Introduction to Radar Signal Processing

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A Few Key Concepts

Complex exponentials are Eigen functions of linear, time-invariant (LTI) systems (of the convolution operator). This is why systems engineers like them so much!

$$\mathrm{H}x = \lambda x$$

Convolution in an LTI relates the input, impulse response, and output

$$y(t) = \int_{-\infty}^{\infty} h(u)x(t-u)du = h * x \qquad \mathrm{Y}(f) = H(f)X(f)$$

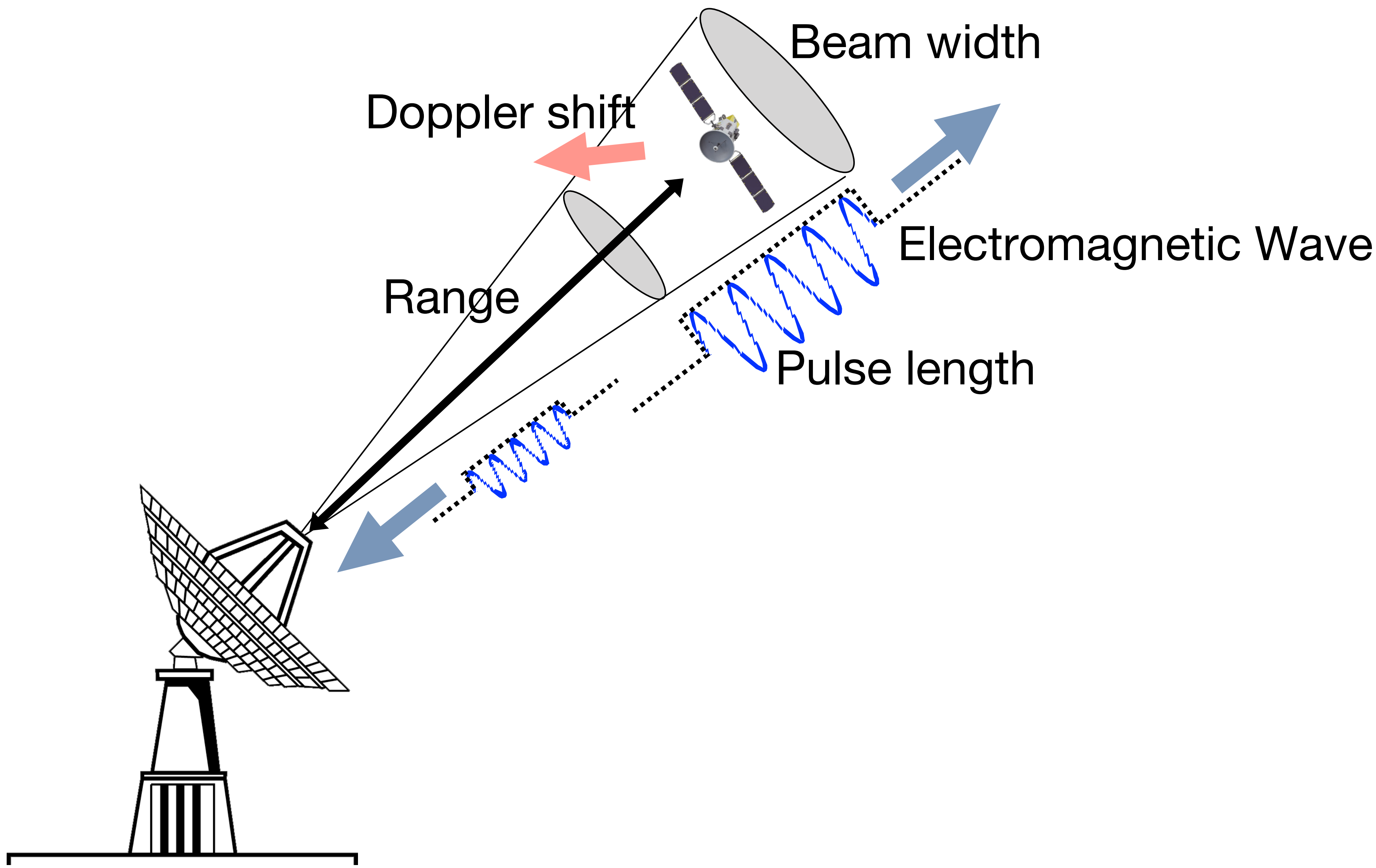
Autocorrelation describes the similarity between observations of a random variable with itself at a different point in time

$$r_{xx}(\tau) = E[x^*(t)x(t+\tau)]$$

Wiener-Khinchin theorem

The Autocorrelation function and the Power spectral density function make a Fourier Transform pair for a wide-sense-stationary random process (even though the Fourier Transform of the process itself does not exist).

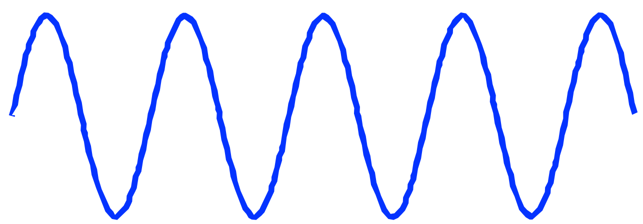
Pulse Doppler Radar

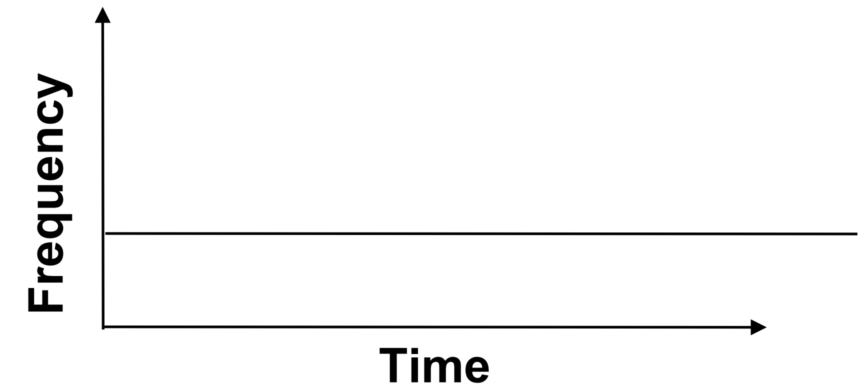


Small ‘simple’ target
Echo is a modified copy of the transmitted waveform
Time delay for the pulse echo to return -> range
Frequency shift of the echo -> velocity component

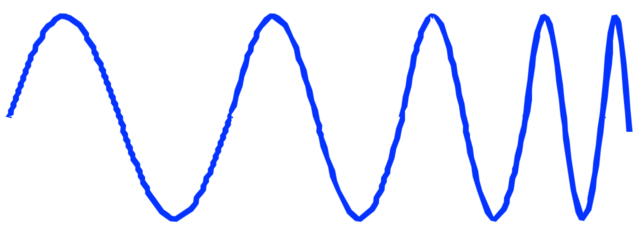
Radar Waveforms

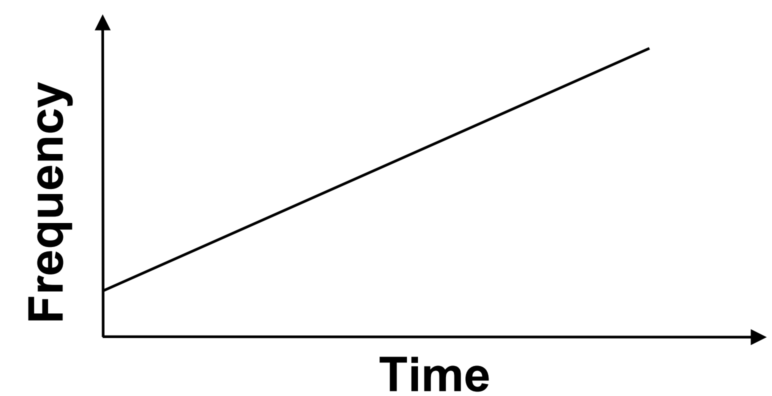
Pulse at single frequency





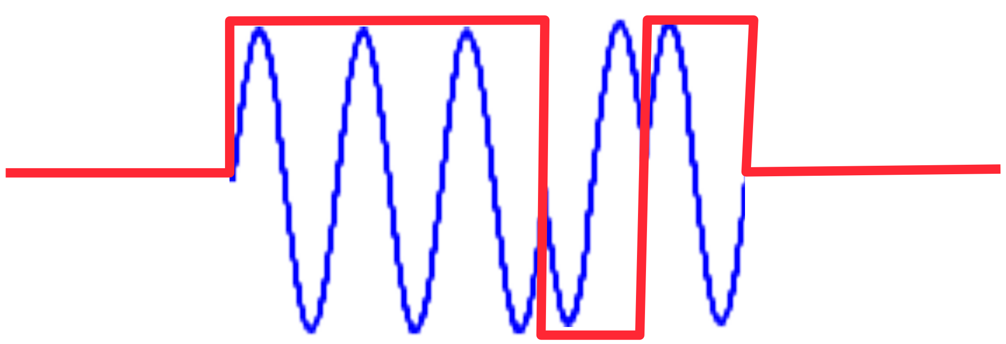
Pulse with changing frequency

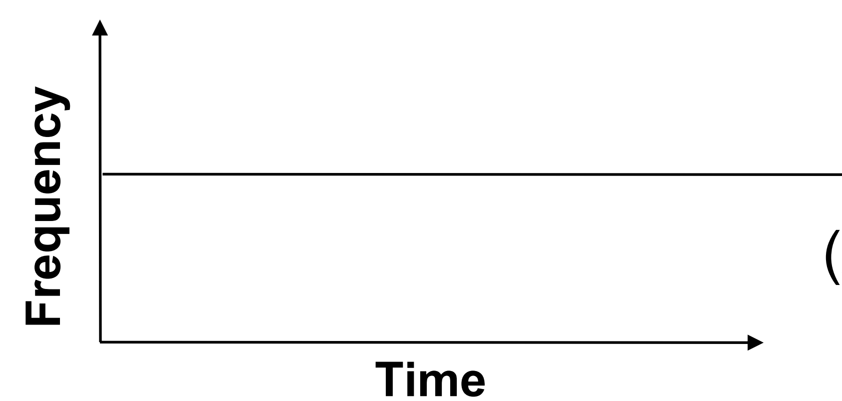




Linear Frequency-Modulated (LFM) Waveform

Pulse at single frequency, but variable phase





Phase-coded Waveform (Alternating codes Barker Codes)

How do you choose what to transmit?

Matched Filtering

- Extracting signal from noise
  - Output of the matched filter maximizes the attainable SNR when the sum of a deterministic signal and white noise are applied to the input
  - Impulse response of the matched filter is the complex conjugate of the time reversed version of the signal

$$h(t) = s^*(t_M - t)$$

$$H(f) = S^*(f)\exp(-j2\pi f t_M)$$

where

$h(t)$ is the impulse response of the matched filter

$s(t)$ is the signal to be detected

t_M is the measurement time

t, f are time and frequency

Ambiguity Functions

- The ambiguity function is defined as the absolute value of the envelope of the output of a matched filter when the input to the filter is a Doppler shifted version of the original signal

$$\left|X(\tau, f)\right| = \left|\int_{-\infty}^{\infty} u(t)\,u^*(t-\tau)\exp(j2\pi f t)\,dt\right|$$

$u(t)$ is the complex envelope of the signal

τ is the additional delay

f is the frequency shift (Doppler)

Ambiguity Functions

For $u(t)$ with unit energy

$$|X(\tau, f)| \le |X(0,0)| = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |X(\tau, f)|^2 \, d\tau df = 1$$

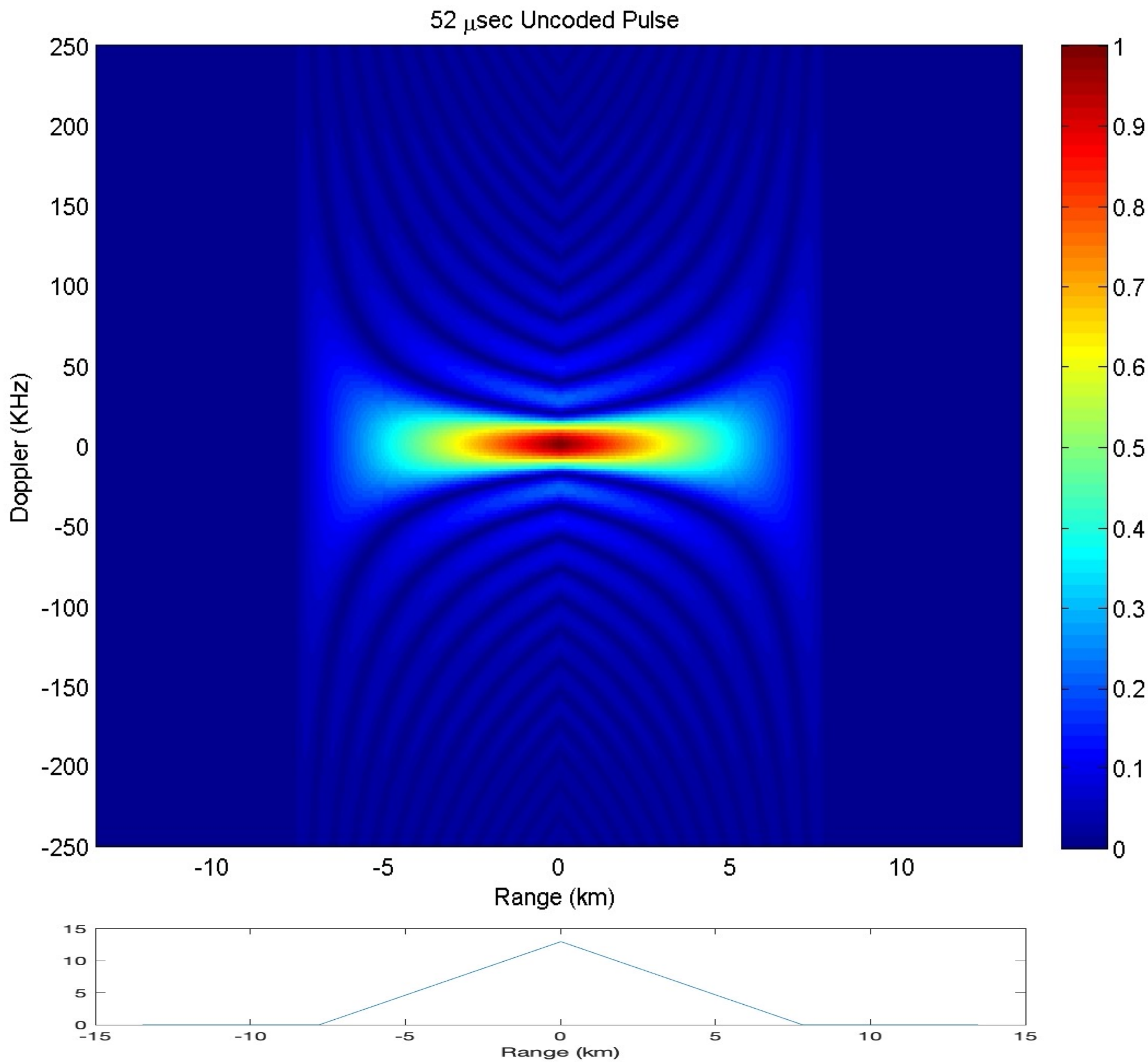
and for all signals

$$|X(-\tau, -f)| = |X(\tau, f)|$$

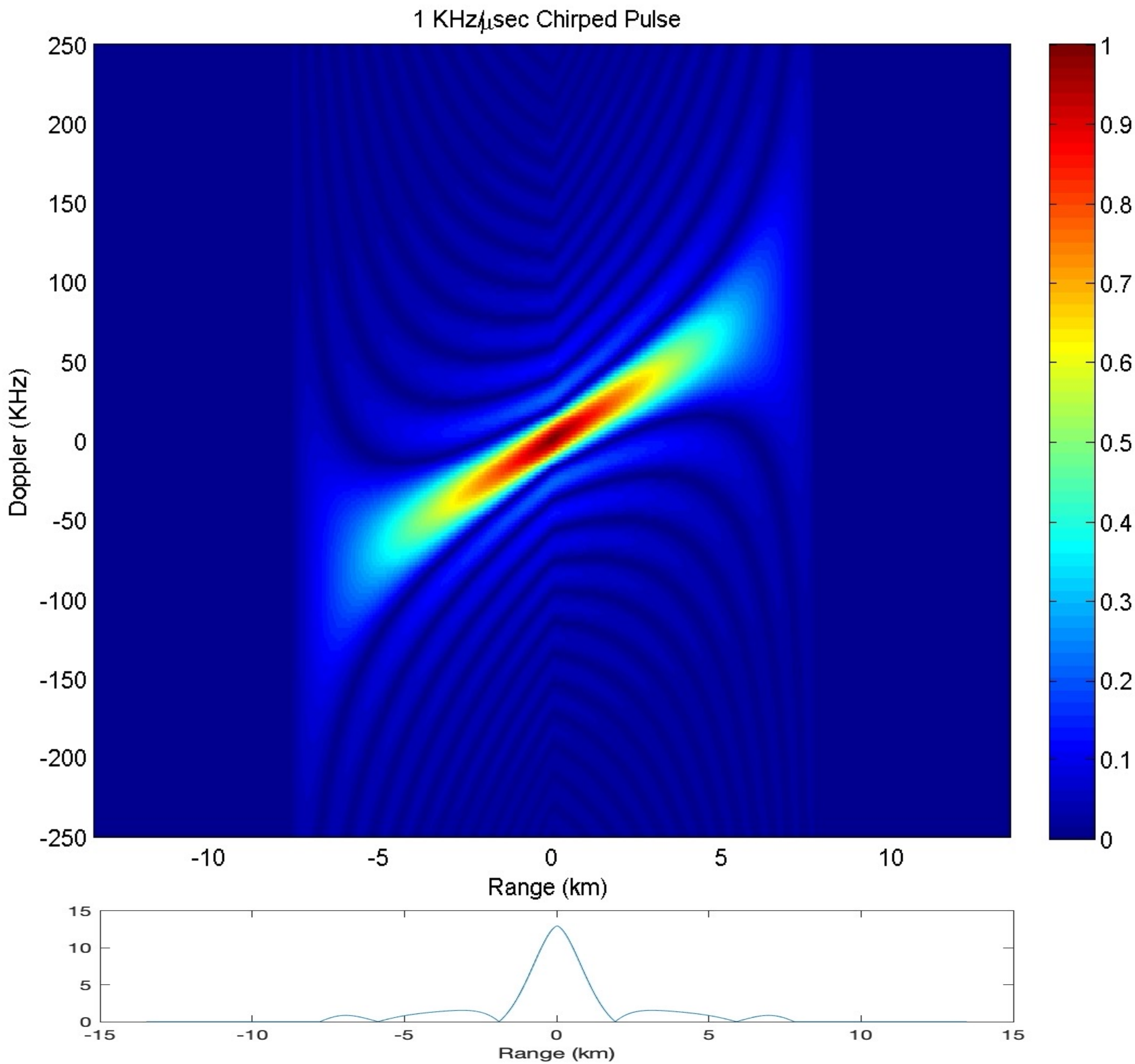
if $u(t) \leftrightarrow |X(\tau, f)|$

then $u(t)\exp\left(j\pi k t^2\right) \leftrightarrow |X(\tau, f + k\tau)|$

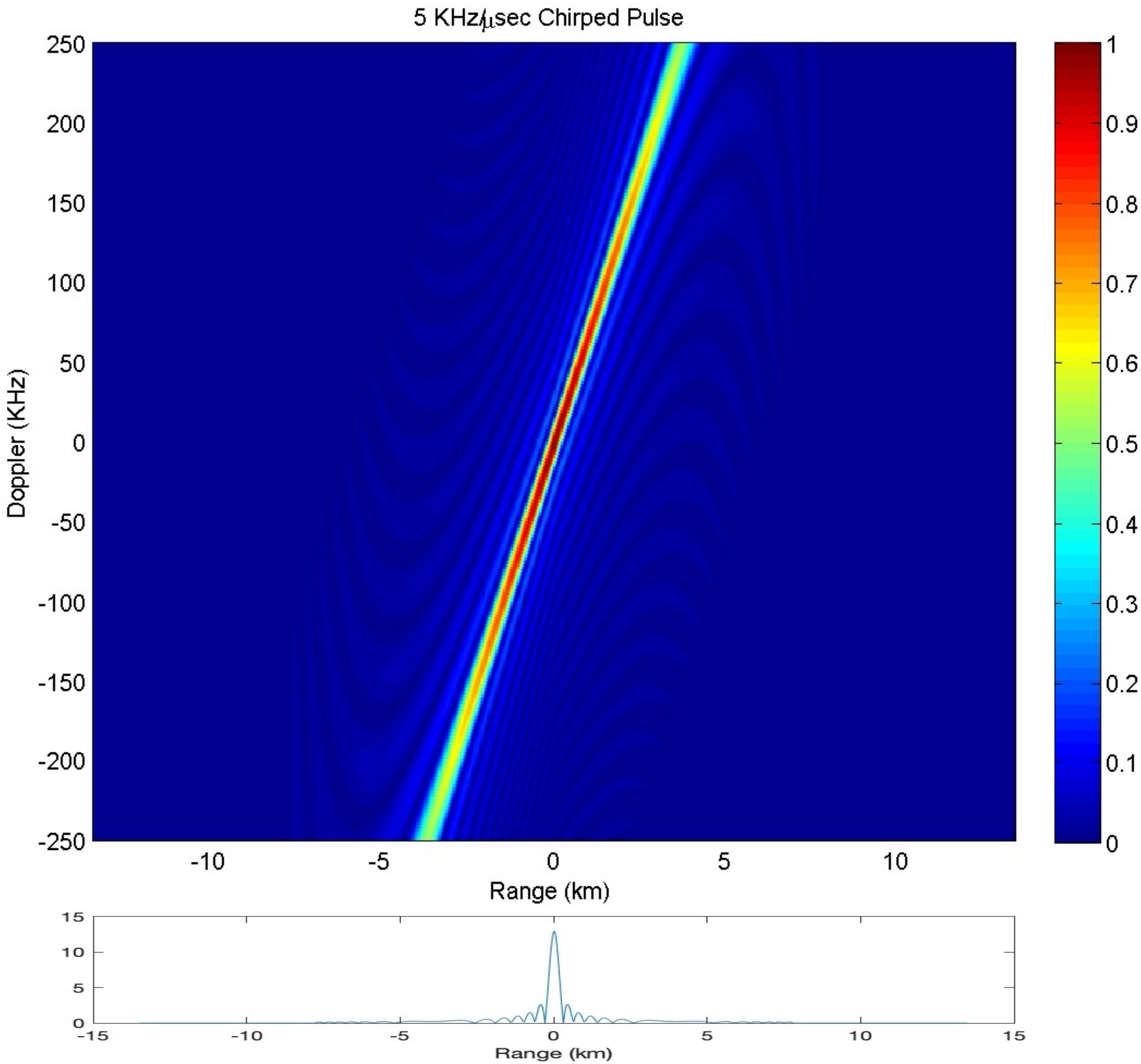
Ambiguity Function



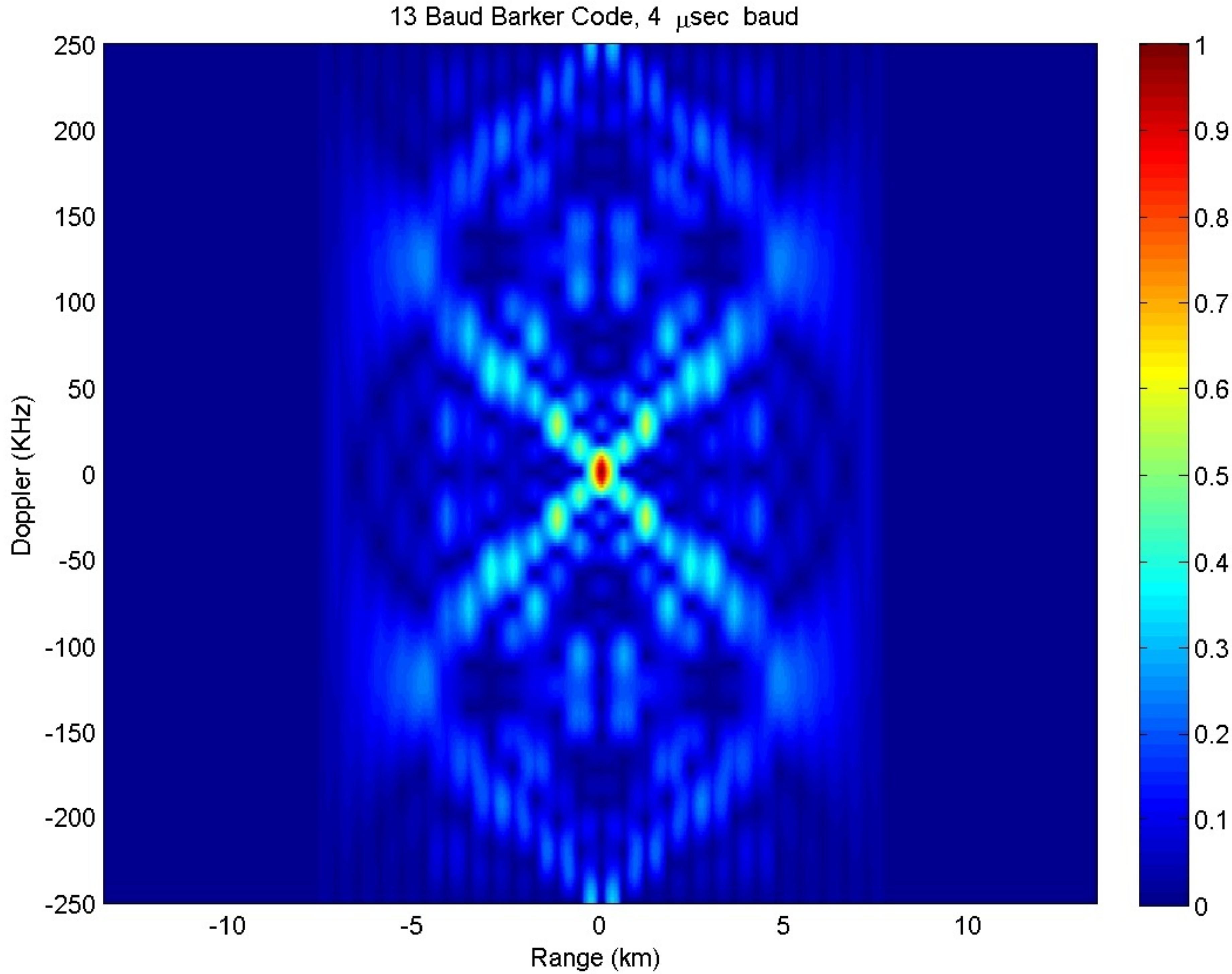
Ambiguity Function

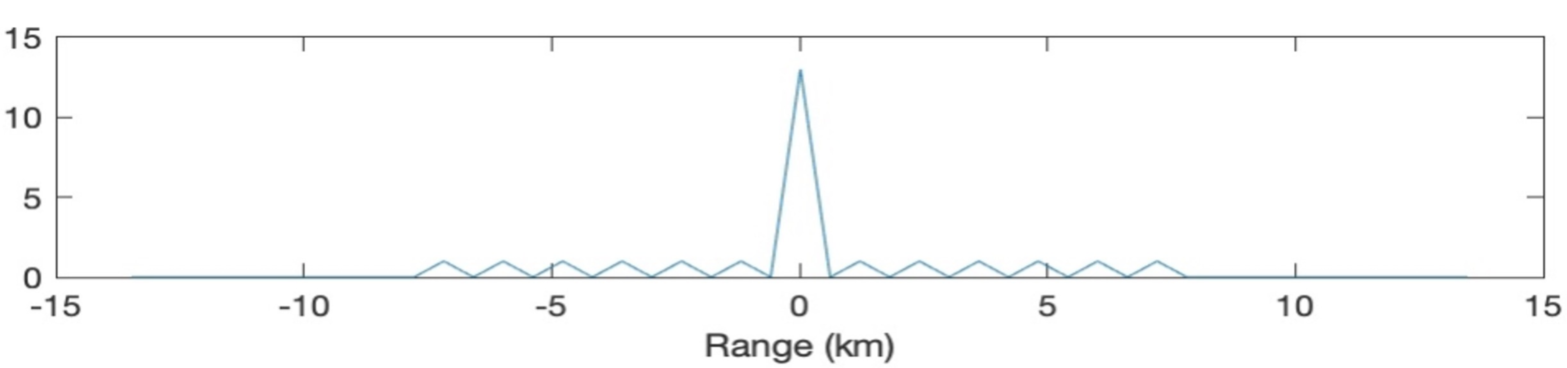


Ambiguity Function



Ambiguity Function





Pulse Doppler Radar
Distributed "Incoherent" Targets

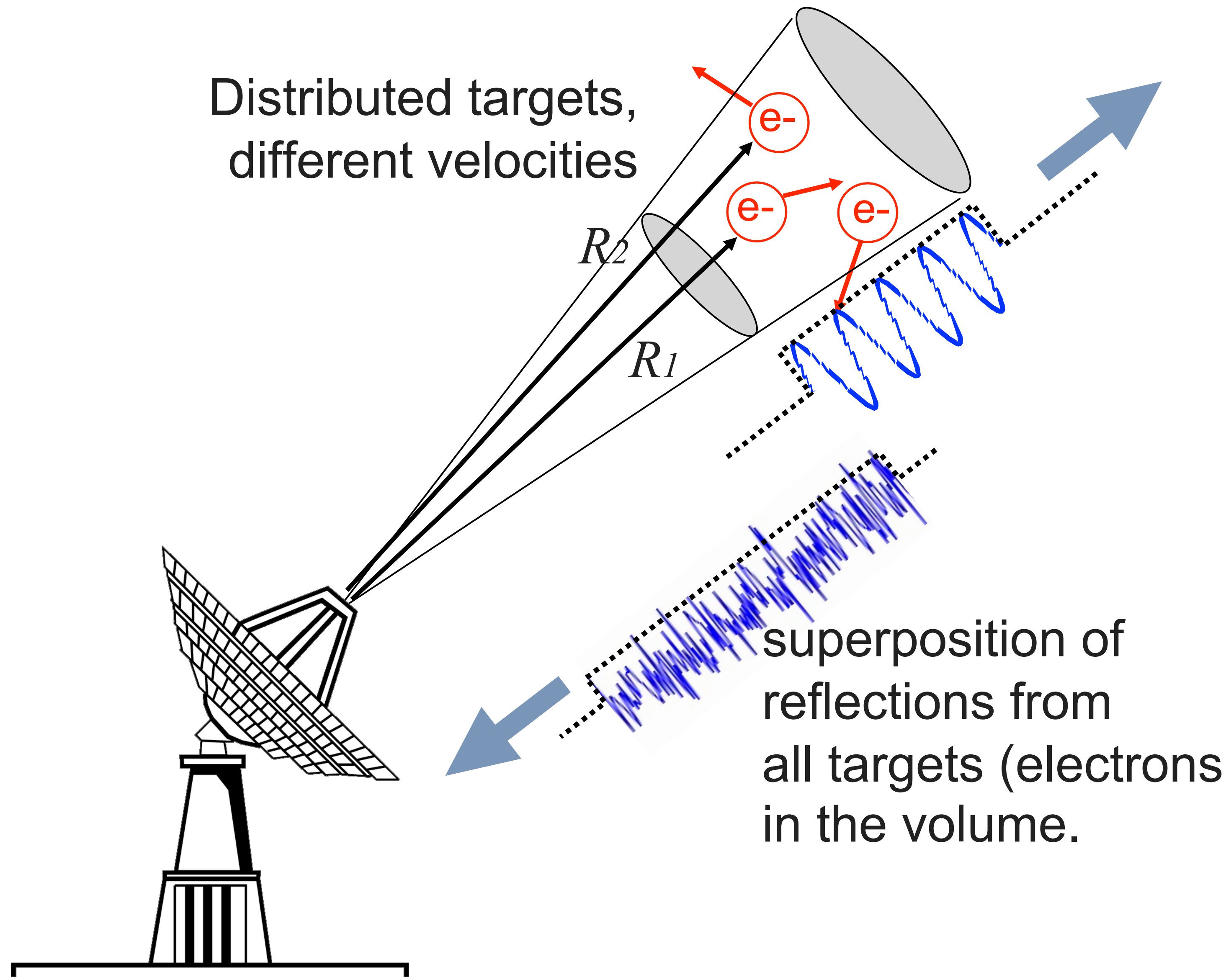
Two key concepts:

Time ⟺ Distance

$$R = \frac{c\Delta t}{2}$$

Frequency ⟺ Velocity

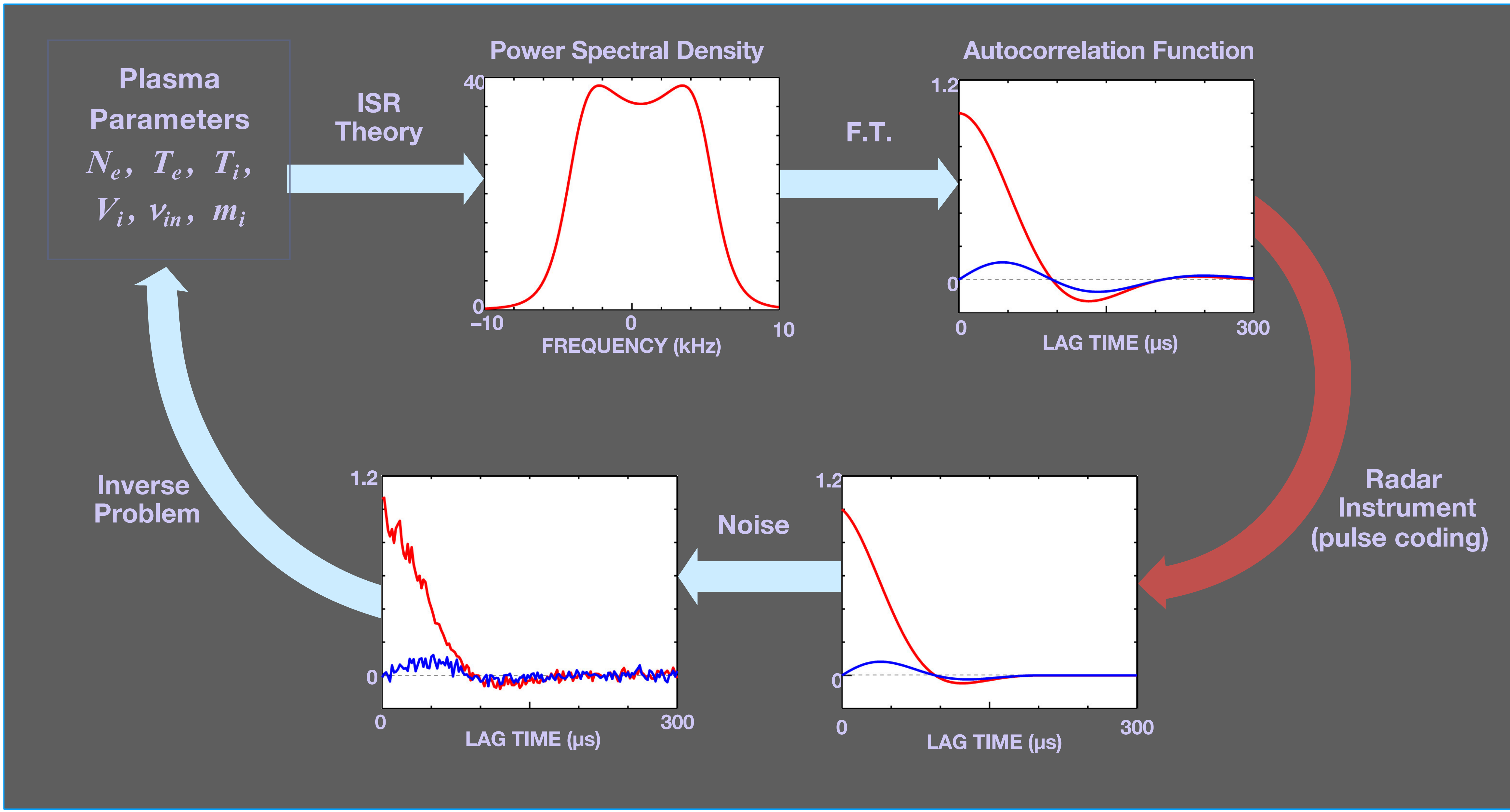
$$f_D = -\frac{2f_o}{c}v_o$$



A Doppler radar measures backscattered power as a function range and velocity.
Velocity is manifested as a Doppler frequency shift in the received signal.

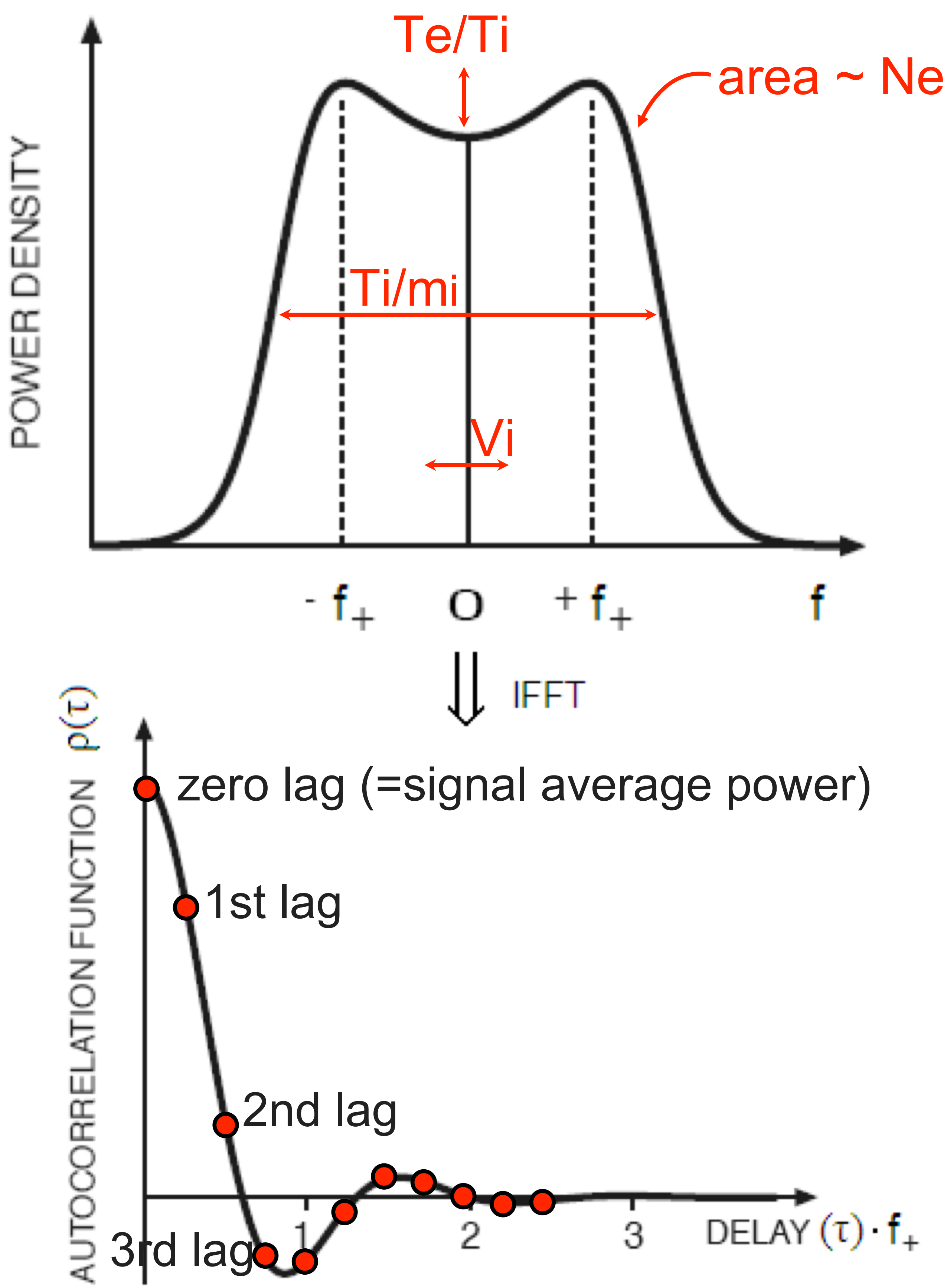
What happens when we have multiple targets in the radar volume, moving at different velocities?

Incoherent Scatter Radar Data Fitting



v3

Autocorrelation function and power spectrum



Ion temperature (Ti) to ion mass (mi) ratio from the width of the spectra

Electron to ion temperature ratio (Te/Ti) from "peak-to-valley" ratio

Electron (= ion) density from total area (corrected for temperatures)

Line-of-sight ion velocity (Vi) from bulk Doppler shift

Our goal is to sample lags with sufficient fidelity to provide meaningful estimates of plasma parameters

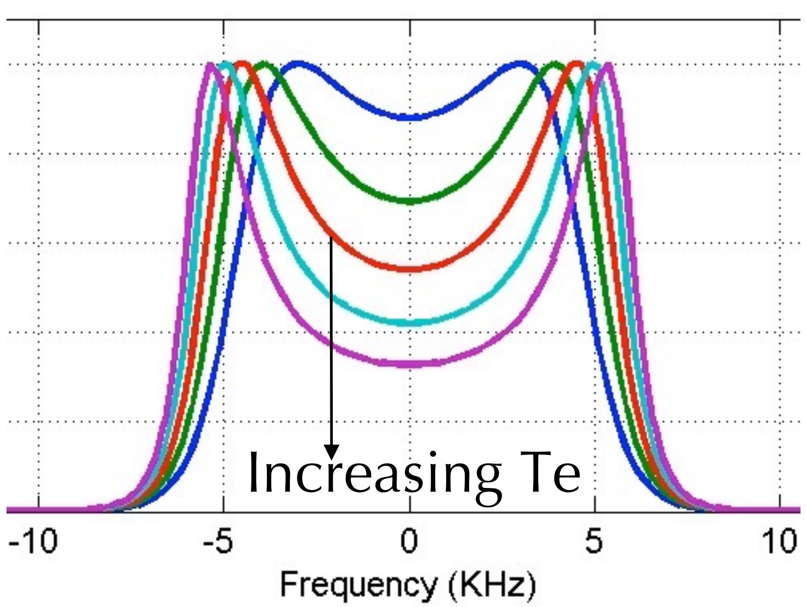
Overspread returns

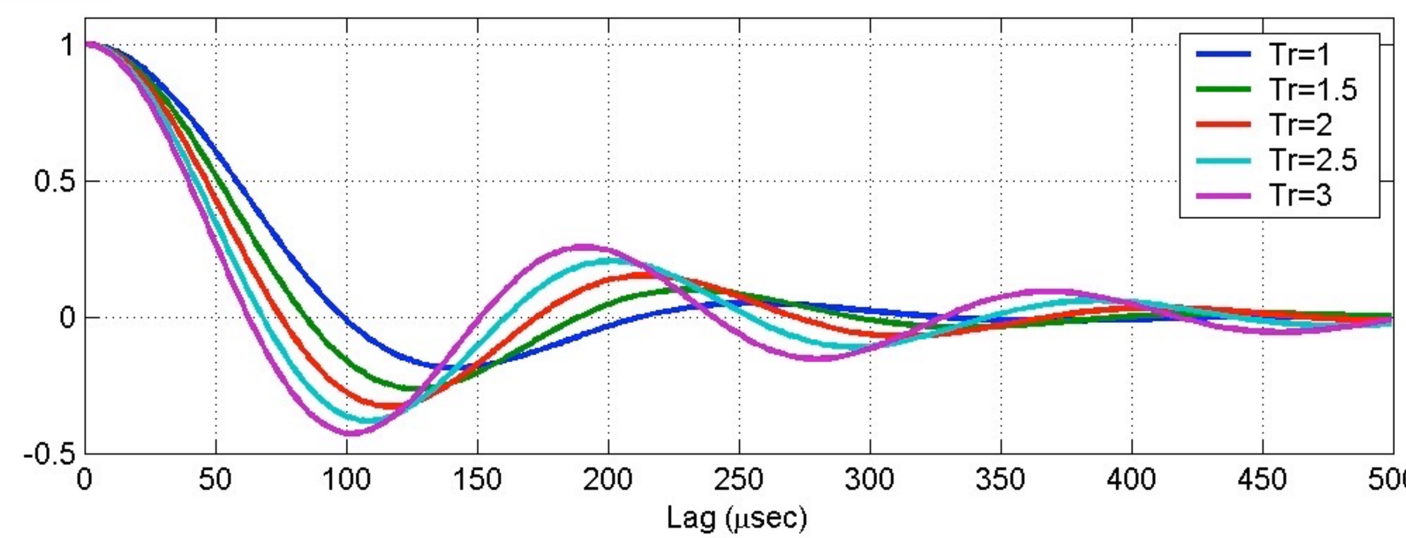
Doppler width at 450 MHz: 10 kHz
de-correlation time (zero crossing): ~1/10kHz = 0.1 ms
Inter-pulse period (IPP) to reach 450 km: $2R/c$ = 3ms

Plasma has de-correlated by the time we send the next pulse.

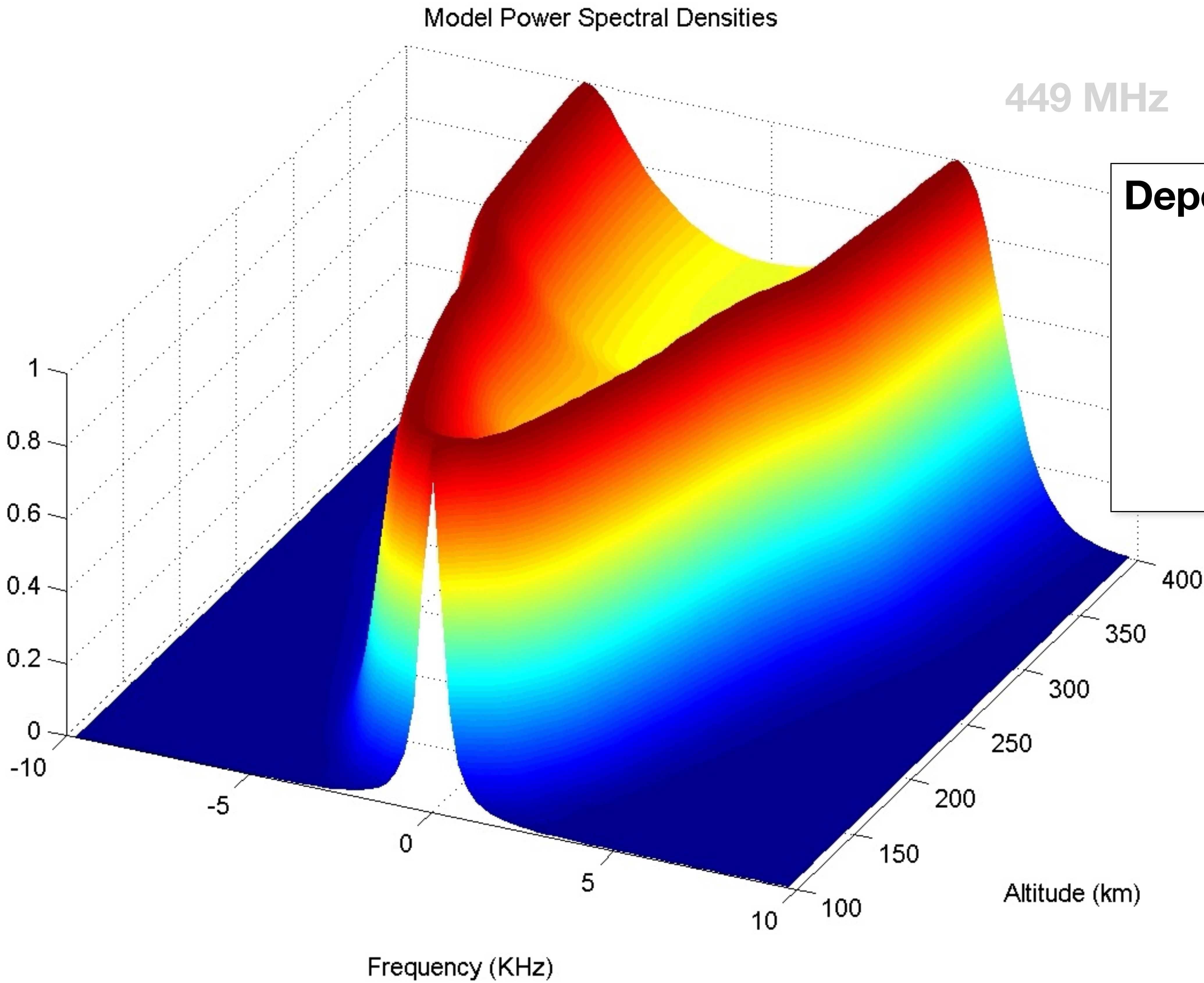
Stated alternately, the Doppler frequency spread of the plasma is much higher than the maximum unambiguous Doppler shift measurable for the pulse-repetition frequency.

ISR spectrum ⟺ Autocorrelation function (ACF)





Incoherent Scatter Power Spectra



Dependencies:

N_e

T_e

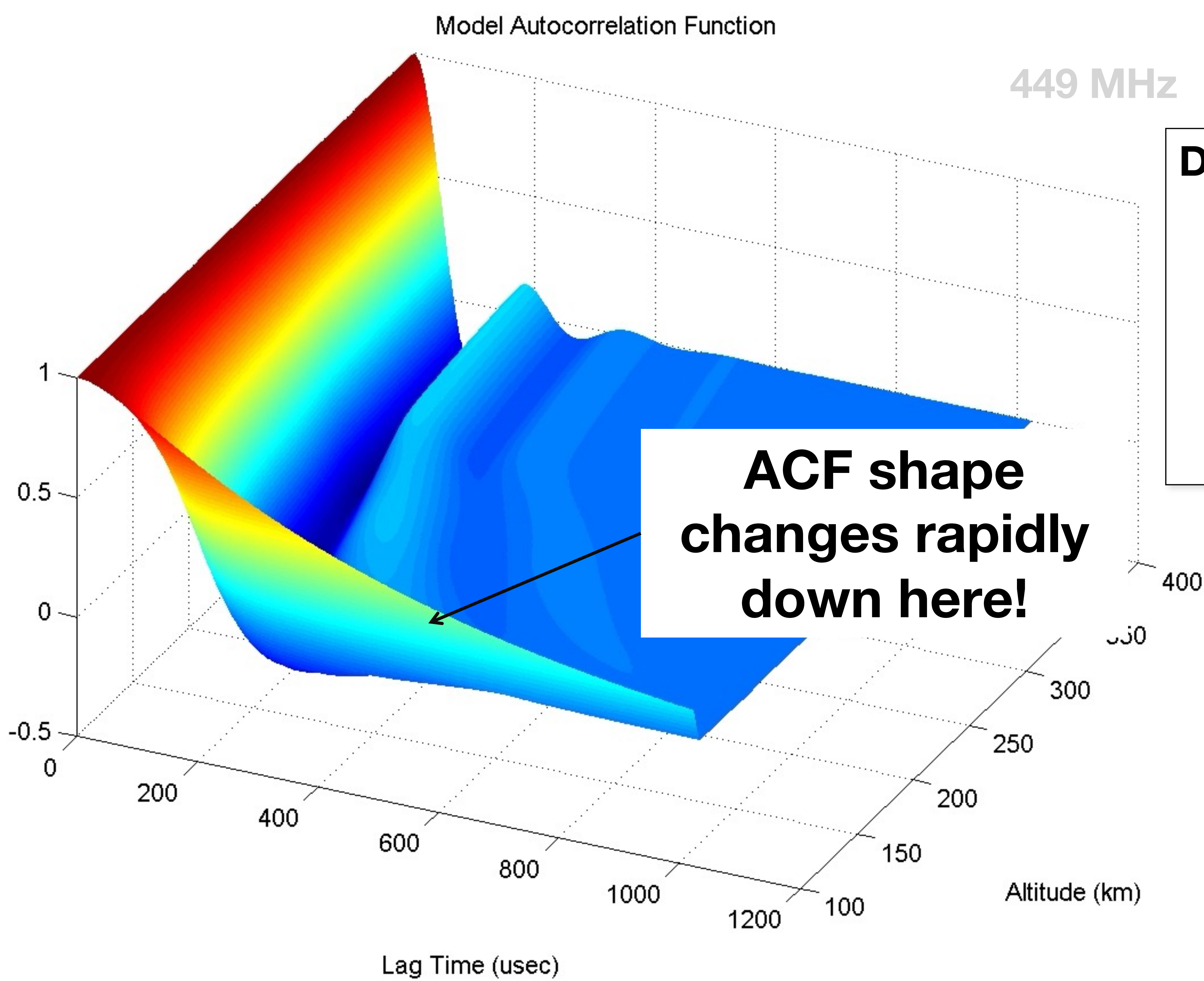
T_i

V_i

m_i

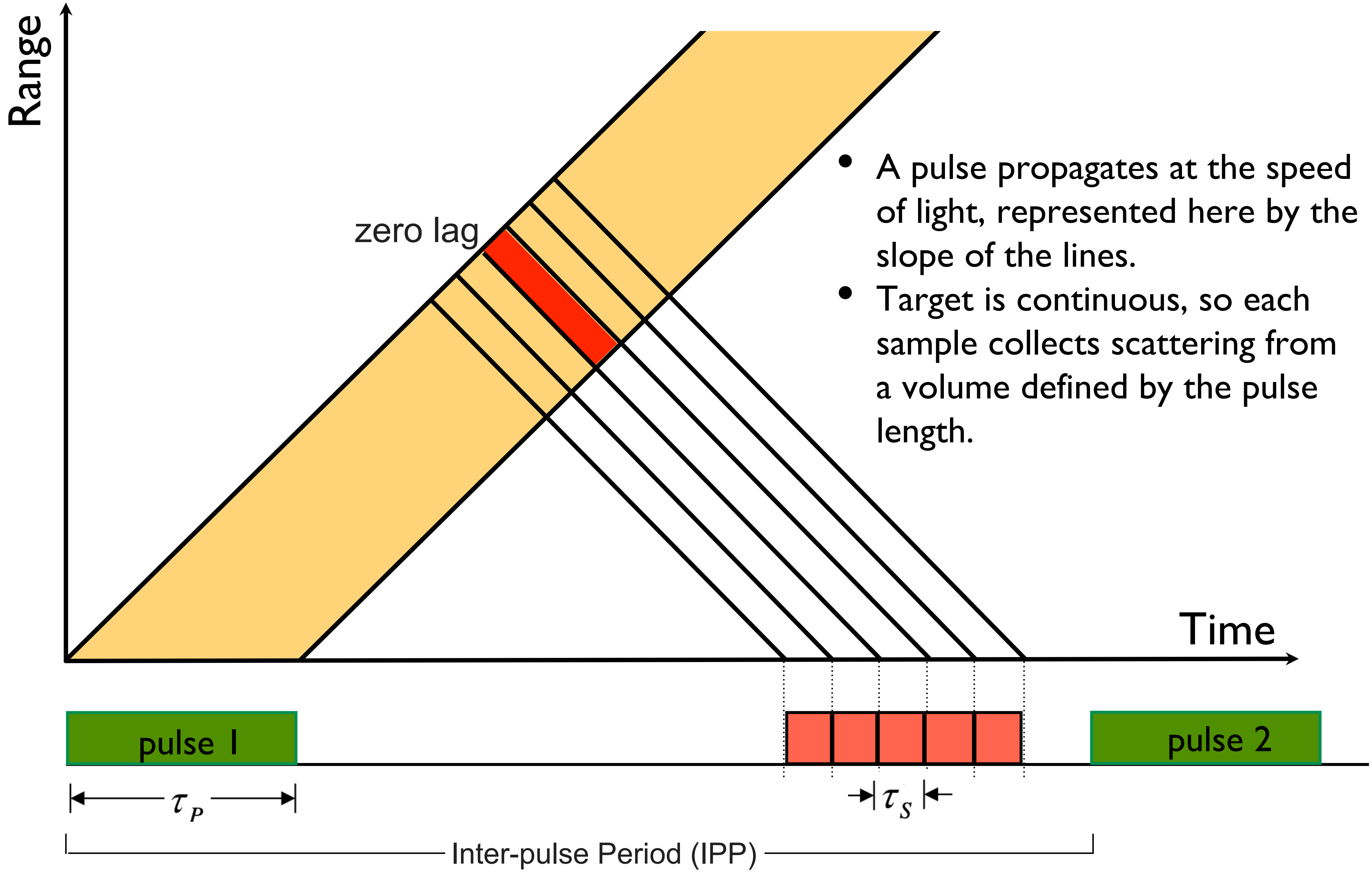
ν_{in}

Incoherent Scatter Autocorrelation Functions



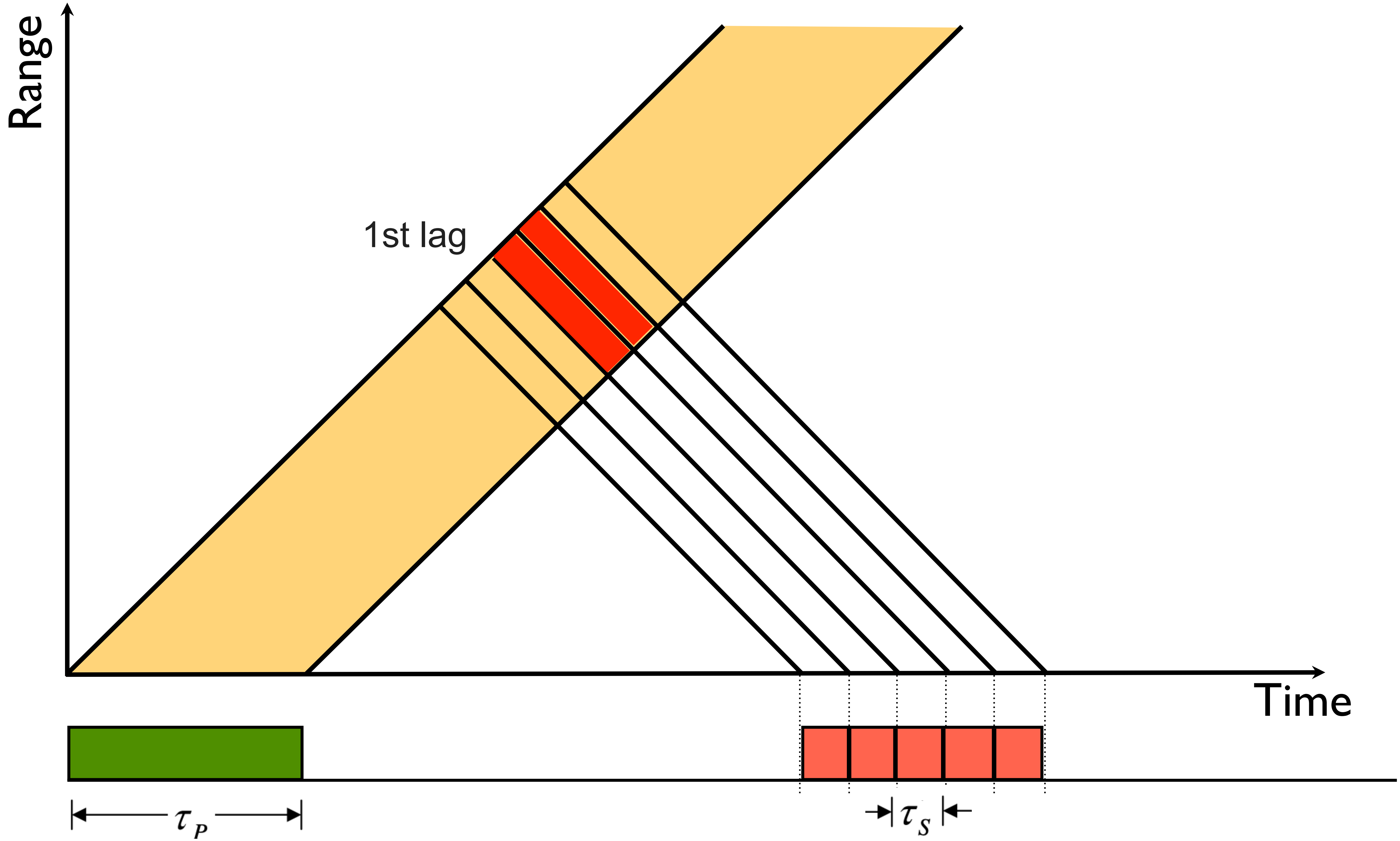
Dependencies:
N_e
T_e
T_i
V_i
m_i
ν_{in}

Computing the ACF (and, hence, spectrum)



- A pulse propagates at the speed of light, represented here by the slope of the lines.
- Target is continuous, so each sample collects scattering from a volume defined by the pulse length.

Computing the ACF (and, hence, spectrum)



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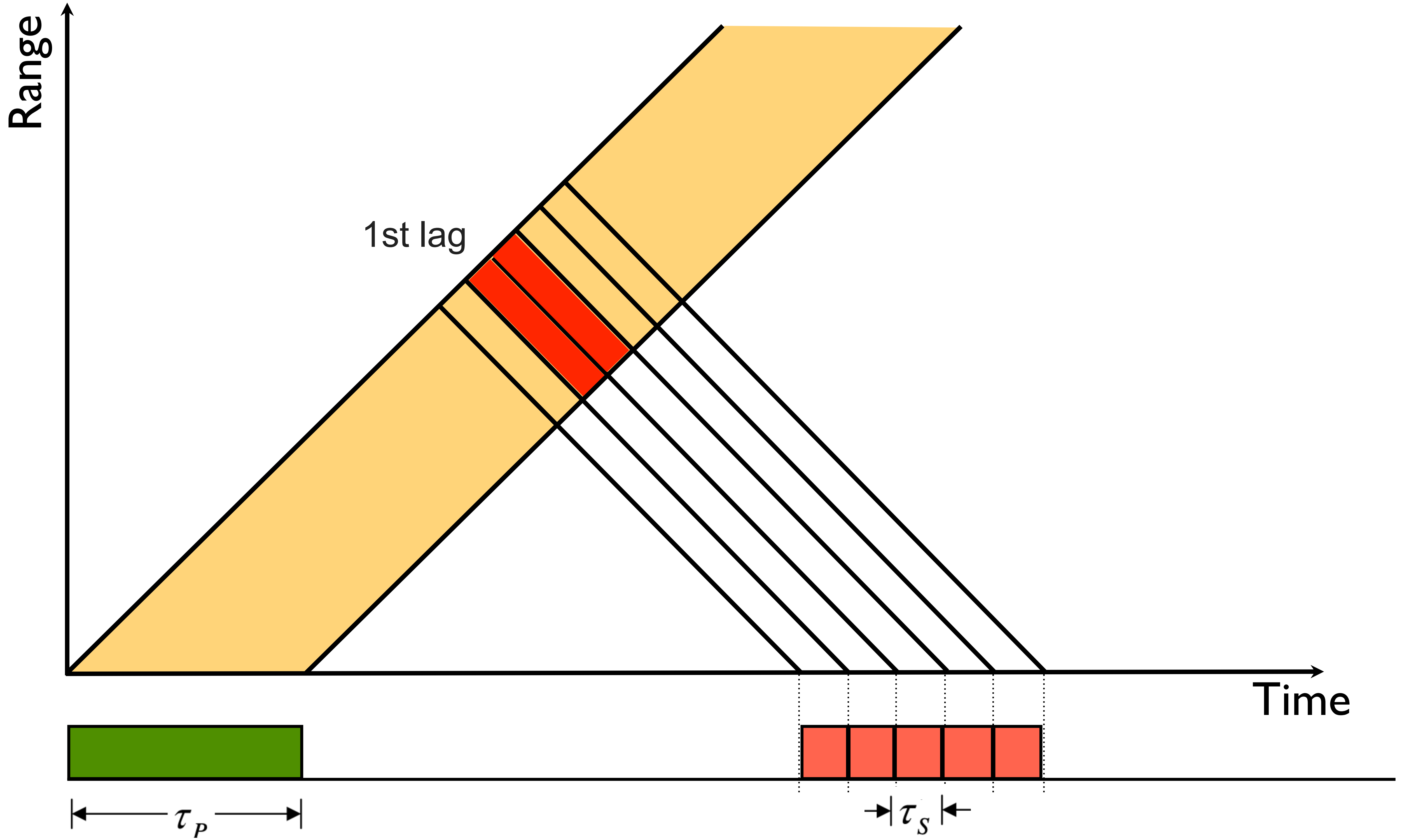

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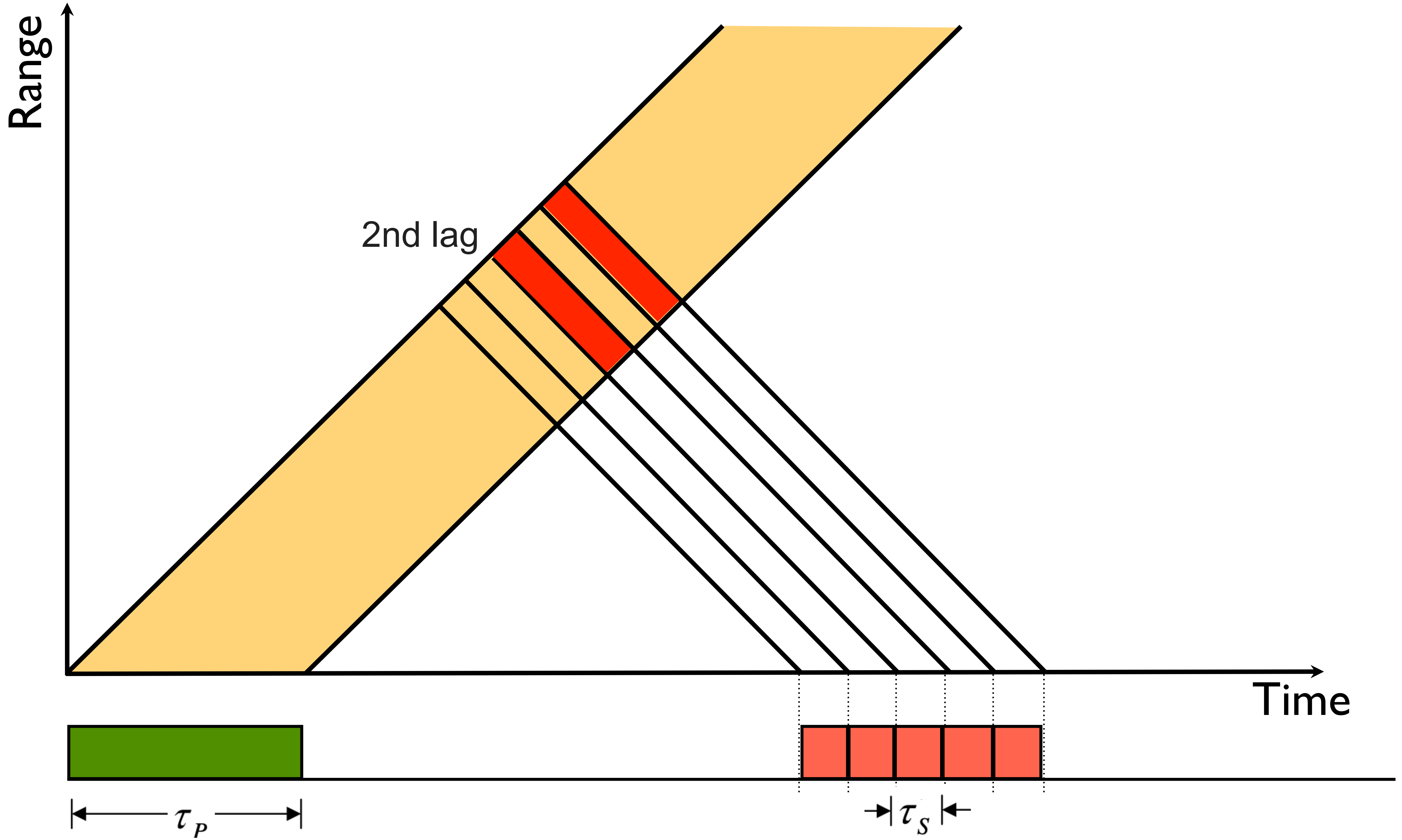
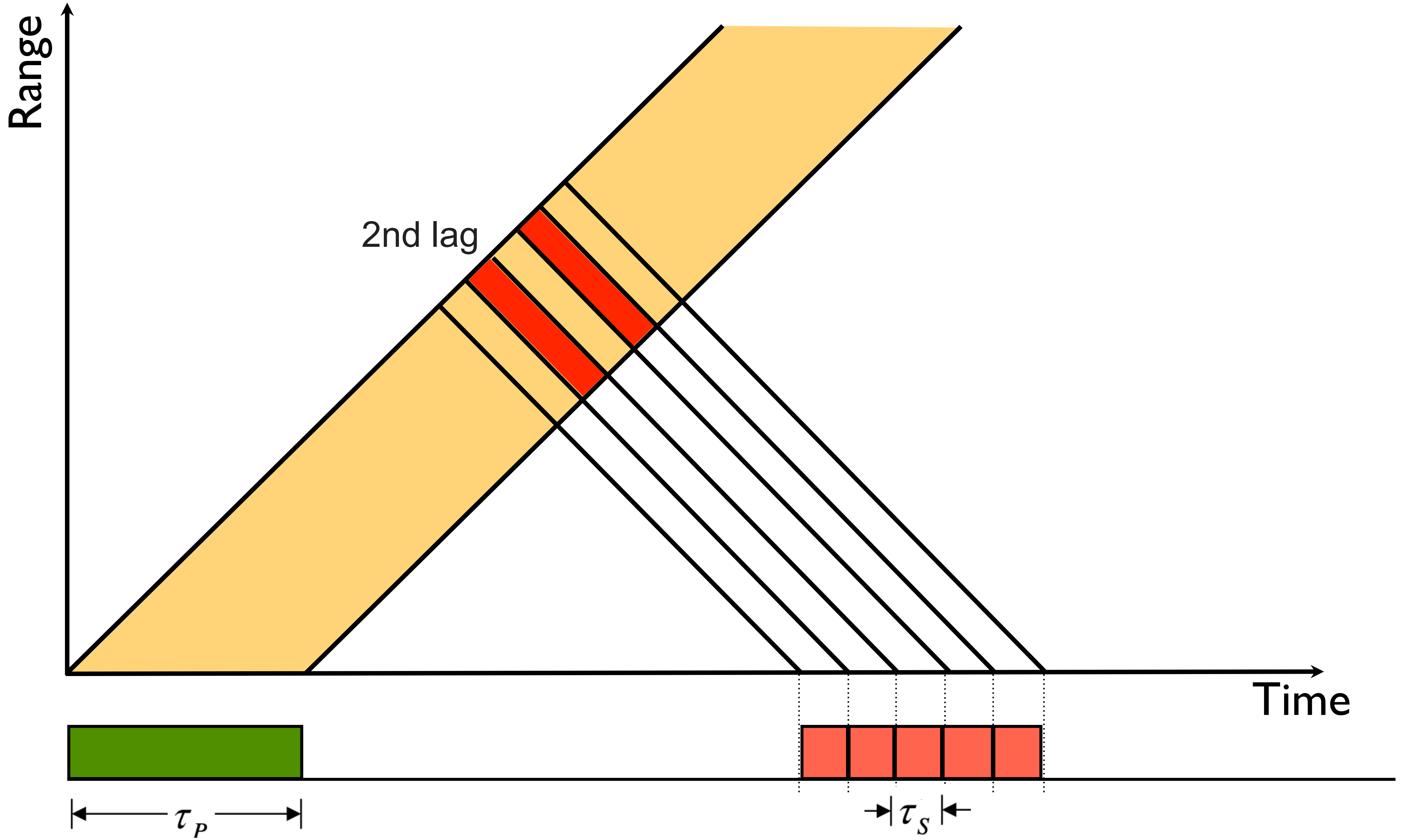
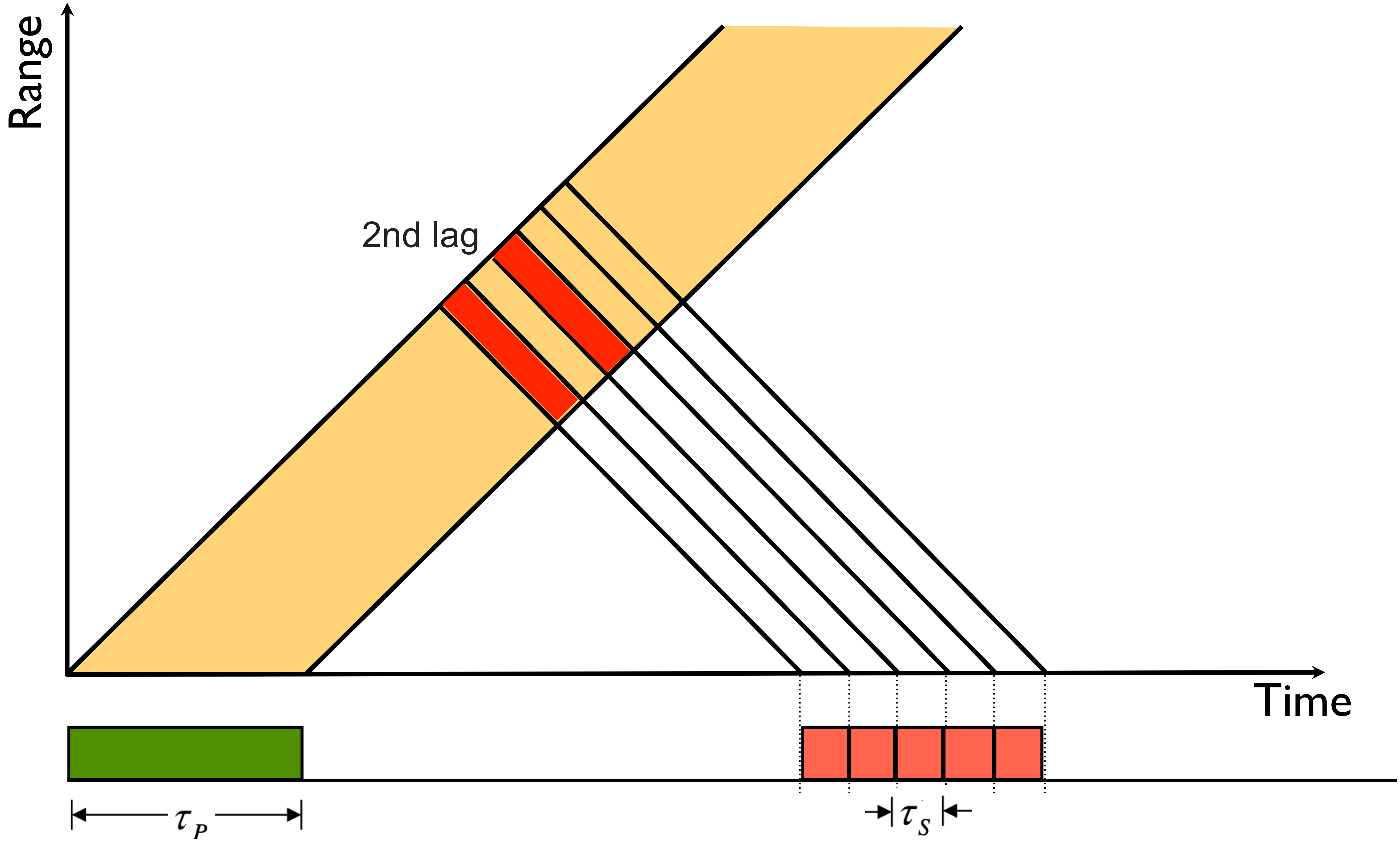


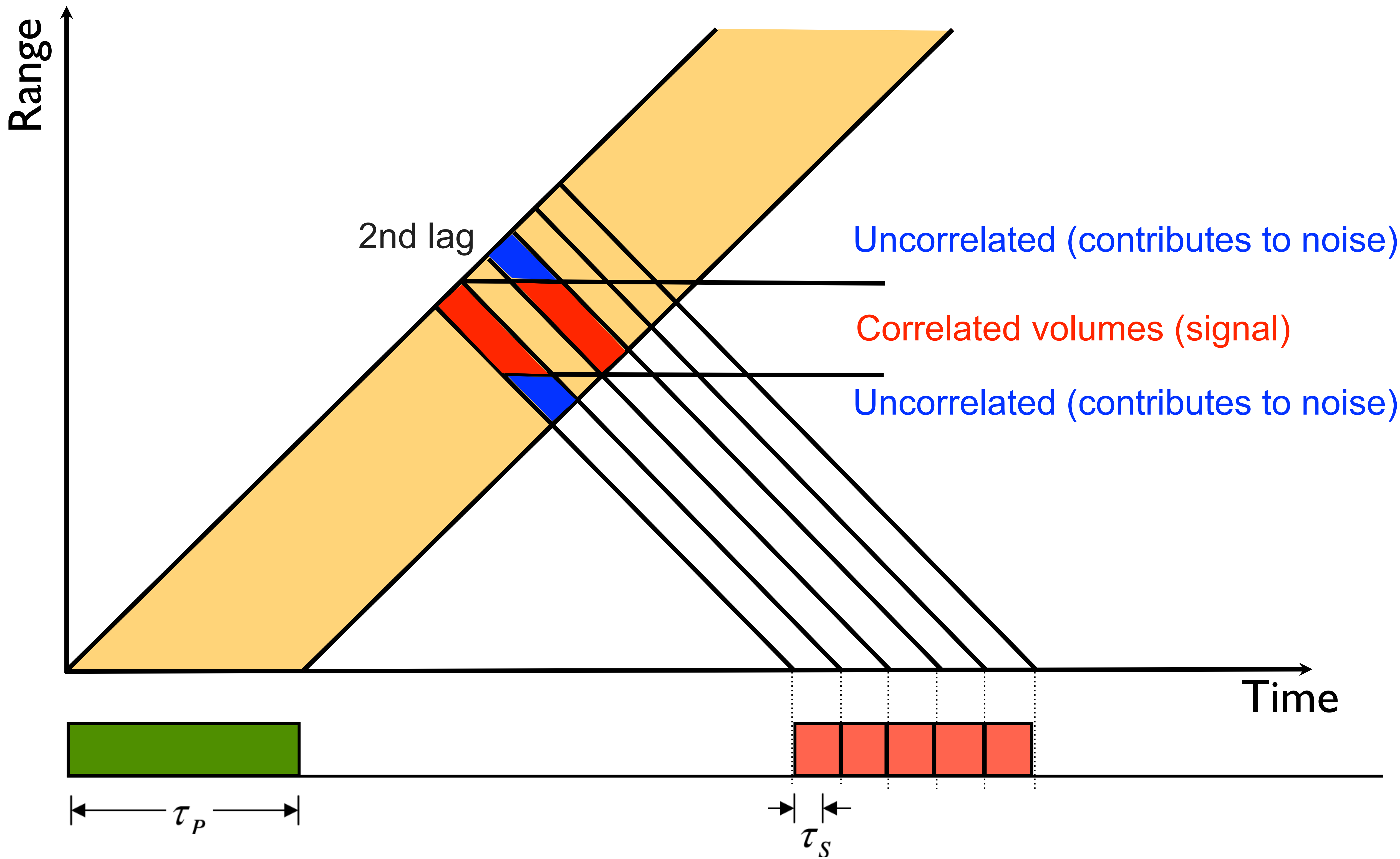
Computing the ACF (and, hence, spectrum)



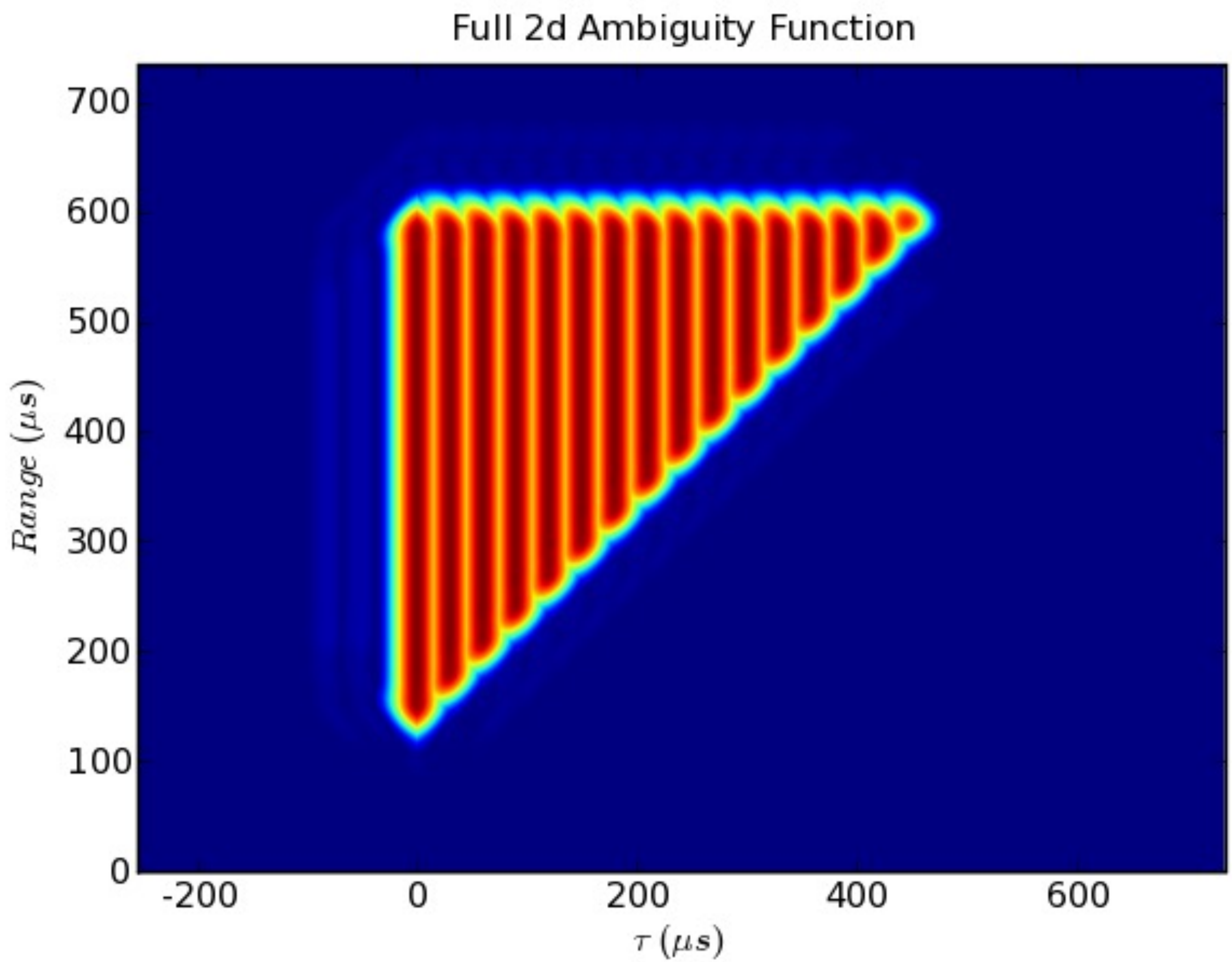
Computing the ACF (and, hence, spectrum)



Computing the ACF (and, hence, spectrum)



Volume Scatter Ambiguity Function (smearing in range and lag)



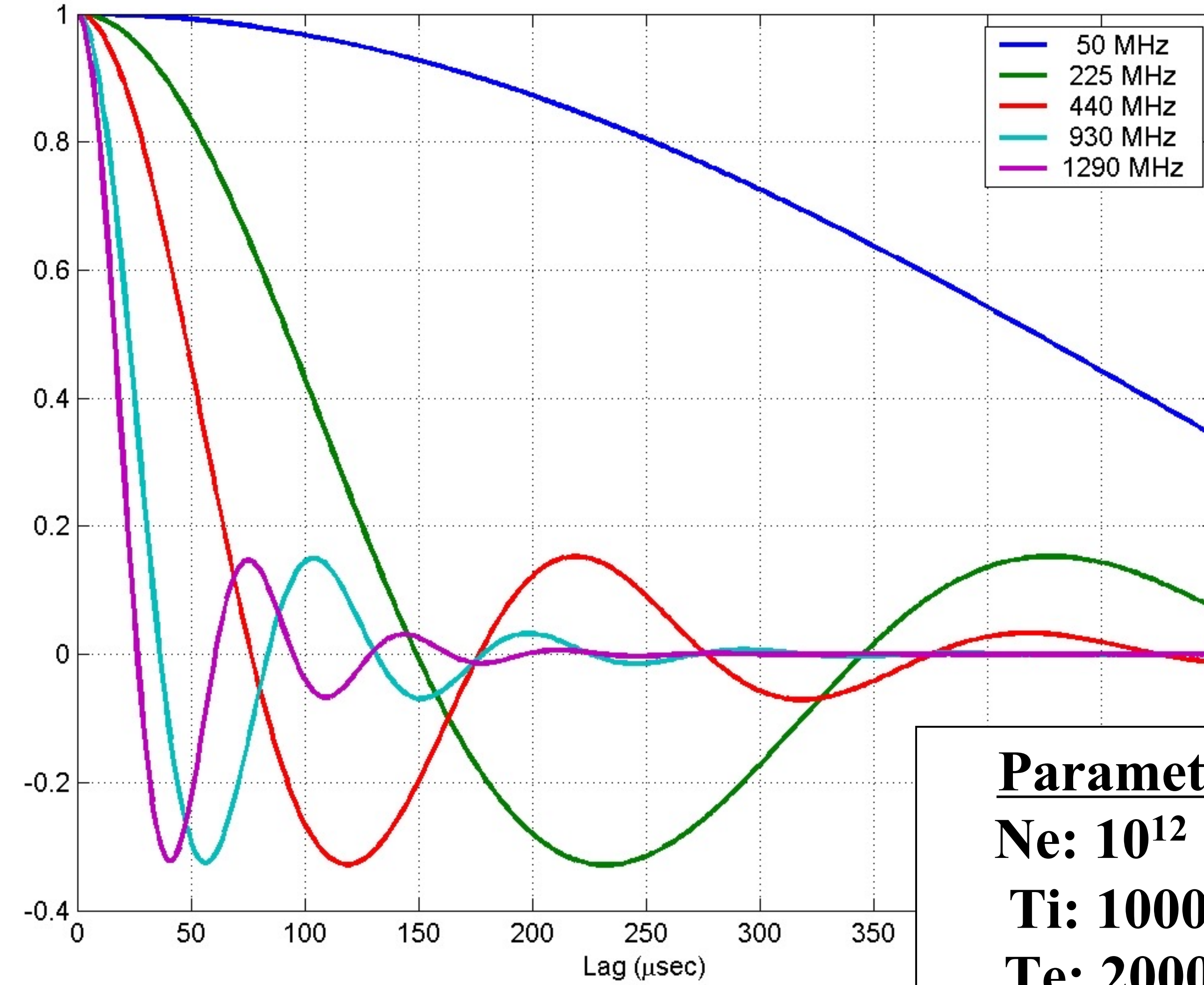
ACF measurements, can we use phase coding?

- Yes, but we must be careful!

- Barker codes, for instance, can be used if the total code length is sufficiently short (less than the correlation time of the medium – Gray and Farley, 1973). Spectral information generally requires correlations between pulses!

- Other classes of modulation are also available that, when incoherently averaged, provide good range resolution at the expense (usually) of increased bandwidth, processing complexity, and self clutter
  - Alternating Codes (Lehtinen and Haggstrom, 1987)
  - Coded Long Pulse (Sulzer, 1986)
  - Compressed Alternating Codes
  - Multipulse (not used much for ISR any more because of the superior performance of other techniques)
  - ”Perfect” codes that reduce range sidelobes
  - Multipurpose codes, processed differently for different altitudes (lag profile inversion)

- Finally, at Arecibo they often had too much SNR and use phase coding to obtain more estimates of the acf.



Parameters

Ne: 10^{12} m^{-3}

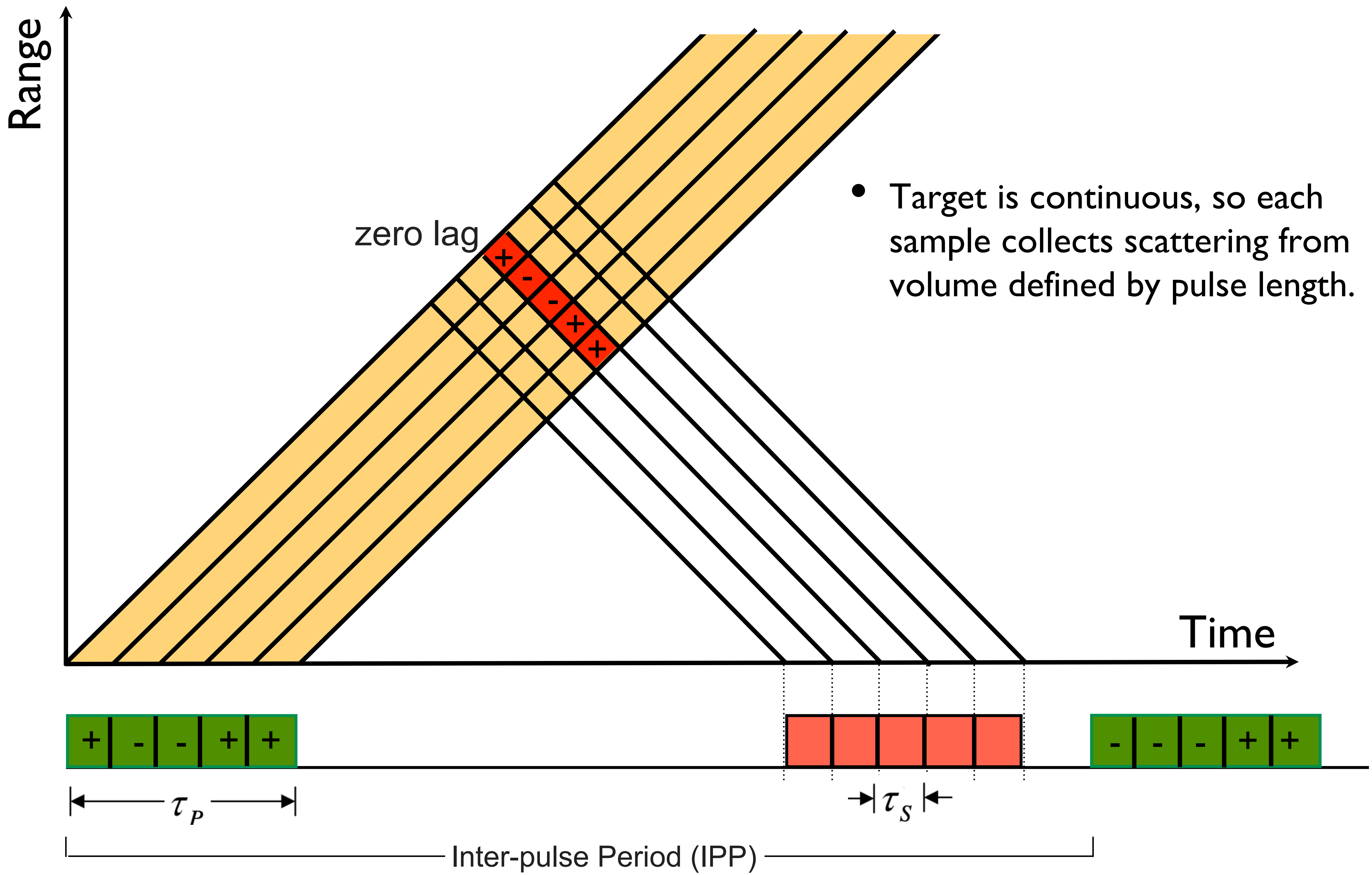
Ti: 1000 K

Te: 2000 K

Comp: 100% O^+

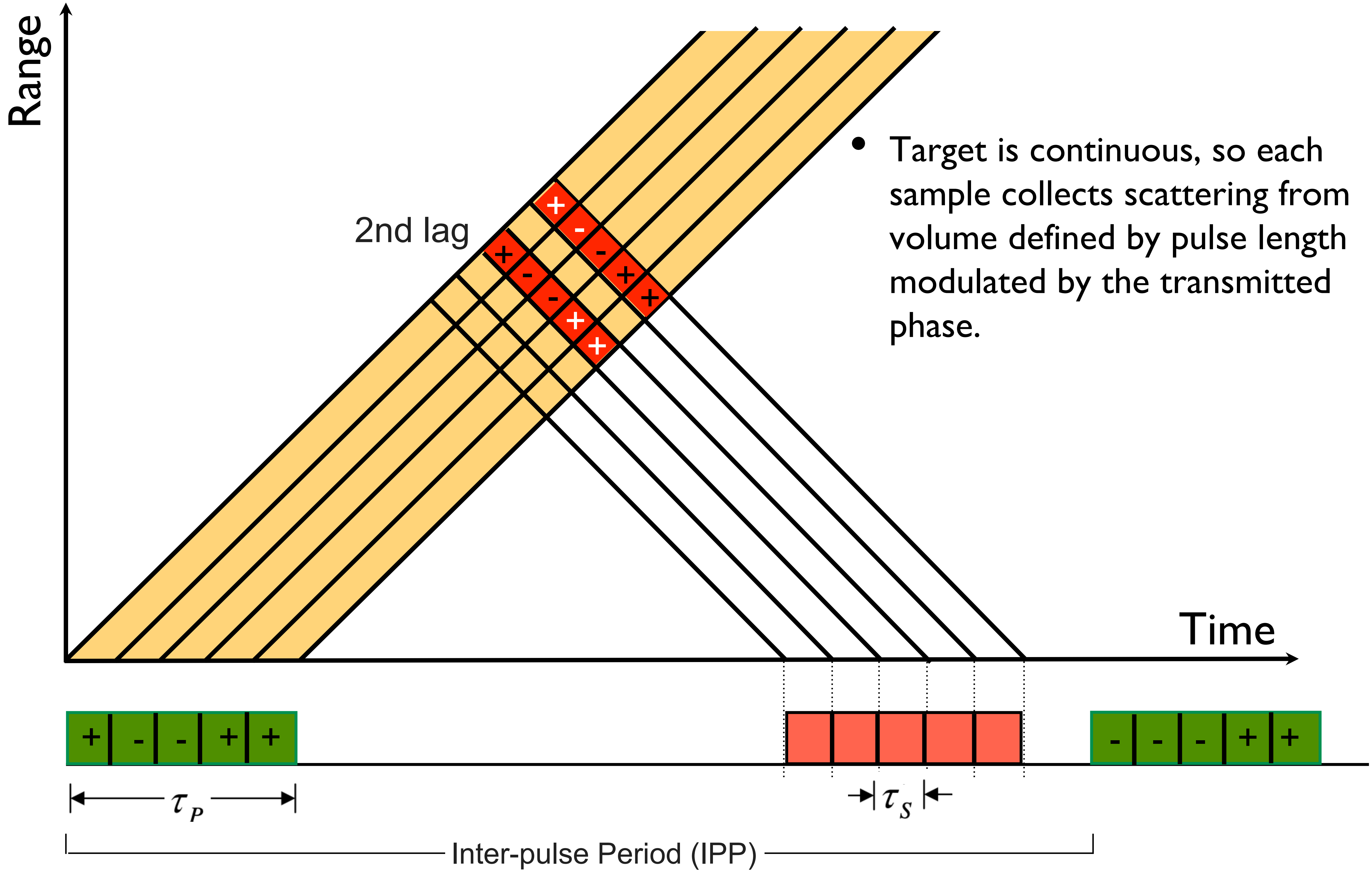
ν_{in}: 10^{-6} KHz

Computing the ACF (and, hence, spectrum)



- Target is continuous, so each sample collects scattering from volume defined by pulse length.

Computing the ACF (and, hence, spectrum)



- Target is continuous, so each sample collects scattering from volume defined by pulse length modulated by the transmitted phase.

Alternating Codes

- 4-baud alternating code example:

++−+

+−++

+−−+

−+++

−+−+

−−++

−−−+

++++

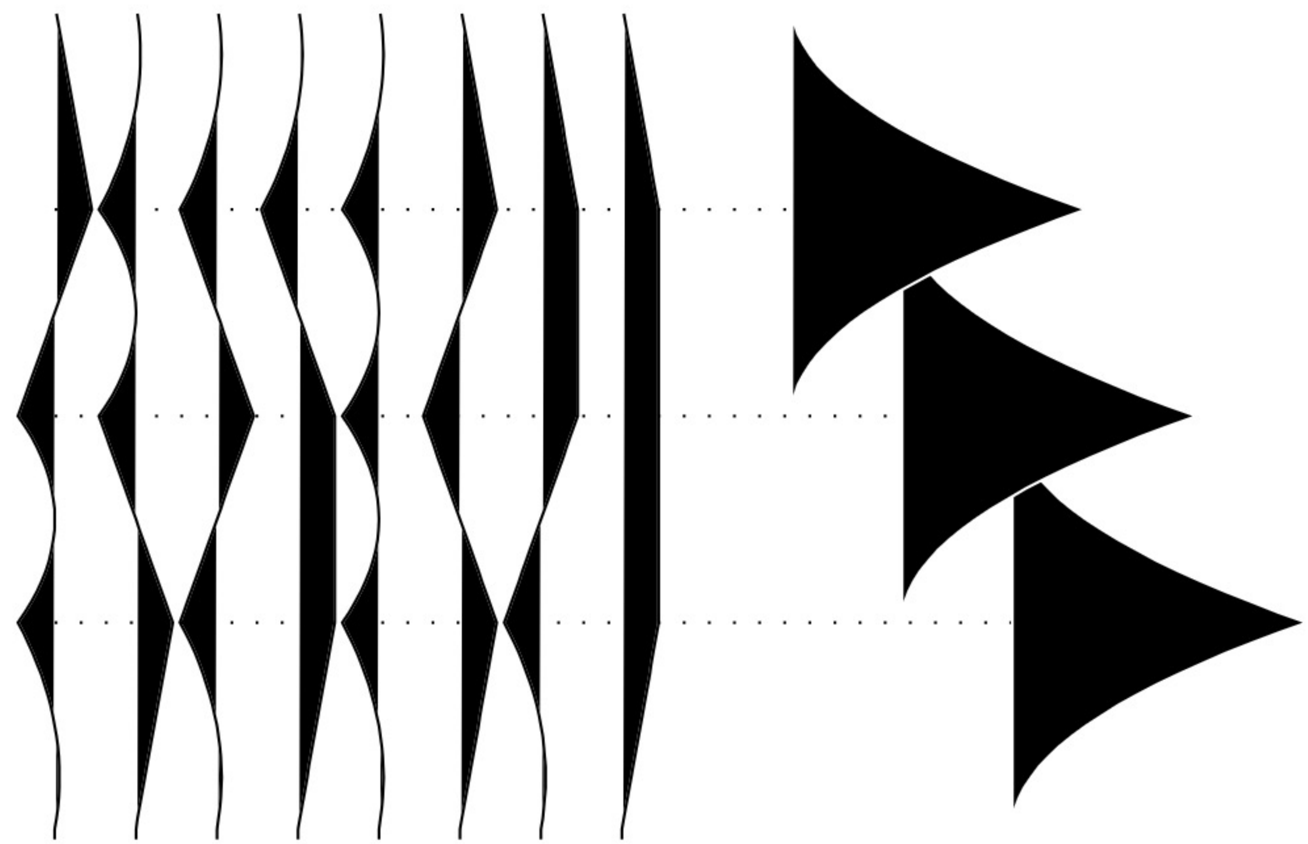
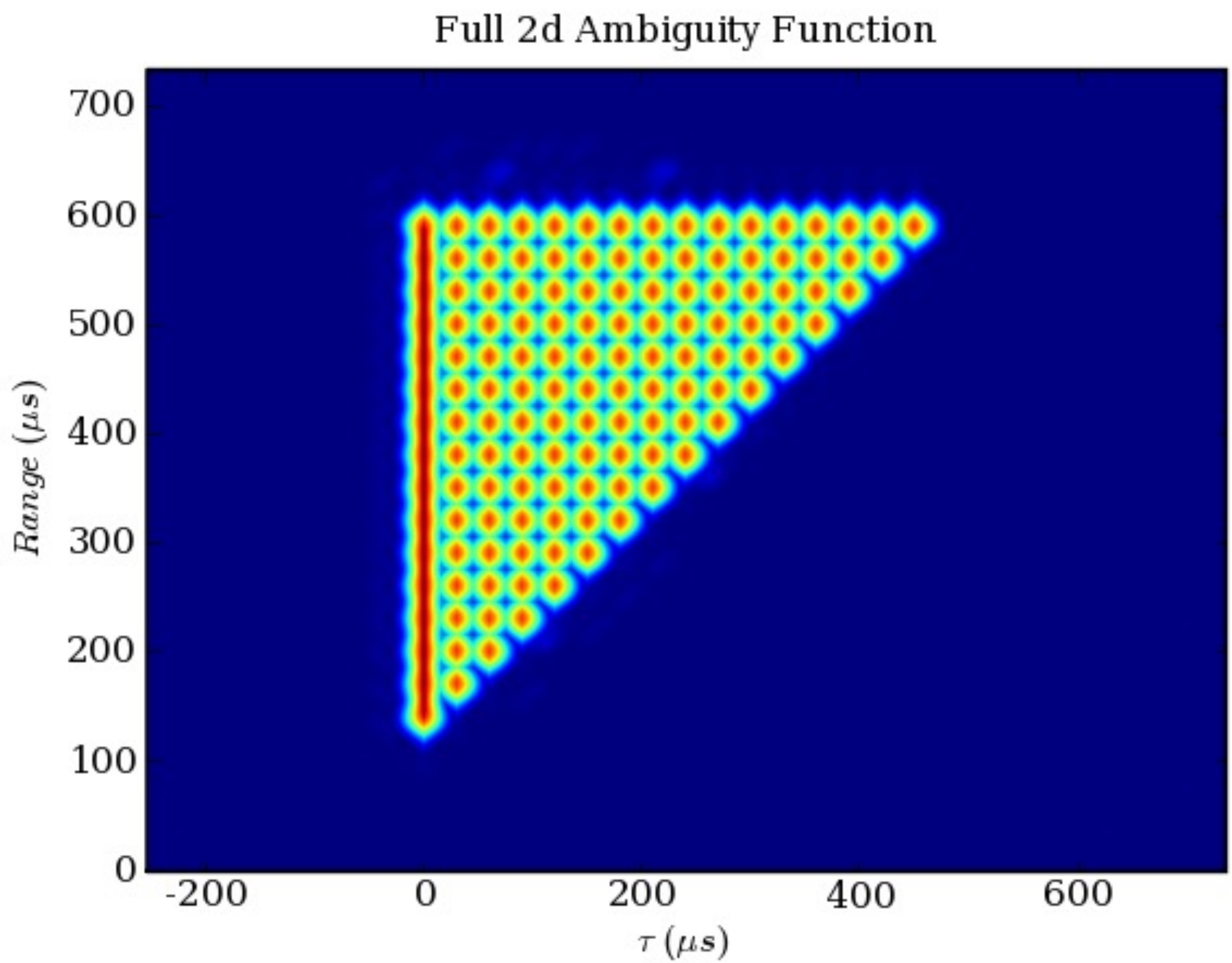


Figure 12. Range ambiguity functions for individual codes and for the decoded measurement

Ambiguity Function Alternating Code (smearing in range and lag)



Example Radar Waveform Set

3.1.3 manda

Version	4.0
Raw data available	Yes
Plasma line	No
Transmitter frequency	929.6 MHz
Integration time	4.8 s
Code	Alternating, 61 bit, 128 subcycles
Baud length	2.4 µs
Sampling rate	1.2 µs
Subcycle length	1.5 ms
Duty cycle	0.098

Ion line Normal

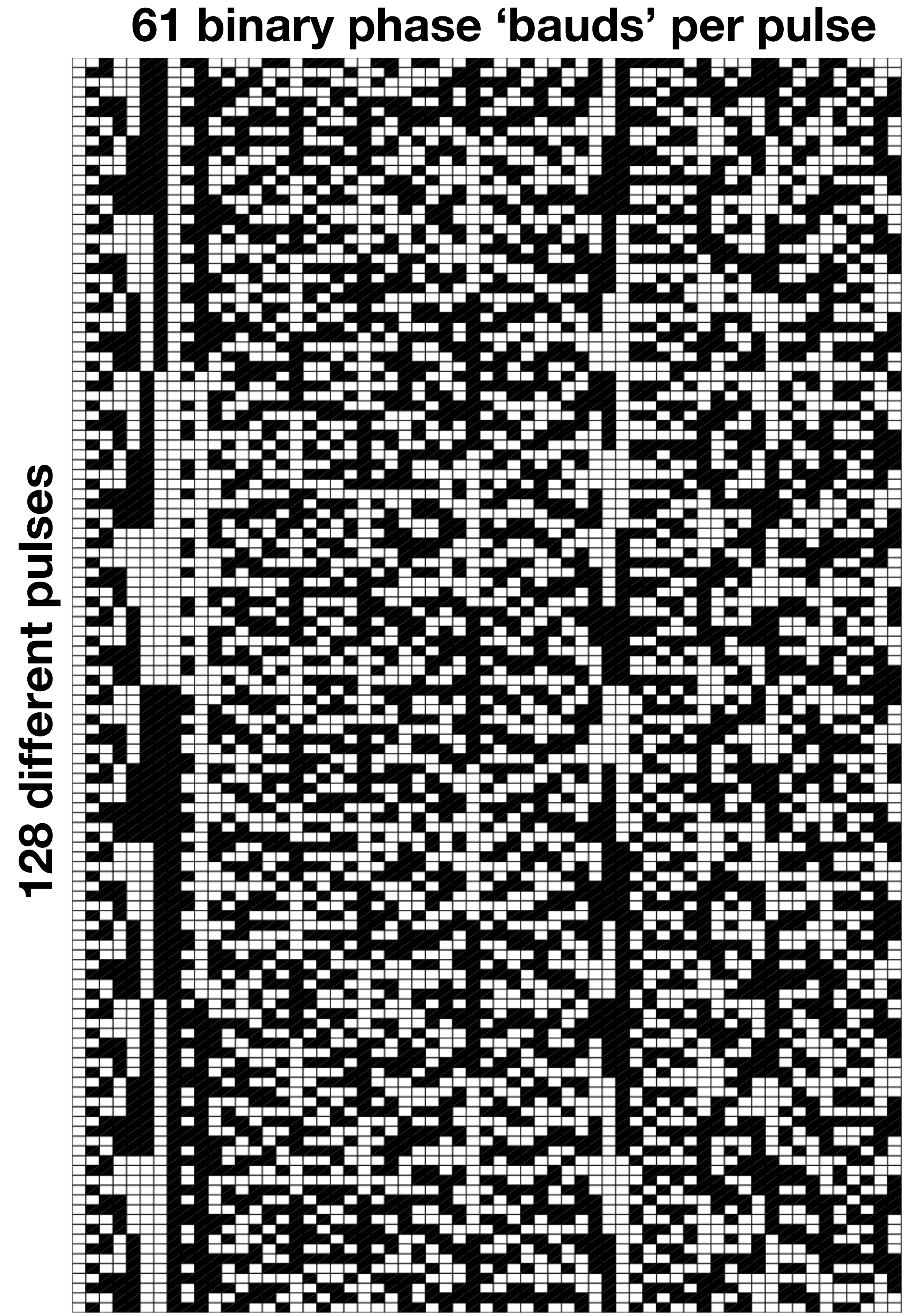
Time resolution	4.8 s
Range span	19 km to 209 km
Range gate size	0.36 km
Spectral range	±417 kHz
Spectral resolution	3.47 kHz
Lag step	1.2 µs
Maximum lag	120 (144 µs)

Ion line D region

Time resolution	4.8 s
Range span	19 km to 109 km
Range gate size	0.36 km
Spectral range	±333 Hz
Spectral resolution	2.62 Hz
Lag step	1.5 ms
Maximum lag	127 (190.5 ms)

Ion line D region, long lags

Time resolution	4.8 s
Range span	19 km to 109 km
Range gate size	0.36 km
Spectral range	±2.6 Hz
Spectral resolution	0.174 Hz
Lag step	192 ms
Maximum lag	15 (2.88 s)



Your Experiments

- Long Pulse
  - F-region measurements
- Alternating Code
  - E-region measurements
- Barker Code
  - D-region measurements (plus PMSE, if present)