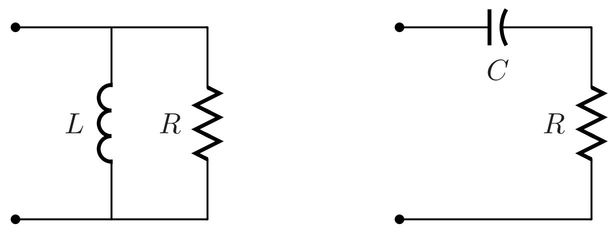
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2008

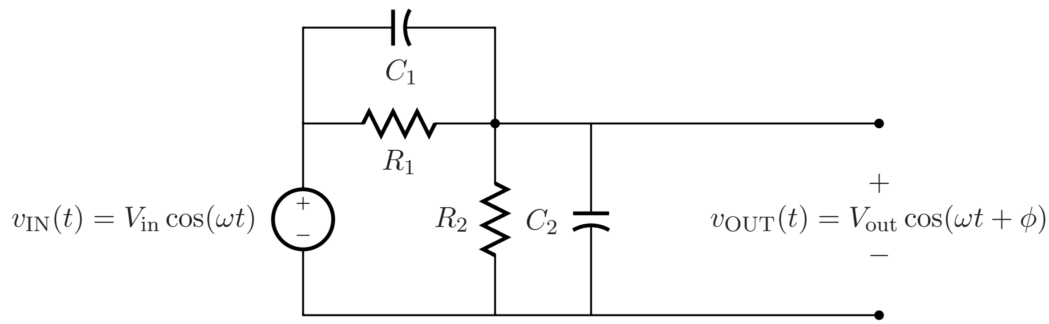
Problem Set #10

Issued 4/16/08 – Due 4/23/08

Exercise 10.1 (1 Point): Determine the impedance of both networks shown below. Also, identify the asymptotic dependence of the impedances on frequency for very low frequencies and for very high frequencies, and explain the dependences physically.

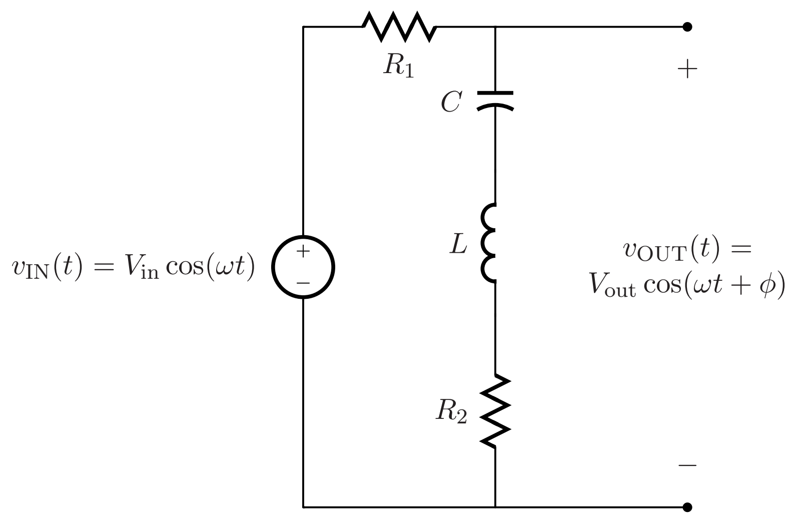


Exercise 10.2 (1 Point): The network shown below models an oscilloscope probe that provides a 10:1 voltage attenuation. Resistor R_1 is a fixed resistor in the probe, resistor R_2 models the input resistance of the oscilloscope, capacitor C_1 is a variable capacitor in the probe, and capacitor C_2 models the combined input capacitance of the oscilloscope and the cable between the probe and the oscilloscope. What relations are required between R_1, R_2, C_1 and C_2 so that $v_{\mathrm{OUT}}(t) = 0.1\ v_{\mathrm{IN}}(t)$ for all ω? That is, what relations are required so that $V_{\mathrm{out}} = 0.1\ V_{\mathrm{in}}$ and $\phi = 0$ for all ω? (Note that the value of C_2 is difficult to guarantee in practice due to variations in cable length and oscilloscope input capacitance, so C_1 is made manually adjustable in the probe.)



Problem 10.1 (2 Points): This problem studies the sinusoidal-steady-state response of the notch filter shown below, both theoretically and experimentally, in three parts. First, you will determine the theoretical response of the filter. Next, you will measure the response of the filter as a function of frequency using the iLab Dynamic Signal Analyzer Lab. Finally, through a comparison of the theoretical and experimental responses, you will determine the values of several of its components.

The notch filter consists of the resistor R_1 in series with the capacitor C, the inductor L and the resistor R_2. The inductor L and resistor R_2 together model a real inductor. The filter is driven by a voltage source, v_{IN}, and the output voltage v_{OUT} is measured across the capacitor, the inductor and the second resistor.



(A) Consider first the theoretical characterization of the notch filter. From the filter description, use the impedance method to determine the theoretical output voltage. That is, determine V_{out} and ϕ in terms of V_{in}, R_1, R_2, L, C and ω.

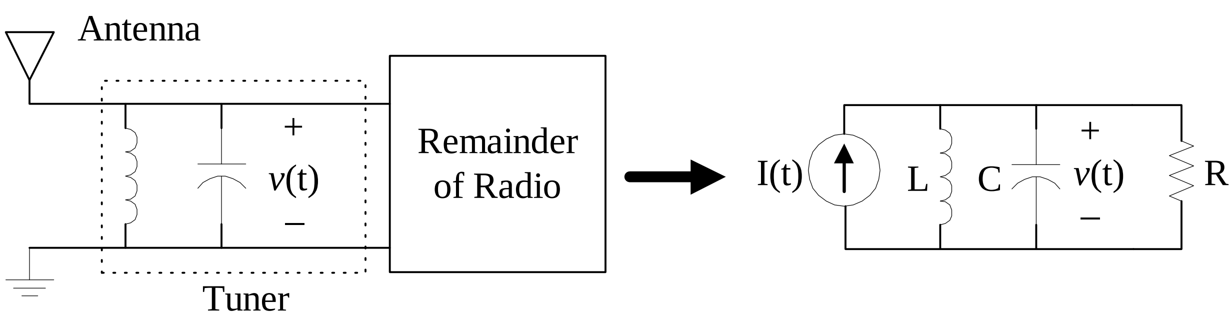
(B) Derive an expression for the Q of the notch filter. Here, Q is defined to be the same as the Q of the equivalent bandpass filter having complex gain $(1 - \tilde{V}_{\mathrm{out}}/V_{\mathrm{in}})$. Note that $\tilde{V}_{\mathrm{out}}$ is the complex output amplitude of the notch filter such that $V_{\mathrm{out}} = |\tilde{V}_{\mathrm{out}}|$ and $\phi = \angle \tilde{V}_{\mathrm{out}}$.

(C) On appropriate axes, sketch and clearly label $V_{\mathrm{out}}/V_{\mathrm{in}}$ and ϕ as functions of ω. In doing so, assume that the notch filter is narrow, that is, assume that $Q \gg 1$.

(D) Consider next the experimental characterization of the notch filter. Log in to iLab, and select the Dynamic Signal Analyzer Lab Client in the usual manner. When you launch the lab client, you will see the filter on the canvas. Run the iLab client to measure the frequency response of the filter using the standard parameters of a 2-V peak-to-peak input, a 1-Hz start frequency and a 50-kHz stop frequency. Take a screen shot of the canvas.

(E) Compare the theoretical characterization determined in Part (A) to the experimental characterization measured in Part (B). Knowing that $L = 98$ mH, determine C, R_1 and R_2. Explain how you do this and give the parameter values.

Problem 10.2 (2 Points): This problem examines the very simple tuner for an AM radio shown below. Here, the tuner is the parallel inductor and capacitor. The injection of radio signals into the tuner by the antenna is modeled by a current source, while the Norton resistance of the antenna in parallel with the remainder of the radio is modeled by a resistor. (You will learn about antenna modeling in 6.013.) The AM radio band extends from 540 kHz through 1600 kHz. The information transmitted by each radio station is constrained to be within ± 5 kHz of its center frequency. (You will learn about AM radio transmission in 6.003.) To prevent frequency overlap of neighboring stations, the center frequency of each station is constrained to be a multiple of 10 kHz. Therefore, the purpose of the tuner is to pass all frequencies within 5 kHz of the center frequency of the selected station, while attenuating all other frequencies.



(A) Assume that $I(t) = I\cos(\omega t)$. Find $V(\omega)$ and $\phi(\omega)$ in $v(t)$ where $v(t) = V\cos(\omega t + \phi)$. Note that $v(t)$ is the output of the tuner, namely the signal that is passed on to the remainder of the radio.

(B) For a given combination of I, C, L and R, at what frequency is V maximized?

(C) Assume that $L = 365$ μH. Over what range of capacitance must C vary so that the frequency of maximum V/I may be tuned over the entire AM band. Note that tuning the frequency of maximum V/I to the center frequency of a particular station tunes in that station.

(D) As a compromise between passing all frequencies within 5 kHz of a center frequency and rejecting all frequencies outside that band, let the design of R be such that $V(1 \text{ MHz} \pm 5 \text{ kHz})/V(1 \text{ MHz}) \approx 0.25$ when the tuner is tuned to 1 MHz. Given this design criterion, determine R.

(E) Given your design for R, determine $V(1 \text{ MHz} \pm 10\text{kHz})/V(1 \text{ MHz})$. Also, determine Q for the tuner and its load resistor when the tuner is tuned to 1 MHz.

(F) Suppose the tuner is first tuned to another station and then quickly tuned to the station broadcasting at 1 MHz. Approximately how long will it take for $v(t)$ to depend primarily on the signal from the station broadcasting at 1 MHz. Assume that both stations broadcast signals of equal strength. Hint: consider the time-domain interpretation of Q.

Problem 10.3 (2 Points): Following Problem 7.1, this problem studies the coupling of successive amplifier stages through capacitors and transformers. Here the focus is on sinusoidal-steady-state behavior.

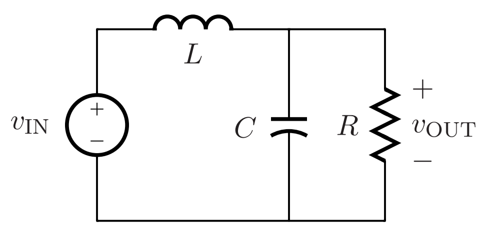
(A) This problem is based on the small-signal circuit model developed for Part (D) of Problem 7.1. Draw the circuit model again, and do not assume that $C_{\mathrm{GS}} = 0$.

(B) Assume that the small-signal input is $v_{\mathrm{in}} = V_{\mathrm{in}} \cos(\omega t)$, and correspondingly that the small-signal output is $v_{\mathrm{out}} = V_{\mathrm{out}} \cos(\omega t + \phi)$. Using the small-signal model from Part A, determine V_{out} and ϕ. Hint: you may find it easiest to first find the v_{gs} from v_{in}, next find v_{out} from v_{gs}, and finally combine the two earlier results find v_{out} from v_{in}.

(C) Define the small-signal amplifier gain G to be $G \equiv V_{\mathrm{out}}/V_{\mathrm{in}}$, and determine G. Also, determine the asymptotic dependence of G on ω for very small ω and for very large ω.

(D) In view of your answer to Part (C), should the input capacitance C and the magnetizing inductance L be made made small or large in order to improve the low-frequency and high-frequency gains? That is, how should C and L be chosen to extend the mid-range bandwidth over which G is approximately constant?

Problem 10.4 (2 Points): In this problem, a low-voltage sinusoidal source is coupled to a resistive load through an inductor-capacitor network as shown below. The role of the network is to boost the voltage at the load.



(A) Derive a second-order differential equation that describes the evolution of v_{OUT} as driven by v_{IN}. You need not solve the differential equation.

(B) Assume that the circuit operates in the sinusoidal steady state with $v_{\mathrm{IN}}(t) = V_{\mathrm{in}} \cos(\omega t)$. Correspondingly, let $v_{\mathrm{OUT}}(t)$ take the form $v_{\mathrm{OUT}} = V_{\mathrm{out}} \cos(\omega t + \phi)$. Determine ϕ, and the voltage gain G defined by $G \equiv V_{\mathrm{out}}/V_{\mathrm{in}}$.

(C) For a given R, L and ω, determine the value of C that maximizes G, and for this value of C, determine G.

(D) Suppose that $v_{\mathrm{IN}}(t)$ is abruptly set to zero in an attempt to remove the voltage at the load. In this case, the amplitude of the load voltage will decay in proportion to $e^{-t/\tau}$. Assuming that C is chosen to maximize G following the result from Part (C), determine τ in terms of G and ω. Hint: can you get the time constant (as a function of C, L and R) from the differential equation found in Part (A)?

(E) In view of the results of Parts (C) and (D), what is the disadvantage of using an inductor-capacitor network to boost the voltage which excites the load?