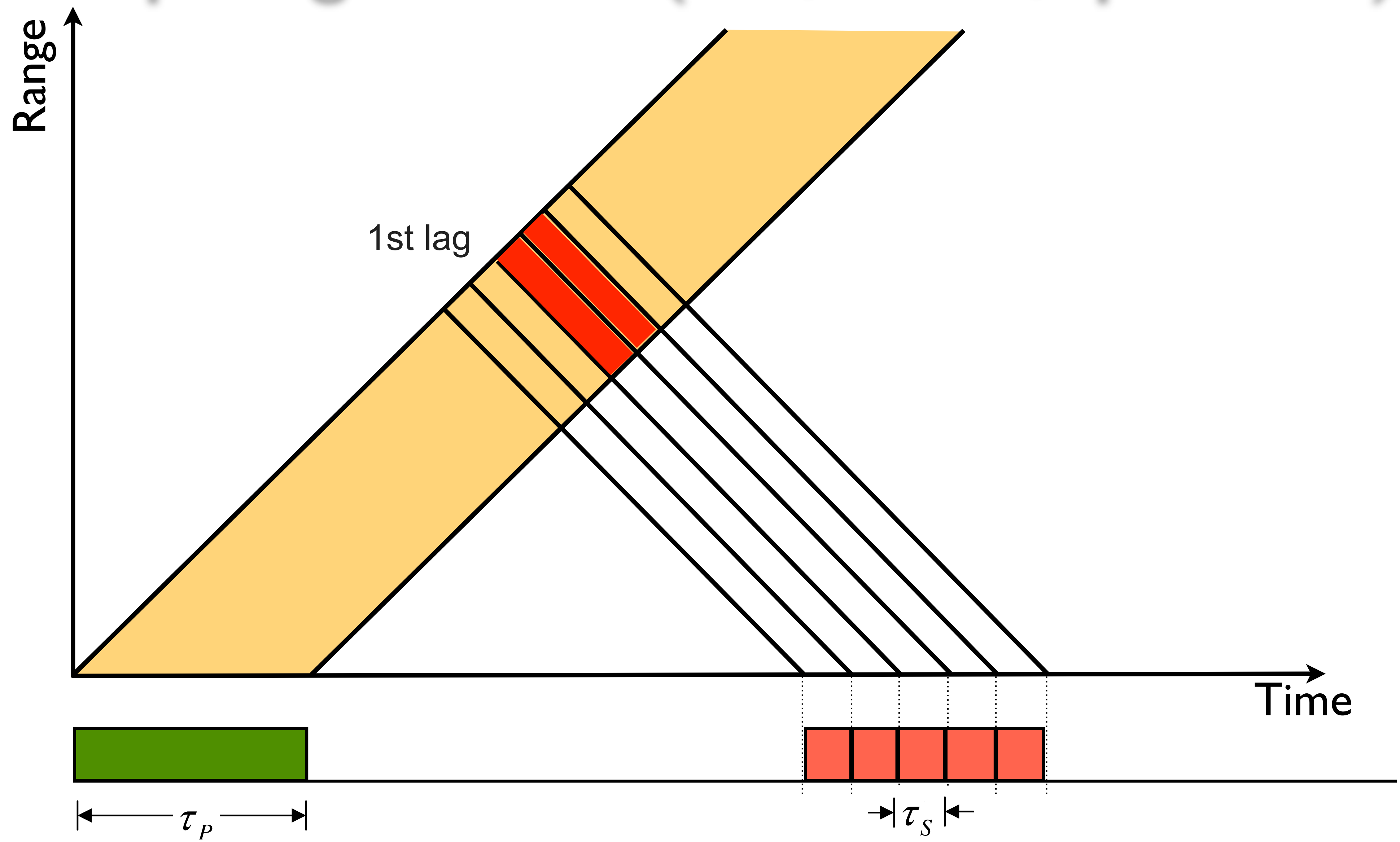


- A pulse propagates at speed of light, represented here by the slope of the lines.
- Target is continuous, so each sample collects scattering from volume defined by pulse length.

τ_P = Length of RF pulse

τ_s = Sample Period (typically ~ 1/10 pulse length)

Computing the ACF (and, hence, spectrum)



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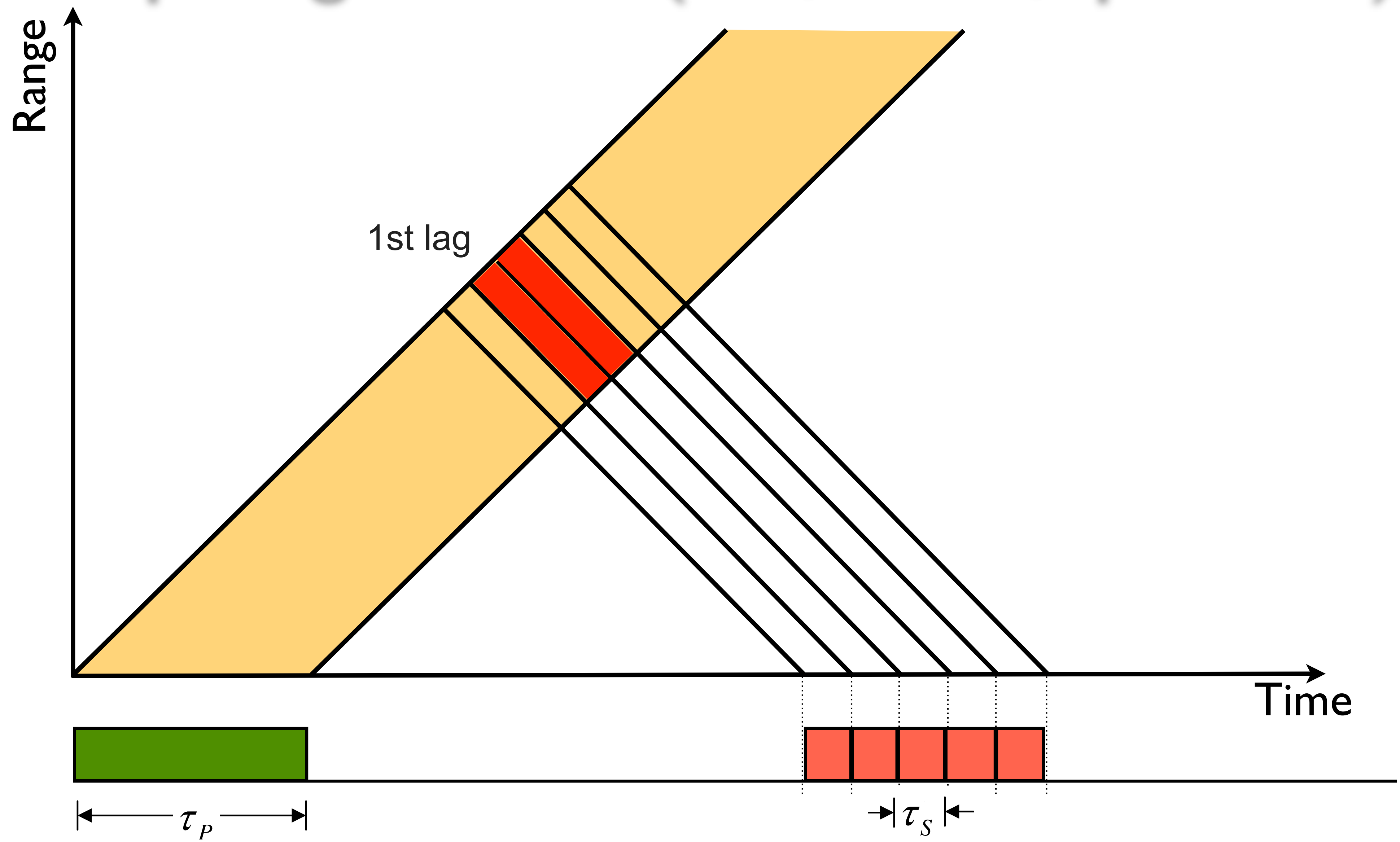

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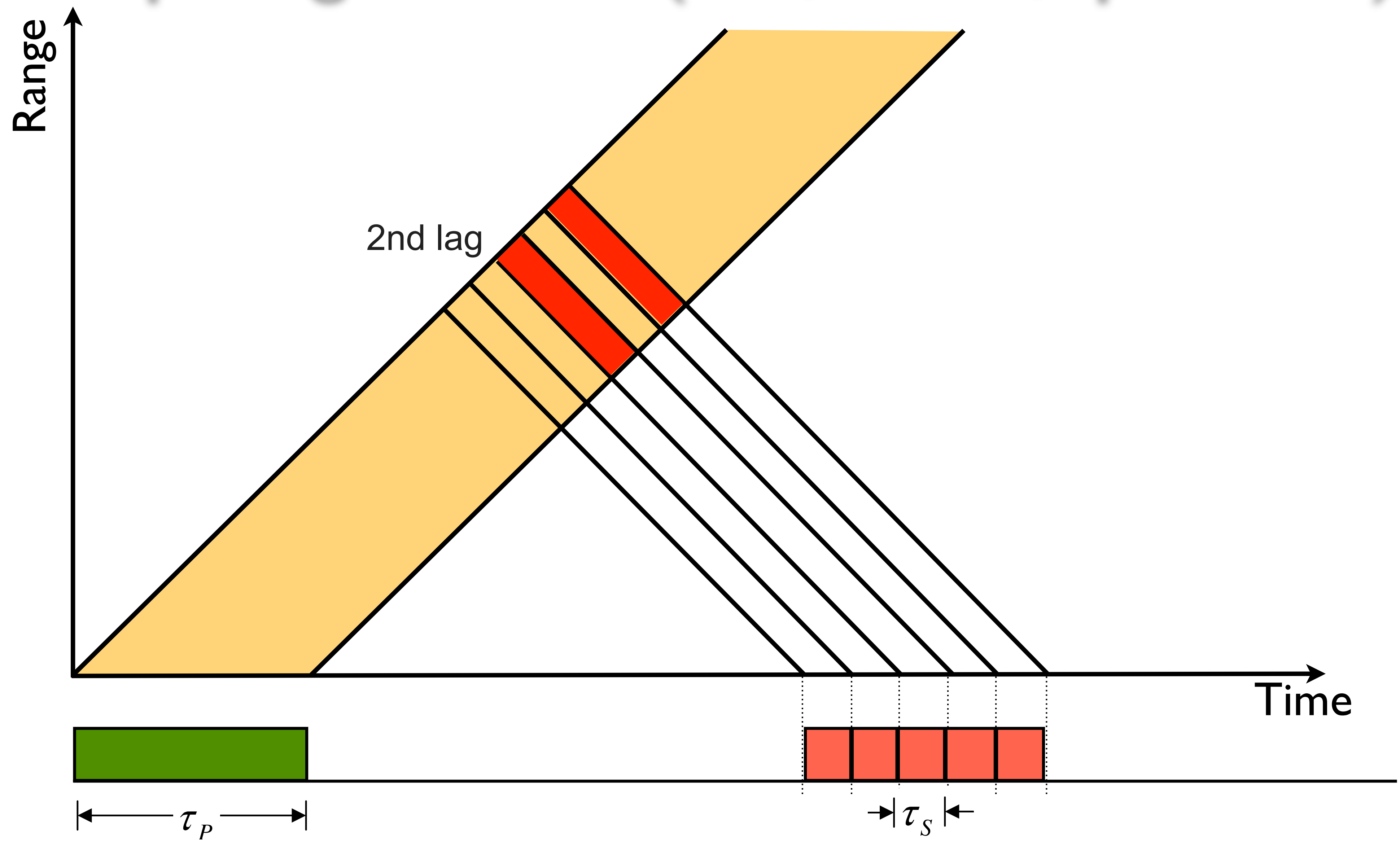

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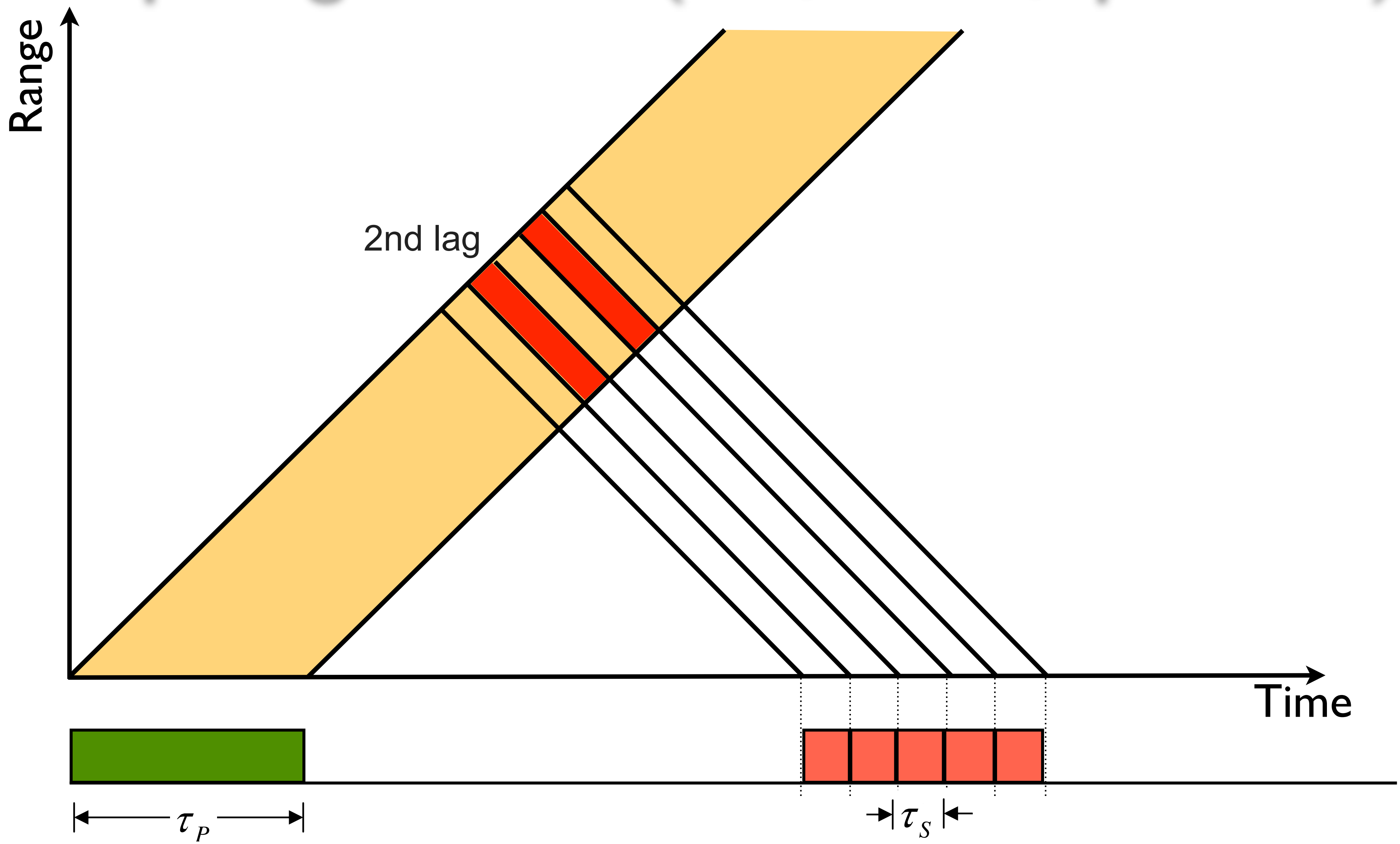

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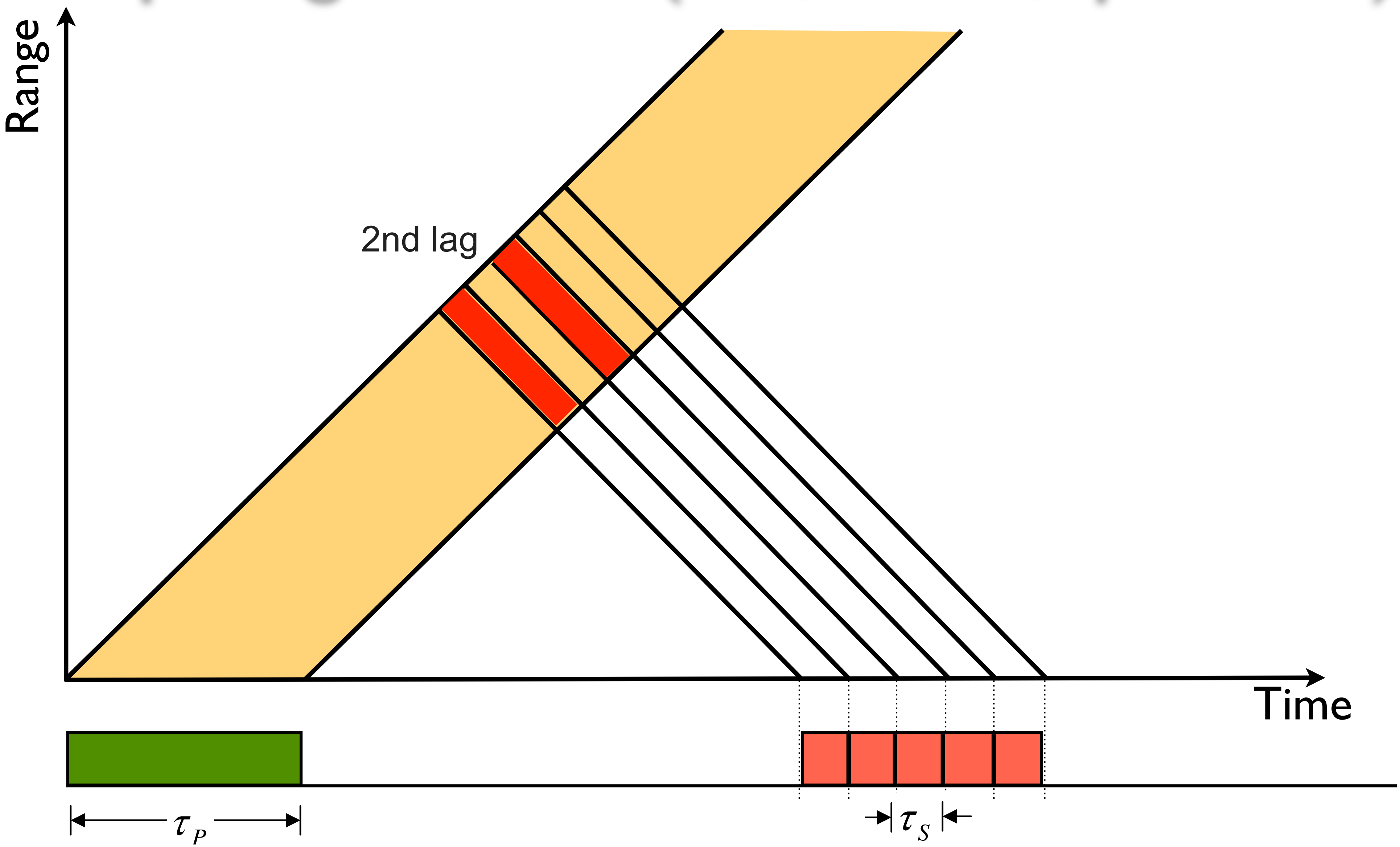

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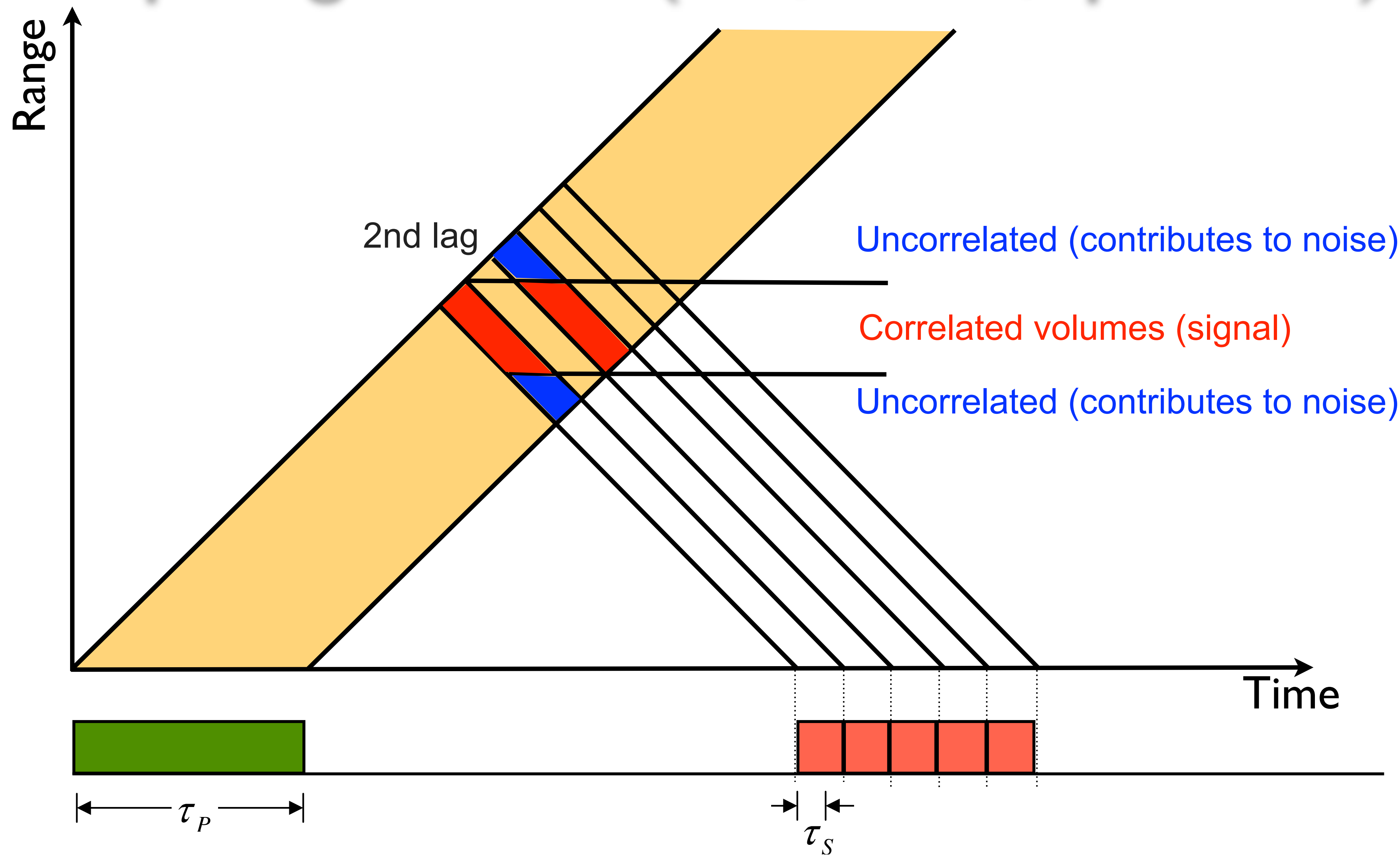

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Lag-product matrix