# Massachusetts Institute of Technology Department of Electrical Engineering and Computer Science 

6.002 - Circuits \& Electronics

Spring 2008
Problem Set \#1
Issued 2/6/08 - Due 2/13/08

Exercise 1.0: Problem 1.4 uses one of the MIT iLabs. The MIT iLabs are physical remoteaccess laboratories that run through the MIT iLab Service Broker. To obtain an iLab account, go to http://ilab.mit.edu, click on the register here link, provide the requested information, group yourself with Spring 2008 6.002 Students, and submit your request for an account. Account approval may take several hours to a day, so register early to avoid any last minute delays.

Exercise 1.1 (1 Point): Find the equivalent resistance, as viewed from its port, of each resistor network shown below.


Exercise 1.2 ( 1 Point): What is the largest-valued resistor that can be fabricated by combining a $1-\Omega$ resistor, a $2-\Omega$ resistor, and a $3-\Omega$ resistor? What is the smallest-valued resistor that can be fabricated by combining the same three resistors? How can one fabricate a $1.5-\Omega$ resistor by combining the same three resistors? Suppose that each individual resistor can dissipate up to 1 W of power before burning up. How much power can the fabricated $1.5-\Omega$ resistor dissipate before burning up?

Problem 1.1 (2 Points): Each network shown below has several of its branch currents or voltages specified numerically. Several other branch currents or voltages are labeled as unknowns. Find all labeled unknown branch currents and voltages. Note that the elements in the networks are drawn as non-specific boxes, indicating that they can each be a source or a resistor.


Consider the five-element network shown (twice) in the first row above. With the restriction that they must satisfy KVL, assign otherwise arbitrary non-zero voltages to the five elements using the sign conventions of the left-hand network. With the restriction that they must satisfy KCL, assign otherwise arbitrary non-zero currents to the five elements using the sign conventions of the righthand network. (Note that the voltage and current conventions of the two networks are consistent.) The product of the voltage and current for each element represents the power absorbed by that element. Sum the five powers, and note that they sum to zero. This remarkable fact, which follows from Tellegen's theorem, illustrates that power is always conserved in an electrical independent of the elements in the network. (Note that the selection of the voltages and currents were carried out independently, and thus not predicated on the behavior of any of the network elements.)

For more on Tellegen's Theorem, see Introductory Network Theory by Bose and Stevens, Harper \& Row, 1965, for example.

Problem 1.2 (2 Points): The network shown below has four elements: two resistors, a current source and a voltage source. The resistance of the resistors and the strengths of the sources are all given. Branch currents $\left(i_{k}\right)$ and voltages $\left(v_{k}\right)$ are also defined for each element.
(A) How many nodes are there in the network? Write a KCL equation for each node in terms of the branch currents $i_{k}$. How many of the KCL equations are independent?
(B) How many loops are there in the network? (For the purposes of this problem, a loop is any direction-independent closed path through the network that passes through each branch no more than once.) Write a KVL equation for each loop in terms of the branch voltages $v_{k}$. How many of the KVL equations are independent?
(C) Write an expression for the $v-i$ constitutive law for each element.
(D) By combining the independent equations from Parts (A) and (B) with the equations from Part (C), you should have a set of eight linear equations, matching in number the set of $i_{k}$ plus $v_{k}$. Solve the equations to find all four $i_{k}$ and all four $v_{k}$. Summarize your findings in a table.
(E) Find all four branch powers $v_{k} i_{k}$. Show that the sum of the four $v_{k} i_{k}$ is zero, and hence that power is conserved in the network. Which branch elements source power and which branch elements sink power?


Problem 1.3 (2 Points): Using the node method, analyze the network below and find all node voltages and then all branch currents. Note the definition of the reference node.


Problem 1.4 (2 Points): This is the first problem to make use of an MIT iLab this semester in 6.002. The MIT iLabs are physical remote-access laboratories that allow students to perform experiments through a Java-enabled web browser. They are located in Building 38. This semester in 6.002 , you will use three different iLabs. They are based on sophisticated equipment that would not otherwise be available to 6.002 students.

In this problem, you will use an Agilent 4155B Semiconductor Parameter Analyzer to make electrical measurements on the two-port three-resistor network shown below. This iLab is called the Microelectronics Device Characterization Laboratory. Each port in the two-port network is connected to one Sense-Measurement Unit (SMU) of the Agilent 4155B. Behind an SMU is sophisticated circuitry that allows it to act as a voltage source, a current source, an open circuit or a short circuit to ground. In all of these configurations, you can measure the current flowing out of the SMU, and the voltage across the SMU, with the polarities shown below.


The objective of this problem is to determine the resistance values of the three resistors in the network by performing and analyzing experiments that you define. You may perform as many experiments as you wish. As you do so, answer the following questions.
(A) With SMU1 set up as a voltage source, and SMU2 set up as an open circuit, what kinds of measurements can you make and what information do they yield?
(B) With SMU1 set up as a voltage source, and SMU2 set up as a short circuit to ground, what kinds of measurements can you make and what information do they yield?
(C) With SMU1 set up as a current source, and SMU2 set up as an open circuit, what kinds of measurements can you make and what information do they yield?
(D) With SMU1 set up as a current source, and SMU2 set up as a short circuit to ground, what kinds of measurements can you make and what information do they yield?
(E) How many experiments must be performed in order to determine $R_{1}, R_{2}$ and $R_{3}$ ? Here, consider one experiment to be a set of measurements with both SMUs programmed appropriately.
(F) Now select a set of measurements to make, perform the corresponding experiments following the guidelines given below, and determine the values of $R_{1}, R_{2}$ and $R_{3}$. Clearly explain your the rationale behind the selected experiments, and your analysis of their results.

To perform your experiments in this problem, carry out the following procedure.

- Start iLab
- Go to http://ilab.mit.edu and login to your account on the MIT iLab Service Broker. If your account does not open up under the My Labs section of iLab, move to that section by clicking on the My Labs tab on the top bar. If you have trouble running iLab on one computer, try switching to another computer, or try moving to an Athena workstation. You can request help and advice from iLab personnel, or report a bug, by clicking on the Help tab on the top bar.
- Select the Microelectronics Device Characterization Lab Client V7.0 by clicking on its link. Afterwards, you can click on the Documentation link for detailed information about the iLab client.
- Start the iLab client by clicking the Launch Lab link.
- Set Up Experiment

Experiments are run through the iLab window that will pop up following the launch of the iLab client.

- From the drop-down Devices menu on the top bar, select the T-shaped Resistor Network. A picture of the circuit similar to the one above appears. You must now program each SMU to be a voltage source, a current source, an open circuit or a short circuit to ground. This is accomplished through the Configuring SMU panel that is accessed by double clicking the SMU icon on the canvas.
- To program SMU1, click on SMU1. Name its voltage and current, and select the download option for both variables. Now program SMU1 to act in one of the following modes.
* Voltage Source: Set Mode to V. If you want a fixed value for the voltage, set Function to CONS. If you want to sweep voltage values, set Function to VAR1 and then make appropriate choices for Scale, Start, Stop, Step and Compliance. You should not exceed a voltage level of 10 V . Compliance refers to the maximum current that you will allow SMU1 to deliver when acting as a voltage source. This is intended to protect the device under test. A choice of 1 mA for compliance is prudent in this experiment.
* Current Source: Set Mode to I. If you want a fixed value for the current, set Function to CONS. If you want to sweep current values, set Function to VAR1 and then make appropriate choices for Scale, Start, Stop, Step and Compliance. You should not exceed a current level of 1 mA . Compliance refers to the maximum voltage that you will allow SMU1 to deliver when acting as a current source. This is intended to protect the device under test. A choice of 10 V is prudent in this experiment.
* Short Circuit To Ground: Set Mode to COMM.
* Open Circuit: Program SMU1 as a current source with zero current. That is, set Mode to I, Function to CONS and Value to 0 A. Set Compliance to 10 V.

Click on $O K$ to complete the programming of SMU1.

- Program SMU2 in a similar manner.
- Be careful to observe the following considerations.
* In a given experiment, at least one (but no more than one) variable must be set as VAR1. That is, you must sweep the value of one variable such as a voltage or a current. (Note that the " 1 " in "VAR1" indicates the "first" swept variable. It does not necessarily indicate an association with SMU1.)
* When sweeping through one or more of the SMU variables, note that the maximum number of distinct data points that the 4155B will allow you to measure during one experiment is 500 .
- Run Experiment \& View Measurements
- From the drop-down Experiment menu on the top bar, select Run Measurement. Your experiment will be queued, and run on a first-come-first-served basis. Hint: if you wait until the last minute like everyone else, you may find the queue and the wait longer than desired.
- The measured data will be displayed on a graph at the bottom of the iLab window. It is probably best viewed with a linear X-axis display. You can read off values from the graph by pressing the Tracking button on the bottom left of the canvas.
- If for any reason you wish to clear your data and run the experiment again, you can first clear the data by selecting Clear Current Result from the Results drop-down menu on the top bar.


## - Download Measurements

You can download the measured voltage and current data to your local computer. For this, select Export to CSV File ... from the Results drop-down menu on the top bar and proceed from there.

If you need help with the mechanics of the iLab, consult the documentation that is available through the Help item on the client menu. If you are stuck, request help from your TA. If nothing seems to work, send an error message through a page that is available in the Help section of the ilab.mit.edu web site.

