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HW #3

Sep 13, 2008

(1) (a) $E = -3600 \left[\cos^2\left(10^4\pi\left(x - 5\times10^{-5}\right)\right)\right] \ \frac{V}{cm}$

$$E = -\frac{d\phi}{dx} \Rightarrow \phi = -\int E\,dx$$

$$\phi = -\int_0^x -3600\left[\cos^2\left(10^4\pi\left(x' - 5\times10^{-5}\right)\right)\right]dx' + \phi_0$$

$$\phi(x) = 3600\int_0^x \left[\cos^2\left(10^4\pi\left(x' - 5\times10^{-5}\right)\right)\right]dx' + \phi_0$$

** double angle formulas : $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$ **

$$\cos^2\theta = \frac{1}{2} + \frac{\cos(2\theta)}{2}$$

let $y = 10^4\pi\left(x' - 5\times10^{-5}\right)$

$dy = 10^4\pi\,dx'$

$$\phi(x) = 3600\int_{-0.5\pi}^{10^4\pi(x - 5\times10^{-5})} (\cos^2 y)\frac{1}{(10^4\pi)}\,dy + \phi_0$$

$$\phi(x) = \frac{0.36}{\pi}\int_{-0.5\pi}^{10^4\pi(x - 5\times10^{-5})}\left[\frac{1}{2} + \frac{\cos(2y)}{2}\right]dy + \phi_0$$

$$\phi(x) = \frac{0.36}{\pi}\left[\frac{1}{2}y + \frac{\sin(2y)}{4}\right]_{-0.5\pi}^{10^4\pi(x - 5\times10^{-5})} + \phi_0$$

$$\phi(x) = \frac{0.36}{\pi}\left[\frac{1}{2}10^4\pi\left(x - 5\times10^{-5}\right) + \frac{\sin\left(2\times10^4\pi\left(x - 5\times10^{-5}\right)\right)}{4} + \frac{\pi}{4}\right] + \phi_0$$

$$\boxed{\phi(x) = \left\{1800x + \frac{0.09}{\pi}\sin\left[2\times10^4\pi\left(x - 5\times10^{-5}\right)\right] + \phi_0\right\}V}$$

$\boxed{\text{plots: see attched}}$

(b) $\frac{dE}{dx} = \frac{\rho}{\varepsilon_s} \Rightarrow \rho = \varepsilon_s \frac{dE}{dx}$

$$\rho = \varepsilon_s \frac{d}{dx}\left\{-3600\left[\cos^2\left(10^4\pi(x-5\times10^{-5})\right)\right]\right\}$$

$$\rho = \varepsilon_s \frac{d}{dx}\left\{-3600\left[\frac{1}{2} + \frac{\cos\left(2\times10^4\pi(x-5\times10^{-5})\right)}{2}\right]\right\}$$

$$\rho = \varepsilon_s \left(-\frac{3600}{2}\right)\frac{\left(-\sin\left(2\times10^4\pi(x-5\times10^{-5})\right)\right)}{2\times10^4\pi}$$

$$\rho = \frac{1800\,\varepsilon_s}{2\times10^4\pi}\sin\left[2\times10^4\pi\,(x-5\times10^{-5})\right]$$

$$\boxed{\rho = \frac{0.09\,\varepsilon_s}{\pi}\sin\left[2\times10^4\pi\,(x-5\times10^{-5})\right]}$$

plots: see attached

** note that $\rho \ll qN_a$, quasi-neutral condition is satisfied!!

(c) $\phi(x=0) = \phi_0 \Rightarrow \phi_0 = -60\,\text{mV}\log\frac{10^{16}}{10^{10}} \Rightarrow \phi_0 = -360\,\text{mV}$

$$\underset{x=1\times10^{-4}\,\text{cm}}{\phi(x=1\,\mu\text{m})} = 1800\,(1\times10^{-4}) + \frac{0.09}{\pi}\sin\left[2\times10^4\pi\,(1\times10^{-4}-5\times10^{-5})\right] - 360\,\text{mV}$$

$$\phi(x=1\,\mu\text{m}) = 0.18 + (-360\,\text{mV})$$

$$\boxed{\phi(x=1\,\mu\text{m}) = -0.18\,\text{V}}$$

$$\phi(x=1\,\mu\text{m}) = -180\,\text{mV} = -60\,\text{mV}\log\frac{p(x=1\,\mu\text{m})}{10^{10}\,\text{cm}^{-3}}$$

$$\boxed{p(x=1\,\mu\text{m}) = 10^{13}\,\text{cm}^{-3}}$$

these can be simply derived following the book's procedure

(2) we know

$$E_o(x) = \begin{cases} \frac{qN_d}{\varepsilon_s}[x-(-x_{no})] & , -x_{no}<x<0 \\ \frac{qN_a}{\varepsilon_s}[x_{po}-x] & , 0<x<x_{po} \end{cases}$$

and

$$\phi_o(x) = \begin{cases} \frac{qN_a}{2\varepsilon_s}(x_{po}-x)^2+\phi_p & , 0<x<x_{po} \\ \phi_n - \frac{qN_d}{2\varepsilon_s}(x-(-x_n))^2 & , -x_{no}<x<0 \end{cases}$$

$$x_{po} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_a}\right)\left(\frac{N_d}{N_d+N_a}\right)} \quad , \quad x_{no} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_d}\right)\left(\frac{N_a}{N_d+N_a}\right)}$$

(a) since $N_d \gg N_a$ (i.e. $10^{20}\,cm^{-3} \gg 10^{16}\,cm^{-3}$)

$x_{po} \gg x_{no} \Rightarrow X_{do} \approx x_{po}$

$$X_{do} \approx x_{po} = \sqrt{\frac{2\varepsilon_s\phi_B}{qN_a}} = \sqrt{\frac{2\times 11.7\times 8.85\times10^{-14}\,F/cm \times 910\,mV}{(1.6\times10^{-19}C)(1\times10^{16}cm^{-3})}}$$

$\phi_B = \phi_n - \phi_p = 550\,mV - (-360\,mV) = \underline{910\,mV}$

$$\boxed{X_{do} \approx x_{po} \approx 3.43\times10^{-5}\,cm = 0.343\,\mu m}$$

$$\phi_o(x=0) = \frac{qN_a}{2\varepsilon_s}(0+x_{po})^2 + \phi_p$$

$$\phi_o(x=0) = \frac{(1.6\times10^{-19}C)(1\times10^{16}cm^{-3})}{2\times 11.7\times 8.85\times10^{-14}\,F/cm}(3.43\times10^{-5}cm)^2 + (-360\,mV)$$

$\phi_o(x=0) = 0.549\,V$

$$\boxed{\phi_o(x=0) = 549\,mV}$$

*** note, this should not surprise you. Since $N_d \gg N_a$, $x_{po} \gg x_{no}$, and $x_{no} \approx 0\,cm$.

$\therefore\ \phi_o(x=0) \approx \phi_o(x=x_{no}) \approx \underline{550\,mV}$

from 550mV

(b) from (a), we know $n_o(x=0) \approx n_o(x=x_n) \approx \underline{\underline{1.47\times10^{19}\,cm^{-3}}}$

and $p_o(x=0) \approx p_o(x=x_n) \approx \frac{1\times10^{20}}{1.47\times10^{19}} = \underline{\underline{6.8\,cm^{-3}}} \approx \underline{\underline{0\,cm^{-3}}}$

let's plug in $\phi(x=0)$ to verify our approximation,

$$n_o(x=0) = n_i \exp\left(\phi(x=0)/V_{th}\right)$$

$$n_o(x=0) = 1\times10^{10}\,cm^{-3} \exp\left(\frac{549\,mV}{26\,mV}\right)$$

$$\boxed{n_o(x=0) = 1.48\times10^{19}\,cm^{-3}}$$

$$p_o(x=0) = \frac{n_i^2}{n_o} = \frac{1\times10^{20}(cm^{-3})^2}{1.48\times10^{19}\,cm^{-3}} \Rightarrow \boxed{p_o(x=0) = 6.8 \approx 0\,cm^{-3}}$$

(c)

$$\phi_o(x) = \frac{qN_a}{2\varepsilon_s}(-x+x_{po})^2 + \phi_p$$

$$0 = \frac{qN_a}{2\varepsilon_s}(-x+x_{po})^2 + \phi_p$$

$$0 = x^2 - 2x\,x_{po} + x_{po}^2 + \frac{2\varepsilon_s\phi_p}{qN_a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{2x_{po} \pm \sqrt{4x_{po}^2 - 4\left(x_{po}^2 + \frac{2\varepsilon_s\phi_p}{qN_a}\right)}}{2}$$

$$x = \frac{2x_{po} \pm \sqrt{-\frac{8\varepsilon_s\phi_p}{qN_a}}}{2}$$

$$x = x_{po} \pm \sqrt{-\frac{2\varepsilon_s\phi_p}{qN_a}}$$

$$x = 3.43\times10^{-5}\,cm \pm \sqrt{\frac{-2\times11.7\times8.85\times10^{-14}\,F/cm \times(-360\,mV)}{(1.6\times10^{-19}\,C)(1\times10^{16}\,cm^{-3})}} \Rightarrow \boxed{x = 1.27\times10^{-5}\,cm}$$

(d)

$$Q = qN_a x_p = qN_a x_{po}\sqrt{1-\frac{V_p}{\phi_B}}$$

$$Q = (1.6\times10^{-19}\,C)(1\times10^{16}\,cm^{-3})(3.43\times10^{-5}\,cm)\sqrt{1-\frac{(-2.5)}{0.91}}$$

$$\boxed{Q = 1.06\times10^{-7}\,\frac{C}{cm^2}}$$

③ (a)

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_P}{\phi_B}}} \quad \text{where} \quad C_{j0} = \frac{\varepsilon_s}{X_{d0}}$$

$$X_{d0} = \sqrt{\frac{2\varepsilon_s \phi_B}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)} \quad , \quad \phi_B = \phi_n - \phi_p = 540\,mV + 360\,mV$$

$$\phi_B = \underline{900\,mV}$$

$$X_{d0} = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}\,\frac{F}{cm} \times 900\,mV}{1.6 \times 10^{-19}\,C}\left(\frac{1}{10^{16}\,cm^{-3}} + \frac{1}{10^{19}\,cm^{-3}}\right)}$$

$$\boxed{\begin{aligned} X_{d0} &= 3.415 \times 10^{-5}\,cm \\ X_{d0} &= 3.415 \times 10^{-7}\,m \end{aligned}}$$

$$C_{j0} = \frac{11.7 \times 8.85 \times 10^{-14}\,\frac{F}{cm}}{3.415 \times 10^{-5}\,cm}$$

$$\boxed{C_{j0} = 3.03 \times 10^{-8}\,F/cm^2}$$

$$f_0(V) = \frac{1}{2\pi\sqrt{L \cdot C(V)}}$$

$$2\pi\,(2.4 \times 10^9) = \frac{1}{\sqrt{(5 \times 10^{-9}\,H)\ C_j \cdot A}}$$

$$2\pi\,(2.4 \times 10^9) = \frac{1}{\sqrt{(5 \times 10^{-9}\,H)\,(3.03 \times 10^{-8}\,\frac{F}{cm^2})\,A}}$$

$$\boxed{A = 2.90 \times 10^{-5}\,cm^2}$$

(b) (c) see plots attached

Channel	1	2	3	4	5	6	7	8	9	10	11
Voltage (V)	-0.0165	-0.024	-0.032	-0.0395	-0.047	-0.055	-0.063	-0.071	-0.0785	-0.08675	-0.095

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$$\text{slope} = \frac{\Delta V}{1 \text{ decade } \Delta \text{ in } I}$$

$$1 \text{ decade } \Delta \text{ in } I = \Delta(\log I)$$

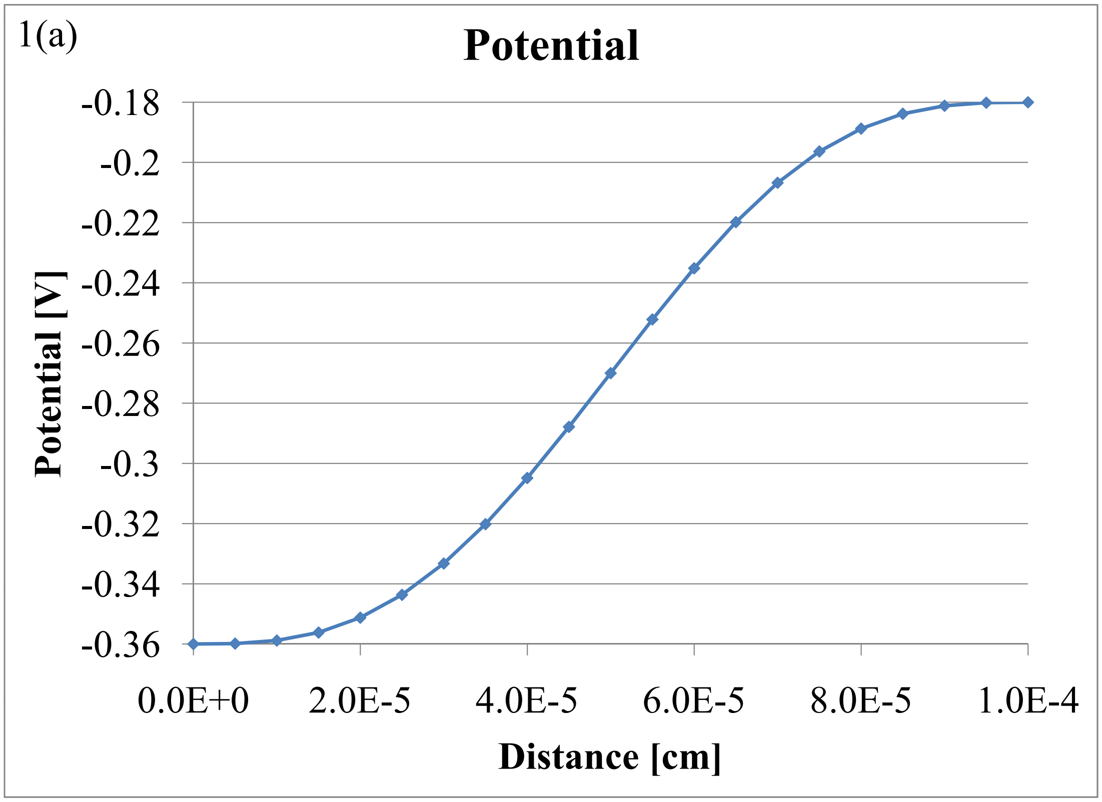
$$\Rightarrow \boxed{\text{slope} = \frac{\Delta V}{\Delta(\log I)}}$$

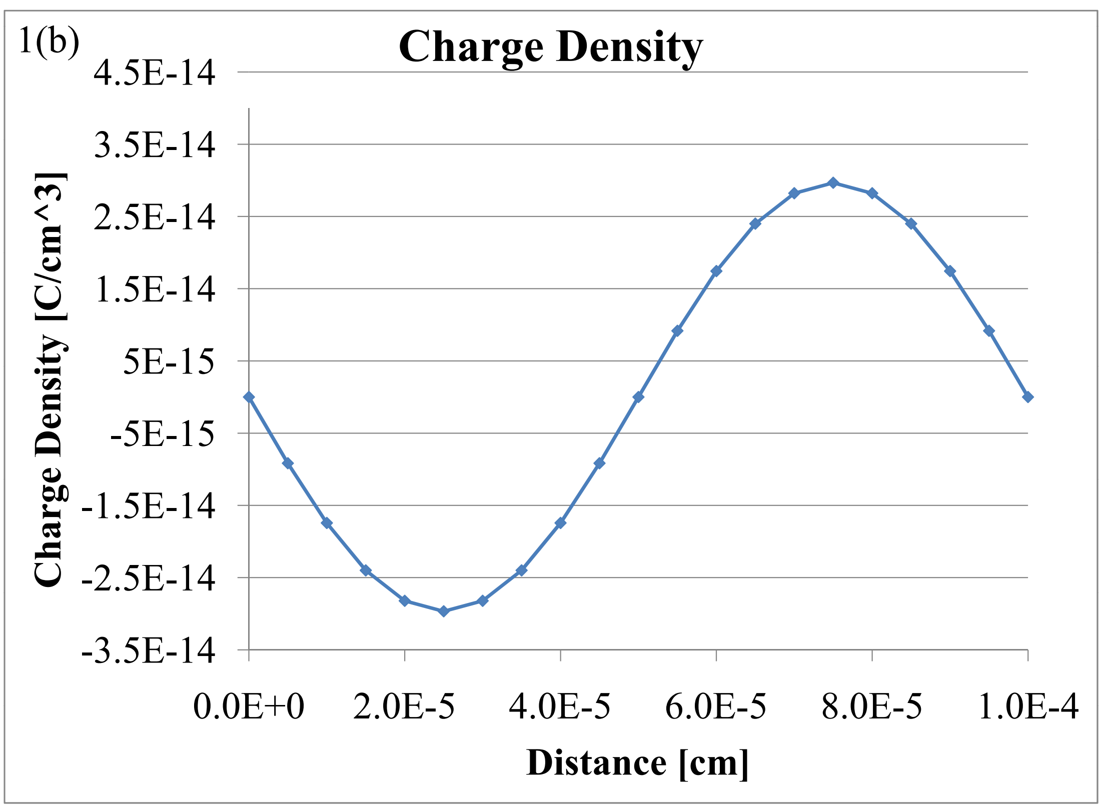
*** to obtain nice slope, simply use smaller step (i.e. $\Delta V = 1\text{mV}$) and approximate slope $= \dfrac{dV}{d(\log I)} \approx \dfrac{\Delta V}{\Delta(\log I)}$ ***

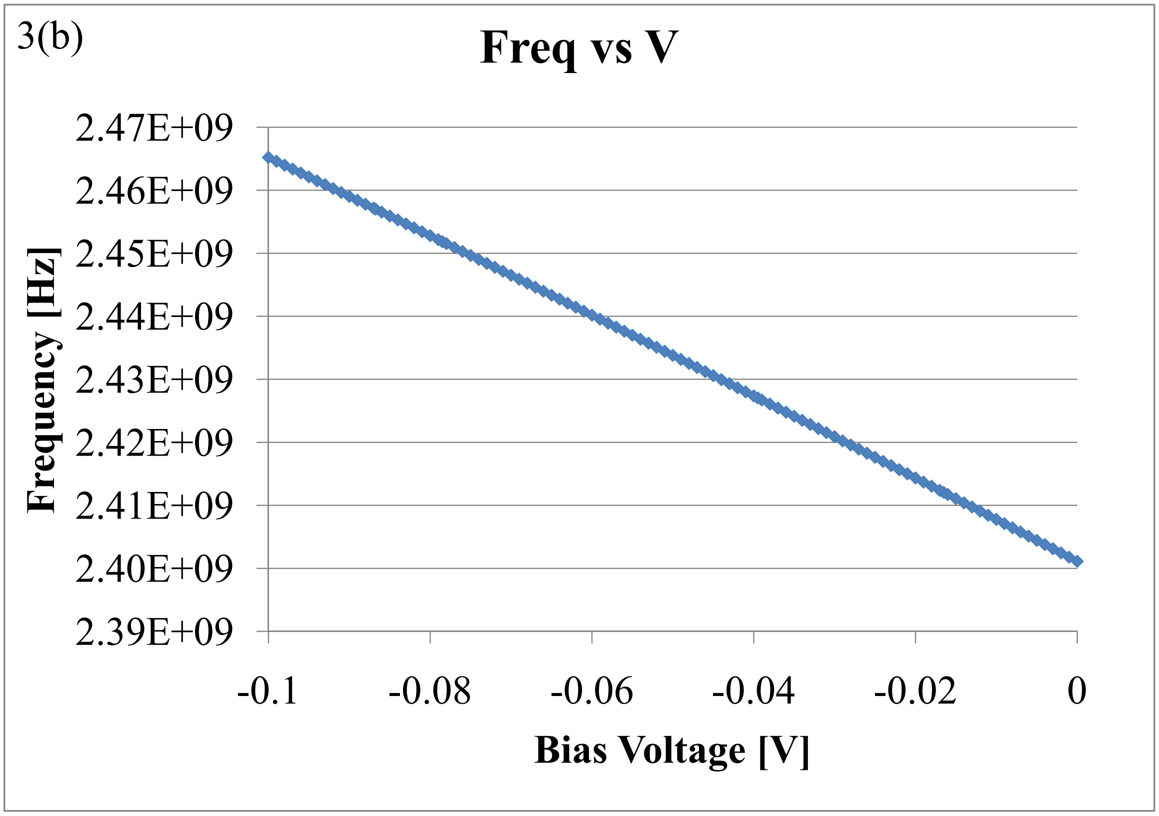
<u>Extra</u>: Why 1N914 Diode behaves differently from 6.012 Diode? Why does 1N914 have higher ΔV/decade? (i.e. why does 1N914 require more voltage to increase the I by a factor of 10?)

Hint: 1N914 has much longer base than 6.012 Diode. (i.e. e^- needs to travel longer distance).

Answer: Come talk to the staffs if you are interested in finding out.

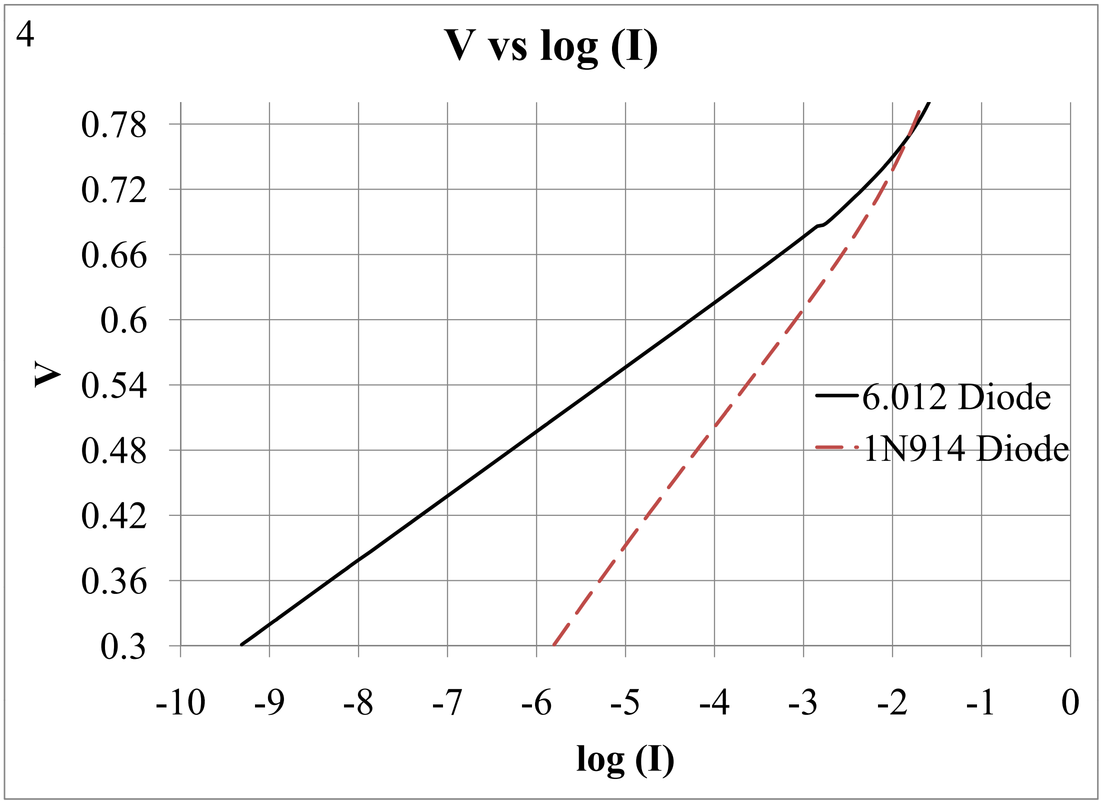

1(a)
Potential
-0.18
-0.2
-0.22
-0.24
-0.26
-0.28
-0.3
-0.32
-0.34
-0.36
Potential [V]
0.0E+0
2.0E-5
4.0E-5
6.0E-5
8.0E-5
1.0E-4
Distance [cm]

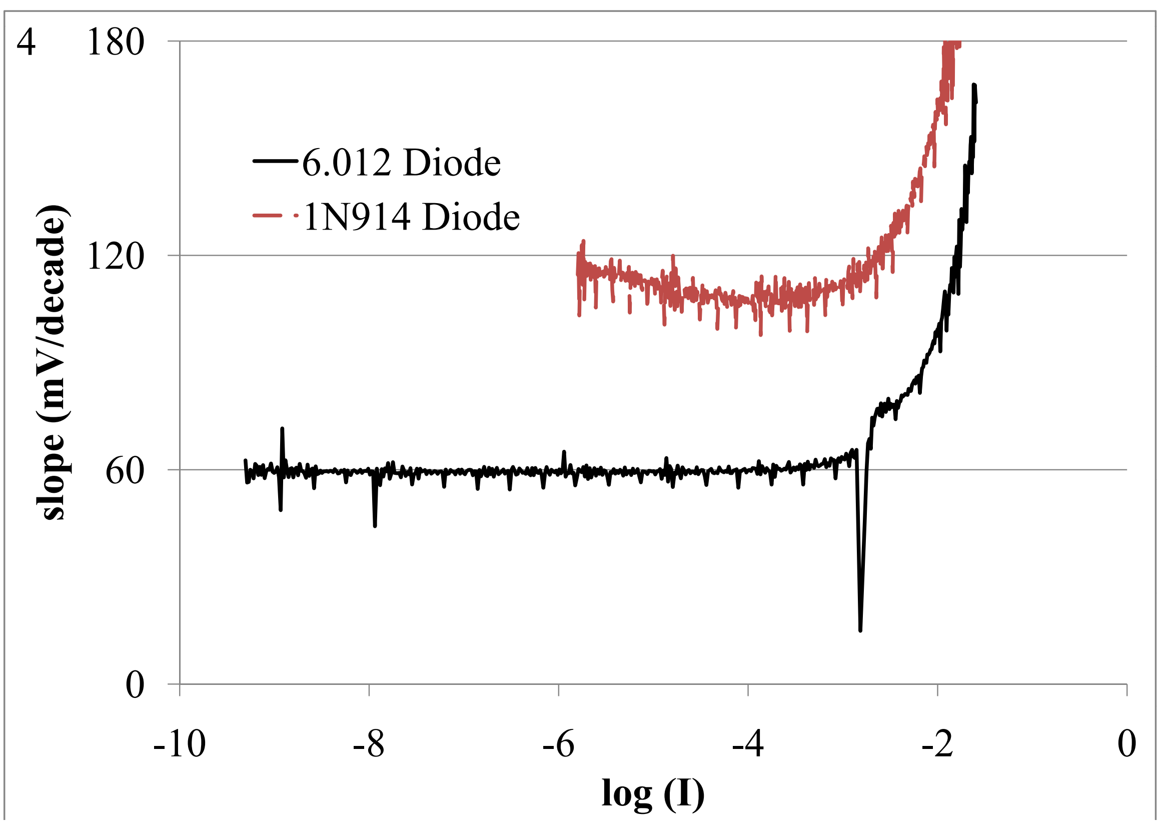

1(b)
Charge Density
4.5E-14
3.5E-14
2.5E-14
1.5E-14
5E-15
-5E-15
-1.5E-14
-2.5E-14
-3.5E-14
Charge Density [C/cm^3]
0.0E+0
2.0E-5
4.0E-5
6.0E-5
8.0E-5
1.0E-4
Distance [cm]



3c

V [V]	L [H]	C [F]	f [Hz]	Channel
-0.0165	5.00E-09	8.708E-13	2.4121E+09	1
-0.024	5.00E-09	8.672E-13	2.4170E+09	2
-0.032	5.00E-09	8.635E-13	2.4222E+09	3
-0.0395	5.00E-09	8.6E-13	2.4270E+09	4
-0.047	5.00E-09	8.566E-13	2.4319E+09	5
-0.055	5.00E-09	8.53E-13	2.4370E+09	6
-0.063	5.00E-09	8.495E-13	2.4421E+09	7
-0.071	5.00E-09	8.46E-13	2.4471E+09	8
-0.0785	5.00E-09	8.427E-13	2.4519E+09	9
-0.08675	5.00E-09	8.392E-13	2.4570E+09	10
-0.095	5.00E-09	8.357E-13	2.4621E+09	11


4
V vs log (I)
0.78
0.72
0.66
0.6
0.54
0.48
0.42
0.36
0.3
V
6.012 Diode
1N914 Diode
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
log (I)


4
180
120
60
0
slope (mV/decade)
6.012 Diode
1N914 Diode
-10
-8
-6
-4
-2
0
log (I)