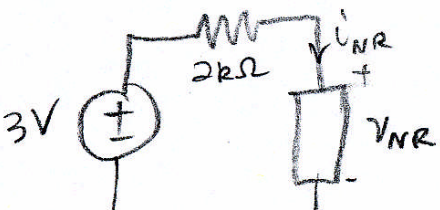
# 6.002 Spring 2008
# Problem Set #6 Solutions

## Exercise 6.1

First, with $i_s = 0$: KVL: $3V - 2k\Omega \, i_{NR} = v_{NR}$

given $v_{NR} = (1V/mA/mA) \, i_{NR}^2$



expect, assume units of $i_{NR}$ to be mA, to cancel out with kΩ for Volts

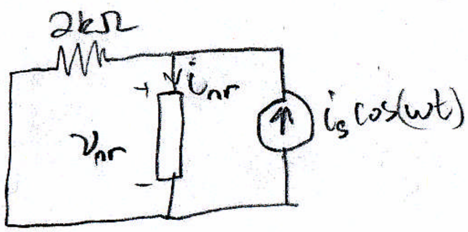
$$3 - 2 i_{NR} = 1 \, i_{NR}^2 \quad \text{OR} \quad i_{NR}^2 + 2 i_{NR} - 3 = 0$$

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)} = \frac{-2 \pm \sqrt{16}}{2} = -1 \pm 2 \text{ mA}$$

the voltage source will drive a clockwise current, so $i_{NR}$ will be positive. and can discard the negative solution

$$i_{NR} = 1\,mA \quad \text{and} \quad v_{NR} = 1 \frac{V}{(mA)^2} (1\,mA)^2 = 1\,V$$

Small Signal Model:

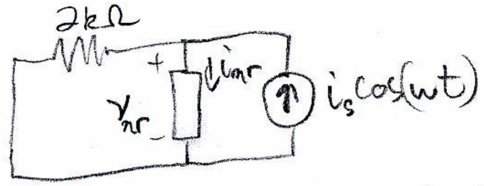


Linearizing the behavior of NR at the bias point:

$$v_{nr} = \underbrace{\left( \frac{d v_{NR}}{d i_{NR}} \right) \Bigg|_{i_{NR} = 1mA}}_{\text{linear slope of nonlinear v-i characteristic at bias point}} \cdot \underbrace{i_{nr}}_{\text{small signal deviation from bias point}} = 2 \frac{V}{(mA)^2} \cdot (1mA) \cdot i_{nr}$$

## Exercise 6.1 (continued)

Small signal model:




Small signal device:

$$v_{nr} = 2\tfrac{V}{mA}\, i_{nr} = (2\cdot 10^{+3})\, i_{nr}$$

KCL: $i_s\cos(wt) = \frac{v_{nr}}{2k\Omega} + i_{nr}$ Solve for $v_{nr}$

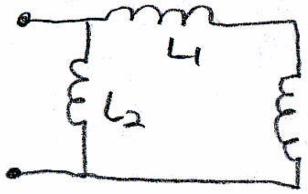
$$i_s\cos(wt) = \frac{v_{nr}}{2k\Omega} + \frac{v_{nr}}{2k\Omega} = \frac{v_{nr}}{1k\Omega}, \quad i_s = 1\mu A$$

$$v_{nr} = (1\times 10^{-6}\, A)(1\times 10^{3}\,\Omega)\cos(wt) \Rightarrow \boxed{v_{nr} = \cos(wt)\ mV}$$

As a sanity check, with $v_{nr} = 2\tfrac{V}{mA})\, i_{nr} = (2\cdot 10^3)\, i_{nr}$, the nonlinear device acts as a 2kΩ resistor to small signal responses around this bias point, so with two 2kΩ resistors in parallel, $i_{nr}$ should be half of the source current. $i_{nr} = \frac{v_{nr}}{2\cdot 10^3\,\Omega} = \frac{1\cdot 10^{-3}\cos(wt)\, V}{2\cdot 10^3\,\Omega}$

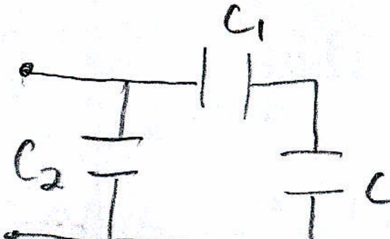
$$i_{nr} = 0.5\times 10^{-6}\cos(wt)\, A$$
$$|i_{nr}| = 0.5\,\mu A = 0.5\, i_s \quad ✓$$

## Exercise 6.2




Inductor combine same as resistors:

$$L_{tot} = L_2 // (L_1 + L_3) = \frac{1}{\frac{1}{L_2} + \frac{1}{L_1+L_3}} = \frac{L_2(L_1+L_3)}{L_1+L_2+L_3}$$




Capacitors combine like conductances

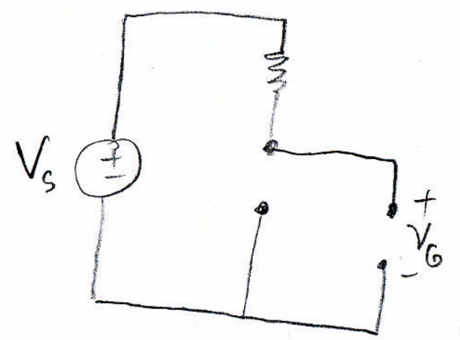
$$C_{tot} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_3}} + C_2$$

$$C_{tot} = \frac{C_1 C_3}{C_1 + C_3} + C_2$$

## Problem 6.1

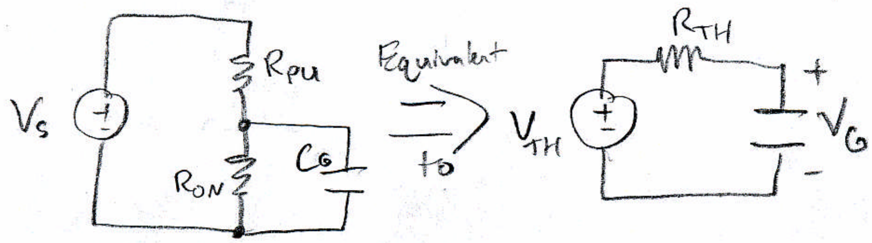
(A) Since the MOSFET was off for a long time before $t=0$, the circuit had reached a steady state equilibrium: with $\frac{dV_G}{dt}=0$ and $i_{C_G}=C\frac{dV_G}{dt}=0$

and $C_G$ modeled as an open circuit:




$$V_G(t=0)=V_S$$

After the MOSFET turns on:
Expect $V_G$ to exponentially drop to $V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$




$$V_{TH}=V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$$

$$R_{TH}=\frac{R_{PU}\cdot R_{ON}}{R_{PU}+R_{ON}}$$

KCL: $\frac{V_{TH}-V_G}{R_{TH}}=i_G=C_G\frac{dV_G}{dt}$ , $\underline{\frac{dV_G}{dt}+\frac{V_G}{R_{TH}C_G}=\frac{V_{TH}}{R_{TH}C_G}}$

$$V_{G_{total}}(t)=V_{G_{particular}}(t)+V_{G_{homogeneous}}(t)$$

$$\underline{V_{G_{part}}=V_{TH}}$$

$$\frac{dV_{G_{homogeneous}}}{dt}+\frac{V_{G_{homogeneous}}}{R_{TH}C_G}=0$$

## Problem 6.1 (continued)

(A) solving for homogeneous part: $\frac{dV_{Gh}}{dt} = \frac{-V_{Gh}}{R_{TH}C_G}$

$$\int \frac{dV_{Gh}}{V_{Gh}} = \int \frac{-dt}{R_{TH}C_G}$$

$$\ln V_{Gh} = \frac{-t}{R_{TH}C_G} + \text{Const} \quad \text{OR} \quad V_{G_h} = e^{-t/R_{TH}C_G} \cdot e^{\text{const}}$$

$$\underline{V_{Gh} = Ae^{-t/R_{TH}C_G}}$$ combining $V_{G_{tot}} = V_{G_{part}} + V_{G_{homogeneous}}$

$$V_G(t) = V_{TH} + Ae^{-t/(R_{TH}C_G)}$$ apply Initial Condition to solve for A

$$V_G(t=0) = V_S = V_{TH} + A(1) \quad A = V_S - V_{TH}$$

$$\boxed{V_G(t) = V_{TH} + (V_S - V_{TH})e^{-t/R_{TH}C_G}}$$

where $V_{TH} = V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$ and $R_{TH} = \frac{R_{ON}\cdot R_{PU}}{R_{ON}+R_{PU}}$

(B) solving $V_G(t) = V_{OL}$ for $t_{fall} \Rightarrow V_{OL} = V_{TH} + (V_S - V_{TH})e^{-t_{fall}/R_{TH}C_G}$

$$\frac{V_{OL}-V_{TH}}{V_S - V_{TH}} = e^{-t_{fall}/R_{TH}C_G} \quad \text{and} \quad -t_{fall} = R_{TH}C_G \ln\left(\frac{V_{OL}-V_{TH}}{V_S - V_{TH}}\right)$$

$$\boxed{t_{fall} = R_{TH}C_G \ln\left(\frac{V_S - V_{TH}}{V_{OL}-V_{TH}}\right)}$$

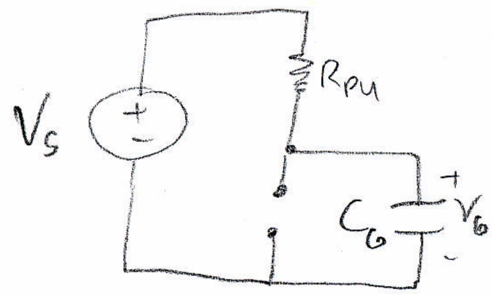
Again with

$$V_{TH} = V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right) \quad \text{and} \quad R_{TH} = \frac{R_{ON}\cdot R_{PU}}{R_{ON}+R_{PU}}$$

## More Problem 6.1

(C) ON for long time $\Rightarrow V_G(t=0) = V_{TH} = V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$

After opens, will eventually rise back to $V_G = V_S$




Similar to (A) but with $V_s$ and $R_{PU}$

KCL: $\frac{V_S - V_G}{R_{PU}} = i_G = C_G \frac{dV_G}{dt}$

$$\frac{dV_G}{dt} + \frac{V_G}{R_{PU} C_G} = \frac{V_S}{R_{PU} C_G}$$

$V_G(t) = V_{G_p}(t) + V_{G_h}(t)$ New Particular Solution $V_{G_P}(t) = V_S$

Same $V_{G_h}(t) = A_c e^{-t/R_{PU}C_G}$ with new $R$ and $A_c$

$V_G(t=0) = V_{TH} = V_S + A_c$ and $A_c = V_{TH} - V_S$

$$\boxed{V_G(t) = V_S + (V_{TH} - V_S)e^{-t/(R_{PU}C_G)} \quad \text{for } t \geq 0}$$

$V_{TH} = V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$ still

(D) Solving new $V_G(t_{rise}) = V_{OH}$ for $t_{rise} \Rightarrow V_{OH} = V_{TH} + (V_{TH} - V_S)e^{-\frac{t_{rise}}{R_{PU}C_G}}$

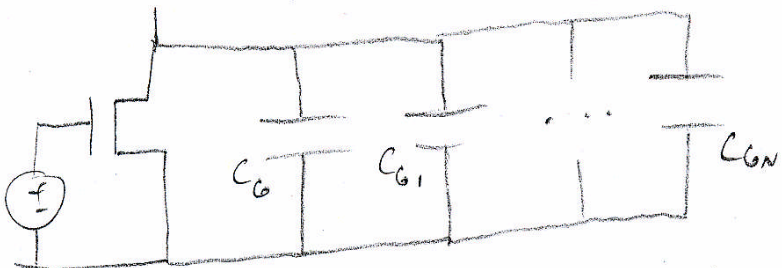
$\frac{V_{OH} - V_{TH}}{V_{TH} - V_S} = e^{-\frac{t_{rise}}{R_{PU}C_G}}$ and $-t_{rise} = R_{PU}C_G \ln\left(\frac{V_{OH} - V_{TH}}{V_{TH} - V_S}\right)$

$$\boxed{t_{rise} = R_{PU} C_G \ln\left(\frac{V_{TH} - V_S}{V_{OH} - V_{TH}}\right)}$$

$V_{TH} = V_S\left(\frac{R_{ON}}{R_{ON}+R_{PU}}\right)$

## Last of Problem 6.1

(E) Connecting more gates to the inverter output will add additional gate capacitors across the output, in parallel with $C_G$:



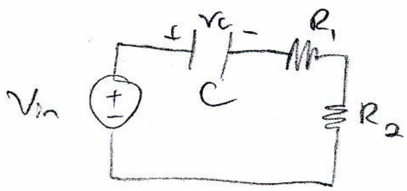

And since parallel capacitors add directly, the total equivalent capacitance will be $C_{tot} = C_G + C_1 + \dots C_N$. This increased total capacitance will increase the RC time constants and

Increase the rise and fall times from parts (B) and (D)

(F) Similarly, decreasing $R_{pu}$ will decrease the rise and fall times. However, the lowest possible output of the inverter is $V_{TH} = V_S \left( \frac{R_{ON}}{R_{ON} + R_{pu}} \right)$, so to obey the static discipline, this minimum must be below $V_{OL}$, and changing $R_{pu}$ will impact delays thru both the time constant and the final values.

## Problem 6.2

(A) With $T \gg RC$, the circuit will reach steady state before switching. In the steady state the capacitor can be modeled by an open circuit, so the steady state voltages will be the source voltage, $\pm V$. And the initial condition will be the steady state from the previous interval, $\mp V$.




KVL: $V_{in} - V_c - i_c(R_1+R_2) = 0 \quad i_c = C\frac{dV_c}{dt}$

$$\frac{dV_c}{dt} + \frac{V_c}{C(R_1+R_2)} = V_{in}$$

$V_c(t) = V_{c_p}(t) + V_{c_h}(t)$ total solution is sum of particular and homogeneous solutions

$V_{cp}(t) = V_{in}C(R_1+R_2)$

Solving $\int \frac{dV_{c_h}}{V_{c_h}} = \int \frac{-dt}{C(R_1+R_2)} \Rightarrow \ln V_{c_h} = \frac{-t}{C(R_1+R_2)} + \text{constant}$

$V_{c_h}(t) = Ae^{-t/C(R_1+R_2)}$ and $\underline{V_c(t) = V_{in} + Ae^{-t/C(R_1+R_2)}}$

Have 2 cases with different initial conditions after each transition $\pm V = V_{in} + A$

And $A_1 = -V - V_{in}$ when $v_{in}(t) = V \Rightarrow A_1 = -2V$

$A_2 = V - V_{in}$ when $v_{in}(t) = -V \Rightarrow A_2 = 2V$

$$\boxed{V_c(t) = \begin{cases} V\left(1 - 2e^{-t/C(R_1+R_2)}\right) & \text{for } v_{in}(t) = V \\ V\left(-1 + 2e^{-t/C(R_1+R_2)}\right) & \text{for } v_{in}(t) = -V \end{cases}}$$

(where $t$ is time since last transition)

## More Problem 6.2

(B) $i_c(t) = C\dfrac{dv_c(t)}{dt}$ using $V_c(t) = \begin{cases} V(1-2e^{-t/\tau}), & v_{in}(t)=V \\ V(-1+2e^{-t/\tau}), & v_{in}(t)=-V \end{cases}$

$$\boxed{i_c(t) = \begin{cases} \dfrac{2CV}{\tau}e^{-t/\tau} & \text{for } v_{in}(t)=V \\ -\dfrac{2CV}{\tau}e^{-t/\tau} & \text{for } v_{in}(t)=-V \end{cases}}$$

(where t is time since last transition)

$\tau = C(R_1+R_2)$

(C) $v_{out}(t) = i_c(t)\cdot R_2$

$$\boxed{V_{out}(t) = \begin{cases} \dfrac{2R_2CV}{\tau}e^{-t/\tau}, & \text{for } v_{in}(t)=V \\ -\dfrac{2R_2CV}{\tau}e^{-t/\tau}, & \text{for } v_{in}(t)=-V \end{cases}}$$

(where t is time since last transition and $\tau = C(R_1+R_2)$)

(D) Multiple ways to do this – reaches steady state in $5\tau \approx 1\text{ms} \Rightarrow \boxed{\tau \approx 250\,\mu\text{sec.}}$

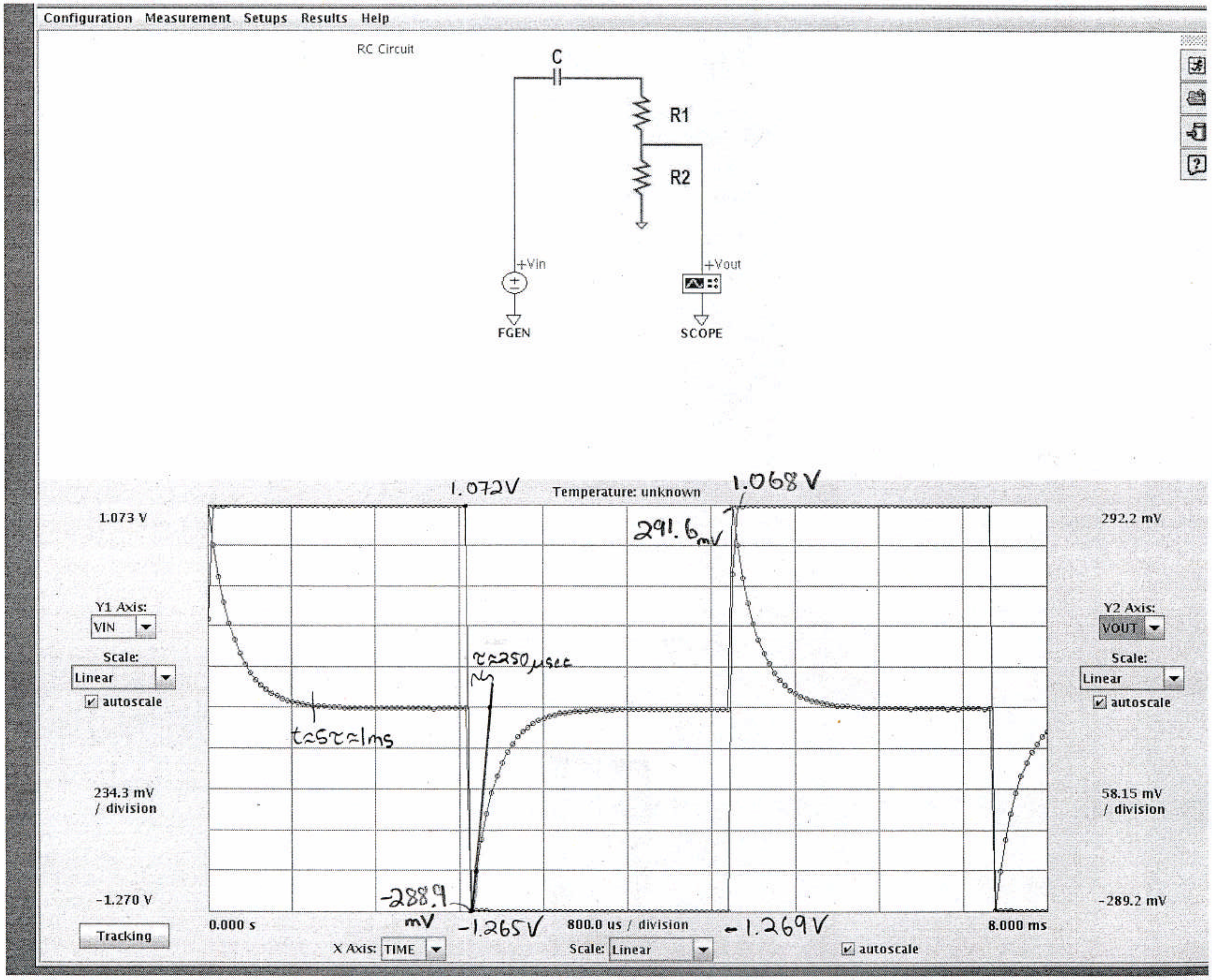
This agreed with value from extending initial slope to 0 crossing and finding $\tau = \Delta t$

(E) from (C) $V_{out\,max} = \dfrac{2R_2\cancel{C}V}{\cancel{C}(R_1+R_2)} = \dfrac{R_2(2V)}{R_1+R_2} = \dfrac{R_2(-V_{in_{high}} + V_{in_{low}})}{R_1+R_2}$ (Since Vin is not very accurate)

$R_2 = 3k\Omega$

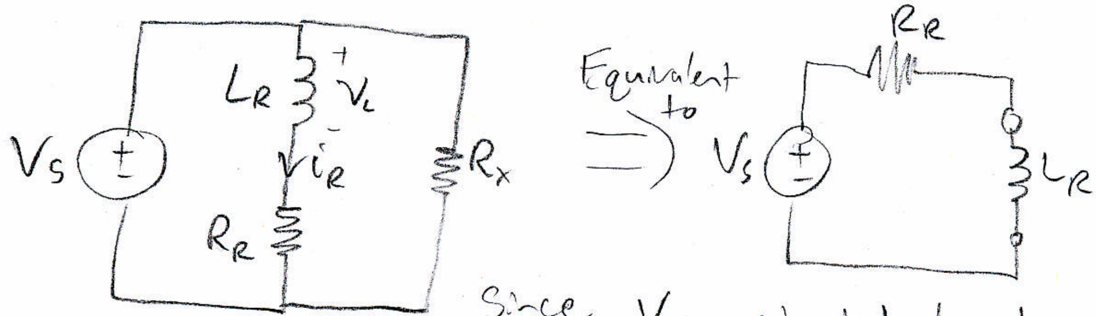
$\dfrac{-288.9\,mV}{(-1{,}265-1{,}072)\,mV}(R_1) = 3k\Omega\left[1-\left(\dfrac{288.9}{1265+1072}\right)\right] \Rightarrow \boxed{R_1 \approx 21\,k\Omega}$

(F) $\tau = C(R_1+R_2) \Rightarrow C \approx \dfrac{\tau}{R_1+R_2} = \dfrac{250\times10^{-6}\text{sec.}}{24\times10^{3}\,\Omega} \Rightarrow \boxed{C \approx 12\,nF}$


Configuration Measurement Setups Results Help
RC Circuit
C
R1
R2
+Vin
+Vout
FGEN
SCOPE
Temperature: unknown
1.072V
1.068V
291.6 mV
1.073 V
292.2 mV
Y1 Axis:
VIN
Scale:
Linear
autoscale
Y2 Axis:
VOUT
Scale:
Linear
autoscale
τ≈250 μsec
t≈5τ≈1ms
234.3 mV / division
58.15 mV / division
-1.270 V
-289.2 mV
-288.9 mV
Tracking
0.000 s
-1.265V
800.0 us / division
-1.269V
8.000 ms
X Axis: TIME
Scale: Linear
autoscale

# Problem 6.3

(A) given $i_R(t=0)=0$, find $i_R(t)$ with MOSFET on:




Since $V_{TH}$ at inductor terminals $= V_s$, $R_{TH} = R_R$, and $I_{sc} = \frac{V_s}{R_R}$.

In this case,
$R_x$ has no impact on the voltage across or current thru $L_R$.

KVL: $V_s - V_L - i_R R_R = 0$ and $V_L = L_R \frac{di_R}{dt}$

$$V_s - L_R \frac{di_R}{dt} - i_R R_R = 0 \text{ or } \underline{\frac{di_R}{dt} + i_R \frac{R_R}{L_R} = \frac{V_s}{L_R}}$$

And the total solution is the combination of a particular and a homogeneous solution:

$$i_R(t) = i_{R_p}(t) + i_{R_H}(t)$$

the particular solution is a constant where

$$i_{R_p} \cdot \frac{R_R}{L_R} = \frac{V_s}{L_R} \Rightarrow \underline{i_{R_p}(t) = \frac{V_s}{R_R}}$$

# Problem 6.3 (continued)

(A) Homogeneous Part: $\frac{di_{R_H}(t)}{dt} + i_{R_H}(t)\frac{R_R}{L_R} = 0$

$$\int \frac{di_{RH}}{i_{R_H}} = -\int \frac{R_R}{L_R}dt \Rightarrow \ln i_{R_H} = -t \cdot \frac{R_R}{L_R} + \text{constant}$$

$$i_{R_H} = e^{-t/\tau} e^{\text{const}} = Ae^{-t/\tau} \quad \text{where } \tau = \frac{L_R}{R_R}$$

total solution $i_R(t) = \frac{V_S}{R_R} + Ae^{-t/\tau}$ use initial condition to find $A$:

$$i_R(t=0) = 0 = \frac{V_S}{R_R} + A \Rightarrow A = -\frac{V_S}{R_R}$$

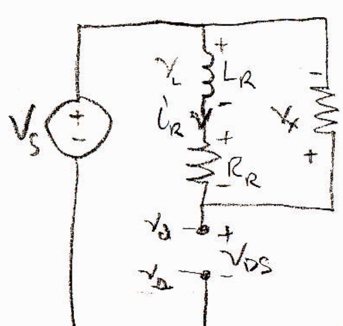
$$\boxed{i_R(t) = \frac{V_S}{R_R}\left(1 - e^{-t \cdot \frac{R_R}{L_R}}\right)} \text{ A for } t \geq 0$$

(B) New "initial" condition at $t=T$: $i_R(t=T) = i_R(t=T)$ from (A)

since $i_R(t)$ must be continuous

$$i_R(t=T) = \frac{V_S}{R_R}\left(1 - e^{-T/\tau_1}\right) = i_0 \quad \text{where } \tau_1 = \frac{L_R}{R_R}$$

With MOSFET OPEN: $L_R$ will supply a current thru the 2 resistors that decays to zero




KVL: $V_L + i_R(R_R + R_X) = 0$

$$L\frac{di_R}{dt} + i_R(R_R + R_X) = 0$$

and for $V_{DS}$: $V_d - V_X - V_S = V_s$ with $V_{DS} = V_d - V_s$

$$V_{DS} = V_X + V_S$$

# More Problem 6.3

(B) Solving $\frac{di_R}{i_R} = \frac{-(R_R+R_x)dt}{L_R}$ there are no source terms, so it's just the homogeneous part, $i_{R_p}(t) = 0$

$$\ln i_R = -t\frac{(R_R+R_x)}{L_R} + \text{constant} \text{ and } i_R = Ae^{-t/\tau_2},\ \tau_2 = \frac{L_R}{R_R+R_x}$$

Apply initial condition to solve for A, but now the "initial" condition and switching occur at $t=T$, not $t=0$, so solution for $i_R$ must be shifted $t \to (t-T)$

$$i_R(t=T) = i_0 = Ae^{-\cancel{(T-T)}/\tau_2} = A,\quad A = i_0$$

$$i_R(t) = i_0 e^{-(t-T)/\tau_2},\quad i_0 = \frac{V_s}{R_R}\left(1-e^{-T\frac{R_R}{L_R}}\right)$$

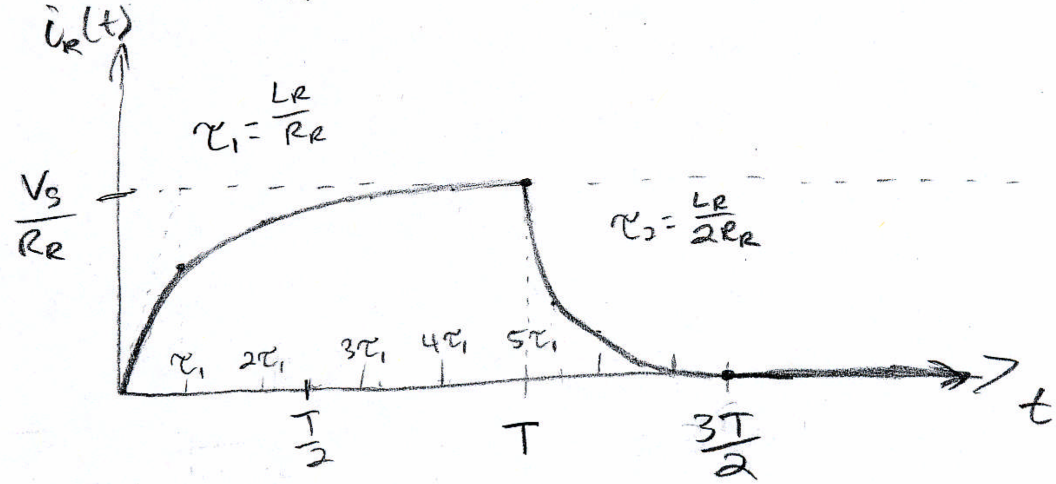
$$\boxed{i_R(t) = \frac{V_s}{R_R}\left(1-e^{-T\frac{R_R}{L_R}}\right)e^{-(t-T)\frac{(R_R+R_x)}{L_R}} \text{ A for } t \geq T}$$

$$V_{DS}(t) = V_s - V_{Rx} = V_s - i_R(t)\cdot R_x$$

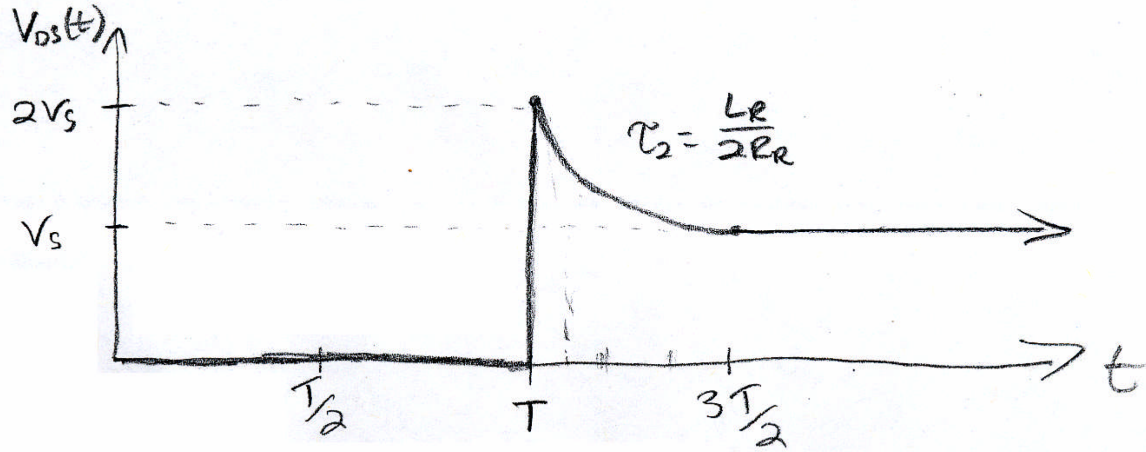
$$\boxed{V_{DS}(t) = V_s - \frac{V_s R_x}{R_R}\left(1-e^{-T\frac{R_R}{L_R}}\right)e^{-(t-T)\frac{(R_R+R_x)}{L_R}} \text{ V for } t \geq T}$$

## More Problem 6.3

(C) $i_R$ starts at zero, rises towards $\frac{V_S}{R_R}$ until $T \approx 5L_R/R_R \approx 5\tau_1$ at 5 time constants $i_R \approx \frac{V_S}{R_R}$, then decays to zero with a time constant $\tau_2 = \frac{L_R}{2R_R}$, half as large, so twice as fast




If the MOSFET is an ideal switch, $V_{DS} = 0$ for $t < T$, then switches up to $V_{DS} = V_S + i_R \cdot R_X = V_S + \frac{V_S}{R_R} \cdot R_R = 2V_S$ then decays down to $V_S$ with the same time constant and curve as $i_R(t)$:



## End of Problem 6.3

(D) A complete circuit loop is needed to provide a current path after the MOSFET opens. The current in the inductor must be continuous so if $R_X$ were removed, as soon as the MOSFET tried to open and instantly switch the inductor current to zero, the inductor voltage would approach infinity according to $V_L = L\frac{di}{dt}$ as the finite $i$ turned off over a very small dt. The rising $V_{DS}$ from the inductor could increase above the rated breakdown voltage of the MOSFET and force it to conduct, destroying the switch.

## Problem 6.4

(A) For the inductor circuit, $R_{TH_L} = R_1 + R_2$ and $I_{SC_L} = I(t)\left(\frac{R_1}{R_1+R_2}\right)$

For the Capacitor Circuit: $R_{TH_C} = \frac{R_1 \cdot R_2}{R_1+R_2}$ and $V_{TH_C} = V(t)\,\frac{R_2}{R_1+R_2}$

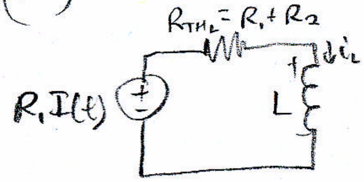
But know $V_L = L\frac{di_L}{dt}$ and $i_C = C\frac{dV_C}{dt}$ so simpler to use a series circuit for the inductor and a parallel circuit for the capacitor

and $V_{TH_L} = I_{SC_L} \cdot R_{TH_L} = R_1 \cdot I(t)$

and $I_{N_C} = \frac{V_{THC}}{R_{THC}} = \frac{V(t)}{R_1}$

# Problem 6.4 (continued)

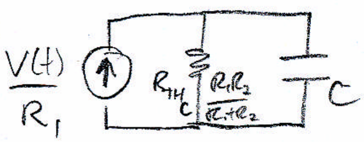
(A) For inductor: KVL: $R_1 I(t) - i_L(R_1+R_2) = V_L = L\frac{di_L}{dt}$




$$\boxed{\frac{di}{dt} + i_L\frac{(R_1+R_2)}{L} = \frac{R_1 I(t)}{L}}$$

For Capacitor:

KCL: $\frac{V(t)}{R_1} = \frac{V_c}{R_{THc}} + i_c \Rightarrow \frac{V(t)}{R_1} = \frac{V_c(R_1+R_2)}{R_1R_2} + C\frac{dV_c}{dt}$




$$\boxed{\frac{dV_c}{dt} + V_c\frac{(R_1+R_2)}{R_1R_2C} = \frac{V(t)}{R_1C}}$$

(B) At $t=\infty$, both circuits will be in steady state, with $\frac{di_L}{dt}=0$ and $\frac{dV_c}{dt}=0$, nothing changing, the inductor acting as a short circuit and the capacitor open: $\boxed{i_c(t=\infty)=0 \text{ and } V_L(t=\infty)=0}$

and the initial charge and flux delivered by the impulse sources has bled out through the resistors

## Problem 6.4 (continued) 

(B) (continued) to find $i_L(t=0^+)$ and $V_c(t=0^+)$, integrate the differential equation from $t=0^-$ to $t=0^+$.

$$\int_{0^-}^{0^+} di + \int_{0^-}^{0^+} i_L(t)\frac{(R_1+R_2)}{L}\,dt = \int_{0^-}^{0^+} \frac{R_1 Q}{L}\,\delta(t)\,dt$$

$$\int_{0^-}^{0^+} dV + \int_{0^-}^{0^+} V_c(t)\frac{(R_1+R_2)}{R_1 R_2 C}\,dt = \int_{0^-}^{0^+} \frac{\Lambda\,\delta(t)\,dt}{R_1 C}$$

If $i_L(t)$ and $V_c(t)$ remain finite, then the integrals of these finite values over zero time go to zero. Even though the $I(t)$ and $v(t)$ from the impulse sources may be infinite values, forcing the inductor voltage and capacitor current to infinite values, we know that the sources still deliver only a finite amount of charge, $\Lambda$ in this case, and flux, $Q$ here, to the inductor and capacitor. With finite charge and flux, the capacitor voltage, $V_c(t)$, and inductor current, $i_L(t)$ must remain finite. So the second terms above $\Rightarrow 0$ and left with:

$$i_L(t=0^+) - i_L(t=0^-) + 0 = \frac{R_1}{L}Q \qquad \text{with } i_L(t=0^-)=0$$

$$V_c(t=0^+) - V_c(t=0^-) + 0 = \frac{\Lambda}{R_1 C} \qquad \text{with } V_c(t=0^-)=0$$

$$\boxed{i_L(t=0^+) = \frac{R_1}{L}Q \quad \text{and} \quad V_c(t=0^+) = \frac{\Lambda}{R_1 C}}$$

# Problem 6.4

(C) From the Thevenin and Norton equivalents,

$$\tau_L = \frac{L}{R_{TH_L}} \quad \text{and} \quad \tau_C = R_{TH_C} C$$

$$\boxed{\tau_L = \frac{L}{R_1 + R_2}} \qquad \boxed{\tau_C = \frac{R_1 R_2}{R_1 + R_2} C}$$

(D) Know initial and final values and time constants for exponential decay: Using values at $t=0^+$, after sources act, for initial values:

$$V_c(t) = V_c(t=0^+)\, e^{-t/\tau_C} \quad \text{and} \quad i_L(t) = i_L(t=0^+)\, e^{-t/\tau_L}$$

$$\boxed{V_c(t) = \frac{\Lambda}{R_1 C} e^{-t/\left(\frac{R_1 R_2}{R_1+R_2}\right)C}} \qquad \boxed{i_L(t) = \frac{R_1 Q}{L} e^{-t/\left(\frac{L}{R_1+R_2}\right)}} \quad t \geq 0$$

(E) Plugging back into differential equation: first for inductor:

$$\frac{i_L(t=0^+)}{-\tau_L} e^{-t/\tau_L} + \frac{i_L(t=0^+) e^{-t/\tau_L}}{\tau_L} = \frac{R_1}{L} Q\,\delta(t) = 0 \quad \text{for all } t > 0$$

and for capacitor:

$$\frac{-V_c(t=0^+)}{\tau_C} e^{-t/\tau_C} + \frac{V_c(t=0^+)}{\tau_C} e^{-t/\tau_C} = \frac{\Lambda\,\delta(t)}{R_1 C} = 0 \checkmark \quad \text{for all } t > 0$$

the first 2 terms cancel out in both, to be true for all $t$, $V_c(t)$ and $i_L(t)$ should both include a step function $u(t)$