6.012 Problem Set # solutions

1)

a) N_type Si $\Rightarrow \rho \simeq \dfrac{1}{q \mu_n n_o}$

$\rho = 10\ \Omega.cm \xrightarrow[\text{resistivity - doping concentration graph in lecture note}]{\text{From}} n_o \simeq N_d \simeq \boxed{5 \times 10^{14}\ cm^{-3}}$ (i)

$p_o = \dfrac{n_i^2}{n_o} = \boxed{2 \times 10^{5}\ cm^{-3}}$ (ii)

$N_d = 5 \times 10^{14}\ cm^{-3} \xrightarrow[\text{mobility - doping concentration graph in lecture note}]{\text{From}} \begin{cases} \mu_n \simeq \boxed{1380\ cm^2/V.s} \\ \mu_p \simeq \boxed{480\ cm^2/V.s} \end{cases}$ (iii)

b) $E = \rho J = 10\ \Omega.cm \times 10^{3}\ A/cm^2 = \boxed{10^{4}\ V/cm}$

c) $\dfrac{J_n^{drift}}{J_p^{drift}} = \dfrac{\mu_n n_o}{\mu_p p_o} = \dfrac{1380\ cm^2/V.s \times 5 \times 10^{14}\ cm^{-3}}{480\ cm^2/V.s \times 2 \times 10^{5}\ cm^{-3}} \simeq \boxed{7 \times 10^{9}}$

d) $v_p = \mu_p E = 480\ cm^2/V.s \times 10^{4}\ V/cm = \boxed{4.8 \times 10^{6}\ cm/s}$

e) $v_n = \mu_n E = 1380\ cm^2/V.s \times 10^{4}\ V/cm = \boxed{1.38 \times 10^{7}\ cm/s}$

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2) a) For each layer with thickness of dx :

$R = \rho \frac{L}{A}$ where ρ is resistivity and A is cross section

$\Rightarrow \frac{1}{R} = \frac{1}{\rho}\frac{A}{L} = q\mu_n n_o(x) \frac{W\,dx}{L}$

for parallel resistances : $\frac{1}{R_{tot}} = \sum_{i=1}^{N} \frac{1}{R_i}$

in this case since there are infinite number of parallel resistances the summation becomes an integral :

$$\frac{1}{R_{tot}} = \int_0^T q\mu_n n_o(x) \frac{W}{L}\,dx = \boxed{q\mu_n \frac{W}{L}\int_0^T n_o(x)\,dx}$$

Therefore R_{tot} depends only on the integral of $n_o(x)$ from $\emptyset$ to T.

b) $R_{sh} = \left(q\mu_n \int_0^T n_o(x)\,dx\right)^{-1}$

$$\int_0^T n_o(x)\,dx = 10^{18}\,cm^{-3}\int_0^T e^{-\left(\frac{x-x_p}{L_x}\right)^2} dx = 10^{18}\,cm^{-3}\int_{-x_p}^{T-x_p} e^{-y^2/L_x} dy$$

where $y \equiv x - x_p$, By approximation we can set the boundaries of the integral from $-\infty$ to ∞ :

$$10^{18}\,cm^{-3}\int_{-\infty}^{+\infty} e^{-y^2/L_x^2} dx = 10^{18}\,cm^{-3}\left(\sqrt{\pi}\,L_x\right)$$

$$\Rightarrow R_{sh} = \left(q \mu_n \, 10^{18} \, cm^{-3} \, \sqrt{\pi} \, L_x \right)^{-1}$$

$$= \left(1.6 \times 10^{-19} \, \frac{V.s}{\Omega} \times 300 \, \frac{cm^2}{V.s} \times 10^{+18} \, cm^{-3} \times \sqrt{\pi} \times 0.25 \times 10^{-4} \, cm \right)^{-1}$$

$$= \boxed{470 \, \Omega}$$

note that the value of μ_n, $300 \, cm^2/V.s$, is based on the mobility-doping graph for doping level of $10^{18} \, cm^{-3}$

c) $R = R_{sh} N_{\square}$ where $N_{\square}$ is the number squares in the path between ponits A and B.

By counting $N_{\square} = 66$ (considering the corner squares as one full square, refer to the textbook for a more accurate way to count corner squares)

$$\Rightarrow R = 470 \, \Omega/\square \times 66 \, \square = \boxed{31 \, k\Omega}$$

3)

$$n(x) = 10^{16}\ cm^{-3} + 10^{13}\ cm^{-3}\ (1 - x) \qquad (x \text{ in units of } \mu m)$$

If you bring x into units of cm, then

$$n(x) = 10^{16}\ cm^{-3} + 10^{13}\ cm^{-3}\ (1 - 10^{4}\ x) \qquad (x \text{ in units of cm})$$

similarly

$$p(x) = 10^{4}\ cm^{-3} + 10^{13}\ cm^{-3}\ (1 - 10^{4}\ x) \qquad (x \text{ in units of cm})$$

a) $$J_p^{diff} = -q D_p \frac{dp}{dx}$$

$$\left.\begin{aligned} D_p &= \frac{kT}{q}\mu_p \\ \mu_p \text{ @ concentration } 10^{16} &= 450\ cm^2/V.s \end{aligned}\right\} \Rightarrow D_p = 11.25\ cm^2/s$$

$$\Rightarrow J_p^{diff} = -(1.6\times10^{-19}\ A.s) \times 11.25\ cm^2/s \times 10^{13}\ cm^{-3}\ \cancel{\times}(-10^{4}\ cm^{-1})$$

$$= \boxed{0.18\ A/cm^2}$$

b) $$\left.\begin{aligned} D_n &= \frac{kT}{q}\mu_n \\ \mu_n \text{ @ concentration } 10^{16} &= 1200\ cm^2/V.s \end{aligned}\right\} \Rightarrow D_n = 30\ cm^2/s$$

$$J_n^{diff} = q D_n \frac{dn}{dx} =$$

$$1.6 \times 10^{19}\ A.s \times 30\ cm^2/s \times 10^{13}\ cm^{-3} \times (-10^{4}\ cm^{-1})$$

$$= \boxed{-0.48\ A/cm^2}$$

c) $J_{tot} = 0 \Rightarrow J^{drift} = -(J_n^{diff} + J_p^{diff})$

$$= -(-0.48\ A/cm^2 + 0.18\ A/cm^2)$$

$$= \boxed{0.3\ A/cm^2}$$

d) $J^{drift} \simeq J_n^{drift} = q\mu_n n E$

$$\simeq (1.6\times10^{-19}\ A.s)\times 1200\ cm^2/V.s \times 10^{16}\ cm^{-3} \times E$$

comparing with part (c):

$$E = \frac{0.3\ A/cm^2}{1.6\times10^{-19}\ A.s \times 1200\ cm^2/V.s \times 10^{16}\ cm^{-3}} \simeq \boxed{0.16\ V/cm}$$

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4)

$$P_o(x) = 10^{+19}\ cm^{-3} \times 10^{-\frac{x}{Lx}} \qquad Lx = 10\mu m\ ,\ 0 < x < 50\mu m$$

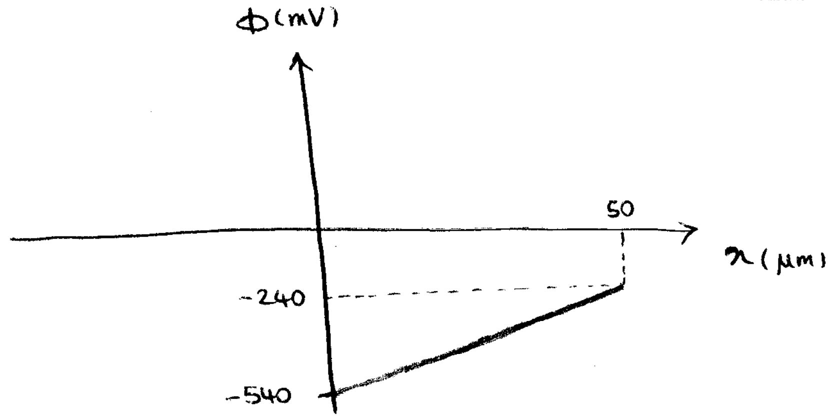
a)

$$\Phi(x) = -60\ mV \log \frac{P_o(x)}{n_i}$$

$$= -60\ mV \log \frac{10^{+19} \times 10^{-\frac{x}{Lx}}}{10^{10}}$$

$$= -60\ mV \left(9 - \frac{x}{Lx}\right)$$

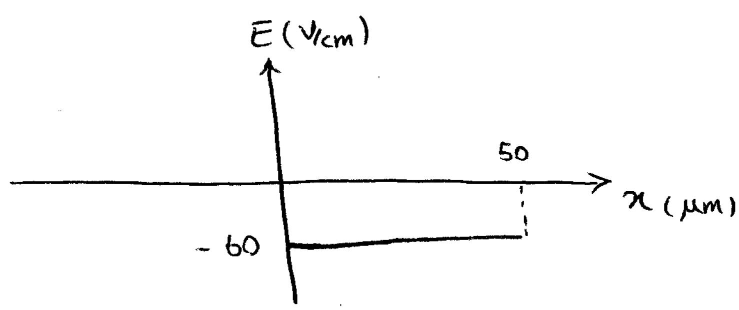
$$= \boxed{-540\ mV + 60\ mV \frac{x}{Lx}}$$



b)

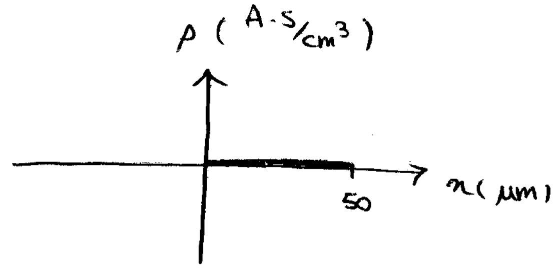
$$E(x) = -\frac{d\Phi}{dx}$$

$$= -\frac{60\ mV}{Lx} = -\frac{60\ mV}{10\ \mu m} = \boxed{-60\ V/cm}$$



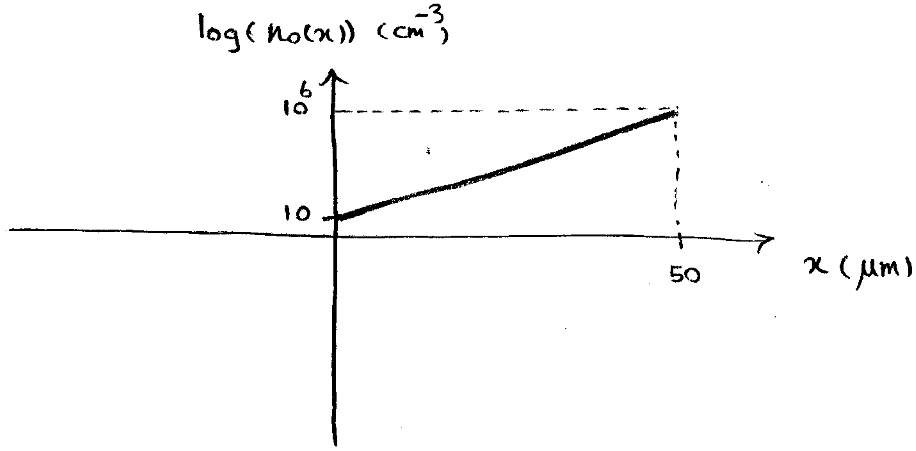
c)

$$\rho = \varepsilon_s \frac{dE}{dx} = \boxed{0}$$

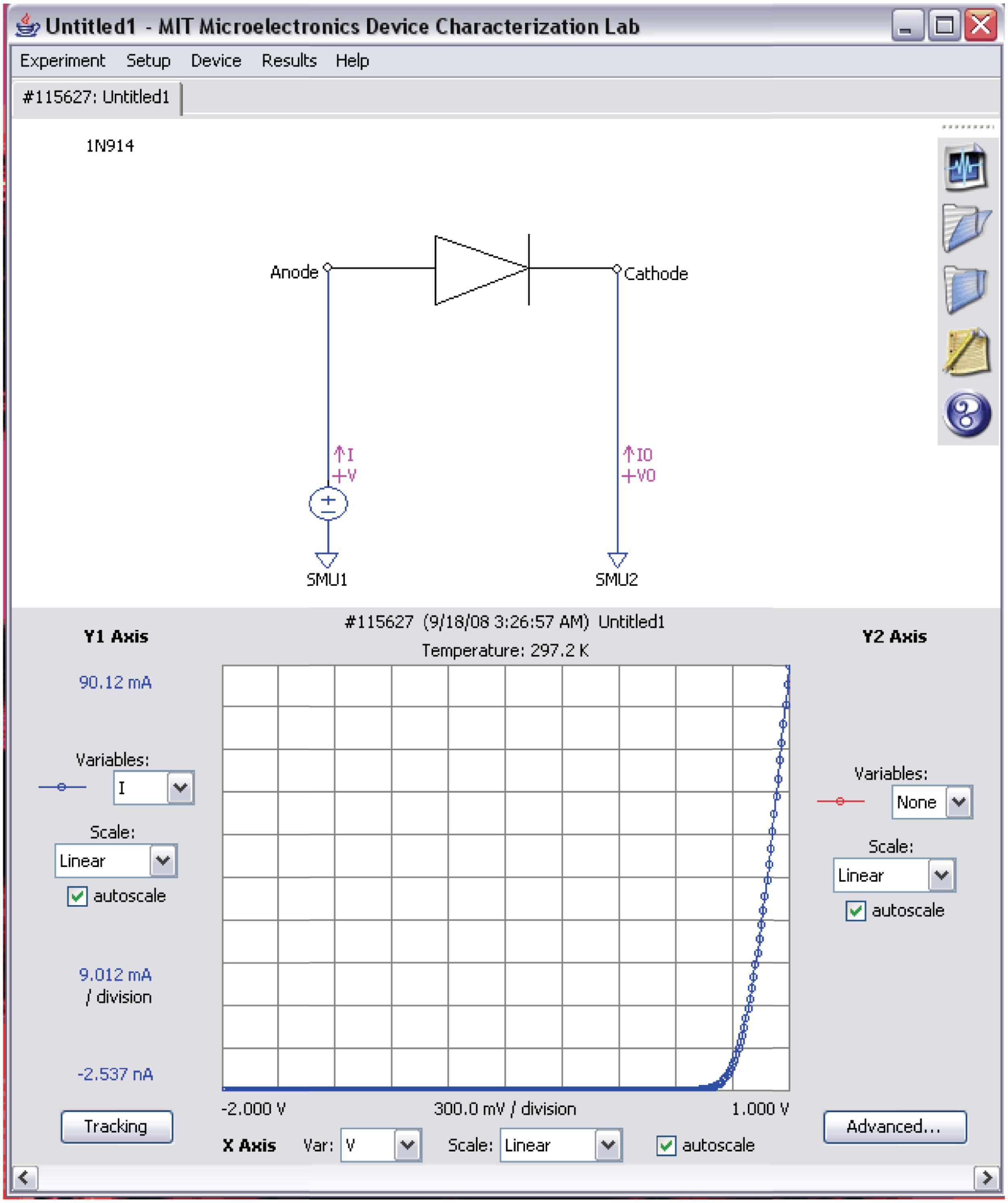


d) $n_o(x) = \dfrac{n_i^2}{P_o(x)} = \dfrac{10^{20}}{10^{19} \times 10^{-x/L_n}}$ cm^{-3}

$$\boxed{= 10 \times 10^{+x/L_n} \text{ cm}^{-3}}$$

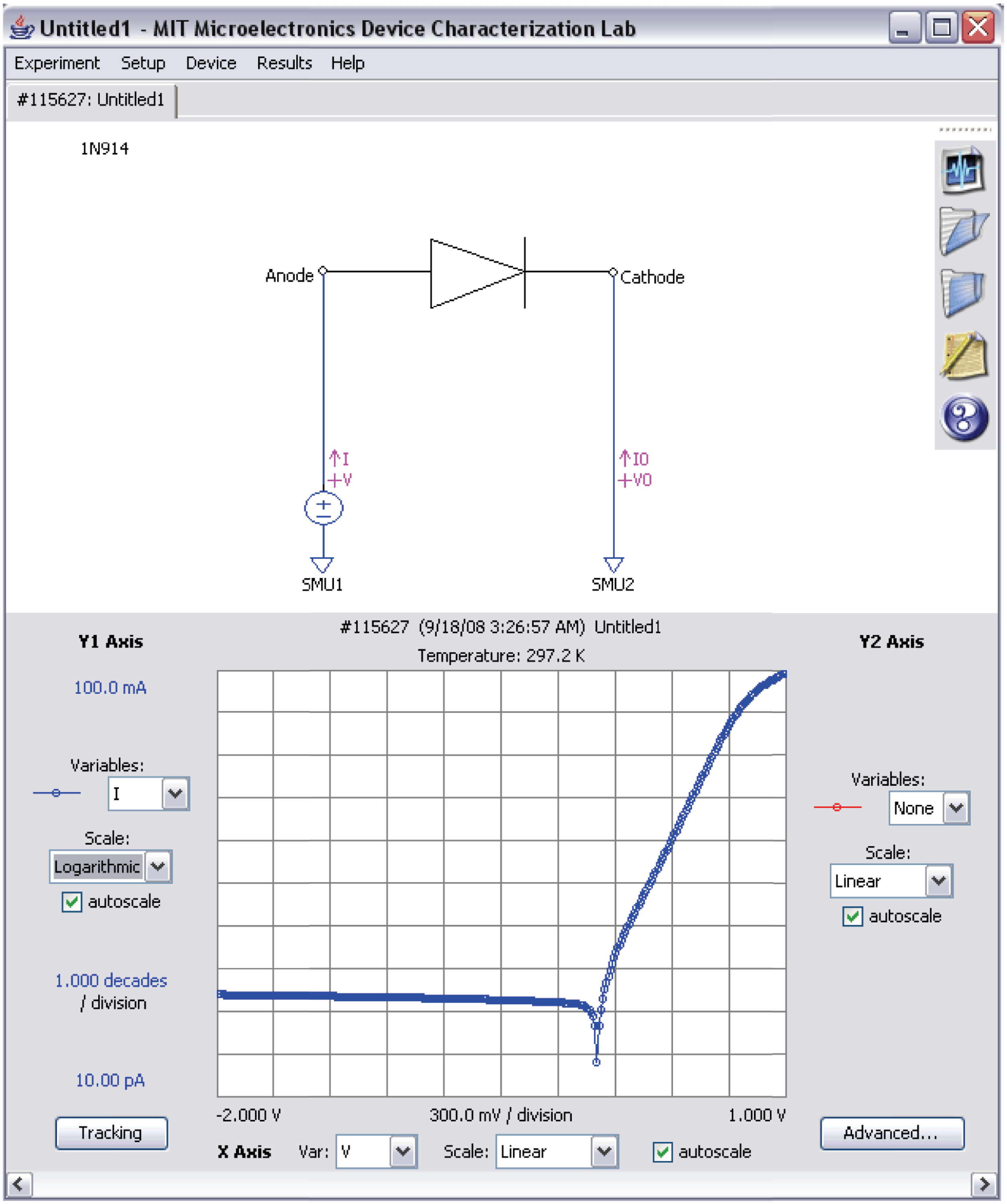


5) Graph 1

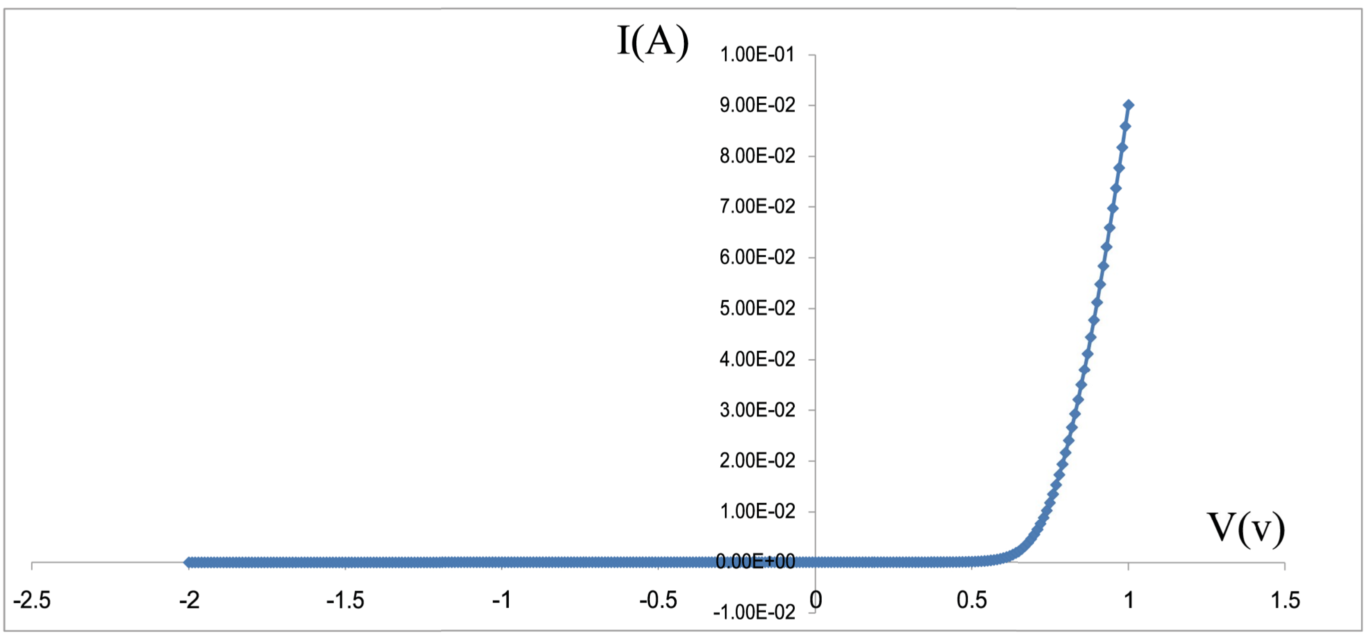


Untitled1 - MIT Microelectronics Device Characterization Lab
Experiment Setup Device Results Help
#115627: Untitled1
1N914
Anode
Cathode
I
+V
I0
+V0
SMU1
SMU2
Y1 Axis
#115627 (9/18/08 3:26:57 AM) Untitled1
Temperature: 297.2 K
Y2 Axis
90.12 mA
Variables:
I
Scale:
Linear
autoscale
9.012 mA
/ division
-2.537 nA
Variables:
None
Scale:
Linear
autoscale
-2.000 V
300.0 mV / division
1.000 V
Tracking
X Axis
Var:
V
Scale:
Linear
autoscale
Advanced...

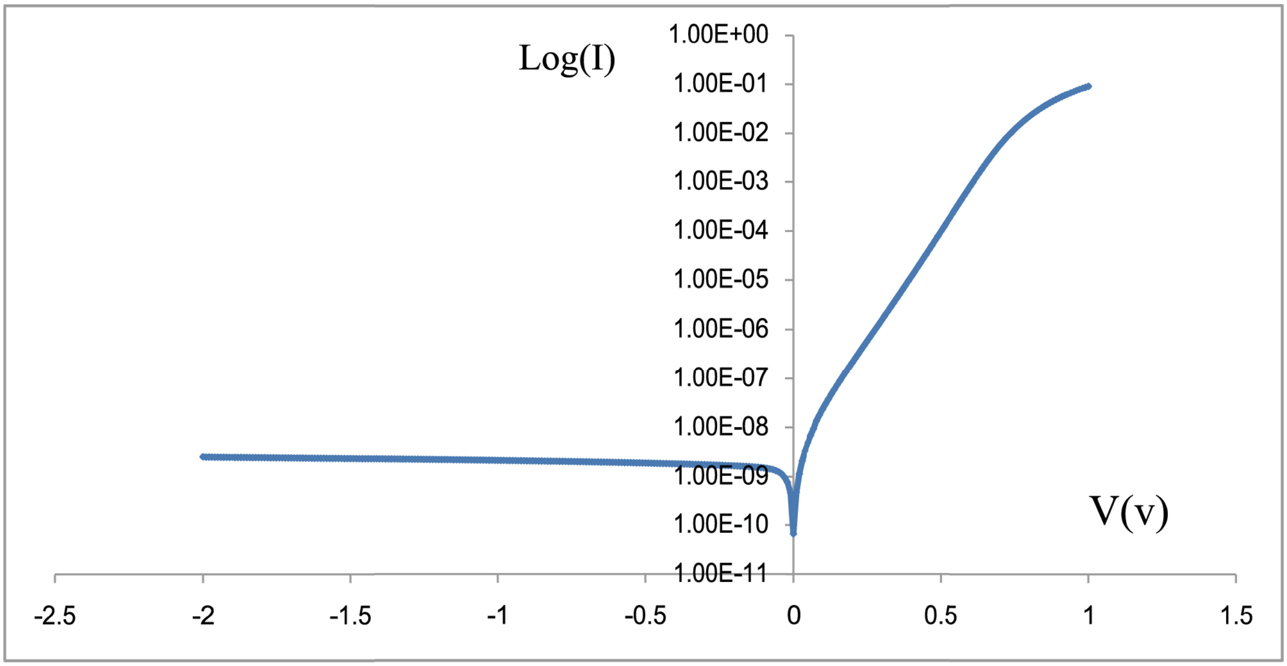
Graph 2


Untitled1 - MIT Microelectronics Device Characterization Lab
Experiment
Setup
Device
Results
Help
#115627: Untitled1
1N914
Anode
Cathode
I
+V
I0
+V0
SMU1
SMU2
#115627 (9/18/08 3:26:57 AM) Untitled1
Temperature: 297.2 K
Y1 Axis
100.0 mA
Variables:
I
Scale:
Logarithmic
autoscale
1.000 decades / division
10.00 pA
Tracking
Y2 Axis
Variables:
None
Scale:
Linear
autoscale
Advanced...
-2.000 V
300.0 mV / division
1.000 V
X Axis
Var:
V
Scale:
Linear
autoscale

Graph 3


I(A)
1.00E-01
9.00E-02
8.00E-02
7.00E-02
6.00E-02
5.00E-02
4.00E-02
3.00E-02
2.00E-02
1.00E-02
0.00E+00
-1.00E-02
V(v)
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5

Graph 4


Log(I)
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
1.00E-07
1.00E-08
1.00E-09
1.00E-10
1.00E-11
V(v)
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5