### 6.002 - Circuits \& Electronics - Fall 2007 <br> Problem Set \#7-SOLUTIONS P7.1 and P7.2

## Problem 7.1

(A) Circuit: Inductor is connected to the voltage source.
$i(t)=1 / \mathrm{L} \int \mathrm{Vdt}=\mathrm{V} / \mathrm{L} \mathrm{t}+\mathrm{K}$ and since $i(0)=0-->\mathrm{K}=0$
$-->i(t)=\mathrm{V} / \mathrm{L} \mathrm{t}$ and $v(\mathrm{t})=0$ for $0 \leq \mathrm{t} \leq \mathrm{T}_{1}$
(B) Circuit: Inductor is connected to the capacitor.
$v(t)=\mathrm{L} \mathrm{d} i / \mathrm{dt}=\mathrm{L} \mathrm{d} / \mathrm{dt}(-\mathrm{C} \mathrm{d} v / \mathrm{dt})=-\mathrm{LC} \mathrm{d}^{2} v / \mathrm{dt}^{2}$
$-->\mathrm{d}^{2} v / \mathrm{dt}^{2}+1 / \mathrm{LC} v=0$
$v(t)=\mathrm{A} \cos \left[\omega\left(\mathrm{t}-\mathrm{T}_{1}\right)\right]+\mathrm{B} \sin \left[\omega\left(\mathrm{t}-\mathrm{T}_{1}\right)\right]$ and $\omega=1 / \sqrt{ }(\mathrm{LC})$
$i(t)=-\mathrm{Cd} v / \mathrm{dt}=\mathrm{CA} \omega \sin \left[\omega\left(\mathrm{t}-\mathrm{T}_{1}\right)\right]-\mathrm{CB} \omega \cos \left[\omega\left(\mathrm{t}-\mathrm{T}_{1}\right)\right]$
We know from before at $\mathrm{t}=\mathrm{T}_{1}$ :
$v\left(\mathrm{~T}_{1}\right)=0$--> A $=0$ and
$i\left(\mathrm{~T}_{1}\right)=\mathrm{V} / \mathrm{L} \mathrm{T}_{1}=-\mathrm{CB} \omega-->\mathrm{B}=-\mathrm{V} / \mathrm{LT}_{1} /(\mathrm{C} \omega)$
Therefore:
$v(\mathrm{t})=-\mathrm{T}_{1} / \sqrt{ }(\mathrm{LC}) \sin \left[\left(\mathrm{t}-\mathrm{T}_{1}\right) / \sqrt{ }(\mathrm{LC})\right]$ and
$\left.i(t)=\mathrm{V} / \mathrm{L} \mathrm{T}_{1} \cos \left[\left(\mathrm{t}-\mathrm{T}_{1}\right) / \sqrt{(L C}\right)\right]$ for $\mathrm{T}_{1} \leq \mathrm{t} \leq \mathrm{T}_{2}$
For $\mathrm{T}_{2}$ it is required that: $i\left(\mathrm{~T}_{2}\right)=0$
--> $\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right) / \sqrt{ }(\mathrm{LC})=\pi / 2 \rightarrow \mathrm{~T}_{2}=\pi / 2 \sqrt{ }(\mathrm{LC})+\mathrm{T}_{1}$
(C) Circuit: All three elements are disconnected.
$v\left(\mathrm{~T}_{2}\right)=-\mathrm{VT}_{1} / \sqrt{ }(\mathrm{LC})$ and $i\left(\mathrm{~T}_{2}\right)=0$
Therefore:
$v(\mathrm{t})=-\mathrm{VT}_{1} / \sqrt{ }(\mathrm{LC})$ and
$i(\mathrm{t})=0$ for $\mathrm{T}_{2} \leq \mathrm{t} \leq \mathrm{T}_{3}$
(D) Circuit: Inductor is connected to the voltage source. Using information from part (A):
$i(t)=\mathrm{V} / \mathrm{L}\left(\mathrm{t}-\mathrm{T}_{3}\right)$ and
$v(\mathrm{t})=v\left(\mathrm{~T}_{3}\right)=v\left(\mathrm{~T}_{2}\right)=-\mathrm{VT}_{1} / \sqrt{ }(\mathrm{LC})$ for $\mathrm{T}_{3} \leq \mathrm{t} \leq \mathrm{T}_{4}$
(E) Circuit: Inductor is connected to the capacitor.

Using information from part (B): $\mathrm{d}^{2} v / \mathrm{dt}^{2}+1 /(\mathrm{LC}) v=0$
$v(t)=A \cos \left[\omega\left(\mathrm{t}-\mathrm{T}_{4}\right)\right]+\mathrm{B} \sin \left[\omega\left(\mathrm{t}-\mathrm{T}_{4}\right)\right]$ and $\omega=1 / \sqrt{ }(\mathrm{LC})$
$i(t)=\mathrm{CA} \omega \sin \left[\omega\left(\mathrm{t}-\mathrm{T}_{4}\right)\right]-\mathrm{CB} \omega \cos \left[\omega\left(\mathrm{t}-\mathrm{T}_{4}\right)\right]$
We know that at T4: $v\left(\mathrm{~T}_{4}\right)=-\mathrm{VT} \mathrm{T}_{1} / \mathcal{}(\mathrm{LC})-->\mathrm{A}=-\mathrm{VT} \mathrm{T}_{1} / \mathcal{V}(\mathrm{LC})$
$i\left(\mathrm{~T}_{4}\right)=\mathrm{V} / \mathrm{L}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)-->\mathrm{V} / \mathrm{L}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)=-\mathrm{CB} \omega-->\mathrm{B}=\mathrm{V} / \mathrm{L}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) /(\mathrm{C} \omega)$
Therefore:
$v(t)=-\mathrm{VT}_{1} / \sqrt{ }(\mathrm{LC}) \cos \left[\left(\mathrm{t}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]-\mathrm{V}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) / \sqrt{ }(\mathrm{LC}) \sin \left[\left(\mathrm{t}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]$ and
$i(t)=-\mathrm{VT}_{1} / \mathrm{L} \sin \left[\left(\mathrm{t}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]+\mathrm{V}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) / \mathrm{L} \cos \left[\left(\mathrm{t}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]$ for $\mathrm{T}_{4} \leq \mathrm{t} \leq \mathrm{T}_{5}$
We know that at $\mathrm{T}_{5}: i\left(\mathrm{~T}_{5}\right)=0$
$-->\mathrm{VT}_{1} / \mathrm{L} \sin \left[\left(\mathrm{T}_{5}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]=\mathrm{V}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) / \mathrm{L} \cos \left[\left(\mathrm{T}_{5}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]$
$\tan \left[1 / \sqrt{ }(\mathrm{LC})\left(\mathrm{T}_{5}-\mathrm{T}_{4}\right)\right]=\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) / \mathrm{T}_{1}$
$-->\mathrm{T}_{5}=\sqrt{ }(\mathrm{LC}) \tan -1\left[\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right) / \mathrm{T}_{1}\right]+\mathrm{T}_{4}$
$v\left(\mathrm{~T}_{5}\right)=-\mathrm{VT} / \sqrt{ }(\mathrm{LC}) \cos \left[\left(\mathrm{T}_{5}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right]-\mathrm{V}\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right) / \sqrt{ }(\mathrm{LC}) \sin \left[\left(\mathrm{T}_{5}-\mathrm{T}_{4}\right) / \sqrt{ }(\mathrm{LC})\right] \ldots$
$v\left(\mathrm{~T}_{5}\right)=-\mathrm{V} \sqrt{ }\left[\mathrm{T}_{1}{ }^{2}+\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)^{2}\right] / \sqrt{ }(\mathrm{LC})$
(F)



## Problem 7.2

(A) $\mathrm{E}_{\mathrm{L}}=1 / 2 \mathrm{~L} i^{2}(\mathrm{t})=1 / 2 \mathrm{~L} i^{2}\left(\mathrm{~T}_{1}\right)=1 / 2 \mathrm{~L}\left(\mathrm{~V} / \mathrm{L} \mathrm{T} \mathrm{T}_{1}\right)^{2}=1 / 2 \mathrm{~V}^{2} \mathrm{~T}_{1}{ }^{2} / \mathrm{L}$
(B) $\quad \mathrm{E}_{\mathrm{L}}=\mathrm{E}_{\mathrm{C}}=1 / 2 \mathrm{C} v^{2}(\mathrm{t})=1 / 2 \mathrm{C} v^{2}\left(\mathrm{~T}_{2}\right)-->1 / 2 \mathrm{~V}^{2} \mathrm{~T}_{1}{ }^{2} / \mathrm{L}=1 / 2 \mathrm{C} v^{2}\left(\mathrm{~T}_{2}\right)-->v\left(\mathrm{~T}_{2}\right)=-\mathrm{VT} 2 / \mathrm{V}(\mathrm{LC})$ Negative solution is selected because of the direction of current $i$.
(C) $\quad \mathrm{E}_{\mathrm{L}}=1 / 2 \mathrm{~L} i^{2}\left(\mathrm{~T}_{4}\right)=1 / 2 \mathrm{LV}^{2} / \mathrm{L}^{2}\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right)^{2}=1 / 2 \mathrm{~V}^{2}\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right)^{2} / \mathrm{L}$
(D) $\mathrm{E}_{\mathrm{L}}+1 / 2 \mathrm{C} v^{2}\left(\mathrm{~T}_{2}\right)=\mathrm{E}_{\mathrm{C}}-->1 / 2 \mathrm{~V}^{2}\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right)^{2} / \mathrm{L}+1 / 2 \mathrm{C} v^{2}\left(\mathrm{~T}_{2}\right)=1 / 2 \mathrm{C} v^{2}\left(\mathrm{~T}_{5}\right)$ $v\left(\mathrm{~T}_{5}\right)=-\mathrm{V} / \sqrt{ }(\mathrm{LC}) \sqrt{ }\left[\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)^{2}+\mathrm{T}_{1}{ }^{2}\right]$
(E) $\quad$ From part (D), we see that if $\left(\mathrm{T}_{4}-\mathrm{T}_{3}\right)=\mathrm{T} 1$ then: $v\left(\mathrm{~T}_{5}\right)=-\mathrm{V} / \sqrt{ }(\mathrm{LC}) \sqrt{ }\left(\mathrm{T}_{1}{ }^{2}+\mathrm{T}_{1}{ }^{2}\right)=-\mathrm{VT} \mathrm{T}_{1} / \sqrt{ }(\mathrm{LC}) \sqrt{ }(2)$ Hence, after $n$ switching cycles: $v\left(\mathrm{nT}_{1}\right)=-\mathrm{VT}_{1} / \sqrt{ }(\mathrm{LC}) \mathrm{s} \sqrt{ }(\mathrm{n})=-\mathrm{VT} \mathrm{T}_{1} \sqrt{ }(\mathrm{n} /(\mathrm{LC}))$

Solution of PS 8
Exercise 8.1
at time $t=\infty$
 $v_{2}(t \rightarrow \infty)=0$
time constant $\tau=\left(R_{1}+R_{2}\right) \cdot C=3 \mathrm{k} \Omega \cdot 3 \mu \mathrm{~F}=9 \mathrm{~ms}$

$$
\begin{aligned}
& \therefore v_{2}(t)=4 \cdot e^{-t / \tau}
\end{aligned}
$$

Excercise 8.2.


$$
\begin{aligned}
& R_{2}=1 \Omega \\
& L=1 H \\
& C=0.5 \mathrm{~F} \\
& R_{1}=2 \Omega
\end{aligned}
$$

From inductor $L \quad i_{1}\left(0^{+}\right)=i_{1}\left(0^{-}\right)=2 \mathrm{~A}$
From inductor $L \quad i_{1}\left(0^{+}\right)=i_{1}(0)=2 \mathrm{~A}$
From capacitor $C \quad v_{4}\left(0^{+}\right)=v_{4}\left(0^{-}\right)=4 \mathrm{~V} \quad \Rightarrow i_{3}\left(0^{+}\right)=\frac{\left.V_{4} 10^{+}\right)}{R_{2}}=4 \mathrm{~A}$
From KVL $\quad v_{1}\left(0^{+}\right)=10-v_{4}\left(0^{+}\right)=6 \mathrm{~V} \Rightarrow i_{2}\left(0^{+}\right)=\frac{v_{1}\left(0^{+}\right)}{R_{1}}=3 \mathrm{~A}$
FromkCL $\quad i_{4}\left(0^{+}\right)=i_{1}\left(0^{+}\right)+i_{2}\left(0^{+}\right)-i_{3}\left(0^{+}\right)$

$$
\begin{aligned}
& =2+3-4 \\
& =1 \mathrm{~A}
\end{aligned}
$$

$$
\therefore \quad \begin{array}{lll}
v_{1}\left(0^{+}\right)=6 \mathrm{~V} & i_{1}\left(0^{+}\right)=2 \mathrm{~A} & i_{2}\left(0^{+}\right)=3 \mathrm{~A} \\
v_{4}\left(0^{+}\right)=4 \mathrm{~V} & i_{3}\left(0^{+}\right)=4 \mathrm{~A} & i_{4}\left(0^{+}\right)=1 \mathrm{~A}
\end{array}
$$

$t=\infty$ Inductor $L=S . C$. Capacitor $C=O . C$.


$$
\begin{aligned}
& i_{1}(\infty)=i_{3}(\infty)=\frac{10 \mathrm{~V}}{R_{2}}=10 \mathrm{~A} \\
& i_{2}(\infty)=i_{4}(\infty)=0 \mathrm{~A} \\
& v_{1}(\infty)=0 \mathrm{~V} \\
& v_{4}(\infty)=10 \mathrm{~V}
\end{aligned}
$$

$$
\therefore \begin{array}{ll}
v_{1}(\infty)=0 \mathrm{~V} & i_{1}(\infty)=i_{3}(\infty)=10 \mathrm{~A} \\
v_{4}(\infty)=10 \mathrm{~V} & i_{2}(\infty)=i_{4}(\infty)=0 \mathrm{~A}
\end{array}
$$


a) From KCL , we have $i_{1}=i_{2}+i \Rightarrow \frac{V_{s}-v}{R_{1}}=\frac{v}{R_{2}}+i$

From inductor $L$, we have $i v=L \frac{d i}{d t}$

Therefore we have differential equation for $v(t)$ \& $i(t)$ :

$$
\left\{\begin{array}{l}
\frac{V_{s}-v(t)}{R_{1}}=\frac{v(t)}{R_{2}}+i(t) \cdots(1) \\
v(t)=1 \frac{d i(t)}{d t}
\end{array} \cdots(2),\right.
$$

We can plug (2) into (1) and colve for it) first.
Rearrange (1) after plugging (2) into (1), we have,

$$
L\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \frac{d i(t)}{d t}+i(t)=\frac{V_{3}}{R_{1}}
$$

Hence the particular solution $i_{p}(t)=\frac{V_{s}}{R_{1}}$
the homogeneous solution $\left(i_{n}(t)=A e^{-i / \tau} \quad \tau=L\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)\right.$
From the initial condition $i\left(0^{-}\right)=i\left(0^{+}\right)=0 \Rightarrow A=-\frac{V_{s}}{R_{1}}$

$$
\begin{aligned}
& \therefore i^{i}(t)=\frac{V_{s}}{R_{1}}\left(1-e^{-t / \tau}\right) \quad t \geqslant 0 \\
& \therefore v(t)=L \frac{d i(t)}{d t}=\frac{-V_{s} R_{2}}{R_{1}+R_{2}} e^{-t / \tau} \quad t \geqslant 0
\end{aligned}
$$

b) Let's say in pant a) $v_{s}(t)=v_{s a}(t)=V_{s} u_{-1}(t)=\left\{\begin{array}{l}0, t<0 \\ v_{s}, t \geqslant 0\end{array}\right.$ in this pant, $\quad v_{s}(t)=v_{s b}(t)=A \mu_{0}(t)$

$$
\because v_{s b}(t)=\Lambda H_{0}(t)=\frac{\Lambda}{V_{s}} \frac{d\left(v_{s a}(t)\right)}{d t}
$$

$\therefore$ The output $v(t)$ sutisfy the similiar relation shown in inputs above, $\left.\begin{array}{r}\text { i.e., } V_{b}(t)=\frac{\Lambda}{V_{s}} \frac{d\left(V_{a}(t)\right)}{d t} \\ v_{b}(t)=V_{s} \frac{R_{2}}{R_{1}+R_{2}} e^{-t / \tau}\end{array}\right\} \Rightarrow \begin{array}{r} \\ V_{b}(t)=-\Lambda\left(\frac{R_{2}}{R_{1}+R_{2}}\right)^{2} \cdot \frac{R_{1}}{L} e^{-t / \tau} \\ t \geqslant 0\end{array}$ ( $v(t)$ in pant $u$ )

$$
\text { c) Similiarly, } V_{s c}(t)=M U-2(t)
$$

$$
\therefore V_{s c}(t)=\frac{M}{V_{s}} \int V_{s} U_{-1}(t) d t
$$



$$
\begin{aligned}
& =\frac{M}{V_{s}} \int V_{s a}(t) d t \\
\therefore V_{c}(t) & =\frac{M}{V_{s}} \int V_{a}(t) d t \quad \& \quad V_{c}\left(0^{+}\right)=V_{c}\left(0^{-}\right)=0
\end{aligned}
$$

$V(t) \frac{\ell}{4}$

$$
\begin{aligned}
\therefore V_{c}(t) & =\left[\frac{M}{V_{s}} \int V_{s} \frac{R_{2}}{R_{1}+R_{2}} e^{-t / \tau} d t\right. \\
& \left.=M \frac{L}{R_{1}}\left(1-e^{-t / \tau}\right)\right]
\end{aligned}
$$




Therefore this input can be decomposed into two ramps and two steps,

$$
\begin{aligned}
V_{s}(t) & =V_{s 1}(t)+V_{s 2}(t)=V_{s i a}(t)-V_{s 16}(t)+V_{s 2 a}(t) V_{v} V_{s 2 b}(t) \\
& =\frac{V_{6}-V_{a}}{\tau}\left(U_{-2}(t)-U_{-2}(t-\tau)+V_{a}\left(U_{-1}(t)-U_{-1}(t-\tau)\right.\right.
\end{aligned}
$$

e) (sing results from $a) \&(C)$, by superposing response $t_{0}$ step and ramp inputs, we can derive $v_{e}(t)(v(t)$ - in pant $e$ ) for $t>0$,

$$
\begin{aligned}
V_{e}(t) & =\frac{R_{2}}{R_{1}+R_{2}} \cdot V_{a} \cdot\left(e^{-t / \tau}-e^{-(t-\tau) / \tau}\right) \\
& -\left(\frac{L^{2}}{R_{1}}\right) \cdot \frac{\left(V_{b}-V_{a}\right)}{\tau} \cdot\left(e^{-t / \tau}-e^{-(t-\tau) \tau}\right) \\
& =\left(\frac{R_{2} V_{a}}{R_{1}+R_{2}}+\frac{L\left(V_{b}-V_{a}\right)}{R_{1} \tau}\right) \cdot\left(e^{-t \tau}-e^{-(f-\tau) / \tau}\right)
\end{aligned}
$$

$\because \tau=L \cdot\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)$ from past a)
$\therefore$ we can further simply $\left(V_{e}(t)=\frac{R_{2}}{R_{1} \neq R_{2}} \cdot\left(2 V_{a}-V_{b}\right) \cdot\left(e^{-t / \tau}-e^{-(t-\tau)}\right)\right.$

a) when $V_{w}=V$, from $K V L$, we have, $V=V_{c}(t)+v_{R_{1}}(t)+v_{R_{2}}(t)$

$$
\begin{aligned}
& \because v_{R_{1}}(t)=R_{1} \cdot i_{c}(t), V_{R_{2}}(t)=R_{2}-i_{c}(t), \text { and } i_{c}(t)=C \frac{d v_{c}(t)}{d t} \\
& \therefore V=v_{c}(t)+\left(R_{1}+R_{2}\right) \cdot C \frac{d v_{c}(t)}{d t} \Rightarrow V=v_{c}(t)+\tau \cdot \frac{d v_{c}(t)}{d t} \text { where } \tau=\left(R_{1}+R_{2}\right) C
\end{aligned}
$$

IF define $t=0$ when source $V_{\text {in }}$ jumps from " $-V$ " to " $V$ ", then we have

$$
\tau \cdot \frac{d v_{c}(t)}{d t}+v_{c}(t)=V \quad \& \quad v_{c}\left(t=0^{-}\right)=-V
$$

Similarly, we define $t$ ' $=0$ when source $V_{i n}$ jumps from " $V$ " to " $-V$ ", then we have $\tau \cdot \frac{d v_{c}(t)}{d t}+v_{c}(t)=-V \quad \& v_{c}\left(t^{\prime}=0^{-}\right)=V$

b) $\because i_{c}(t)=c \frac{d V_{c}(t)}{d t} \quad \therefore i_{c}(t)= \begin{cases}\frac{2 V c}{\tau} e^{-t / \tau} & V_{N}=V\left(\operatorname{or}, \frac{2 V}{R_{1}+R_{2}} e^{-t / \tau}\right) \\ \frac{-2 V C}{\tau} e^{-t^{\prime} / \tau} & V_{N N}=-V\left(\text { or } \frac{-2 v}{R_{1}+R_{2}} e^{-t / \tau}\right)\end{cases}$

d) For $V_{R 2}$, we have $V_{R 2}\left(0^{+}\right)=206 \mathrm{mV}, \quad V_{R 2}(1.1 \mathrm{~ms})=10.3 \mathrm{mV}=0.05 \times V_{R_{2}} 10^{\dagger}$ eePaz10 for xyperimential data wed
$\therefore 3 \tau=1.1 \mathrm{~ms} \Rightarrow \tau \approx 0.367 \mathrm{~ms}$ (Don't need to be so accurate net tally. $2,37 \mathrm{~ms}$ is fine)
import ge, e) Before the input switches from positive to negative, $v_{c}=1,19 v=V_{\text {in }}$, offer the input switches to restive, $v_{c}$ keeps the same as 119 V while $V_{i N}=-1.14 \mathrm{~V}$,
So we have, $\frac{-1.19-1.14}{R_{1}+R_{2}}=\frac{-190 \times 10^{-3}}{R_{2}} \Rightarrow R_{1} \times 34 K$ (given $R_{2}=3 k \rightarrow$

$$
\text { f) } \begin{aligned}
\tau=\left(R_{1}+R_{2}\right) \cdot C \Rightarrow C=\frac{\tau}{R_{1}+R_{2}}=\frac{0.36) \mathrm{ms}}{34 k+3 k} & \approx 9.9 \mathrm{fF} \\
& =9.9 \times 10^{-9} \mathrm{~F}
\end{aligned}
$$

problem 8.3.

a) $V_{c}(t)=0$, because capacitor behaves like short cirmit @ $t=0$ $i(0)=0$. because inductor behaves like open civanit $0 t=0$

$$
\begin{aligned}
& \therefore V_{R}(0)=i(0)-R=0 \\
& \because V_{L}(0)=V_{0} u(0)-V_{R}(0)-V_{C}(0)=V_{0} \\
& \because V_{L}=L \frac{d i}{d t} \\
& \therefore \frac{d i}{d t} V_{t=0}=\frac{V_{L}(0)}{L}=\frac{V_{0}}{L}
\end{aligned}
$$

6) Because capacitor behaves like open circuit a $, t=\infty, \therefore i(t=\infty)=0$
b) $v_{c}(t)=V_{0}\left(1-\cos \left(\omega_{0} t\right)\right) \quad \omega_{0}=\frac{1}{\sqrt{L C}}, t>0$
$i(t)=\sqrt{\frac{C}{L}} V_{0} \sin \left(\omega_{0} t\right)$
d) From $k V L$, we hame, $V_{0}=v_{L}(t)+v_{12}(f)+v_{c}(t)$

$$
\begin{aligned}
& =L \frac{d i(t)}{d t}+i(t) \cdot R+\frac{1}{c} \int i(t) d t \\
& \Rightarrow 0=L \frac{d^{2} i^{\prime}(t)}{d t^{2}}+R \frac{d i(t)}{d t}+\frac{1}{c} i(t) \\
& \Rightarrow \frac{d^{2} i(t)}{d t^{2}}+\frac{R}{L} \frac{d i(t)}{d t}+\frac{1}{L C} i(t)=0 \\
& \frac{d^{2} i(t)}{d t^{2}}+\frac{R}{L} \frac{d i(t)}{d t}+\frac{1}{L C} i(t)=0 \Rightarrow s^{2}+\frac{R}{L} s+\frac{1}{L C}=0 \\
& \therefore S=-\frac{R}{2 L} \pm \sqrt{\frac{R^{2}}{4 L^{2}}-\frac{1}{L C}} \\
& =-\alpha \pm \sqrt{\alpha^{2}-\omega_{0}^{2}} \quad \therefore \alpha=\frac{R}{2 L}, w_{c}=\frac{1}{\sqrt{L C}} \\
& =-\alpha \pm j \omega_{d} \quad \therefore \omega_{d}=\sqrt{\omega_{0}^{2}-\alpha^{2}} \quad\left(\omega_{0}>2\right) \\
& \left.\dot{c}_{t}\right)=e^{-\alpha t}\left(A \cos \left(\omega_{d t}\right)+B \sin \left(\omega_{d} t\right)\right) \\
& \because i(\sigma)=0 \quad \therefore \quad A=0 \\
& \therefore i(t)=B e^{-\alpha t} \sin \left(\omega_{a} t\right) \\
& \frac{d_{i}(t)}{d t}=-\alpha B e^{-\alpha t} \sin \left(\omega_{0 t}\right)+B e^{-\alpha t} \cdot \omega_{t} \cdot \cos \left(\omega_{d} t\right) \\
& \left.\because \frac{\text { dict }}{d t}\right|_{t=0}=\frac{V_{0}}{L} \Rightarrow B \omega_{d}=\frac{V_{0}}{L} \Rightarrow B=\frac{V_{0}}{\omega_{d} L} \\
& \because i(t)=\frac{V_{0}}{\omega_{0} L} e^{-\alpha t} \sin \left(\omega_{d} t\right)=I e^{-\alpha t} \sin \left(\omega t+\phi_{-}\right)
\end{aligned}
$$

where $I=\frac{V_{0}}{L \sqrt{\frac{1}{L C}-\left(\frac{R}{2 L}\right)^{2}}}, \omega=\omega_{0}=\sqrt{\frac{1}{E C}-\left(\frac{R}{2 L}\right)^{2}}, \phi=0, \alpha=\frac{R}{2 L}$

20 k sampling rate, 5 ms duration

time constant is about 0.367 ms
R1 is about 34 Kohm
C is about $9.9 \mathrm{nF}=9.9 \times 10^{-9} \mathrm{~F}$

| $\begin{aligned} & \text { TIME, VIN, VOUT } \\ & \text { s, V, V } \end{aligned}$ | $\begin{aligned} & \text { TIME, VIN, VOUT } \\ & \mathrm{s}, \mathrm{~V}, \mathrm{~V} \end{aligned}$ |
| :---: | :---: |
| $0,512.032192234953 \mathrm{e}-3,206.099181203987 \mathrm{e}-3$ | $2.95 \mathrm{e}-3,-1.14805397227293,-38.0029255723793 \mathrm{e}-3$ |
| $50 \mathrm{e}-6,1.182187934163,179.048278663754 \mathrm{e}-3$ | 3e-3, -1.14837600601138, -32.8503763883594e-3 |
| $100 \mathrm{e}-6,1.18379811410142,153.607551136429 \mathrm{e}-3$ | $3.05 \mathrm{e}-3,-1.14869803974978,-28.9859644501187 \mathrm{e}-3$ |
| $150 \mathrm{e}-6,1.18476422207144,132.353274082149 \mathrm{e}-3$ | 3.1e-3, -1.14869803974978, -25.4435868021231e-3 |
| $200 \mathrm{e}-6,1.18605236603962,113.67527424834 \mathrm{e}-3$ | $3.15 \mathrm{e}-3,-1.14902007348815,-21.901209117733 \mathrm{e}-3$ |
| $250 \mathrm{e}-6,1.18701847402186,97.8955856072151 \mathrm{e}-3$ | $3.2 \mathrm{e}-3,-1.14934210722647,-19.3249344152606 \mathrm{e}-3$ |
| $300 \mathrm{e}-6,1.18766254601292,84.6921732706709 \mathrm{e}-3$ | $3.25 \mathrm{e}-3,-1.14966414096476,-17.7147627163963 \mathrm{e}-3$ |
| $350 \mathrm{e}-6,1.18830661800632,72.7768991940769 \mathrm{e}-3$ | $3.3 \mathrm{e}-3,-1.149986174703,-15.4605223252762 \mathrm{e}-3$ |
| $400 \mathrm{e}-6,1.18862865400389,62.7938320656817 \mathrm{e}-3$ | 3.35e-3, -1.14966414096476, -13.8503506082418e-3 |
| 450c-6, 1.18895069000205, 54.7429717032384e-3 | $3.4 \mathrm{e}-3,-1.14966414096476,-12.2401788836208 \mathrm{e}-3$ |
| $500 \mathrm{e}-6,1.1895947620001,47.9802491530912 \mathrm{e}-3$ | 3.45e-3, -1.14934210722647, -11.2740758452027e-3 |
| $550 \mathrm{e}-6,1.18927272600078,41.2175267430959 \mathrm{e}-3$ | 3.5e-3, -1.14966414096476, -9.66390410842352e-3 |
| $600 \mathrm{e}-6,1.19023883400048,36.0649764291233 \mathrm{e}-3$ | 3.55e-3, -1.149986174703, -9.0198354115813e-3 |
| $650 \mathrm{e}-6,1.1908829060032,31.234460583027 \mathrm{e}-3$ | 3.6e-3, -1.149986174703, -8.37576671352073e-3 |
| $700 \mathrm{e}-6,1.19023883400048,27.6920823407948 \mathrm{e}-3$ | $3.65 \mathrm{e}-3,-1.15030820844121,-7.08762931374214 \mathrm{e}-3$ |
| $750 \mathrm{e}-6,1.1908829060032,24.1497041364478 \mathrm{e}-3$ | 3.7e-3, -1.149986174703, -6.76559496303509e-3 |
| $800 \mathrm{e}-6,1.1908829060032,21.5734291024798 \mathrm{e}-3$ | 3.75e-3, -1.149986174703, $-6.44356061202293 \mathrm{e}-3$ |
| $850 \mathrm{e}-6,1.19056087000155,18.9971540884594 \mathrm{e}-3$ | 3.8e-3, -1.149986174703, -5.79949190908297e-3 |
| $900 \mathrm{e}-6,1.19120494200543,16.4208790943484 \mathrm{e}-3$ | 3.85e-3, -1.15063024217937, -5.79949190908297e-3 |
| 950e-6, 1.19120494200543, 14.8107072331223e-3 | 3.9e-3, -1.149986174703, -5.79949190908297e-3 |
| $1 \mathrm{e}-3,1.1908829060032,12.5564666404284 \mathrm{e}-3$ | 3.95e-3, -1.149986174703, -5.15542320492167e-3 |
| $1.05 \mathrm{e}-3,1.19152697800825,11.5903635339193 \mathrm{e}-3$ | $4 \mathrm{e}-3,-1.149986174703,-4.51135449953844 \mathrm{e}-3$ |
| 1.1e-3, 1.1908829060032, 10.3022260629046e-3 | 4.05e-3, -1.15063024217937, -4.51135449953844e-3 |
| $1.15 \mathrm{e}-3,1.19120494200543,9.33612296288898 \mathrm{e}-3$ | $4.1 \mathrm{e}-3,-1.14966414096476,-3.86728579293267 \mathrm{e}-3$ |
| $1.2 \mathrm{e}-3,1.1908829060032,8.04798550052501 \mathrm{e}-3$ | $4.15 \mathrm{e}-3,-1.15063024217937,-3.86728579293267 \mathrm{e}-3$ |
| $1.25 \mathrm{e}-3,1.19120494200543,8.04798550052501 \mathrm{e}-3$ | $4.2 \mathrm{e}-3,-1.149986174703,-3.86728579293267 \mathrm{e}-3$ |
| $1.3 \mathrm{e}-3,1.19056087000155,7.08188240699204 \mathrm{e}-3$ | $4.25 \mathrm{e}-3,-1.149986174703,-3.86728579293267 \mathrm{e}-3$ |
| $1.35 \mathrm{e}-3,1.19184901401164,6.75984804309779 \mathrm{e}-3$ | $4.3 \mathrm{e}-3,-1.14966414096476,-3.86728579293267 \mathrm{e}-3$ |
| $1.4 \mathrm{e}-3,1.1908829060032,6.43781367951178 \mathrm{e}-3$ | $4.35 \mathrm{e}-3,-1.149986174703,-3.22321708510377 \mathrm{e}-3$ |
| $1.45 \mathrm{e}-3,1.1908829060032,6.11577931623395 \mathrm{e}-3$ | $4.4 \mathrm{e}-3,-1.15063024217937,-2.90118273073046 \mathrm{e}-3$ |
| $1.5 \mathrm{e}-3,1.19120494200543,5.14967622824875 \mathrm{e}-3$ | $4.45 \mathrm{e}-3,-1.15030820844121,-2.90118273073046 \mathrm{e}-3$ |
| $1.55 \mathrm{e}-3,1.1908829060032,4.82764186620286 \mathrm{e}-3$ | 4.5e-3, -1.14966414096476, $-3.54525143917115 \mathrm{e}-3$ |
| $1.6 \mathrm{e}-3,1.1908829060032,4.82764186620286 \mathrm{e}-3$ | $4.55 \mathrm{e}-3,-1.15030820844121,-3.22321708510377 \mathrm{e}-3$ |
| $1.65 \mathrm{e}-3,1.19056087000155,4.18357314303441 \mathrm{e}-3$ | $4.6 \mathrm{e}-3,-1.149986174703,-3.22321708510377 \mathrm{e}-3$ |
| $1.7 \mathrm{e}-3,1.19120494200543,4.50560750446477 \mathrm{e}-3$ | $4.65 \mathrm{e}-3,-1.149986174703,-2.90118273073046 \mathrm{e}-3$ |
| $1.75 \mathrm{e}-3,1.19152697800825,4.18357314303441 \mathrm{e}-3$ | $4.7 \mathrm{e}-3,-1.14966414096476,-2.57914837605114 \mathrm{e}-3$ |
| $1.8 \mathrm{e}-3,1.19120494200543,4.18357314303441 \mathrm{e}-3$ | $4.75 \mathrm{e}-3,1.1805777542391,205.777146635314 \mathrm{e}-3$ |
| $1.85 \mathrm{e}-3,1.19184901401164,4.18357314303441 \mathrm{e}-3$ | $4.8 \mathrm{e}-3,1.18283200613662,176.472002360183 \mathrm{e}-3$ |
| $1.9 \mathrm{e}-3,1.19120494200543,3.21747006058892 \mathrm{e}-3$ | $4.85 \mathrm{e}-3,1.18412015009085,151.997378580386 \mathrm{e}-3$ |
| $1.95 \mathrm{e}-3,1.19184901401164,3.53950442109656 \mathrm{e}-3$ | $4.9 \mathrm{e}-3,1.18508625806262,130.743101638189 \mathrm{e}-3$ |
| $2 \mathrm{e}-3,1.19120494200543,3.53950442109656 \mathrm{e}-3$ | $4.95 \mathrm{e}-3,1.18605236603962,112.065101901535 \mathrm{e}-3$ |
| $2.05 \mathrm{e}-3,1.19152697800825,3.53950442109656 \mathrm{e}-3$ | $5 \mathrm{e}-3,1.18701847402186,96.2854133415102 \mathrm{e}-3$ |
| $2.1 \mathrm{e}-3,1.19184901401164,3.21747006058892 \mathrm{e}-3$ |  |
| $2.15 \mathrm{e}-3,1.19120494200543,3.53950442109656 \mathrm{e}-3$ |  |
| $2.2 \mathrm{e}-3,1.19152697800825,3.21747006058892 \mathrm{e}-3$ |  |
| $2.25 \mathrm{e}-3,1.19184901401164,3.21747006058892 \mathrm{e}-3$ |  |
| $2.3 \mathrm{e}-3,1.19152697800825,3.53950442109656 \mathrm{e}-3$ |  |
| $\begin{aligned} & 2.35 \mathrm{e}-3,1.19217105001562,2.8954357003887 \mathrm{e}-3 \\ & 2.4 \mathrm{e}-3,-1.13968109505891,-190.325127692427 \mathrm{e}-3 \end{aligned}$ |  |
| $2.45 \mathrm{e}-3,-1.14161329749541,-163.596289087623 \mathrm{e}-3$ |  |
| $2.5 \mathrm{e}-3,-1.14257939871309,-139.765756723489 \mathrm{e}-3$ |  |
| $2.55 \mathrm{e}-3,-1.14386753366942,-120.765736601765 \mathrm{e}-3$ |  |
| $2.6 \mathrm{e}-3,-1.14515566862509,-104.664023875416 \mathrm{e}-3$ |  |
| 2.65e-3, -1.14579973610267, -89.8504475438291e-3 |  |
| $2.7 \mathrm{e}-3,-1.1461217698414,-77.2911106152867 \mathrm{e}-3$ |  |
| $2.75 \mathrm{e}-3,-1.1461217698414,-67.3080476154098 \mathrm{e}-3$ |  |
| $2.8 \mathrm{e}-3,-1.14708787105735,-58.613121548975 \mathrm{e}-3$ |  |
| $2.85 \mathrm{e}-3,-1.14740990479592,-50.2402295793757 \mathrm{e}-3$ |  |
| $2.9 \mathrm{e}-3,-1.14740990479592,-44.4436119452075 \mathrm{e}-3$ |  |

