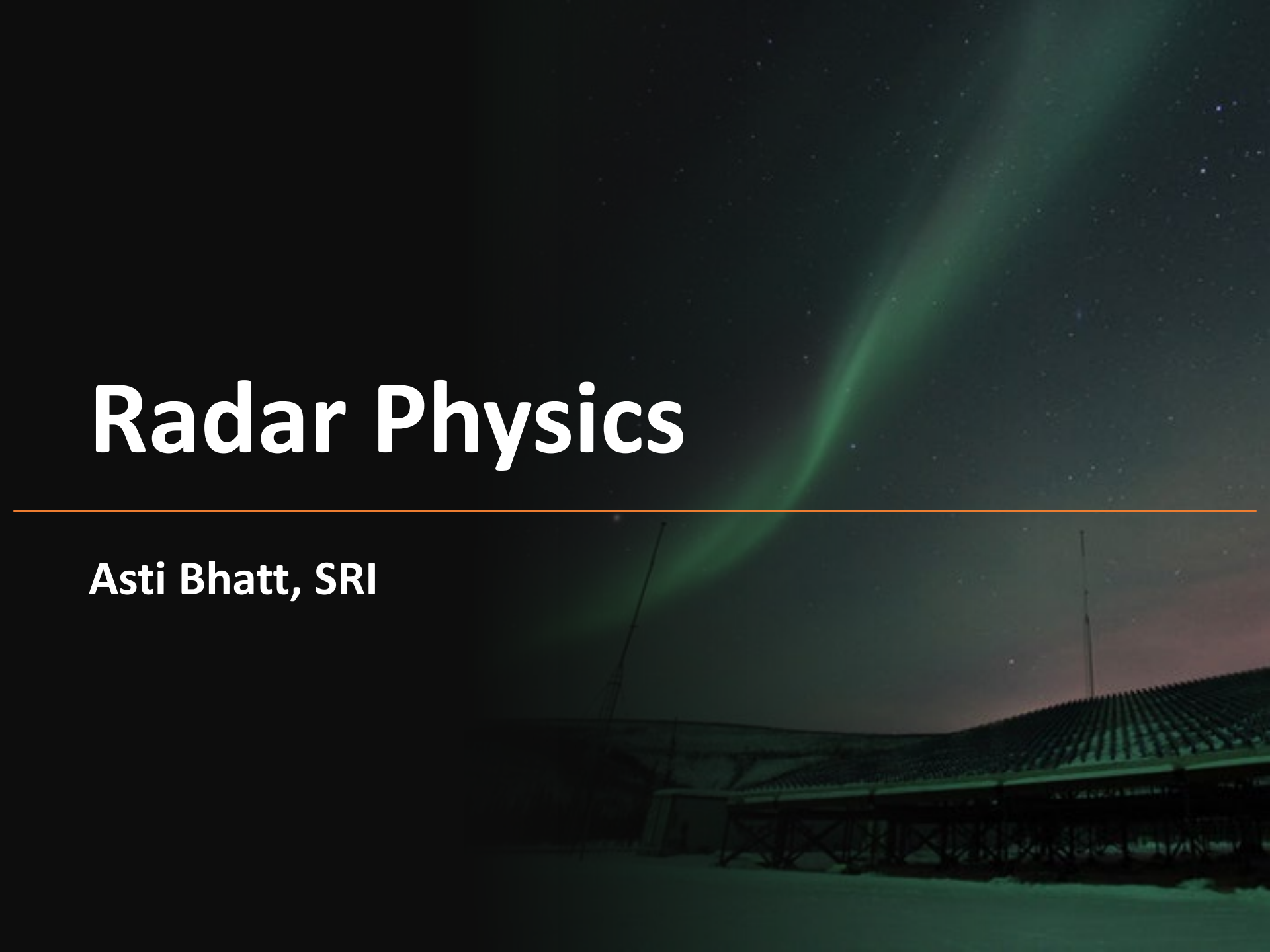


Radar Physics

Asti Bhatt, SRI



Outline - Radar Basics

Electromagnetic spectrum

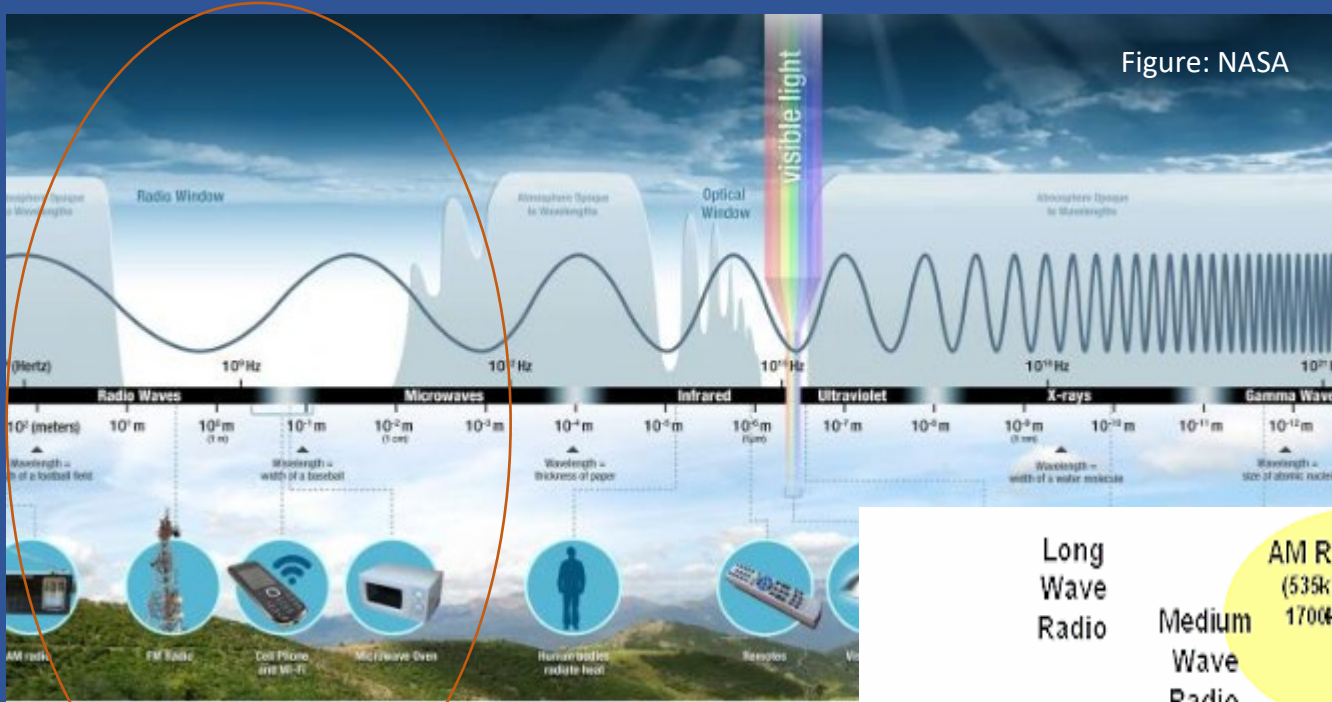
Radio waves and propagation

Radar equation

Range resolution and pulsed radars

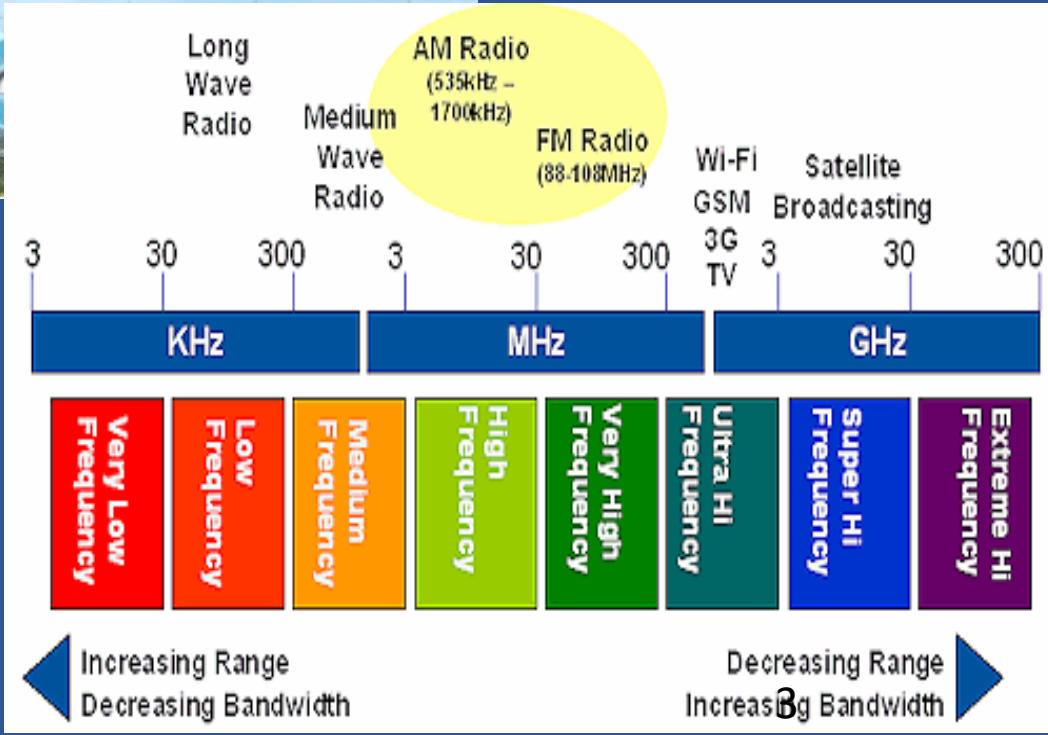
Doppler

The Electromagnetic Spectrum



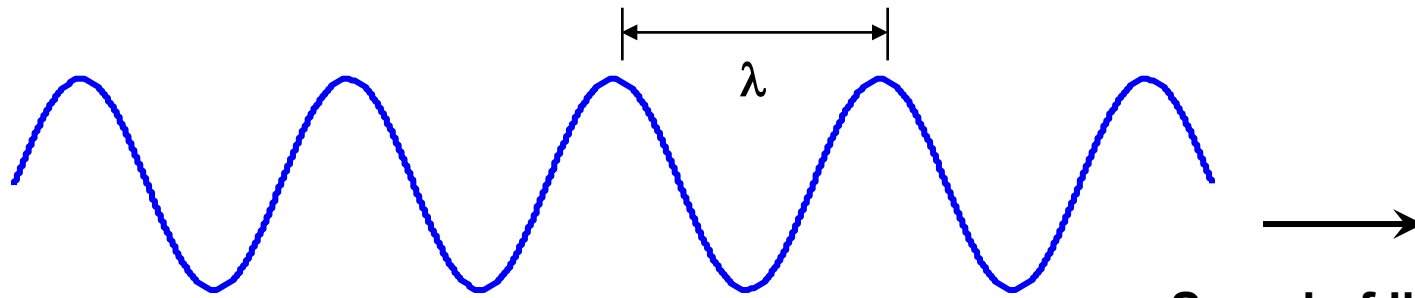
Remote sensing using radio waves:

Just light we can't see without tools.



Properties of Waves

Relationship Between Frequency and Wavelength



Speed of light, c
 $c = 3 \times 10^8 \text{ m/sec}$
 $= 300,000,000 \text{ m/sec}$

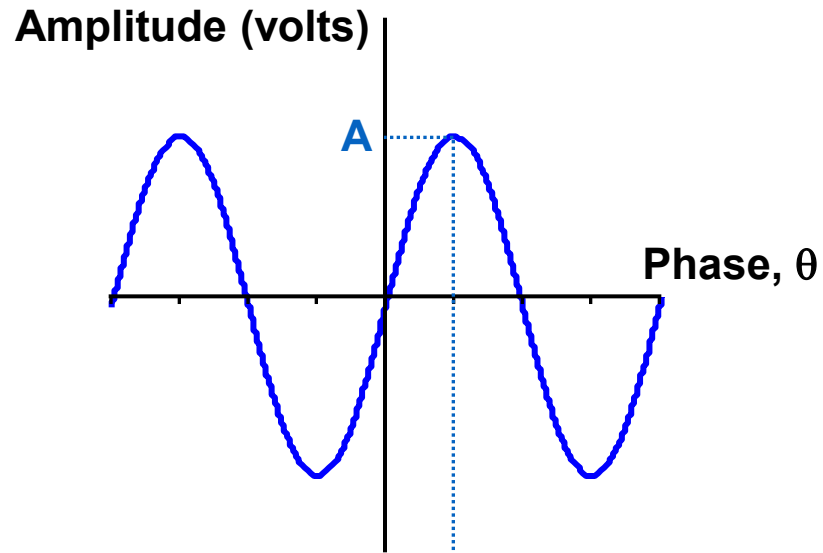
$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

Examples:

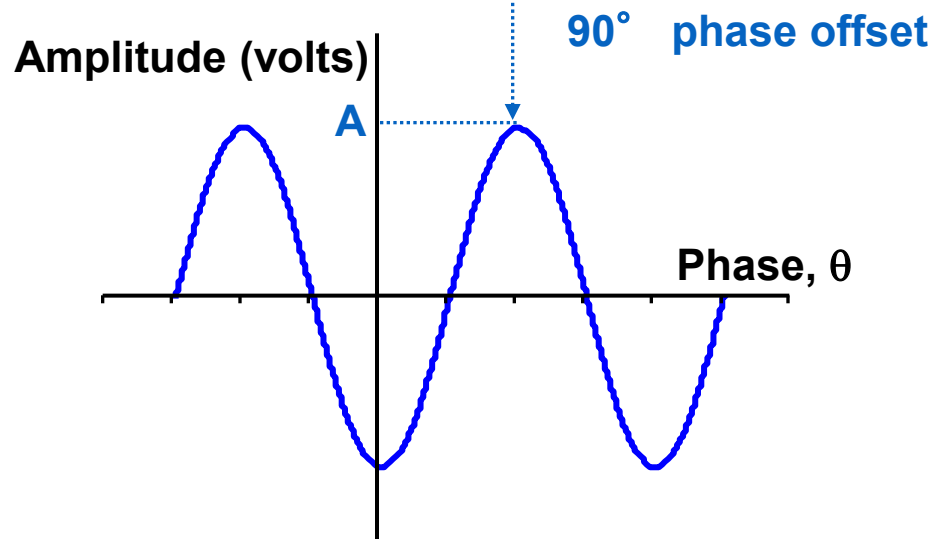
<u>Frequency</u>	<u>Wavelength</u>
100 MHz	3 m
1 GHz	30 cm
3 GHz	10 cm
10 GHz	3 cm

Properties of Waves

Phase and Amplitude



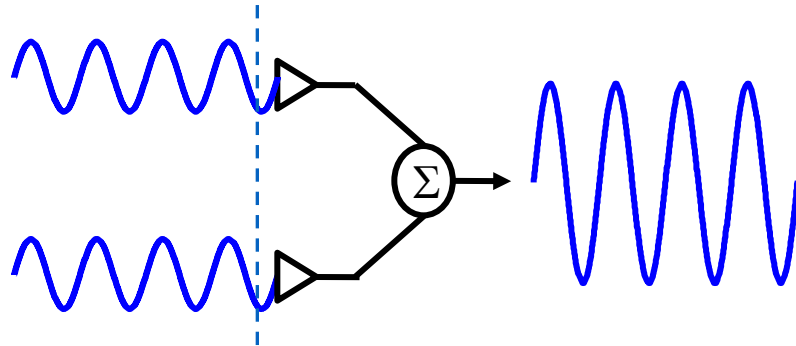
$$A \sin(\theta)$$



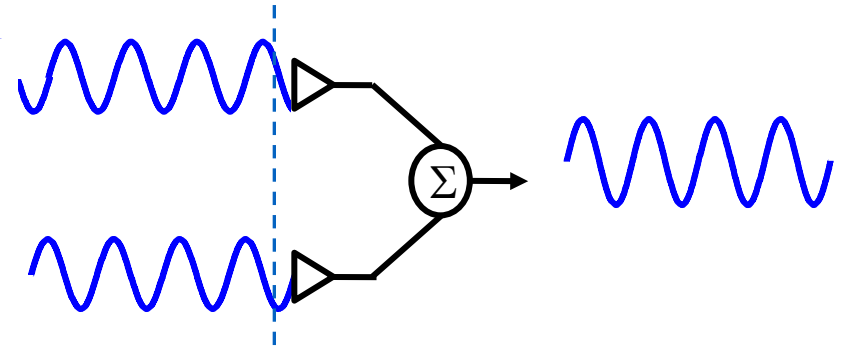
$$A \sin(\theta - 90^\circ)$$

Properties of Waves

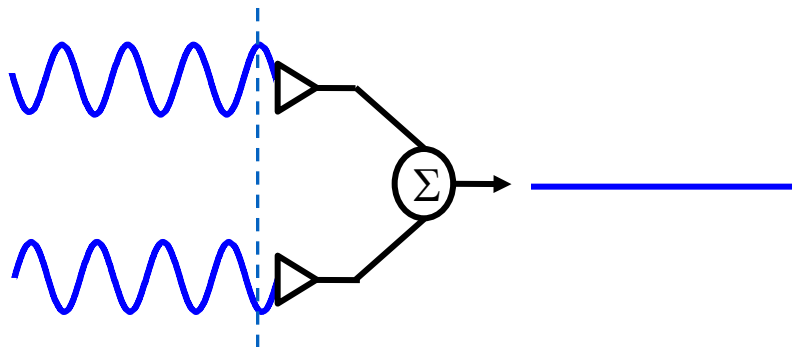
Constructive vs. Destructive Addition



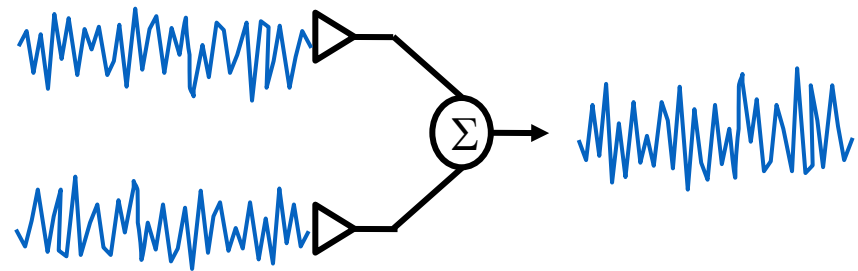
Constructive
(in phase)



Partially Constructive
(somewhat out of phase)



Destructive
(180° out of phase)



Non-coherent signals
(noise)

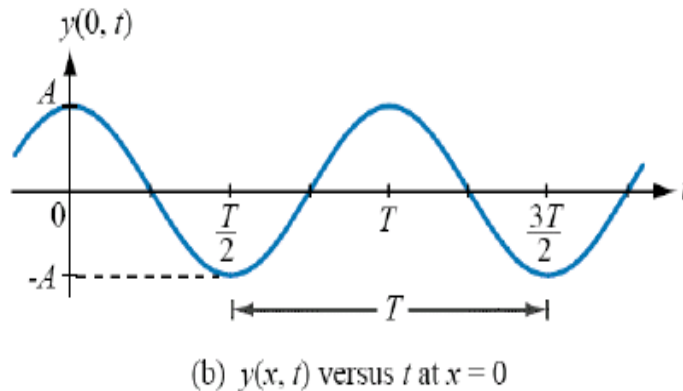
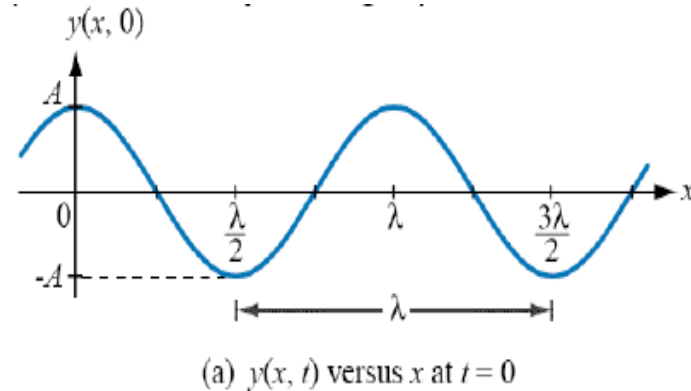
Radio Waves

$$y(x, t) = A \cos(\omega t - kx + \phi_0)$$

Angular frequency
 $\omega = 2\pi f = 2\pi/T$

Wavenumber
 $k = 2\pi/\lambda$

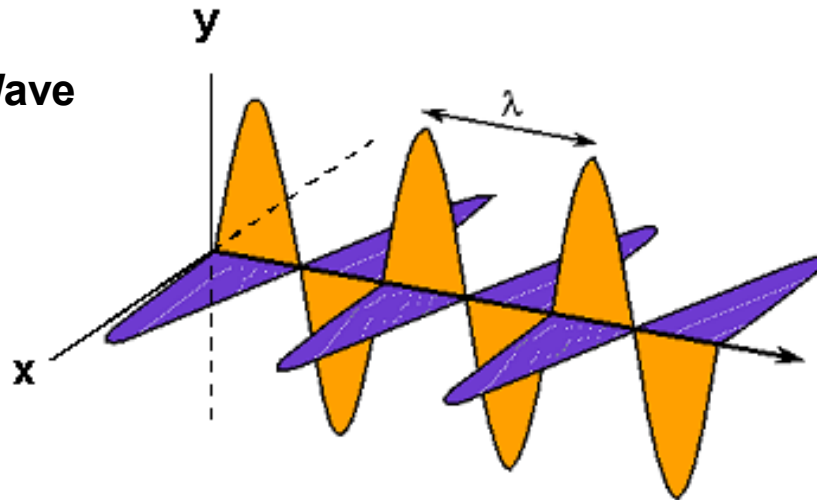
Wave phase velocity
 $c = f\lambda = \omega/k = 3 \times 10^8 \text{ m/s}$



$$\text{Frequency (1/s)} = \frac{\text{Speed of light (m/s)}}{\text{Wavelength } \lambda \text{ (m)}}$$

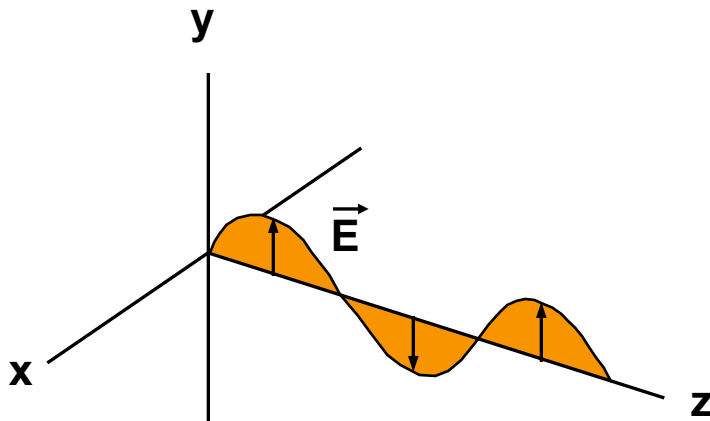
Polarization

Electromagnetic Wave

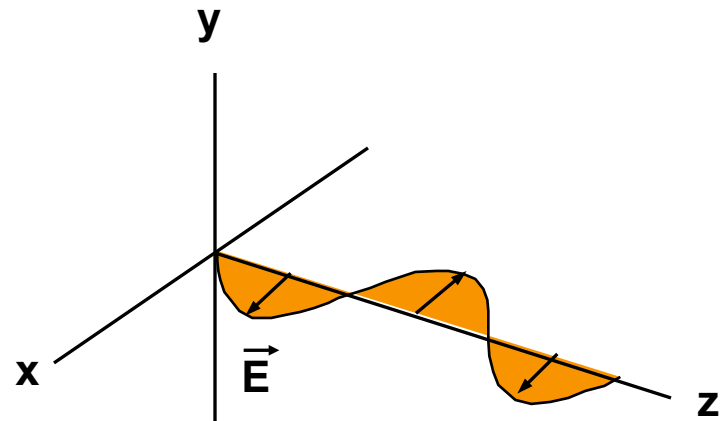


-  Electric Field
-  Magnetic Field

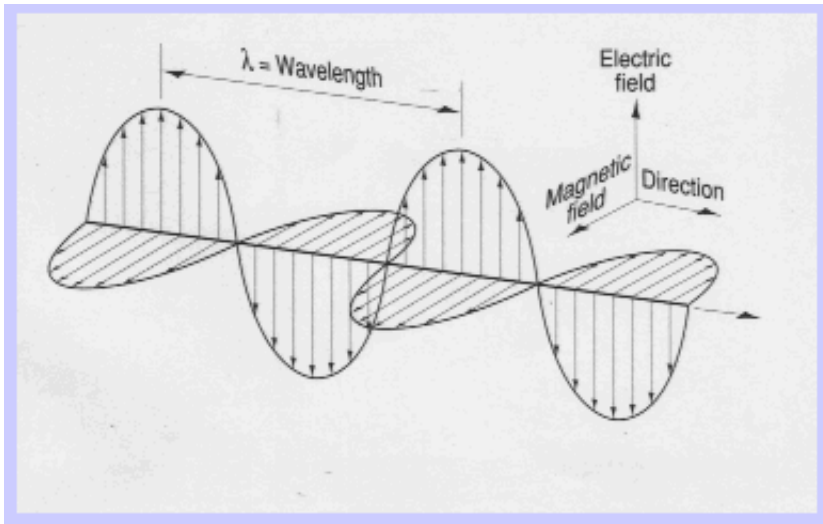
Vertical Polarization



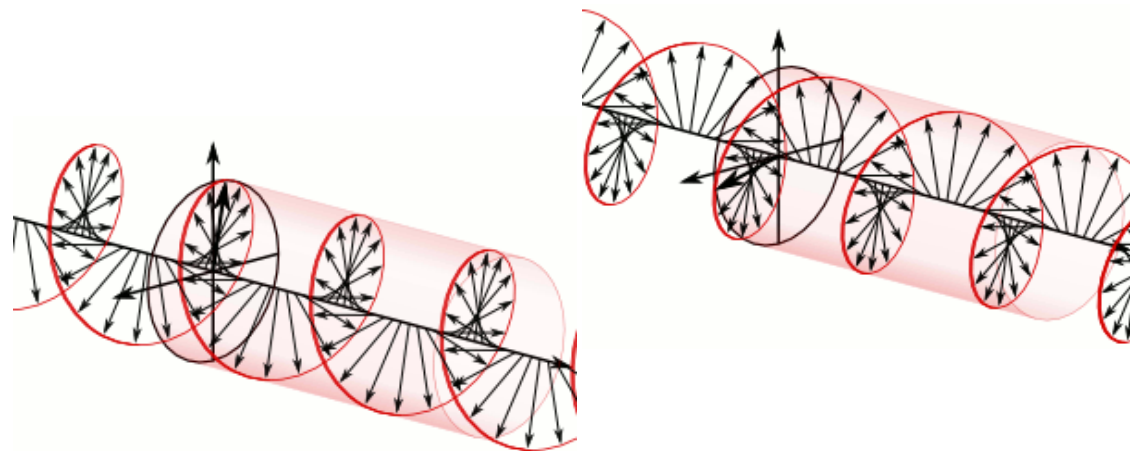
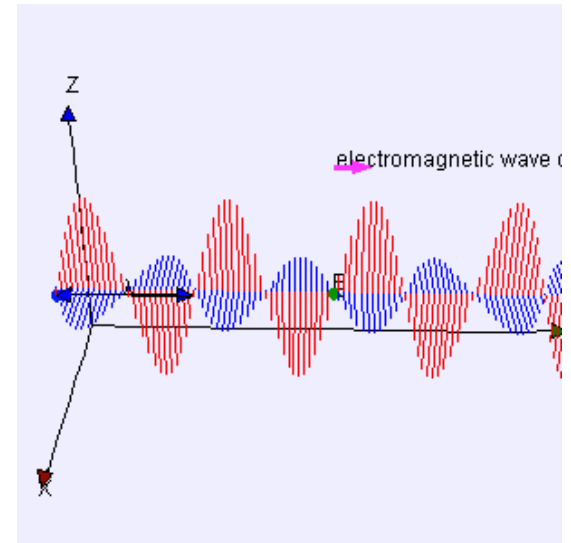
Horizontal Polarization



TEM Waves: *Transverse electromagnetic (TEM) modes* neither electric nor magnetic field in the direction of propagation



Electromagnetic waves in free space propagate in TEM mode



Phase Velocity, Group Velocity, Index of Refraction

$$v_p = \frac{\omega}{k}$$

$$v_g \equiv \frac{\partial \omega}{\partial k}$$

$$n = \frac{c}{v_p}$$



Refraction and Dispersion

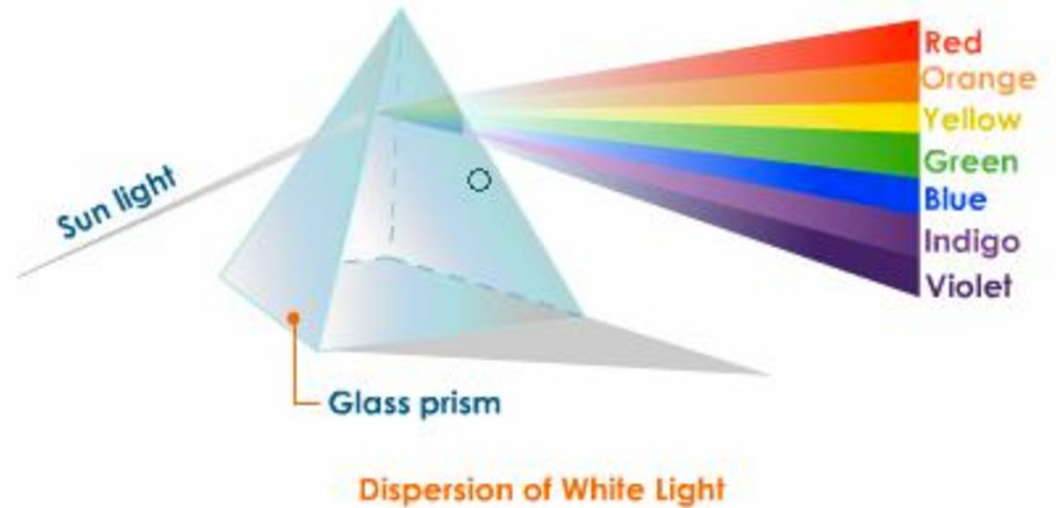
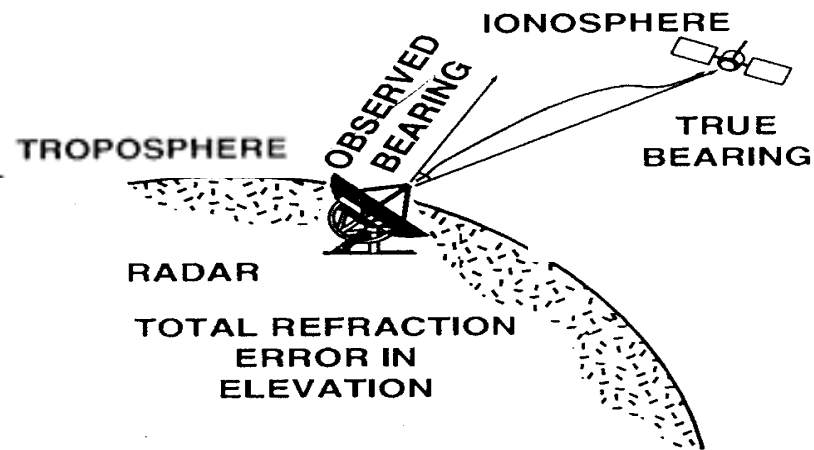
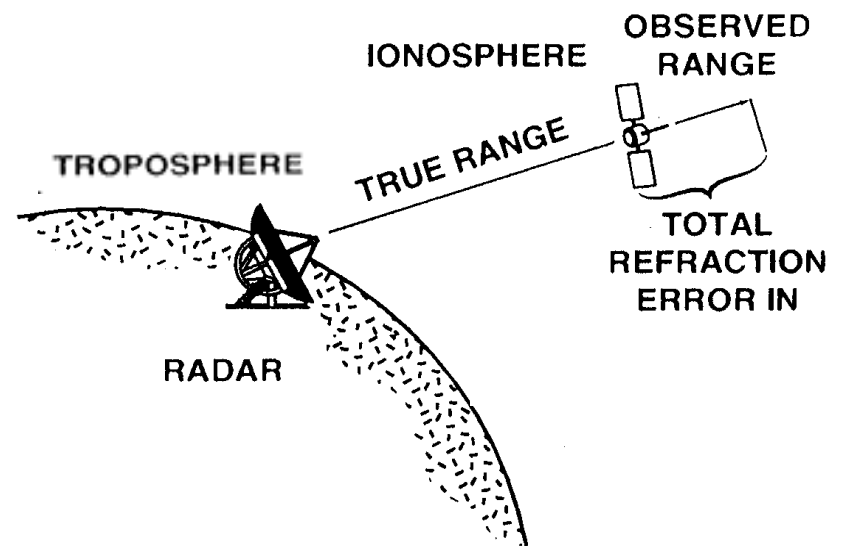


Illustration of Atmospheric Effects

Elevation Refraction



Range Delay



Index of Refraction

$$n = \frac{c}{v_p}$$

in the Ionosphere

is a function of frequency

$$n^2 = 1 - \frac{X}{1 - iZ - \frac{\frac{1}{2}Y^2 \sin^2 \theta}{1 - X - iZ} \pm \frac{1}{1 - X - iZ} \left(\frac{1}{4}Y^4 \sin^4 \theta + Y^2 \cos^2 \theta (1 - X - iZ)^2 \right)^{1/2}}$$

n is the index of refraction

$$X = \frac{\omega_N^2}{\omega^2} \quad Y = \frac{\omega_H}{\omega} \quad Z = \frac{\nu}{\omega} \quad \omega_N = \left(\frac{Ne^2}{\epsilon_0 m_e} \right)^{1/2} \quad \omega_H = \frac{e|B|}{m_e}$$

ω = the angular frequency of the radar wave,

$Y_L = Y \cos \theta$, $Y_T = Y \sin \theta$,

θ = angle between the wave vector $\bar{k}$ and $\bar{B}$,

$\bar{k}$ = wave vector of propagating radiation,

$\bar{B}$ = geomagnetic field, N = electron density

e = electronic charge, m_e = electron mass, ν = electron collision frequency

and ϵ_0 = permittivity constant.

[Appleton-Hartree equation]

Dispersion relation: the concept

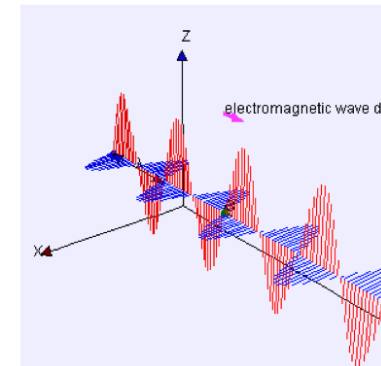
Key concept for wave behavior within a propagation medium.

Describes the relationship between SPATIAL frequency (wavelength) and TEMPORAL frequency.

Some wave modes relate wavelength to frequency **linearly**, but waves in most media have **nonlinear** relation between wavelength and frequency.

Linear dispersion example:

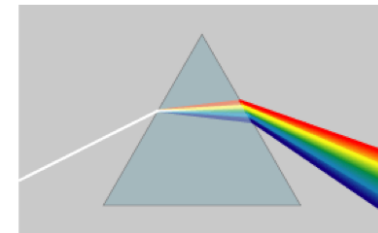
EM radiation propagation through free space



<http://weelookang.blogspot.com/2011/10/ejs-open-source-propagation-of.html>

Nonlinear dispersion example:

splitting of light through a prism
(effective speed of light depends on wavelength due to glass' non-unity index of refraction)



Wikipedia CC-3.0

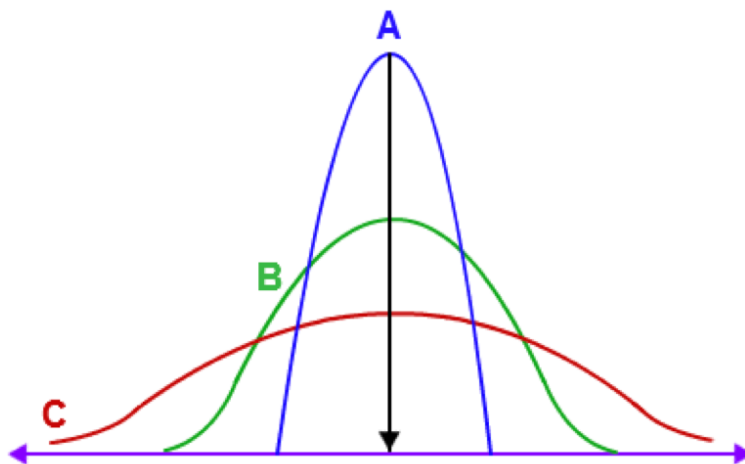
Dispersion relation: the concept

Simple linear case: uniform phase velocity

$$\omega(k) = c k$$

Most propagation speeds depend nonlinearly on the wavelength and/or frequency.

NB: for a **nonlinear** dispersion relation, the pulse will typically spread in either spatial frequency or temporal frequency as a function of time.



Example of pulse spreading spatially from time A to B to C.

Radio Propagation in the Ionosphere

Index of Refraction (no **B** field)

$$n^2 = \frac{c^2 k^2}{\omega^2}$$
$$= 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma Frequency

$$\omega_p^2 = \frac{n_0 e^2}{m \epsilon_0}$$

Phase Velocity

$$V_{ph} = \frac{\omega}{k}$$

Topside
F region peak
E region
D region

Absorption

Significantly Above Critical Frequency

Above Critical Frequency

Ducting

Multiple Hops

Below Critical Frequency



Outline - Radar Basics

Electromagnetic spectrum

Radio waves and propagation

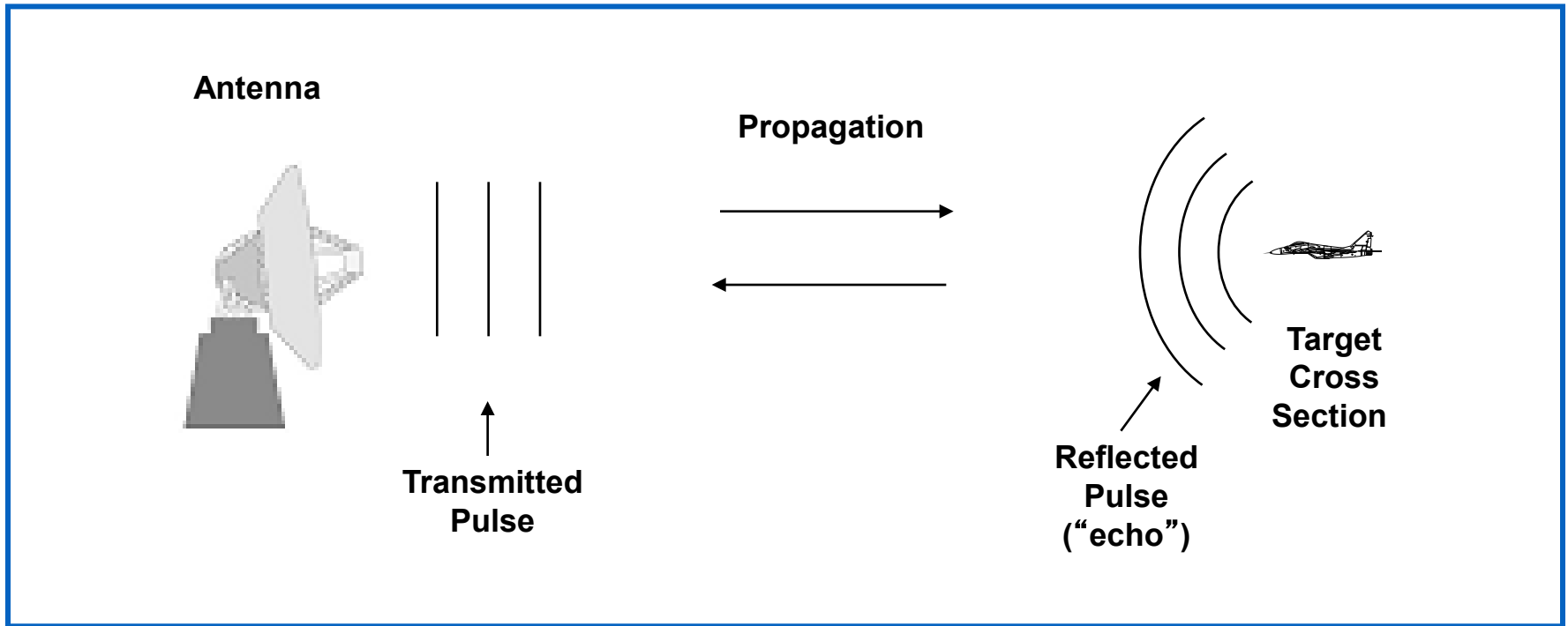
Radar equation

Range resolution and pulsed radars

Doppler

RADAR

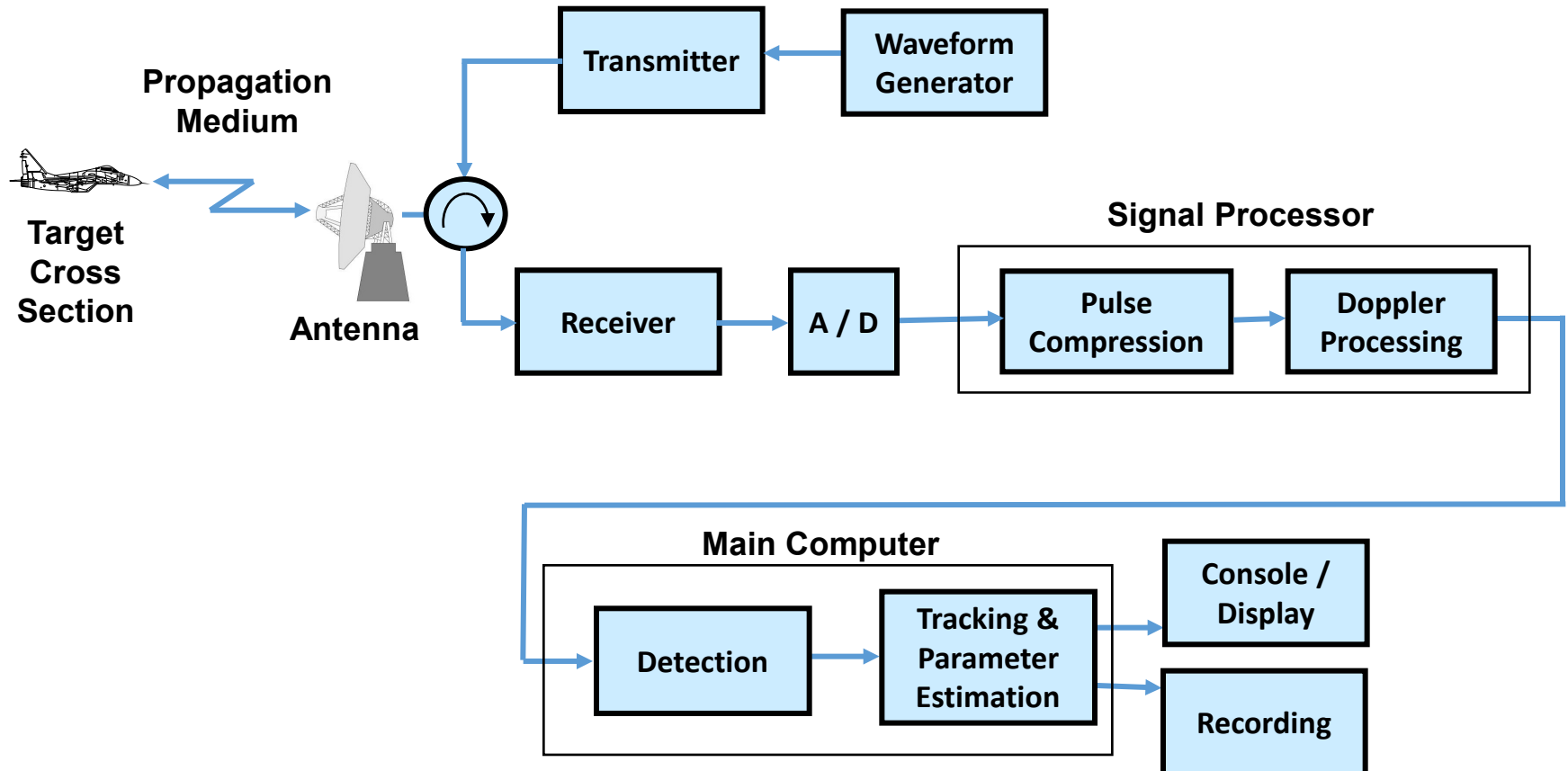
Radio Detection And Ranging



Radar observables:

- Target range
- Target angles (azimuth & elevation)
- Target size (radar cross section)
- Target speed (Doppler)
- Target features (imaging)

Radar Block Diagram



Radar Equation

Antenna Aperture A

Transmit Power P_T

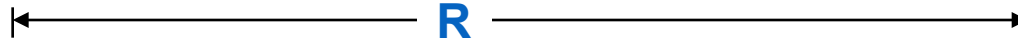
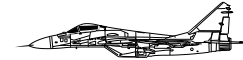


Transmitted Pulse



Received Pulse

Target Cross Section σ

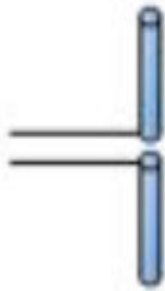


R

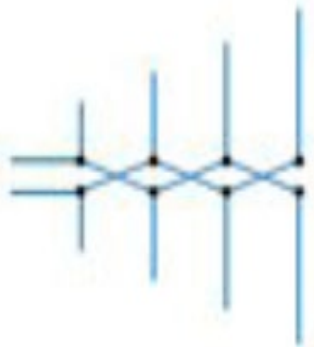
Received Signal Energy	=	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
		$[P_T]$	$\left[\frac{4\pi A}{\lambda^2} \right]$	$\left[\frac{1}{4\pi R^2} \right]$	$\left[\frac{1}{L} \right]$	$[\sigma]$	$\left[\frac{1}{4\pi R^2} \right]$	$[A]$	$[\tau]$

Antennas

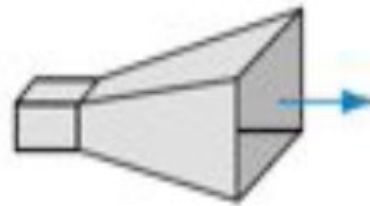
Most basic form of antennas – a wire element with a time varying current flowing in it



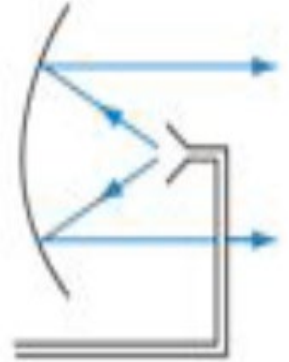
Dipole antenna



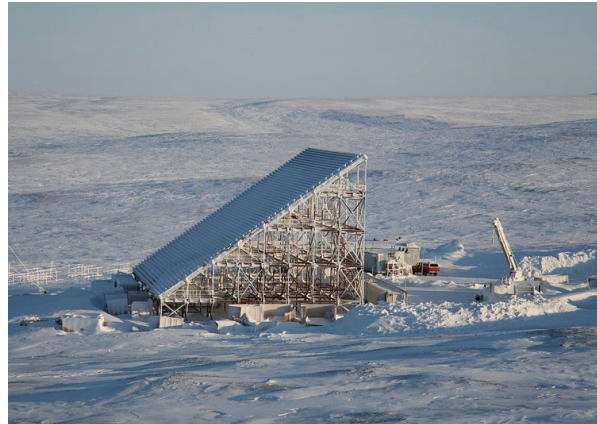
Log periodic



Horn antenna



Parabolic dish
Reflector antenna

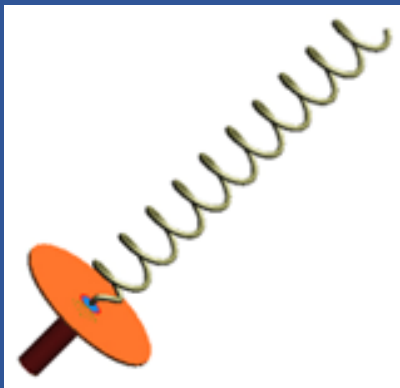


Examples of Antennas

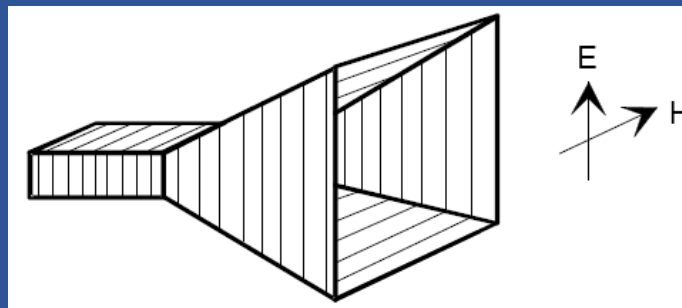
Antennas

Four primary functions of an antenna for radar applications:

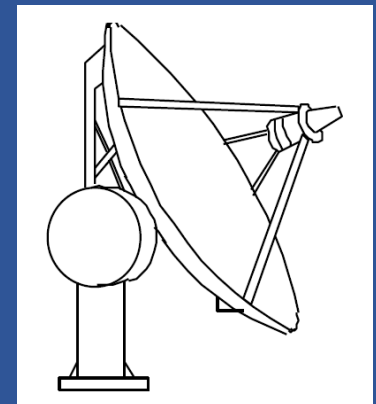
- Impedance transformation (free-space intrinsic impedance to transmission-line characteristic impedance)
- Propagation-mode adapter (free-space fields to guided waves)
- Spatial filter (radiation pattern – direction-dependent sensitivity)
- Polarization filter (polarization-dependent sensitivity)



Helical antenna



Horn antenna



Parabolic reflector antenna

Impedance transformer

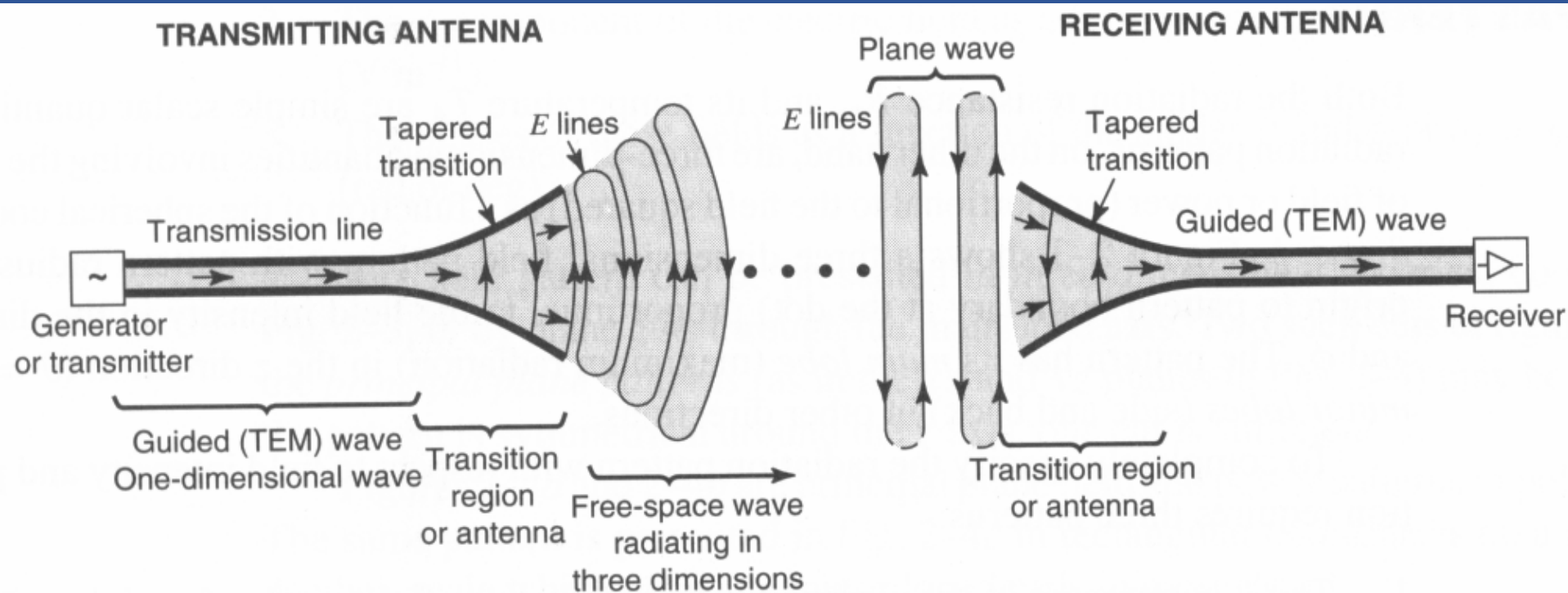
- Intrinsic impedance of free-space, $\eta_0 \equiv E/H$ is

$$\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120 \pi \cong 376.7 \Omega$$

- Characteristic impedance of transmission line,
 $Z_0 = V/I$
- A typical value for Z_0 is 50Ω .
- Clearly there is an impedance mismatch that must be addressed by the antenna.

Propagation-mode adapter

During both transmission and receive operations, the antenna must provide the transition between these two propagation modes.

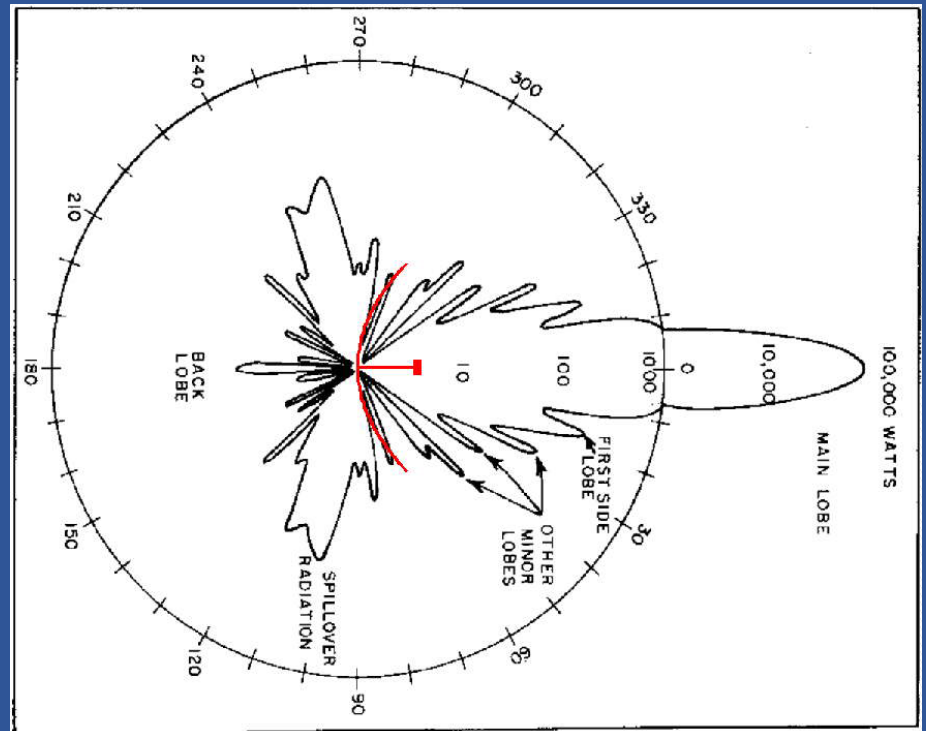


Spatial filter

Antennas have the property of being more sensitive in one direction than in another, which provides the ability to spatially filter signals from its environment.



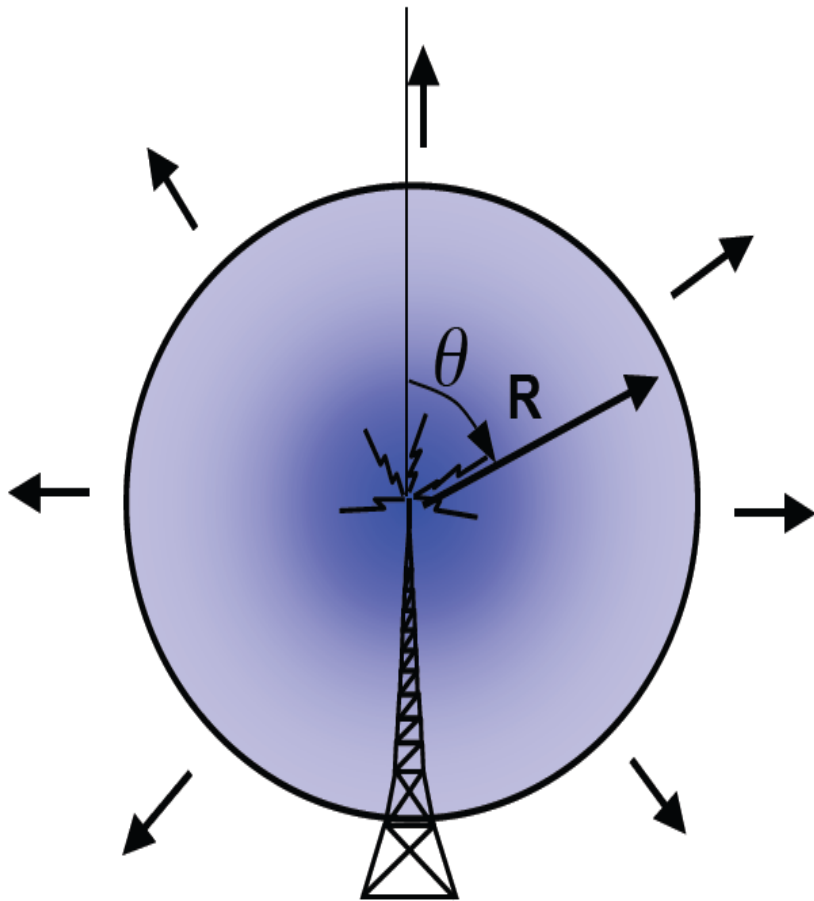
Directive antenna.



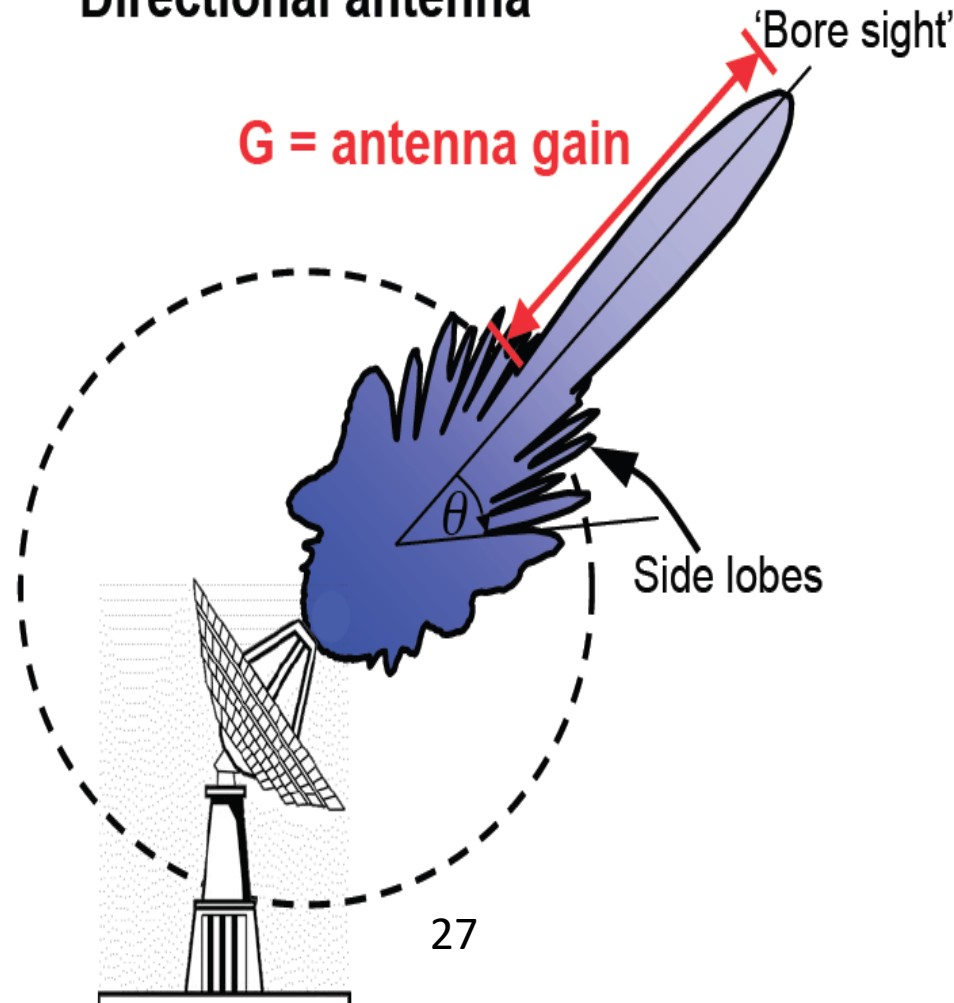
Radiation pattern of directive antenna.

Radiation pattern – antenna gain

Isotropic antenna



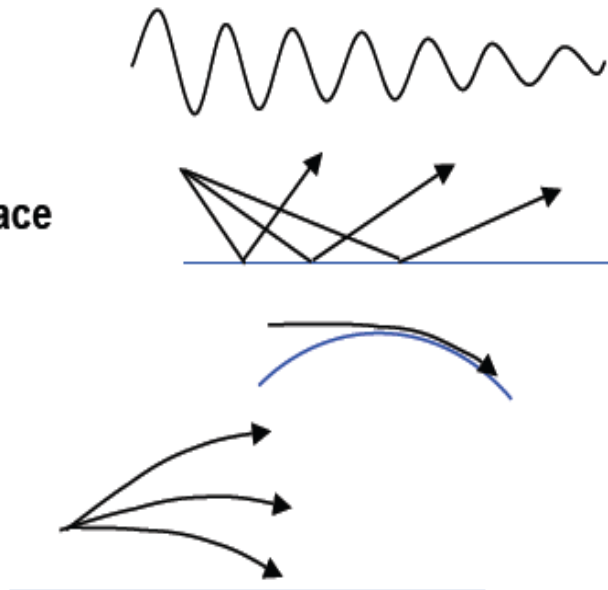
Directional antenna



Propagation medium – losses

Radio waves are affected by the medium they propagate in, and get reflected off of surfaces, all of which cause some losses in the signal between transmit and receive

- **Atmospheric attenuation**
- **Reflection off of earth's surface**
- **Over-the-horizon diffraction**
- **Atmospheric refraction**



Attenuation usually measured in dB

$$\text{SNR dB} = 10 \log_{10} \frac{\text{signal power}}{\text{noise power}}$$

dB value	times by
+30 dB	1000
+20 dB	100
+3 dB	2
-10 dB	0.1
-20 dB	0.01

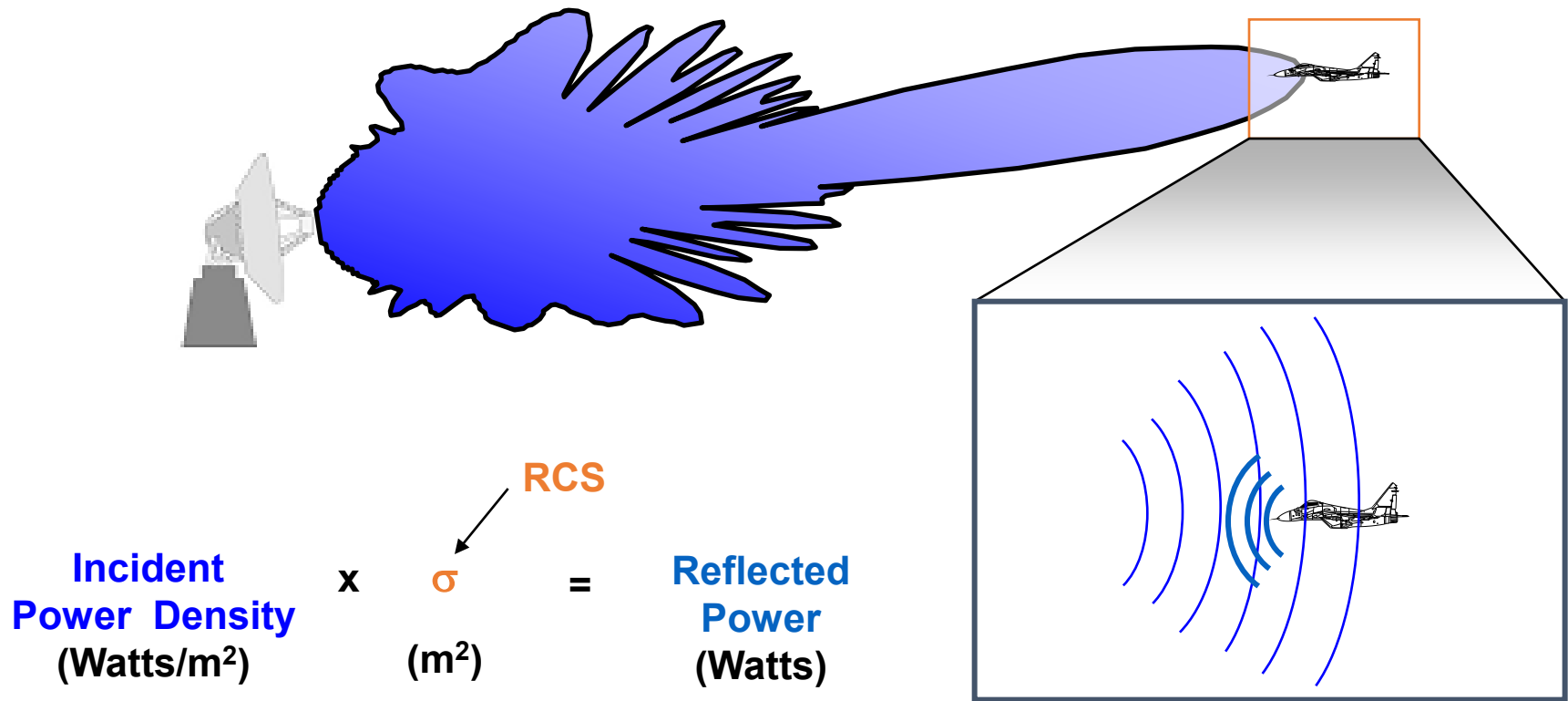
Radar equation

$$\begin{array}{cccccccc} & \text{Transmit} & \text{Transmit} & \text{Spread} & \text{Losses} & \text{Target} & \text{Spread} & \text{Receive} & \text{Dwell} \\ & \text{Power} & \text{Gain} & \text{Factor} & & \text{RCS} & \text{Factor} & \text{Aperture} & \text{Time} \\ \text{Received Signal} & & & & & & & & \\ \text{Energy} & = & [P_T] & \left[\frac{4\pi A}{\lambda^2} \right] & \left[\frac{1}{4\pi R^2} \right] & \left[\frac{1}{L} \right] & [\sigma] & \left[\frac{1}{4\pi R^2} \right] & [A] & [\tau] \end{array}$$

Radar cross section tells us about the target properties

It is the effective target cross section as seen by the radar

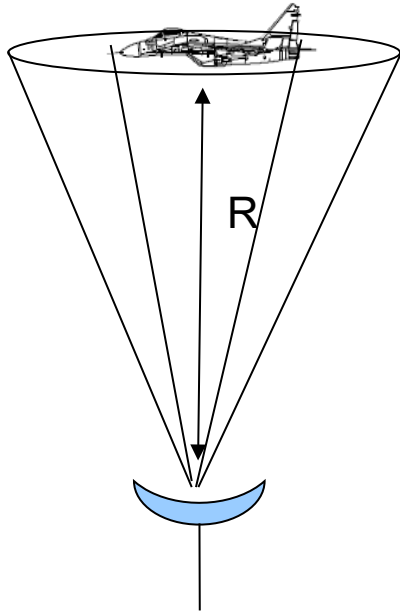
Radar Cross Section (RCS)



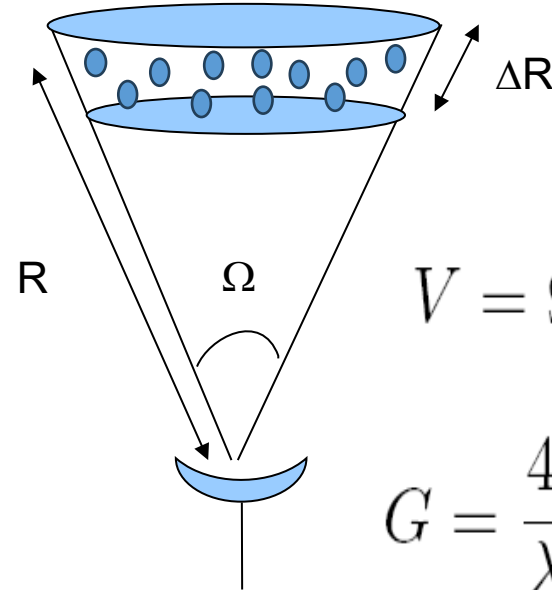
Radar Cross Section (RCS, or σ) is the effective cross-sectional area of the target as seen by the radar

measured in m^2 , or dBm^2

Hard targets vs. Soft targets



vs.



$$V = \Omega R^2 \Delta R$$

$$G = \frac{4\pi}{\lambda^2} A = \frac{4\pi}{\Omega}$$

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad \text{red arrow pointing to } R^4$$

$$P_r = \frac{P_t A \sigma_v \Delta R}{4\pi R^2} \quad \text{red arrow pointing to } R^2$$

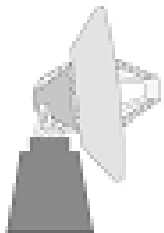
Volume scattering - Ionosphere

- Volume scattering cross section σ_v has units of area/volume
- Signal is proportional to range resolution
- What about the ionosphere ?
 - Cross section of a single electron = 10^{-28} m^2
 - Cross section of a bunch of electrons in a 10 km^3 volume in the ionosphere assuming electron density = $10^{12} / \text{m}^3$, is $10^{10} \times 10^{12} \times 10^{-28} = 10^{-6} \text{ m}^2$!!)
 - **CAN be measured by an incoherent scatter radar.**

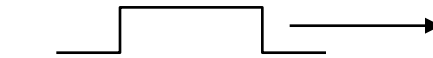
Radar Range Equation

Antenna Aperture A

Transmit Power P_T

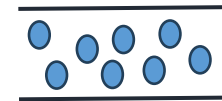


Transmitted Pulse



Received Pulse

Target Cross Section σ



R

	Transmit Power	Transmit Gain	Spread Factor	Losses	Target RCS	Spread Factor	Receive Aperture	Dwell Time
Received Signal Energy	$[P_T]$	$\left[\frac{4\pi A}{\lambda^2} \right]$	$\left[\frac{1}{4\pi R^2} \right]$	$\left[\frac{1}{L} \right]$	$[\sigma]$	$\left[\frac{1}{4\pi R^2} \right]$	$[A]$	$[\tau]$

Outline - Radar Basics

Electromagnetic spectrum

Radio waves and propagation

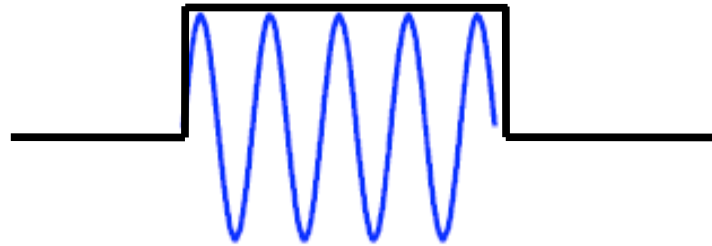
Radar equation

Range resolution and pulsed radars

Doppler

What the radar transmits:

Pulses and waves



How many cycles in a pulse?

PFISR frequency = 449 MHz

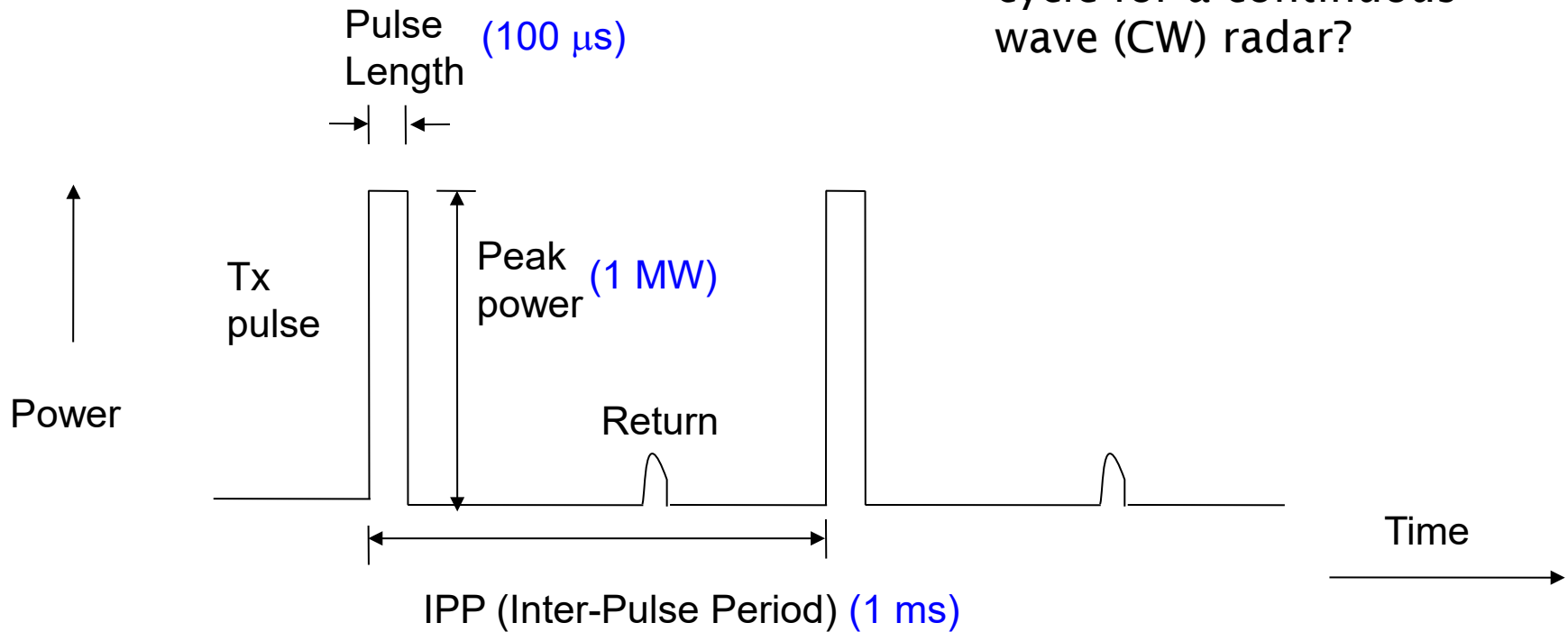
Long pulse length = 330 μ s

of cycles = 148170

Radar waveforms
modulate the waves
with on-off sequence

Pulsed Radar

What would be the duty cycle for a continuous wave (CW) radar?



Duty cycle = Pulse Length/IPP (10%)

PRF (Pulse Repetition Frequency) = 1/IPP (1kHz)

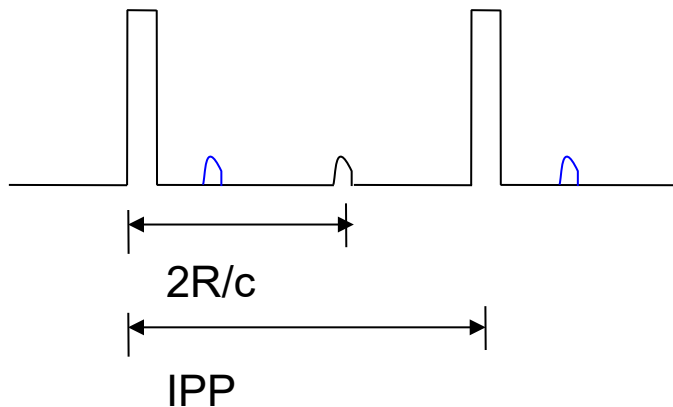
Average power = Peak power x Duty cycle (100 kW)

Range Resolution

Range resolution is set by pulse length

Pulse length = τ_p , Range resolution = $c\tau_p/2$ for a single target.

Maximum unambiguous range



$$\text{MUR} = c \cdot \text{IPP} / 2$$

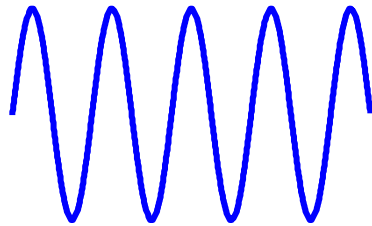
Pulse duration vs. Range resolution

Pulse Duration	Range Resolution
0.1 nsec	1.5 cm
1.0 nsec	15 cm
10 nsec	1.5 m
100 nsec	15 m
1 μ sec	150 m
10 μ sec	1.5 km
100 μ sec	15 km
1 msec	150 km

What is a typical F region ISR pulselength?

Radar Waveforms

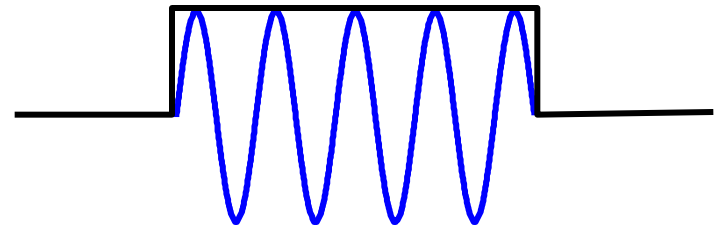
What do radars transmit?



Waves?



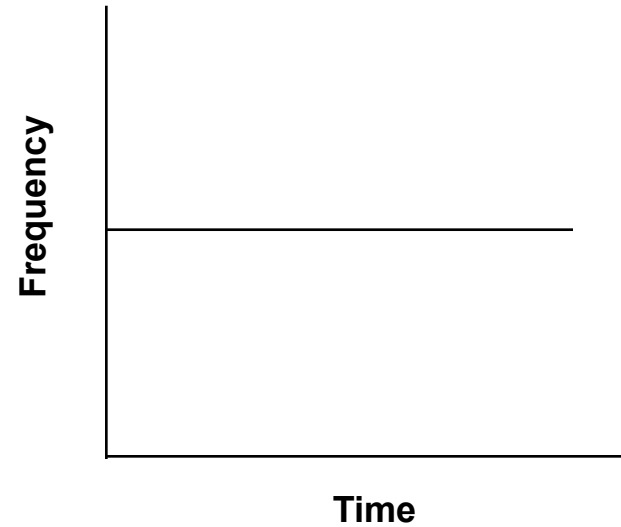
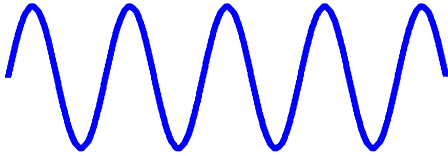
or Pulses?



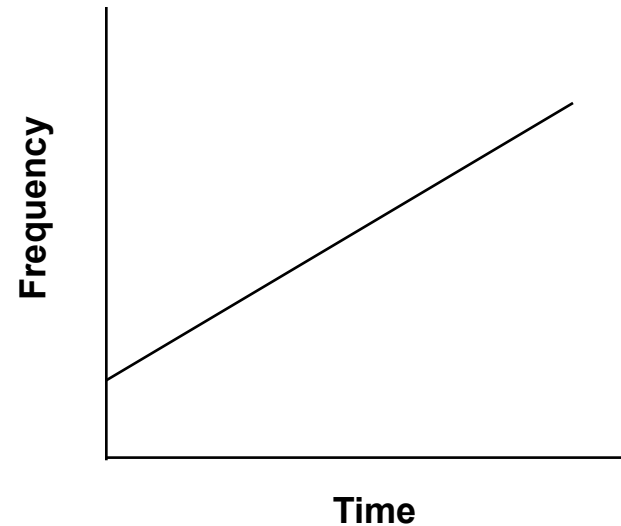
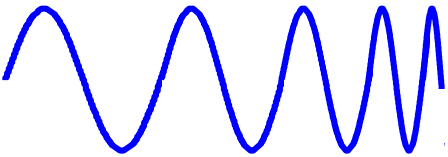
**Waves, modulated
by “on-off” action of
pulse envelope**

Radar Waveforms

Pulse at single frequency



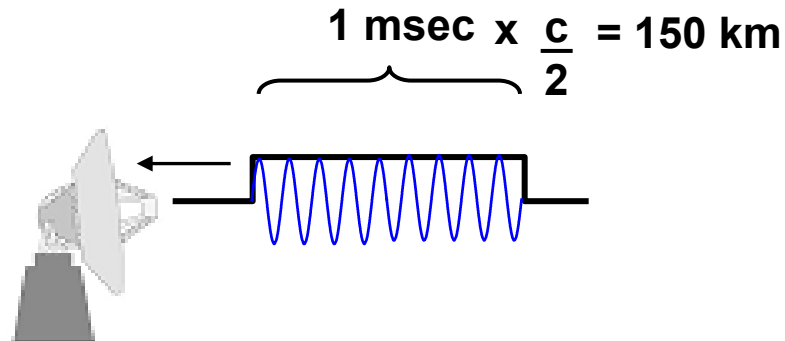
Pulse with changing frequency



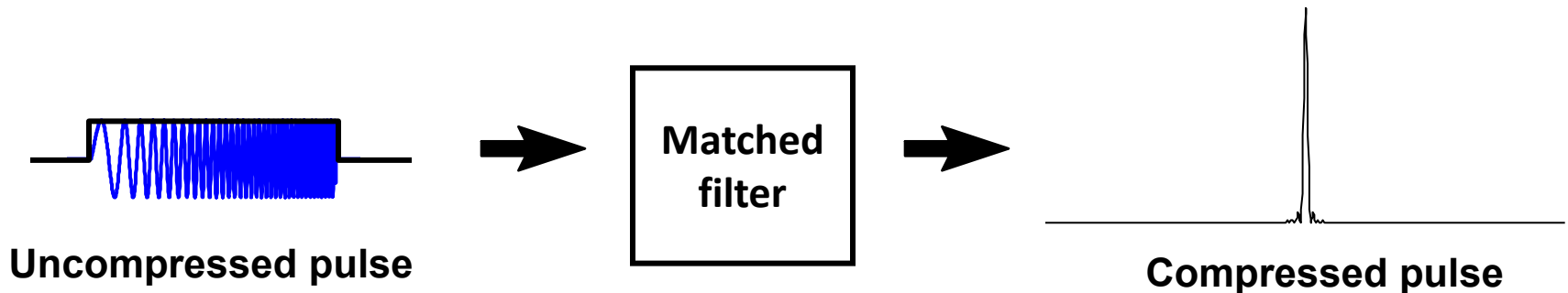
**Linear
Frequency-
Modulated
(LFM)
Waveform**

Range Resolution and pulse compression

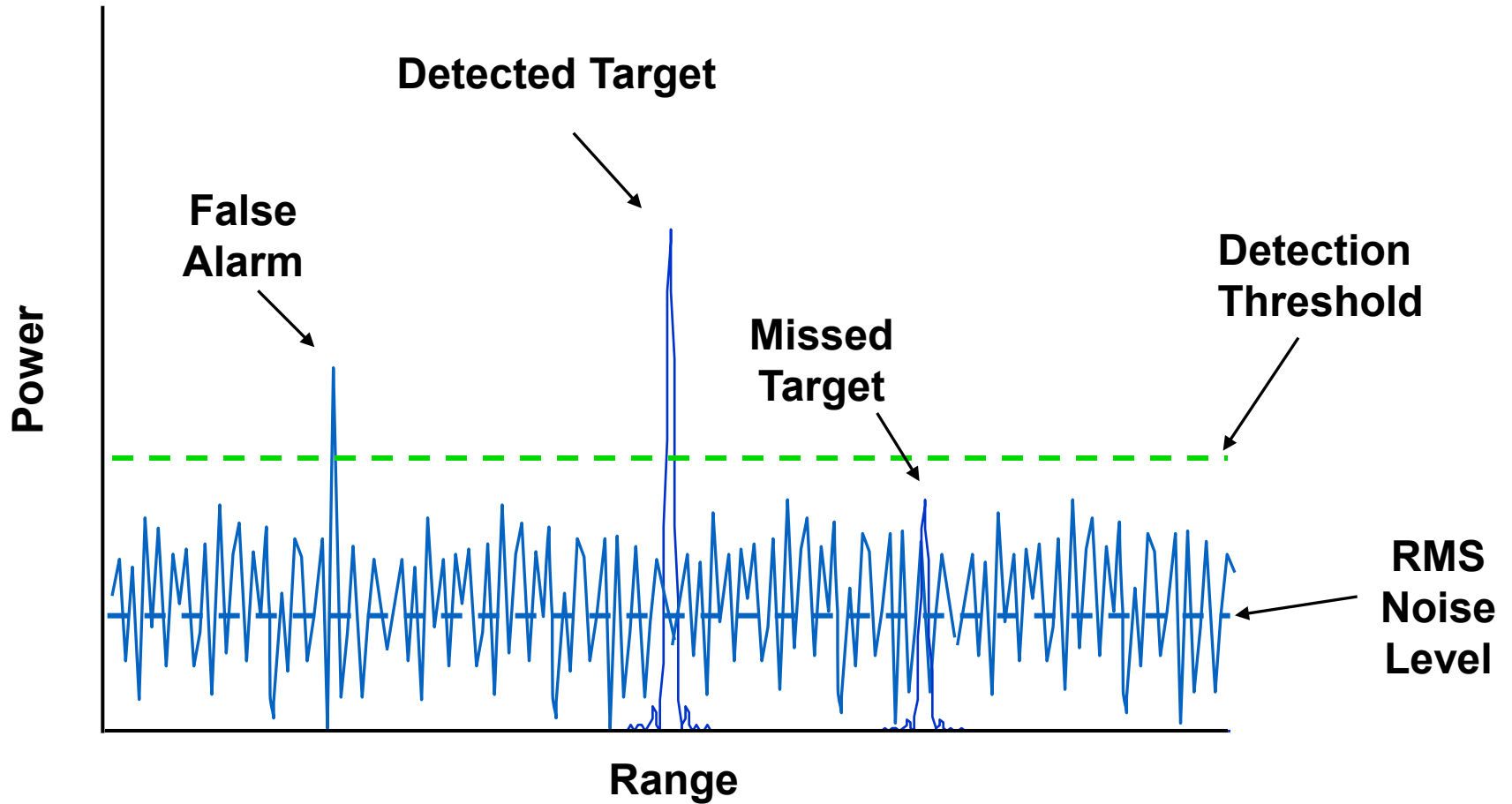
Problem: Pulse can be very long; does not allow accurate range measurement



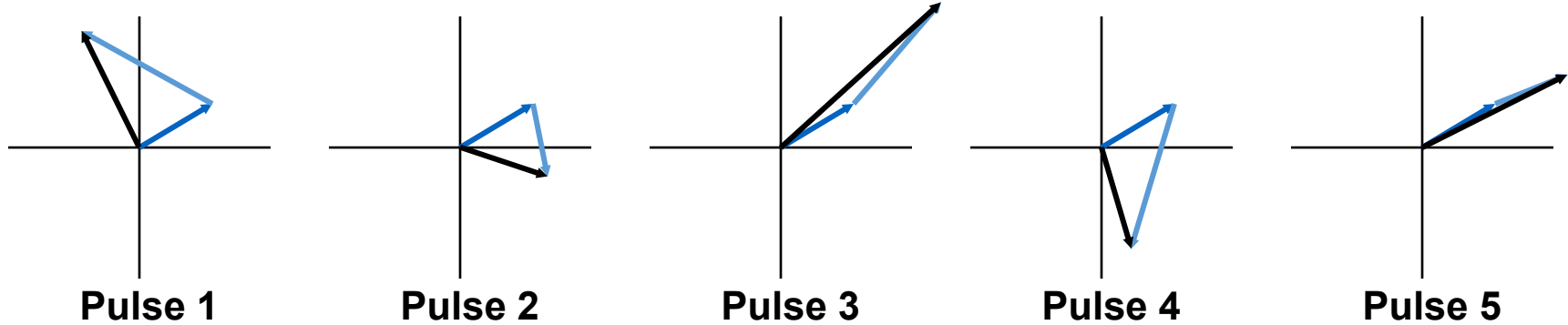
Solution: Use pulse with changing frequency and signal process using “matched filter”



Detection of Signals in Noise



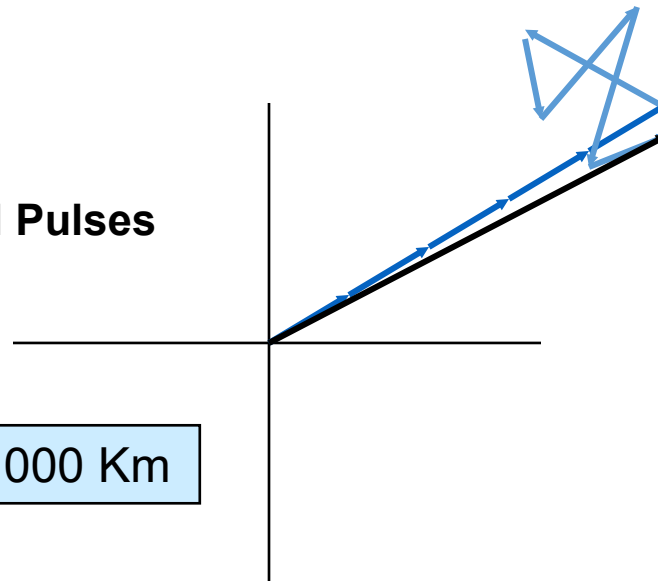
Coherent Integration



- Coherent target returns
- Noise samples at low SNR

• Resultant signal

Coherently Integrated Pulses



Deep space targets at 30,000 – 40,000 Km

Outline - Radar Basics

Electromagnetic spectrum

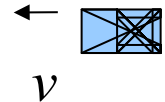
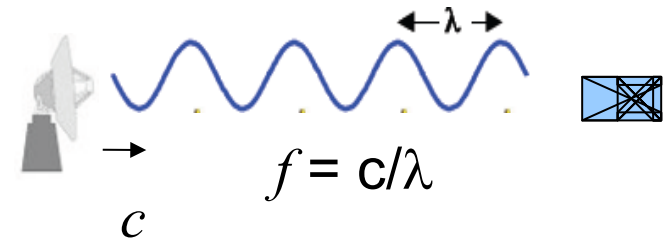
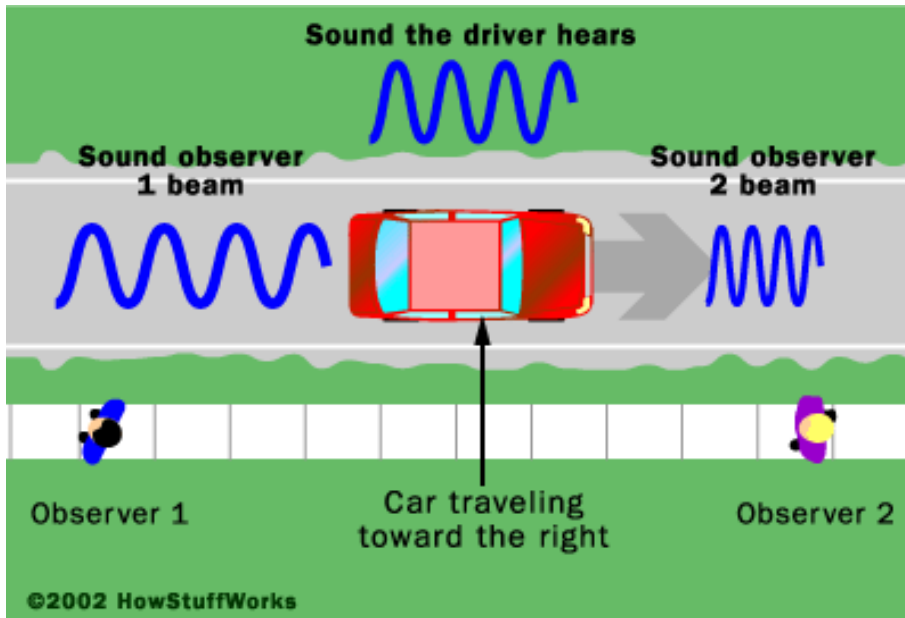
Radio waves and propagation

Radar equation

Range resolution and pulsed radars

Doppler

Moving target: Doppler



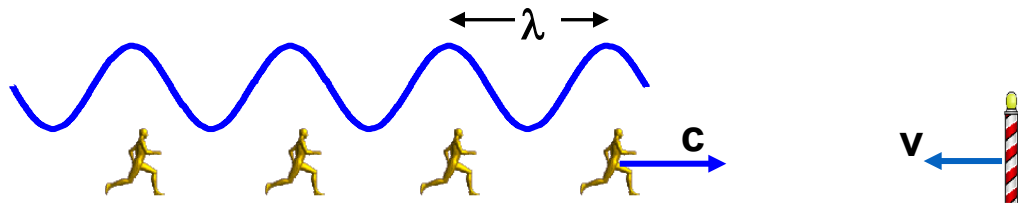
$$f' = f \pm \frac{2v}{\lambda}$$

Doppler shift

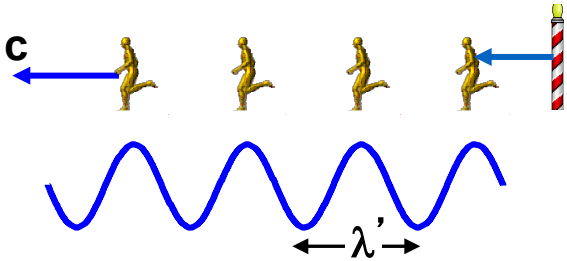
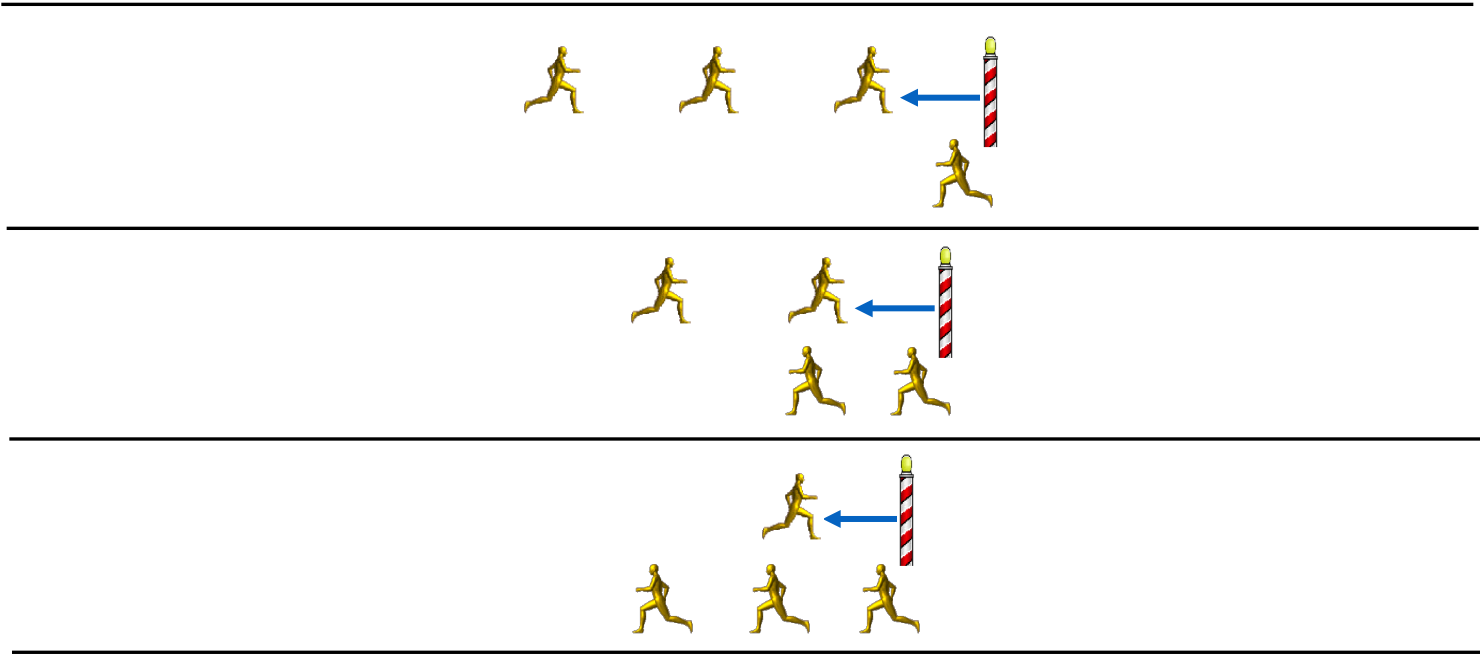
Positive Doppler = target moving **toward** the observer

Negative Doppler = target moving **away** from the observer

Doppler Shift Concept



$$f = \frac{c}{\lambda}$$



$$f' = f \pm (2v/\lambda)$$

Doppler shift

Sign conventions

*The Doppler frequency is **negative** (lower frequency, **red shift**) for objects moving **away** from the radar*

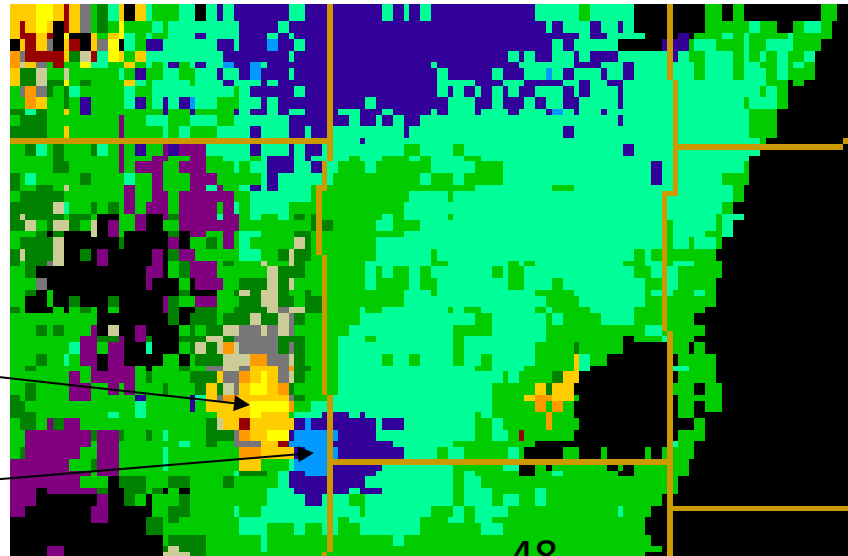
*The Doppler frequency is **positive** (higher frequency, **blue shift**) for objects moving **toward** the radar*

These “color” shift conventions are typically also used on radar displays of Doppler velocity

(for AMISR, the sign convention is opposite – positive velocity is moving away from the radar and negative is moving towards the radar)

Away from radar

Toward radar



Doppler shift frequency

Tx signal: $\cos(2\pi f_o t)$

Return from a moving target: $\cos[2\pi f_o(t + 2R/c)]$

If target is moving with a constant velocity, such that $R = R_o + v_o t$

then,

Return: $\cos[2\pi(f_o + \underbrace{f_o 2v_o/c}_{\text{Doppler frequency}})t + 2\pi f_o R_o/c]$

Doppler frequency:

$$-2f_o v_o/c = -2v_o/\lambda_o$$

Pulsed Doppler Radar system

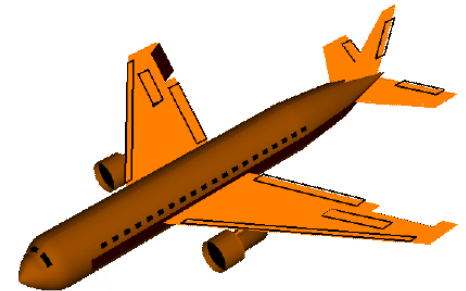
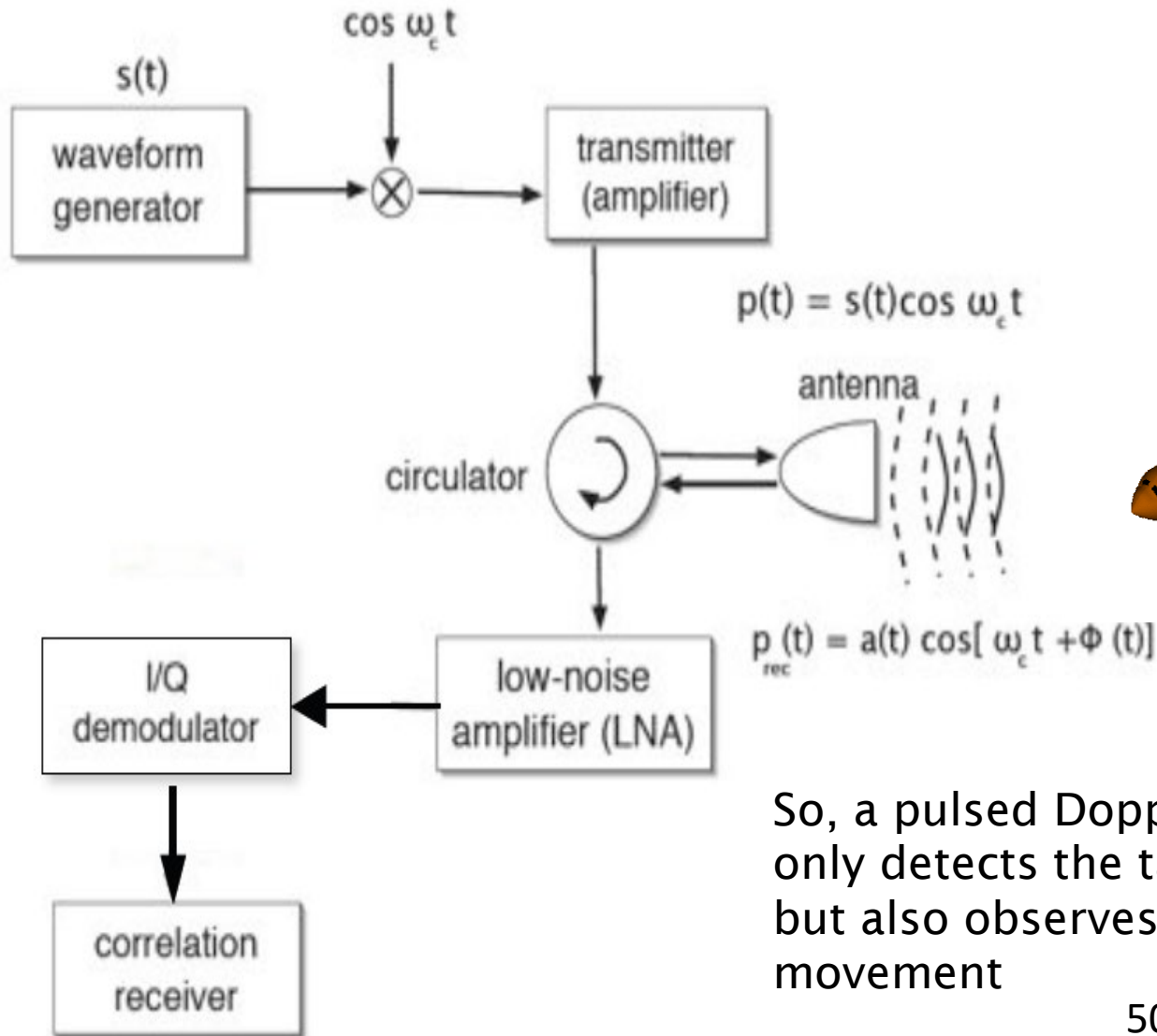


Image courtesy
of NASA

So, a pulsed Doppler radar not only detects the target location, but also observes the target movement