

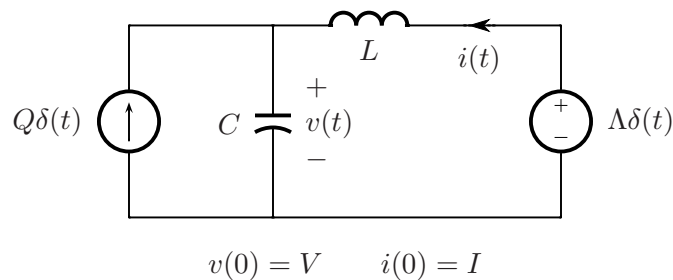
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2008

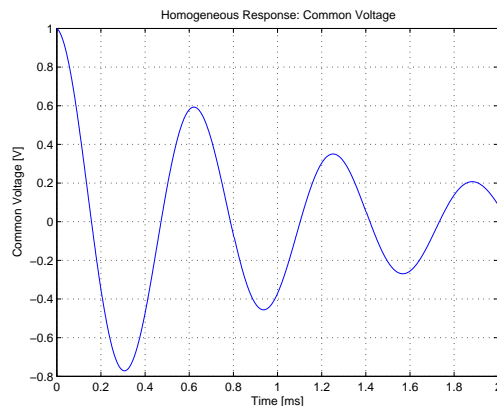
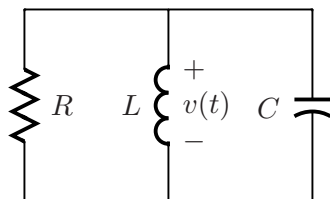
Problem Set #9

Issued 4/9/08 – Due 4/16/08

Exercise 9.1 (1 Point): Determine $i(t)$ and $v(t)$ for $t \geq 0^+$ in the network shown below. Note that the inductor and capacitor both have nonzero states at $t = 0$, and that both sources are impulse sources. Hint: use superposition to establish initial conditions at $t = 0^+$.

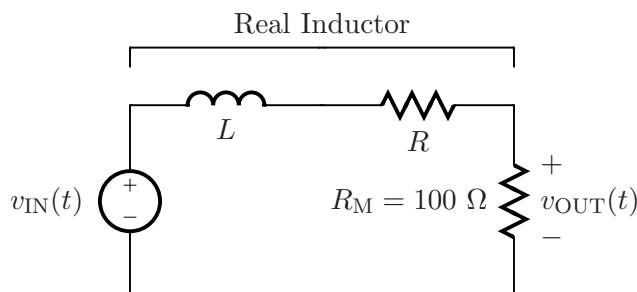


Exercise 9.2 (1 Point): Shown below is a parallel resistor-inductor-capacitor network. Also shown below is a graph of the common network voltage $v(t)$ as a function of time; the voltage is the homogeneous response of the network to some set of initial conditions. From the voltage graph, determine the inductance of the inductor and the capacitance of the capacitor given that the resistance of the resistor is 600Ω . *Note: the next to last page of this problem set contains a larger graph of the common voltage. It can be turned in with your problem set solutions.*



Problem 9.1 (2 Points): This problem studies the sinusoidal-steady-state response of a resistor-inductor network, both experimentally and theoretically, in three parts. First, you will determine the theoretical response of the network. Next, you will measure the response of the network at different frequencies using the ELVIS iLab. Finally, through a comparison of the theoretical and experimental responses, you will determine the inductance and parasitic resistance of the inductor.

The network, shown below, consists of a real inductor in series with a $100\text{-}\Omega$ measurement resistor, R_M . The real inductor is modeled as an ideal inductor, L , in series with a parasitic resistor, R , which models the resistance of the winding in the real inductor. This network is fed by a voltage source, v_{IN} , and the output voltage v_{OUT} is measured across R_M .



When dealing with a sinusoidal signal, one must be careful to distinguish between its frequency f measured in Hz, or cycles per second, and its frequency ω measured in rad/s. The two are related through $\omega = 2\pi f$. Often we simply refer to “frequency” in general. However, the units of that frequency make it perfectly clear which frequency is under discussion, and you should be aware of this at all times to avoid confusion. In this problem, the theoretical analysis is carried out using the frequency ω , while the experiments on the ELVIS iLab are carried out using the frequency f .

- (A) Consider first the theoretical analysis of the sinusoidal-steady-state response. To begin, let $v_{\text{IN}}(t) = V_{\text{in}} \sin(\omega t)$. Correspondingly, let $v_{\text{OUT}}(t) = V_{\text{out}} \sin(\omega t + \phi)$. Now, determine V_{out} and ϕ as functions of ω through the following steps.
- (1) Express v_{IN} and v_{OUT} in the form of the complex exponentials $V_{\text{in}}e^{j\omega t}$ and $\hat{V}_{\text{out}}e^{j\omega t}$, respectively.
 - (2) For the network shown above, derive a differential equation for v_{OUT} driven by v_{IN} .
 - (3) Using the complex-exponential forms for v_{IN} and v_{OUT} from Step (1), solve the differential equation from Step (2), and derive an expression for the complex amplitude \hat{V}_{out} in terms of V_{in} , ω , and the values of the network elements.
 - (4) From the result of Step (3), express the real amplitude V_{out} , and the phase ϕ , of v_{OUT} in terms of V_{in} , ω , and the values of the network elements.
 - (5) Finally, sketch and clearly label two plots. The first plot is $V_{\text{out}}/V_{\text{in}}$ on a logarithmic scale versus ω on a logarithmic scale. The second plot is ϕ on a linear scale versus ω on a logarithmic scale. Identify the low-frequency and high-frequency asymptotes on these plots.
- (B) Consider next the experimental characterization of the network using the ELVIS iLab. To do so, take the following steps.

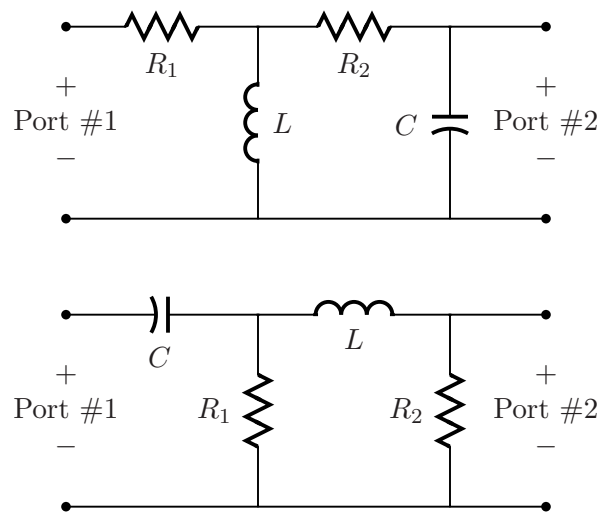
- (1) Log in to iLab, and select the ELVIS Lab Client in the usual manner. When you launch the lab client, you will see the network on the canvas.
- (2) First, select the signal generator (FGEN), and set its parameters to WaveForm = SINE, Amplitude = 1 V, and Offset = 0 V. With these common settings, you will carry out measurements as a function of frequency. Take measurements at two frequencies per decade from 10 Hz to 100 kHz. That is, take measurements at 10 Hz, 30 Hz, 100 Hz, 300 Hz, 1 kHz, and so on. This corresponds to measurements at a total of nine frequencies; feel free to take more measurements if you want. For each frequency, you must select the output measurement unit (SCOPE) and specify a suitable sampling rate that will allow you to see at least one full cycle of the output waveform with adequate resolution. Remember that the system will only allow you to take a maximum of 201 data samples at the output. When you are set, run each experiment. When the data returns, select v_{IN} for the Y1 axis and v_{OUT} for the Y2 axis, and use linear axes for both. You should obtain a nice sinusoidal signal for both v_{IN} and v_{OUT} .
- (3) For each experiment at a given frequency of the input signal, you must extract three parameters. Organizing these parameters in a table will be helpful.

First, read the amplitude of the v_{IN} and v_{OUT} sinusoids. Notice that even though you have set the amplitude of v_{IN} to 1 V, what actually shows up at the network input can be different due to imperfections in the instrument. So, read the amplitude of both v_{IN} and v_{OUT} . The easiest way to do this is to keep the autoscale boxes checked for both axes, and subtract the bottom of each range from the top of that range, and then divide the difference by two.

Second, read the phase of v_{OUT} with respect to v_{IN} , and express this phase in degrees. To do so, first calculate the common period of the v_{IN} and v_{OUT} waveforms. Next, measure the difference in time between two consecutive zero crossings of the v_{IN} and v_{OUT} waveforms. Since the period corresponds to 360 degrees, the time difference divided by the period, then multiplied by 360 degrees, is the phase shift between v_{IN} and v_{OUT} in degrees. Make sure that the sign of the result is consistent with the definition of ϕ .

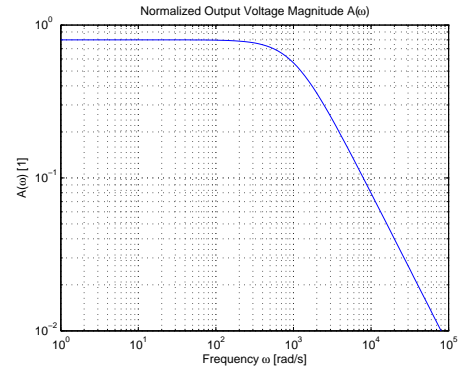
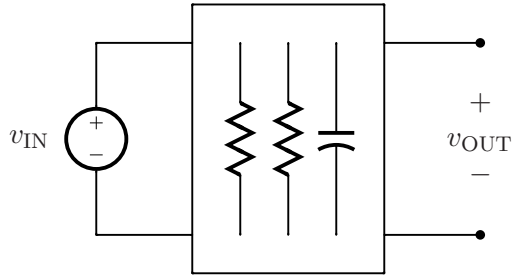
- (4) From the measured data, graph the amplitude ratio and the phase shift as functions of frequency. For the first plot, graph the amplitude ratio on a logarithmic scale versus the frequency ω on a logarithmic scale. Notice that you will need to translate the frequency f used by the ELVIS iLab into the frequency ω , as discussed above. For the second plot, graph the phase difference on a linear scale versus the frequency ω on a logarithmic scale. Comment on the results.
- (C) Compare the model of the network derived in Part (A) to the experimental characterization acquired in Part (B), and determine the inductance L and parasitic resistance R of the inductor. Explain how you do this and give the parameter values.

Problem 9.2 (2 Points): This problem concerns the sinusoidal-steady-state behavior of the networks shown below, both of which have two ports.

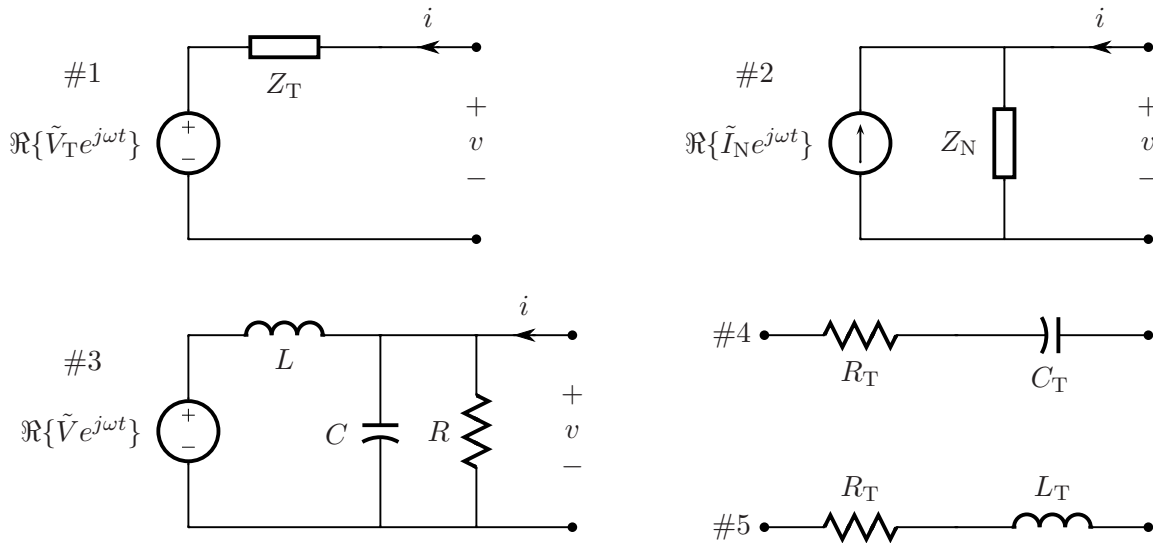


- (A) Determine the impedance of each network as viewed into Port #1 under the assumption that Port #2 is open.
- (B) Assume that Port #1 of each network is driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is open. Determine the current into the positive terminal of each network at Port #1. Express the current in the form $I_1 \cos(\omega t + \phi_1)$ where I_1 is an amplitude and ϕ_1 is a phase angle.
- (C) Assume that Port #1 of each network is again driven in sinusoidal steady state by the voltage $V_1 \cos(\omega t)$, and that Port #2 is again open. Determine the voltage that appears at Port #2. Express the voltage in the form $V_2 \cos(\omega t + \phi_2)$ where V_2 is an amplitude and ϕ_2 is a phase angle.

Problem 9.3 (2 Points): A two-port network is fabricated using two resistors and one capacitor as shown below. For $v_{\text{IN}}(t) = V_{\text{in}} \cos(\omega t)$, it is desired that $v_{\text{OUT}}(t) = V_{\text{in}} A(\omega) \cos(\omega t + \phi(\omega))$, where $A(\omega)$ is as shown graphically below. Determine the topology of the network, and, assuming that the capacitor is a $0.1\text{-}\mu\text{F}$ capacitor, determine the resistances of the two resistors. Also, determine $\phi(\omega)$. *Note: the last page of this problem set contains a larger graph of $A(\omega)$. It can be turned in with your problem set solutions.*

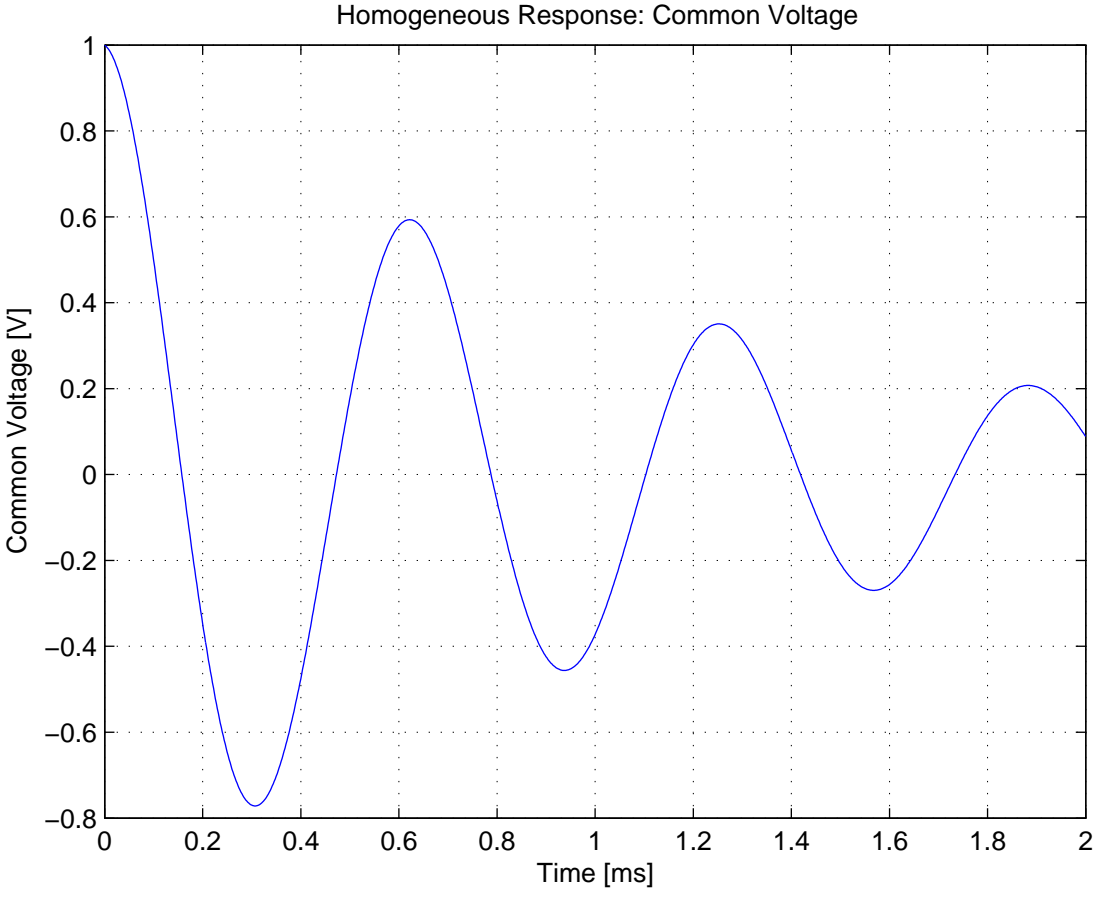


Problem 9.4 (2 Points): This problem explores the Thevenin and Norton equivalence of networks operating in sinusoidal steady state. All networks considered here are comprised of linear resistors, capacitors and inductors, and voltage and current sources all operating at the same frequency ω . Therefore, all branch currents and voltages operate at the frequency ω .



- (A) Determine the relations between \tilde{V}_T , Z_T , \tilde{I}_N and Z_N which must exist for the i - v relations at the terminals of Networks #1 and #2 to be identical when operating in sinusoidal steady state.
- (B) Review the arguments for networks involving only linear resistors and sources, and then briefly explain why Networks #1 and #2 may serve as the Thevenin and Norton equivalents, respectively, of an arbitrary linear network operating in sinusoidal steady state.
- (C) Determine \tilde{V}_T and Z_T in the Thevenin equivalent of Network #3.
- (D) Suppose Z_T in Part C is implemented with Network #4 or #5. Determine R_T and C_T in Network #4, and R_T and L_T in Network #5, in terms of R , L , C and ω . Under what circumstances is Network #4 preferred over Network #5, and vice versa?

Exercise 9.2



Problem 9.3

