Massachusetts Institute of Technology
**Department of Electrical Engineering and Computer Science**

**6.002 – Circuits & Electronics**
Spring 2008

Problem Set #7

Issued 3/19/08 – Due 4/2/08

Exercise 7.1 (1 Point):   Each network shown below has a non-zero initial state at $t = 0$, as indicated. Find the network states for $t \geq 0$. Note that the impulse inputs are delayed. Hint: you may find it intuitively easier to use superposition, and Thevenin and Norton equivalents.
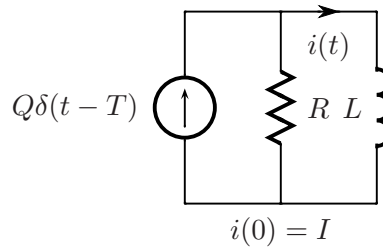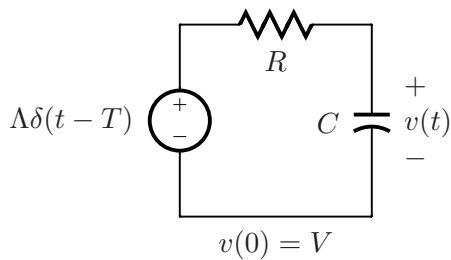

Exercise 7.2 (1 Point):   The network shown below contains a 1-mA current source, a resistor and an inductor all in parallel. The network has been assembled for a long time. At $t = 0$ the current source turns off, after which the common voltage $v(t)$ is measured as shown below. From the measured voltage, determine the resistance of the resistor and the inductance of the inductor.
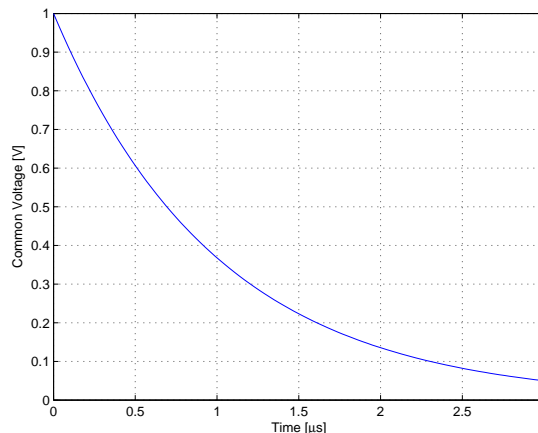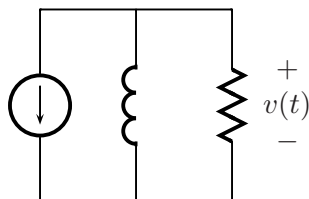
Problem 7.1 (2 Points): This problem illustrates how capacitors and transformers can be used to couple successive amplifier stages, or other stages, in a way that permits each stage, including the input and output, to have independent biasing. In the circuit shown below, an input source is coupled to an amplifier through a capacitor, and the amplifier is in turn coupled to a resistive load through a transformer. Alternative designs could use a transformer to couple the input, a capacitor to couple the output, or both.

A model for the transformer is also shown below. The transformer is modeled as an ideal transformer using two dependent sources, combined with a magnetizing inductor. To learn more about ideal transformers, see Section 9.3.4 in A&L. To learn more about non-ideal transformers, see Section 13:764d-e in A&L; this section is on the web.

(A) Assume that the MOSFET operates in its saturation region, and that it has a non-zero C_{GS}. Let $v_{IN} = V_{IN}$, its large-signal bias component. Determine the large-signal bias components of v_{GS}, i_D and v_{OUT}. Hint: consider the behavior of the capacitors and the magnetizing inductor after the circuit has been turned on for a very long time, and all turn-on transients have died out.

(B) In view of your answer to Part (A), is it necessary, or even useful, to bias the input with a nonzero V_{IN}? Also, is it necessary to remove the large-signal bias component from v_{OUT}.

(C) Assume that the MOSFET threshold voltage V_T is positive. In this case, how must R_1 and R_2 be chosen so that the MOSFET is biased in the saturation region.

(D) Assume that the amplifier is designed so that the MOSFET operates in its saturation region. In this case, develop a small-signal model for the amplifier that can be used to determine the small-signal component $v_{out}(t)$ from the small-signal component $v_{in}(t)$.

(E) Let $v_{in}(t) = Vu(t)$, where $u(t)$ is the unit step function, and determine $v_{out}(t)$ for $t > 0$. In doing so, assume that the circuit has been at rest for a very long time prior to $t = 0$. To simplify this part, let $C_{GS} = 0$. Hint: you may find it easiest to first find $v_{gs}(t)$.

(F) Using your answer to Part (E) determine the period of time over which v_{out} is approximately a step. Use reasonable engineering judgement.

Non-Ideal 1:N Transformer

Ideal 1:N Transformer
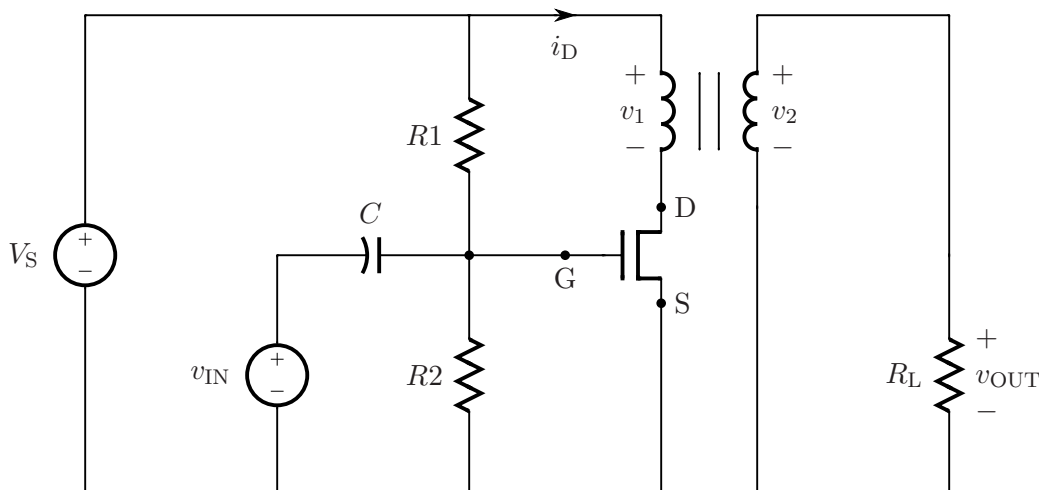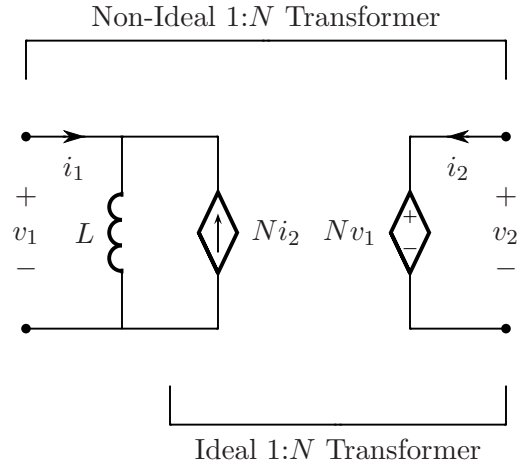
Problem 7.2 (2 Points):   This problem makes use of the ELVIS iLab to study the large-signal and small-signal response of a common-source MOSFET-resistor inverter/amplifier that is loaded with a large capacitor at its output. The circuit is shown below. It is the same circuit studied in Problem 5.1, with the addition of an output capacitor. In this problem, you will measure the large-signal and small-signal step responses of the inverter/amplifier and compare the time constants which characterize those responses to predictions based on the models developed in class.
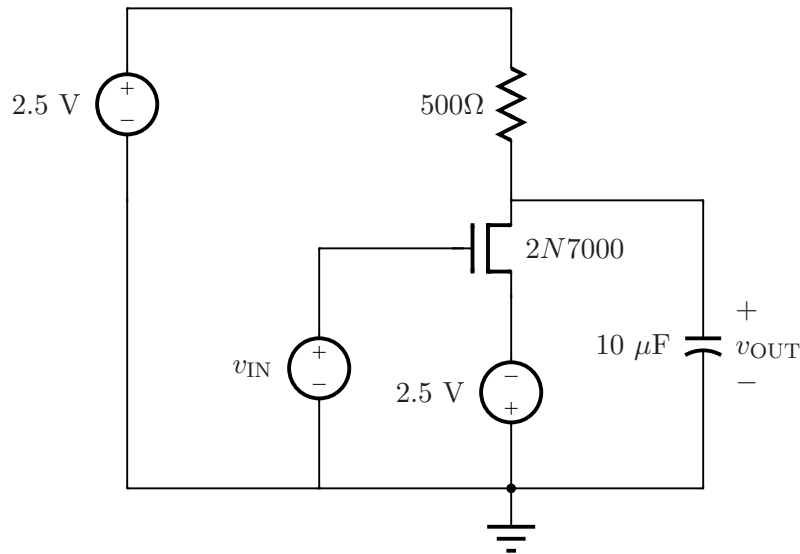
As discussed often in class, the MOSFET-resistor circuit shown above behaves both as an inverter and as an amplifier, depending on how it is used. When this circuit is loaded with a capacitor, it exhibits interesting dynamics. To study those dynamics you will first measure the large-signal step response of the circuit as an inverter, and extract the relevant time constants. You will then measure the small-signal step response of the circuit acting as an amplifier, and again extract the relevant time constants. Finally, using the models developed in class, you will estimate the values of the time constants, and compare those estimates with the experimentally-measured time constants.

The measurements are carried out in Parts (A) and (B) below. To carry out the measurements, log in to iLab and launch the ELVIS Lab Client. When you do, the circuit shown above will appear on the canvas.

(A) Experimentally characterize the large-signal switching behavior of the capacitor-loaded inverter. To do so, first select the signal generator (FGEN), and set its parameters to WaveForm = SQUARE, Frequency = 10 Hz, Amplitude = 1 V, and Offset = 0 V. Second, select the output measurement unit (SCOPE) and choose a suitable sampling rate that will allow you to see at least one full cycle of the output waveform with adequate resolution. Note that the system will allow you to take a maximum of 201 data samples at the output. Third, run the experiment. Finally, select v_{IN} for the Y1 axis and v_{OUT} for the Y2 axis, and use linear axes for both. When the figure resembles what you expect, capture a screen shot.

From the measured data, extract the pull-up (rising output) and pull-down (falling output) time constants. For this, you may find it useful to zoom in on the respective edges of v_{OUT} to clearly observe the exponential portion of the transition, and use the graphical technique discussed in class to estimate the time constant. This might require additional measurements. Turn in any marked-up screen shots that you produce.

(B) Consider now the small-signal behavior of the circuit acting as an amplifier. Reduce the Amplitude to 120 mV. Select an offset that gives you an output bias point close to 0 V. You may need to try different values of the input offset to achieve the desired bias. An output bias within ±300 mV of 0 V is acceptable. Now, measure the square-wave response of the circuit. Capture a screen shot and extract the time constants of the response to a rising step input and a falling step input. Turn in any marked-up screen shots that you produce.

(C) Based on the analysis discussed in class, derive large-signal models for the pull-up (rising output) and pull-down (falling output) time constants of the MOSFET-resistor inverter driving a capacitor. Note that: (1) a typical value of R_{ON} is 10 Ω for the 2N7000 MOSFET; (2) the resistance of the pull-up resistor is 500 Ω; and (3) the capacitance of the load capacitor is 10 μF. Quantitatively estimate the time constants that you expect for this inverter and compare the estimated time constants to those obtained experimentally. Comment on any discrepancies.

(D) Based on the analysis discussed in class, derive models for the time constant associated with the small-signal response of the MOSFET-resistor amplifier driving a capacitor. For this, you should derive a small-signal equivalent circuit model for the amplifier and then derive the time constant for the step response. In doing so, assume that the 2N7000 MOSFET is characterized by the typical values of $K = 0.14$ A/V^2 and $V_{\text{T}} = 1.9$ V. Quantitatively estimate the time constant that you expect for the step response of the amplifier and compare the estimated time constant to those determined experimentally. Comment on any discrepancies.

Problem 7.3 (2 Points):   Consider the two-input digital-logic OR gate from Problem 3.3. Model each MOSFET with a switch-resistor model having on-state resistance R_{ON}. Assume that both pull-up resistors have the same resistance R_{PU}, and that the resistances satisfy $R_{ON} \ll R_{PU}$.

(A) Suppose the inputs IN1 and IN2 cycle through the four combinations 00, 01, 00, and 10 in that order. Assume further that each input combination is held for the same period T. Under these assumptions, determine the average static power dissipated by the logic circuit; do so analytically, rather than using the numerical device values given in Problem 3.3. Make appropriate simplifications based on the inequality for R_{ON} and R_{PU}. Hint: see your solutions to Problem 3.3.

(B) Assume now that MOSFET M3 has the gate-to-source capacitance C_{GS}, and that the output of the OR gate is loaded with a down-stream capacitance N times bigger. The capacitive loading represents the loading by subsequent gates. Assume further that each input combination described above is held for the period T such that $T \gg R_{PU}C_{GS}$. Under these assumptions, determine the average dynamic power dissipated by the logic circuit. Make appropriate simplifications based on the inequalities for R_{ON}, R_{PU}, C_{GS} and T.

Note that the energy stored in the gate-to-source capacitances of M1 and M2 is not considered here. That energy is dissipated in the upstream gates represented by the input sources v_{IN1} and v_{IN2}. Thus, it is not counted in the dissipation of the OR gate.

(C) Assuming that N is not much larger that one, which is greater, the average static dissipation or the average dynamic dissipation of the OR gate? Hint: make use of the inequalities for R_{ON}, R_{PU}, C_{GS} and T.

Problem 7.4 (2 Points):   The network shown below contains a capacitor and an inductor. The network states are observed to oscillate with a frequency of 10^7 rad/s. Further, the peak value of v_C is observed to be 100 mV, and the peak value of i_L is observed to be 10 mA.

(A) What are the values of C and L?

(B) How much energy is stored in the network?

(C) Suppose a resistor is placed in parallel with the inductor and capacitor. What resistance would it have if the energy stored in the network is then observed to decay by the factor of $1/e$ in 10 μs?