

Phased Arrays for Atmospheric and Geospace Science

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Superposition Principle

Maxwell's Equations are Linear:

$$\mathbf{J}_1 = \frac{1}{\mu_0} \nabla \times (\mathbf{B}_1) - \epsilon_0 \frac{\partial}{\partial t} (\mathbf{E}_1)$$

$$0 = \nabla \times (\mathbf{E}_1) + \frac{\partial}{\partial t} (\mathbf{B}_1)$$

$$\mathbf{J}_2 = \frac{1}{\mu_0} \nabla \times (\mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t} (\mathbf{E}_2)$$

$$0 = \nabla \times (\mathbf{E}_2) + \frac{\partial}{\partial t} (\mathbf{B}_2)$$

$$\mathbf{J}_1 + \mathbf{J}_2 = \frac{1}{\mu_0} \nabla \times (\mathbf{B}_1 + \mathbf{B}_2) - \epsilon_0 \frac{\partial}{\partial t} (\mathbf{E}_1 + \mathbf{E}_2)$$

$$0 = \nabla \times (\mathbf{E}_1 + \mathbf{E}_2) + \frac{\partial}{\partial t} (\mathbf{B}_1 + \mathbf{B}_2)$$

Phasor Notation

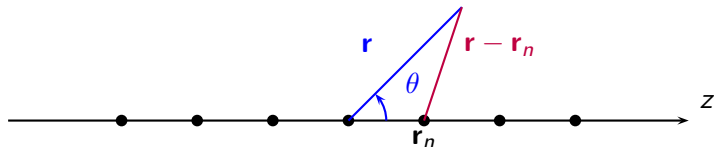
Narrowband assumption

$$\mathbf{E}(\mathbf{r}, t) = \Re \left\{ \mathbf{E}(\mathbf{r}) e^{j\omega t} \right\}$$

Electrical engineering sign convention:
Plane wave propagating in direction $\mathbf{k}$

$$\mathbf{E}(\mathbf{r}, t) \propto \Re \left\{ e^{-j\mathbf{k} \cdot \mathbf{r} + j\omega t} \right\}$$

Superposition Applied to Antenna Arrays



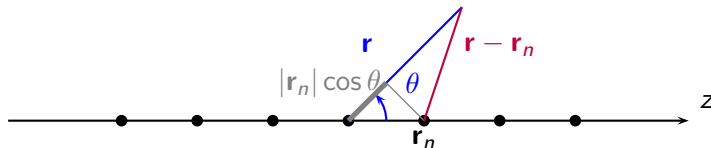
Fields radiated by single element at the origin with applied current I_0 :

$$\mathbf{E} = \mathbf{E}_0 I_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}$$

Fields radiated by entire array:

$$\mathbf{E} = \mathbf{E}_0 \sum_{n=0}^{N-1} I_n \frac{e^{-jk|\mathbf{r} - \mathbf{r}_n|}}{|\mathbf{r} - \mathbf{r}_n|}$$

Far Field Approximation (Fraunhofer Zone)



If $\mathbf{r}$ and $\mathbf{r} - \mathbf{r}_n$ are almost parallel lines:

$$\mathbf{r} - \mathbf{r}_n \approx \mathbf{r} - |\mathbf{r}_n| \cos \theta \hat{\mathbf{r}}$$

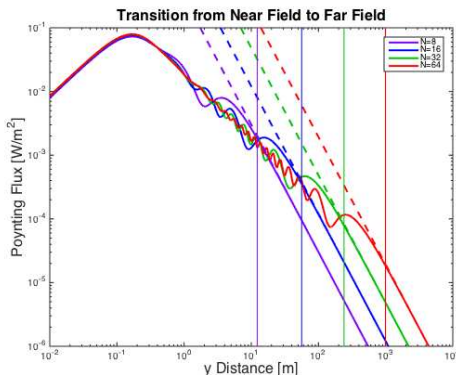
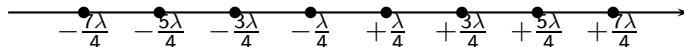
Assume $|\mathbf{r}_n| \ll |\mathbf{r}|$:

$$|\mathbf{r} - \mathbf{r}_n| \approx |\mathbf{r}| \text{ for demoninator terms}$$

$$-jk |\mathbf{r} - \mathbf{r}_n| \approx -jk |\mathbf{r}| + jk |\mathbf{r}_n| \cos \theta$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk|\mathbf{r}_n| \cos \theta}}_{\text{Array Factor}}$$

Distance to Far Field: Fresnel Numbers



Fresnel Number:

$$\frac{L^2}{r\lambda} \ll 1 \rightarrow \text{Far Field}$$

$$\frac{L^2}{r\lambda} > 1 \rightarrow \text{Near Field}$$

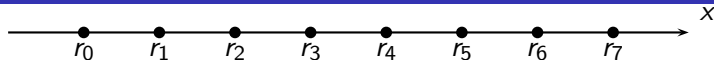
L = Array length

λ = wavelength

AMISR: $L = 32 \text{ m}$ $\lambda = 0.67 \text{ m}$ $L^2/\lambda = 1.5 \text{ km}$

Arecibo: $L = 305 \text{ m}$ $\lambda = 0.70 \text{ m}$ $L^2/\lambda = 133 \text{ km}$

1-D Linear Phased Array



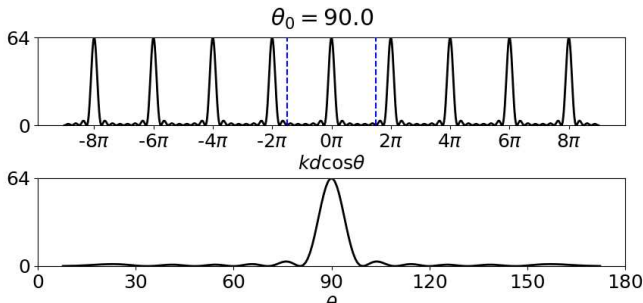
$$|\mathbf{r}_n| = nd \quad I_n = e^{jn\alpha}$$

Array Factor:

$$\begin{aligned}
 F &= \sum_{n=0}^{N-1} e^{jn\alpha} e^{jknd \cos \theta} \\
 &= \frac{1 - e^{jN\alpha + jNkd \cos \theta}}{1 - e^{j\alpha + jkd \cos \theta}} \\
 &= e^{j\frac{(N-1)}{2}(kd \cos \theta + \alpha)} \frac{\sin \left[\frac{N}{2} (kd \cos \theta + \alpha) \right]}{\sin \left[\frac{1}{2} (kd \cos \theta + \alpha) \right]} \\
 |F|^2 &= \frac{\sin^2 \left[\frac{N}{2} (kd \cos \theta + \alpha) \right]}{\sin^2 \left[\frac{1}{2} (kd \cos \theta + \alpha) \right]}
 \end{aligned}$$

1-D Linear Phased Array Cont.

$$|F|^2 = \frac{\sin^2 \left[\frac{N}{2} (kd \cos \theta + \alpha) \right]}{\sin^2 \left[\frac{1}{2} (kd \cos \theta + \alpha) \right]}$$

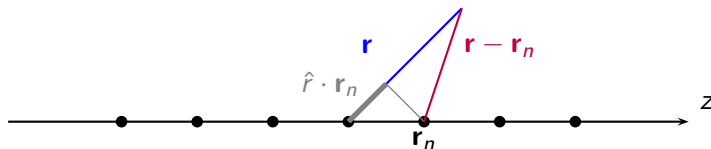


Peak appears when $kd \cos \theta = -\alpha \rightarrow \alpha = -kd \cos \theta_0$

Additional peaks could appear when $kd \cos \theta = -\alpha + 2\pi m$ (Grating Lobes)

Visible Region: $0 < \theta < \pi \rightarrow -kd < kd \cos \theta < kd$

Multi-Dimensional Arrays



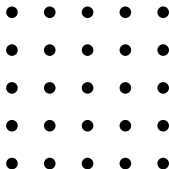
$$-jk |\mathbf{r} - \mathbf{r}_n| \approx -jk |\mathbf{r}| + jk (\hat{\mathbf{r}} \cdot \mathbf{r}_n)$$

$$\mathbf{E} \approx \underbrace{\mathbf{E}_0 \frac{e^{-jk|\mathbf{r}|}}{|\mathbf{r}|}}_{\text{Element Factor}} \underbrace{\sum_{n=0}^{N-1} I_n e^{jk(\hat{\mathbf{r}} \cdot \mathbf{r}_n)}}_{\text{Array Factor}}$$

In spherical coordinates:

$$\hat{\mathbf{r}} \cdot \mathbf{r}_n = x_n \cos \phi \sin \theta + y_n \sin \phi \sin \theta + z_n \cos \theta$$

2-D Rectangular Array



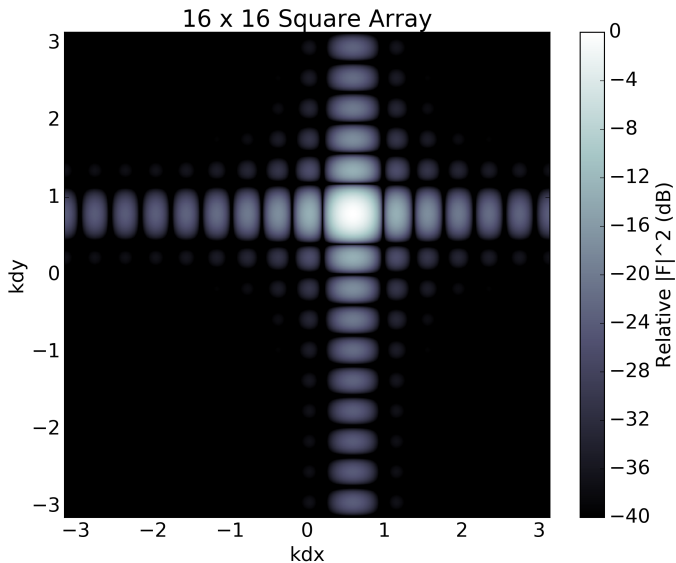
$$\mathbf{r}_{nm} = nd_x \hat{\mathbf{x}} + md_y \hat{\mathbf{y}} \quad I_{nm} = e^{j(n\alpha + m\beta)}$$

Array Factor:

$$|F(\theta, \phi)|^2 = \left| \sum_{n=0}^{N_x-1} \sum_{m=0}^{N_y-1} e^{j(nkd_x \cos \phi \sin \theta + n\alpha + mkd_y \sin \phi \sin \theta + m\beta)} \right|^2$$

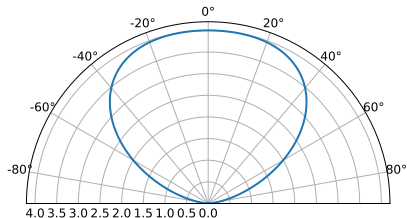
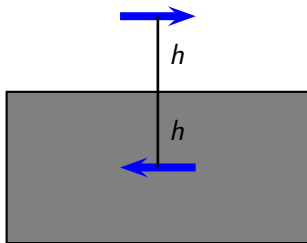
$$= \frac{\sin^2 \left[\frac{N_x}{2} (kd_x \cos \phi \sin \theta + \alpha) \right]}{\sin^2 \left[\frac{1}{2} (kd_x \cos \phi \sin \theta + \alpha) \right]} \frac{\sin^2 \left[\frac{N_y}{2} (kd_y \sin \phi \sin \theta + \beta) \right]}{\sin^2 \left[\frac{1}{2} (kd_y \sin \phi \sin \theta + \beta) \right]}$$

2-D Rectangular Array



Ground Planes: Method of Images

Antenna above conducting ground plane



$$F = I_0 e^{jkh \cos \theta} - I_0 e^{-jkh \cos \theta}$$

$$= 2jI_0 \sin(kh \cos \theta)$$

$$|F|^2 = 4 |I_0|^2 \sin^2(kh \cos \theta)$$

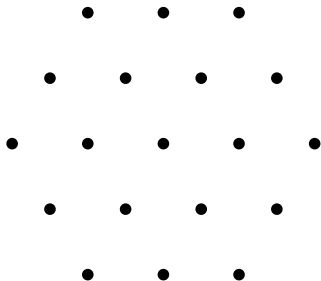
$$\text{When } h = \frac{\lambda}{4} \rightarrow kh = \frac{\pi}{2}$$

$$|F|^2 = 4 |I_0|^2 \sin^2\left(\frac{\pi}{2} \cos \theta\right)$$

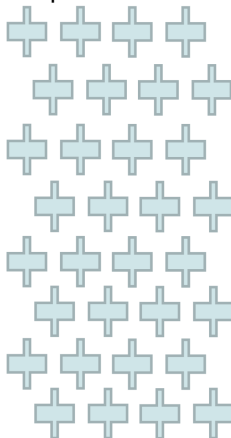
Peaks at $\theta = 0$ (upwards).

Hexagonal Spacing

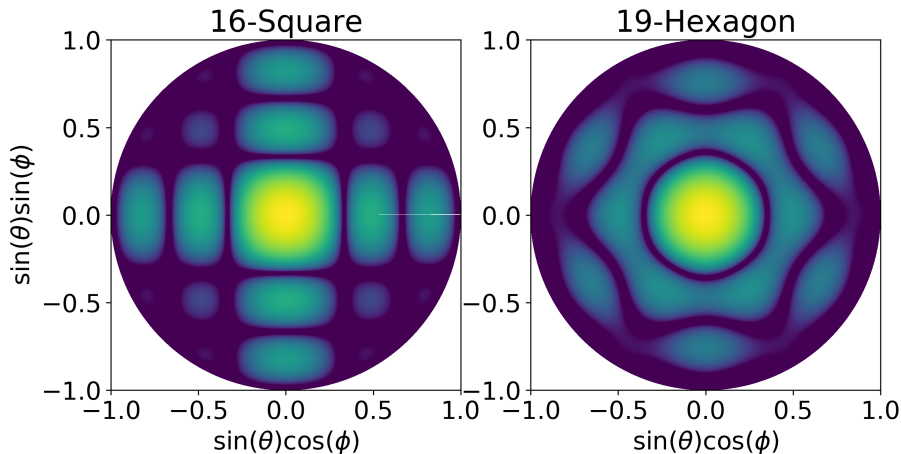
Hexagon



Honeycomb Rectangular Array
One AMISR panel:



Sidelobe Patterns for Squares vs Hexagons



A “Circular” Array: MU Radar



Fourier Analogy

Array factor looks like a Discrete-Space Fourier Transform

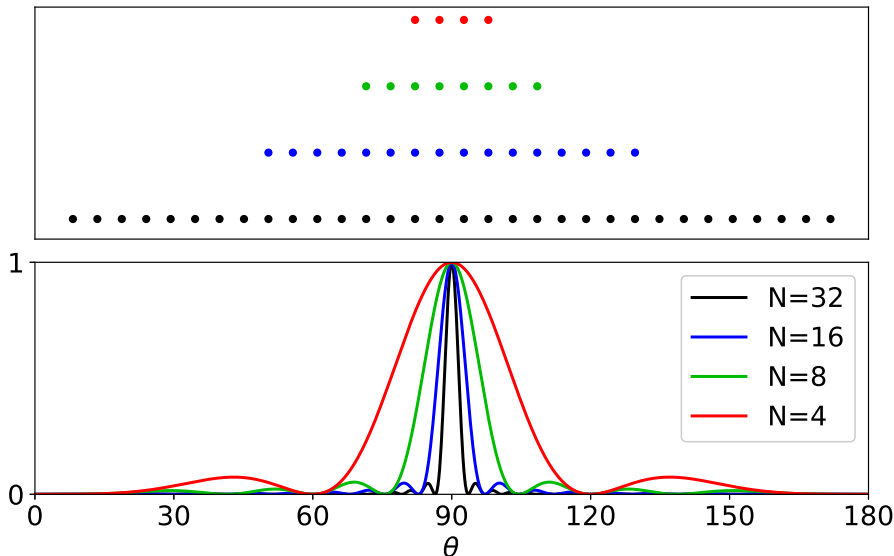
$$F(\theta) = \sum_{n=0}^{N-1} I_n e^{jnk d \cos \theta}$$
$$F(\gamma) = \sum_{n=0}^{N-1} I_n e^{jn\gamma} \quad \gamma \equiv kd \cos(\theta)$$

Inverse Transformation

$$I_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(\gamma) e^{-jn\gamma} d\gamma$$

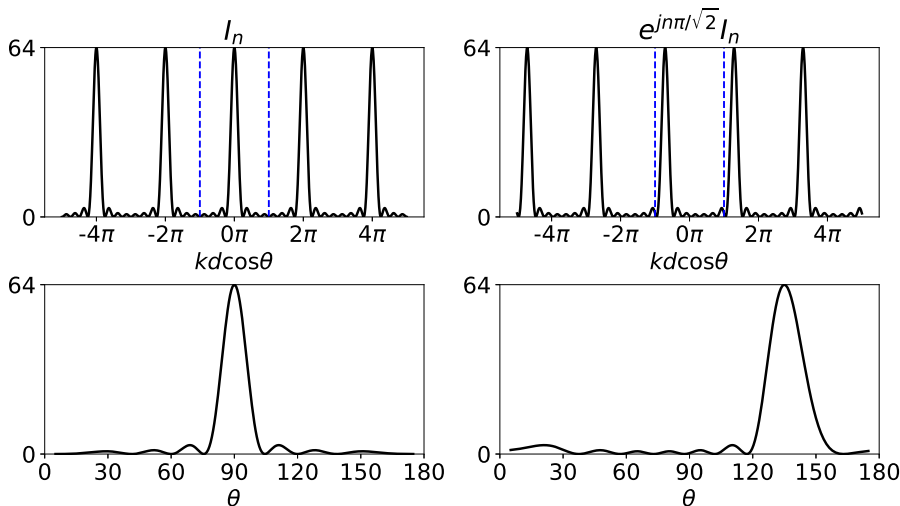
Designing I_n to approximate a desired $F(\gamma)$ is similar to designing FIR filters.

Wide in Space $\leftrightarrow$ Narrow Beamwidth



Shift Theorem

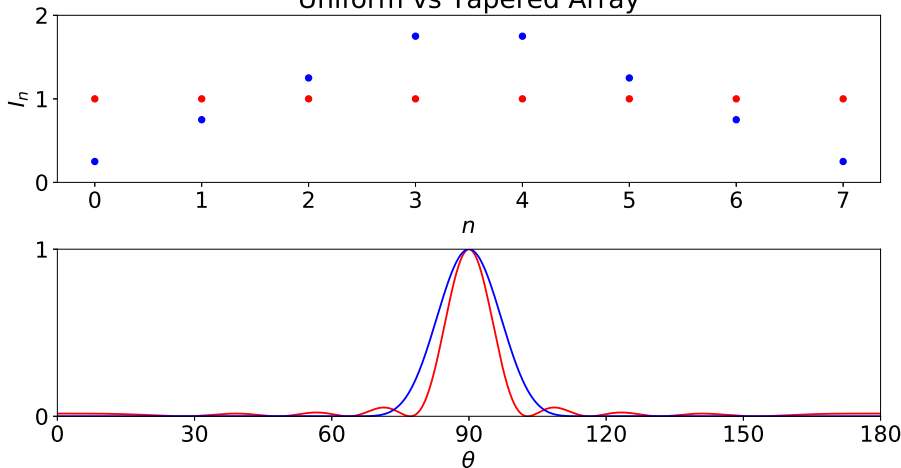
$$F(\gamma) \rightarrow F(\gamma - \gamma_0) \Leftrightarrow I_n \rightarrow e^{jn\gamma_0} I_n$$



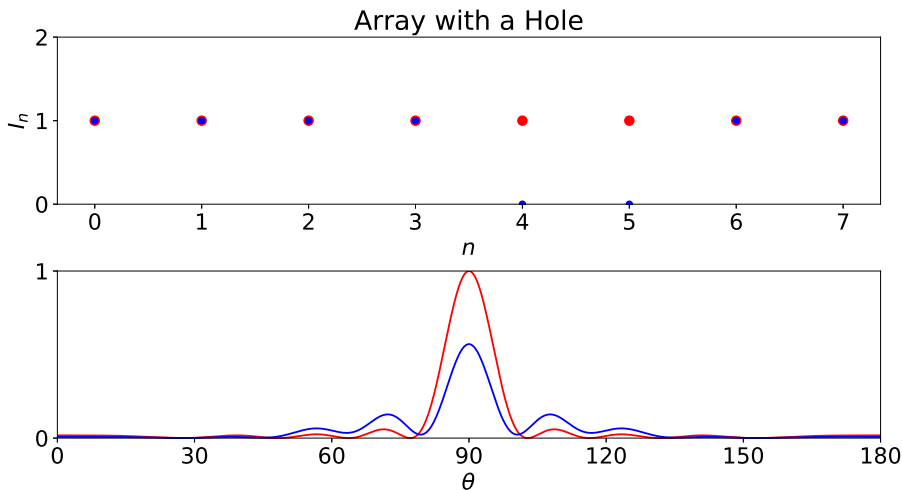
Array Tapers and Windowing

$$I_n \rightarrow w_n I_n \Leftrightarrow F(\gamma) \rightarrow W(\gamma) * F(\gamma)$$

Uniform vs Tapered Array



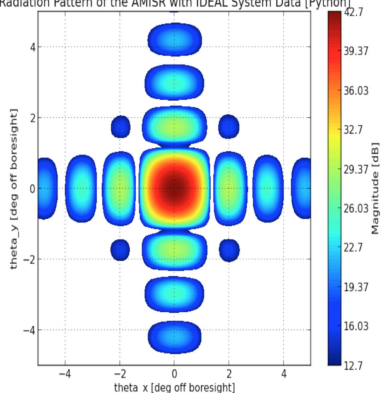
Arrays with Missing Elements



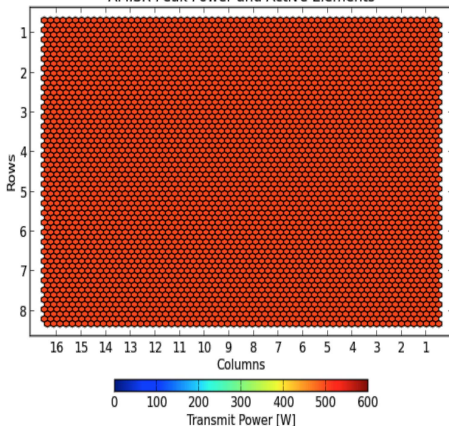
Missing elements reduces power in main beam and raises sidelobes

Ideal AMISR Radiation Pattern

Radiation Pattern of the AMISR with IDEAL System Data [Python]

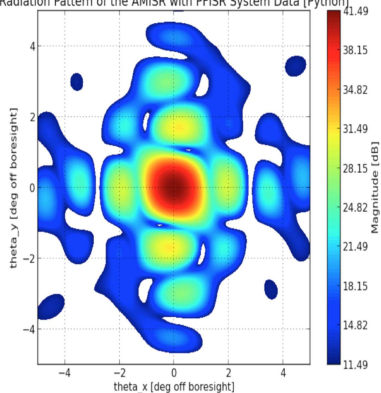


AMISR Peak Power and Active Elements

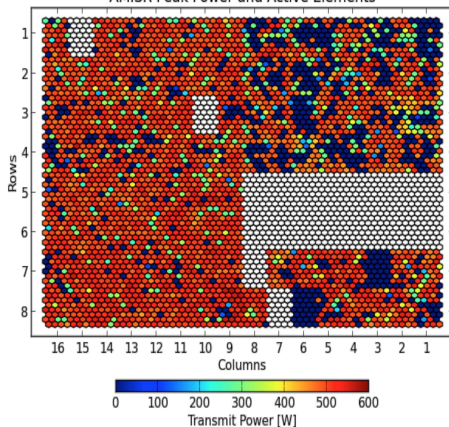


AMISR Graceful Degradation

Radiation Pattern of the AMISR with PFISR System Data [Python]

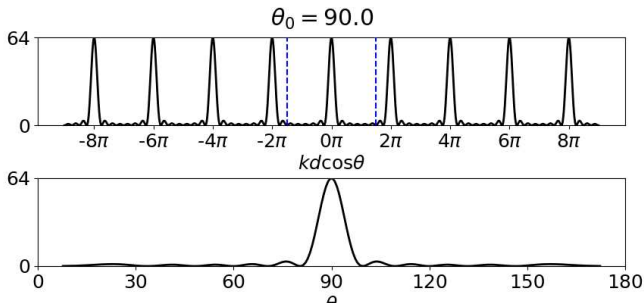


AMISR Peak Power and Active Elements



Visible Region

$$|F|^2 = \frac{\sin^2 \left[\frac{N}{2} (kd \cos \theta + \alpha) \right]}{\sin^2 \left[\frac{1}{2} (kd \cos \theta + \alpha) \right]}$$



Peak appears when $kd \cos \theta = -\alpha \rightarrow \alpha = -kd \cos \theta_0$

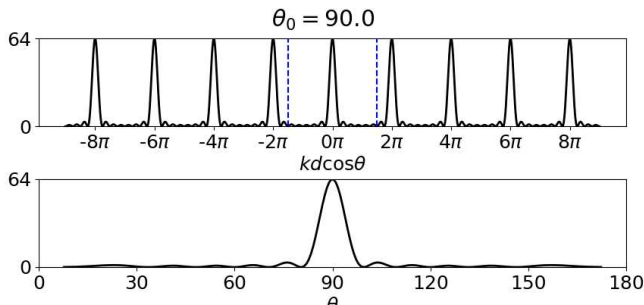
Additional peaks could appear when $kd \cos \theta = -\alpha + 2\pi m$ (Grating Lobes)

Visible Region: $0 < \theta < \pi \rightarrow -kd < kd \cos \theta < kd$

Grating Lobes

- $d < \lambda/2 \rightarrow kd < \pi$: No grating lobes will ever appear
- $\lambda/2 < kd < \lambda \rightarrow \pi < kd < 2\pi$: Grating lobes will only appear at some steering angles
- $d > \lambda \rightarrow kd > 2\pi$: Grating lobes will always appear

Example of linear array with $d = 3\lambda/4$ spacing



- Maximum steering angle (for a linear array):

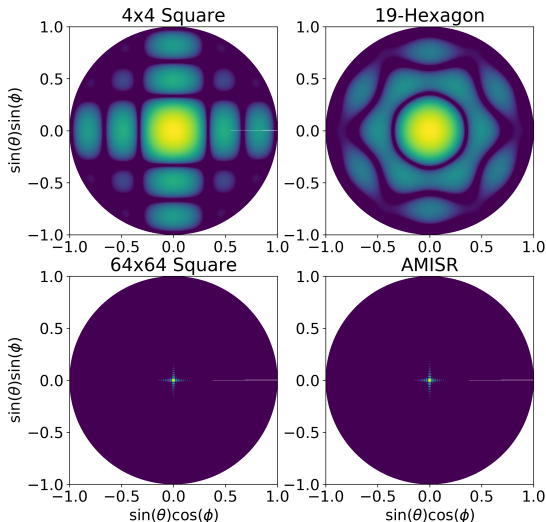
$$\Delta\theta_{\max} = \left| \sin^{-1} \left(1 - \frac{2\pi}{kd} \right) \right|$$

Issues Affecting Antenna Spacing

- Mutual Coupling Between Antennas
 - The true element factor for antennas in an array is different from the same type of antennas in isolation
 - Scattering off of neighboring antennas
 - Inductive coupling involving antenna near-fields
 - Impedance of an antenna in an array is different from the impedance of that antenna in isolation
- Cost
 - Antenna gain is determined by the area of the array
 - Larger distance between antennas $\rightarrow$ fewer elements to fill a given area

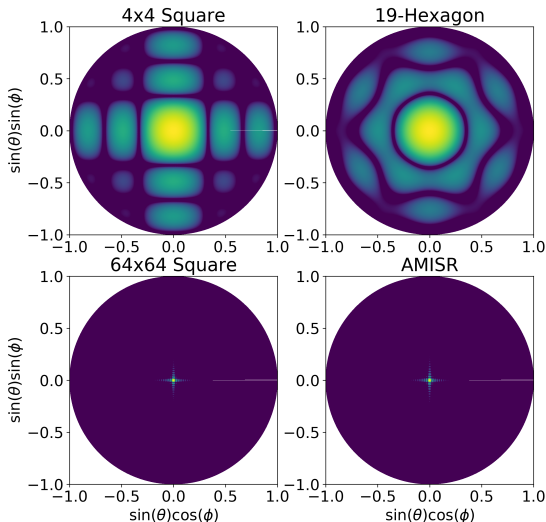
Examples Steering 2-D Arrays ($d = 3\lambda/4$)

$$\theta = 0.0 \quad \phi = 0.0$$



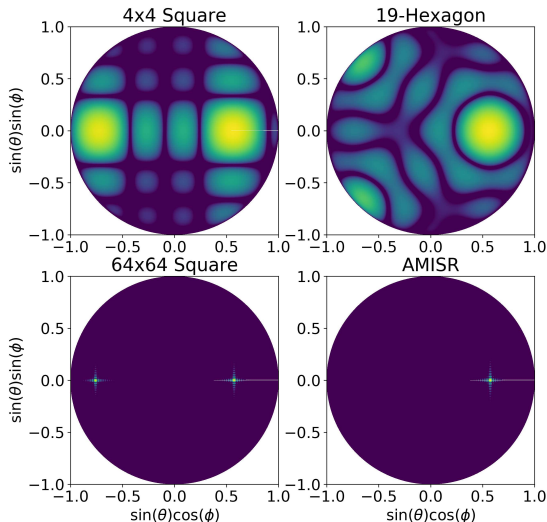
Examples Steering 2-D Arrays ($d = 3\lambda/4$): Elevation Scan

$$\theta = 0.0 \quad \phi = 0.0$$



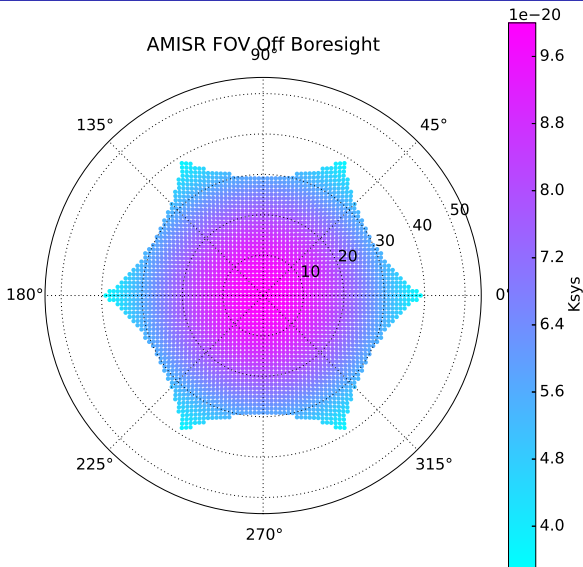
Examples Steering 2-D Arrays ($d = 3\lambda/4$): Azimuth Scan

$$\theta = 35.0 \quad \phi = 0.0$$



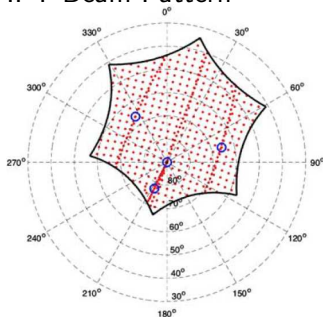
AMISR Antenna Properties

- Hexagonal spacing with $d \approx 3\lambda/4$
- FOV limited by grating lobe limit $\sim 30^\circ - 40^\circ$
- Antenna gain decreases with steering angle off of boresight
- Antenna works best within $\sim 25^\circ$ off of boresight



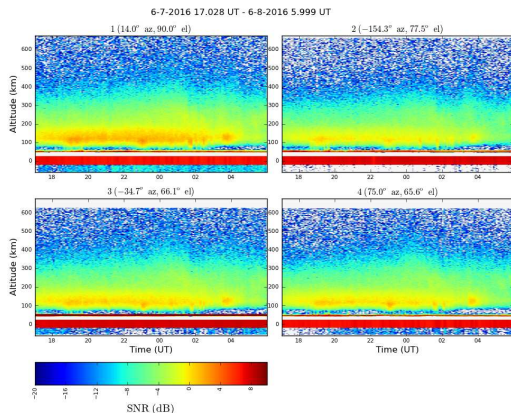
The PFISR Up-B Compromise

IPY Beam Pattern



The Up-B beam is close to the grating lobe limit, and therefore has reduced sensitivity.

Reduced SNR in Up-B (Beam 2)



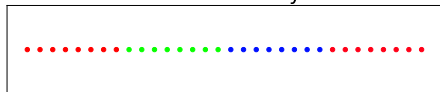
Phasing Subarrays

What if you can only control phases on subarrays instead of every individual antenna?

Phase Each Antenna



Phase Subarrays

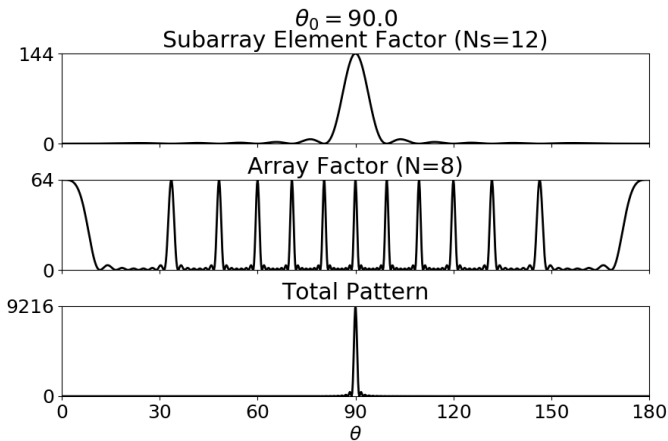


View the whole array as an array of subarrays, and use the subarray pattern as the “element factor.”

Jicamarca Example:

- CoCo antennas spaced by $\lambda/2$
- 18,432 dipoles (9216 per polarization)
- 96×96 square array
- Only able to control phases on 64^{th} s of the array
- 8×8 array of subarrays
- Each subarray is 12×12

Steering Subarrays



Passive Phased Arrays: Jicamarca

- One transmitter feeds entire array



Passive Phased Arrays: Jicamarca



Active Electronically Steerable Phased Arrays

The AMISR UHF System

AMISR AEU = Tx/Rx Unit

- 500 W solid state transmitter
- Phasing control
- Status monitoring
- 4096 AEU's/AMISR radar face

Antenna Element Unit (AEU)



32/panel

AMISR Panel

- 32 Antenna Element Units arranged in hexagonal pattern
- 3.5 x 2 meters; 19.8 dBi / panel
- 16 kW peak power per panel
- Basic system building block for AMISR
- Embedded linux controller



Panel (with PCU)

Face



Utility Distribution Unit (UDU)



AMISR Control System (ACS)

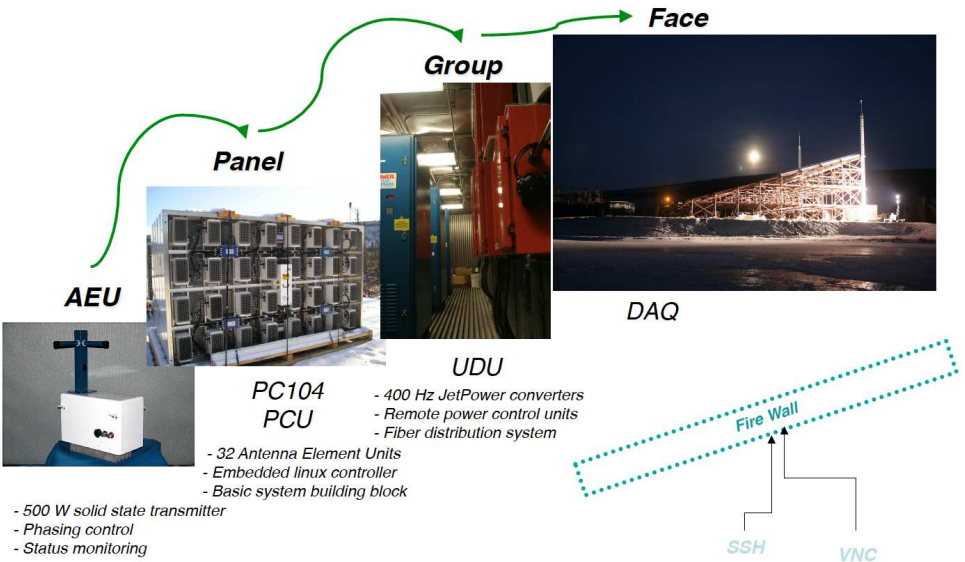
AMISR UDU

- 400 Hz JetPower converters
- Remote power control units
- Fiber distribution system

AMISR ACS

- Flexible transmit and receive system
- Completely remotely controlled
- Experiments run off a scheduler

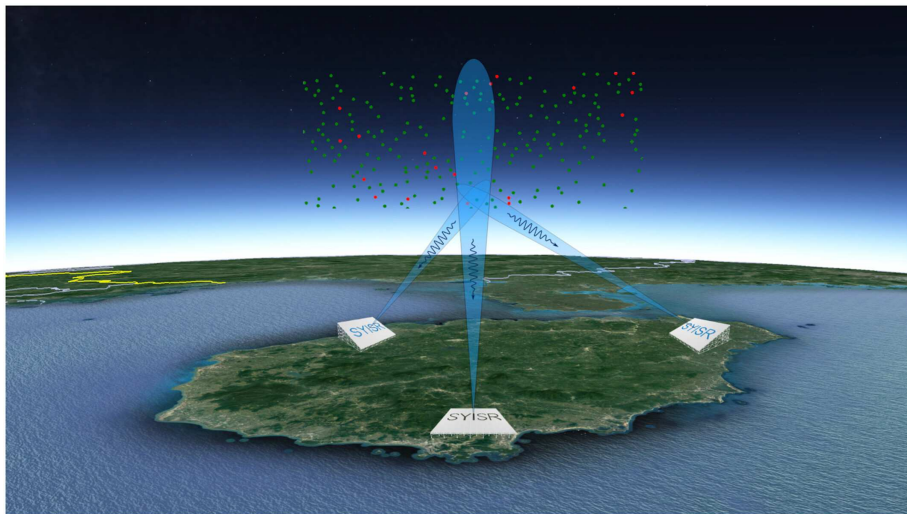
Advanced Modular Incoherent Scatter Radar



Sanya Incoherent Scatter Radar



Sanya Tristatic System



MU Radar



MU Radar Cabling



MU Radar Cabling

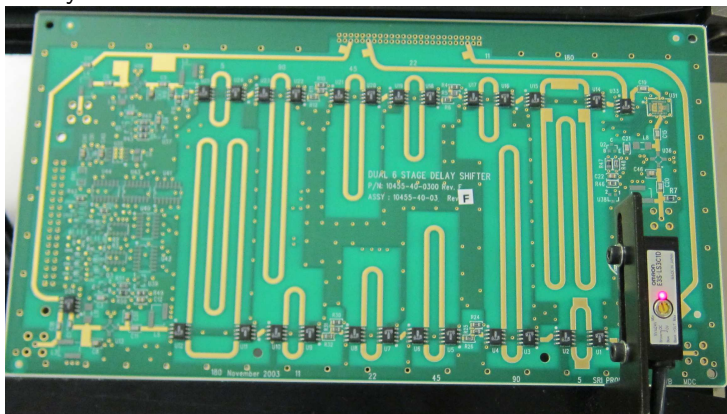


MU Radar Power Amplifiers



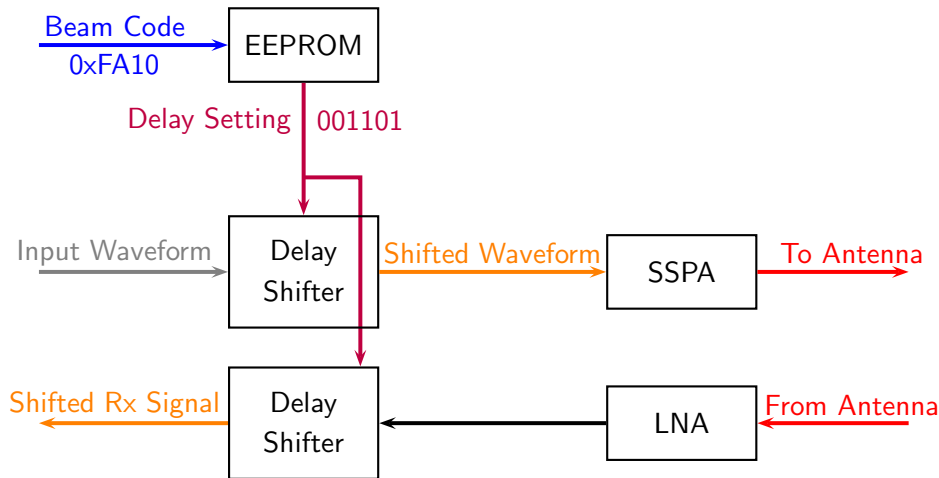
Electronic Steering with Delay Shifters

AMISR Delay Shifter

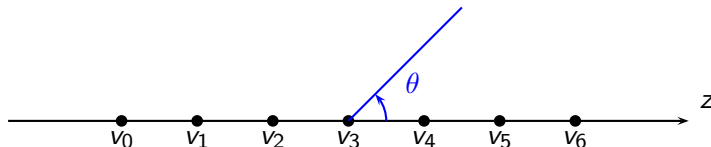


- $2^6 = 64$ steps spaced by $\pi/32 = 5.625^\circ$

Conceptual Diagram of Steering with AMISR



Digital Beam Forming



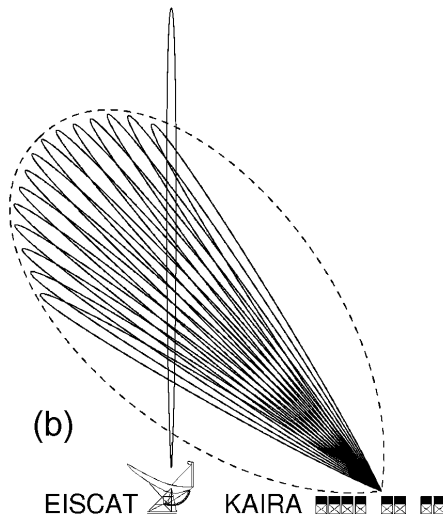
On reception:

- Digitize the signals v_n on every antenna (**expensive!**)
- Synthesize any beams you want by forming different linear combinations in software/firmware (**computationally intense!**)
- Allows you to form custom beam patterns → **look at signals of interest while nulling interference.**
- Allows you to form any number of different radiation patterns → **look in multiple directions at once.**

Hybrid approach:

- Digitize signals from subarrays instead of individual antennas
- Accept limited range of steering on reception

Digital Beam Forming in Multi-static Radar Experiments

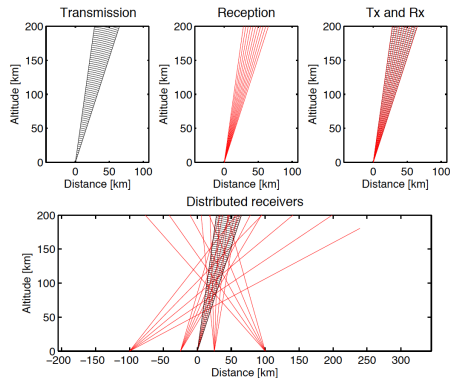
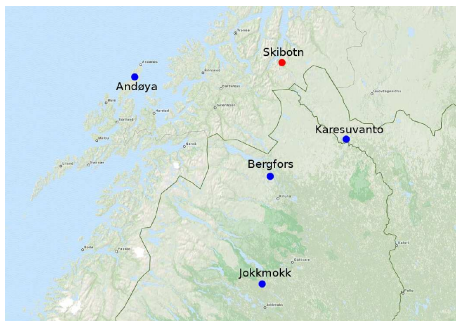


KAIRA = The Kilpisjärvi
Atmospheric Imaging
Receiver Array

McKay et al. (2015)
10.1109/TGRS.2014.2342252

EISCAT_3D

Major planned facility:



EISCAT_3D Array



EISCAT_3D Antennas to Scale



Introduction to Phased Array Summary

- Phased arrays work due to the superposition principle
- Focusing is achieved in the far field
- Gain pattern is dominated by the array factor
- Gain pattern typically has a main lobe and many smaller side lobes
- Side lobe pattern depends on the shape of the array (rectangular vs “circular”)

Fourier Analogy Summary

- In Fraunhofer diffraction the far field radiation pattern is a Fourier transform of the aperture illumination
- Phased array far field radiation patterns are a discrete-space Fourier transform of element currents
- Fourier analogy provides useful intuition for designing radiation patterns by reusing concepts from signal processing
- Shift theorem provides recipe for steering beams

Phased Array Steering Summary

- Limitations on phased array steering are determined by
 - Inter-element spacing
 - Pattern of elements (e.g. rectangular vs hexagonal)
 - Granularity of phase control (individual elements vs subarrays)
- In general phased arrays lose antenna gain as they are steered further from boresight

Practical results for ISR users

- Explains 6-pointed star patterns for AMISR FOV
- Explains why Jicamarca can only steer $\pm 3^\circ$

Summary of Types of Phased Arrays

- Transmitter architecture
 - Passive: One high power transmitter feeds whole array
 - Active: Small transmitters for each element distributed throughout the array
- Phasing architecture
 - Manual changing of cables
 - Electronic steering with delay shifters
 - Digital beam forming