

**Problem Set #3**

Assigned: September 17, 2008  
 Due: September 24, 2008 at recitation

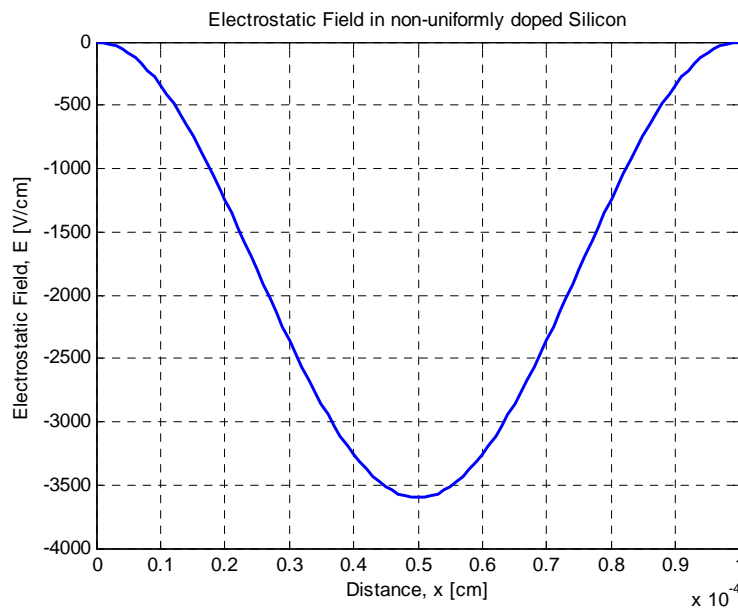
<b>Reading Assignments:</b>	09/16/06	Sections 3.3-3.4 of Howe & Sodini
	09/18/06	Sections 3.5-3.6 of Howe & Sodini
	09/23/06	Sections 3.6-3.7 of Howe & Sodini

PLEASE WRITE YOUR RECITATION SESSION TIME ON YOUR PROBLEM SET SOLUTION

**Problem 1.** [30 points]      *Electrostatics of non-uniformly doped Si*

In a p-type semiconductor in thermal equilibrium at room temperature, there is a spatial electric field distribution as shown in Figure 3.1. E is given by

$$E = -3600 \times \left[ \cos^2 \left( 10^4 \pi \left( x - 5 \times 10^{-5} \right) \right) \right] V / cm$$

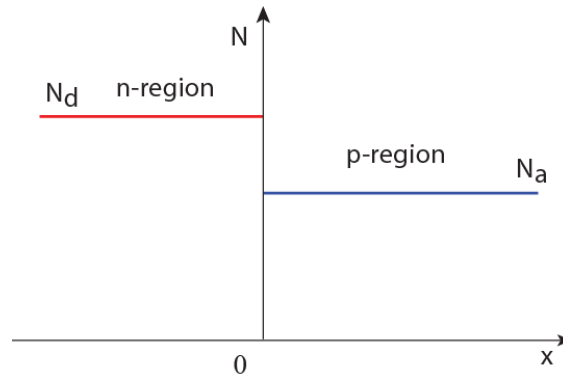


**Figure P3.1:** Electric Field in a non-uniformly doped Silicon

- Derive an analytical expression and sketch the electrostatic potential distribution in the region of the semiconductor  $0 \leq x \leq 1 \mu m$ .
- Derive an analytical expression and sketch the charge density in this region of the semiconductor  $0 \leq x \leq 1 \mu m$ .
- If the equilibrium hole concentration at  $x=0$  is  $p_0=10^{16} \text{ cm}^{-3}$ , calculate the equilibrium hole concentration at  $x=1 \mu m$ .

**Problem 2.** [30 points] *Electrostatics of pn junction*

Consider an abrupt pn junction with  $N_d = 10^{20} \text{ cm}^{-3}$  and  $N_a = 10^{16} \text{ cm}^{-3}$  as sketched below in Figure P3.2.

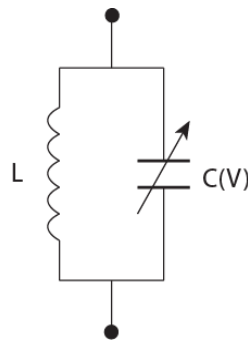


**Figure P3.2.** Doping profile for an abrupt pn junction.

- What is the electrostatic potential at  $x=0$ ,  $\phi(0)$ , in thermal equilibrium?
- What is  $n_o$  and  $p_o$  at  $x=0$  in thermal equilibrium?
- What is the value of  $x$  at which  $n_o = p_o = n_i$  in thermal equilibrium?
- What is the total amount of charge per unit area on the p-type side of the junction when a reverse bias voltage of 2.5 V is applied to the pn junction?

**Problem 3** [30 points] *Voltage Controlled Capacitor – Varactor*

Voltage controlled oscillators are used in most RF circuits (such as wireless LANs) to select a particular frequency (channel) to receive. They are made with tank circuits (parallel LC circuits) where the inductor value is fixed and the capacitor value is controlled by changing the voltage,  $V$  as in Figure P3.4. pn junction diodes are often in these circuits as voltage variable capacitors.



**Figure P3.3:** LC tank circuit with voltage variable capacitor

This problem examines the application of the voltage controlled capacitor (varactor) as the channel selector in Wireless LAN Network that uses the IEEE 802.11g standard which in the US & Canada has 11 channels. Note that the nominal frequencies for IEEE 802.11g Wireless LAN Channels 1 through 11 are Channel 1 – 2412 MHz; Channel 2 – 2417 MHz; Channel 3 – 2422

MHz; Channel 4 – 2427 MHz; Channel 5 – 2432 MHz; Channel 6 – 2437 MHz; Channel 7 – 2442 MHz; Channel 8 – 2447 MHz; Channel 9 – 2452 MHz; Channel 10 – 2457 MHz and Channel 11 – 2462 MHz.

- a) Given an inductance of 5 nH and a pn junction diode with doping of  $N_a = 10^{16} \text{ cm}^{-3}$  and  $N_d = 10^{19} \text{ cm}^{-3}$ . Calculate the area of the capacitor to make the resonant frequency 2.4 GHz when the voltage across the diode is 0 V. The resonant frequency is given by

$$f_o(V_D) = \frac{1}{2\pi\sqrt{L \bullet C(V_D)}}$$

where  $f_o$  is the resonant frequency,  $L$  is the inductance and  $C(V_D)$  is the capacitance.

- b) Plot the tank circuit frequency versus the tuning voltage to cover the range of frequencies required for all 11 channels.
- c) Determine the bias voltages you would use for each channel. **Note that you can use the plot you generated in part (b) of this problem.**

**Problem 4.** [10 points] *I-V characteristics of pn diode (cont.)*

This problem is a continuation of the pn diode characterization exercises of Problem Set #2. In this homework you examine to what extent the ideal-looking portion of the forward branch of the current-voltage characteristics of the diode follows the Boltzmann law.

As you know, the ideal I-V characteristics of a pn diode has an equation:

$$I_D = I_S \left[ \exp\left(\frac{qV_D}{kT}\right) - 1 \right]$$

Under sufficient forward bias, this equation can be simplified to:

$$I_D \approx I_S \exp\left(\frac{qV_D}{kT}\right)$$

Notice that this equation looks a lot like the Boltzmann relation for electrons derived in lecture:

$$n_o = n_i \exp\left(\frac{q\phi}{kT}\right)$$

In the discussion around the Boltzmann relation in lecture, we derived the "60 mV rule". This states that at 300 K and in thermal equilibrium, every change in the electron concentration in a decade (factor of 10) brings along a change in the electrostatic potential of about 60 mV. In lecture, we also showed that, remarkably, the current through a BJT (which in essence consists of two back-to-back pn diodes) in the ideal-looking portion of the forward branch behaves in a

similar way: every increase of 60 mV brings along an increase in the current of a factor of 10x (provided that the diode sits at around 300 K). This is a direct manifestation of Boltzmann law right out of your instrument! This simple exercise is about demonstrating experimentally that this is indeed the case in a more rigorous way than it was done in class.

What you have to do is to measure the I-V characteristics of the pn diode (***Please choose the 6.012 pn diode from the Devices Menu***), such that  $200\text{ mV} \leq V_D \leq 800\text{ mV}$  using 5 mV steps and compute the slope (it should perhaps be called "inverse slope" but it is usually referred to simply as the slope) of the ideal forward branch in units of V/dec ("volts per decade of current"). You can do this in two ways:

- One way is to work with the downloaded data and write a piece of code in MATLAB, EXCEL, or similar program and derive this slope point by point for all points of the forward branch of the pn diode.
- Another way that is faster is to take advantage of the user defined functions of the Microelectronics WebLab and have the instrument do the computation for you. You can then graph the results in the right-hand y axis of the WebLab canvas and simply print out this graph.

In either case, you have to come up with a simple mathematical algorithm to obtain this slope.

You have to turn in:

- A description of the mathematical algorithm that allows you to obtain the slope of the forward branch of the diode in units of V/dec.
- A graph showing the slope of the forward branch of the pn diode currently placed in WebLab as a function of  $V$ . In the same graph or in a separate graph, plot the semilog I-V characteristics of the diode.
- A brief discussion of the range of  $V$  for which a Boltzmann-like behavior is observed in the diode.

If you decide to have WebLab do the computation of the slope for you, you should read about the user defined functions in Section 1.6 and Appendix A of the WebLab manual (accessible from the Applet launch page in <http://ilab.mit.edu>).