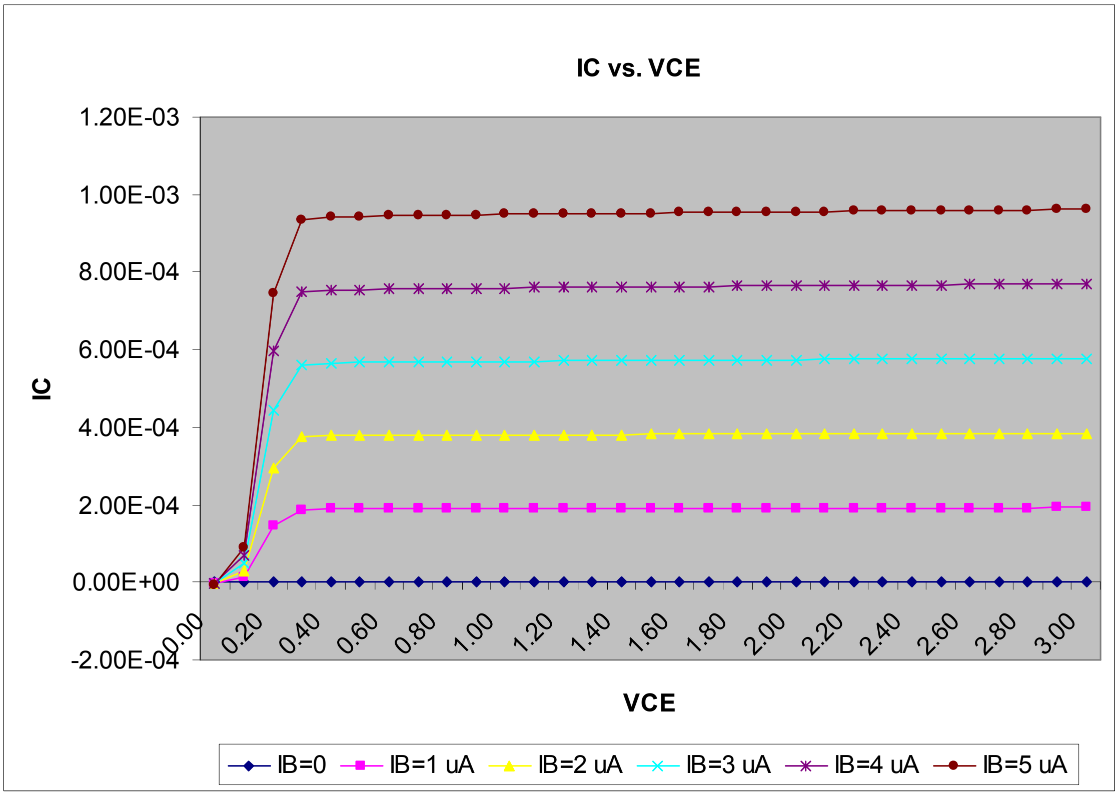
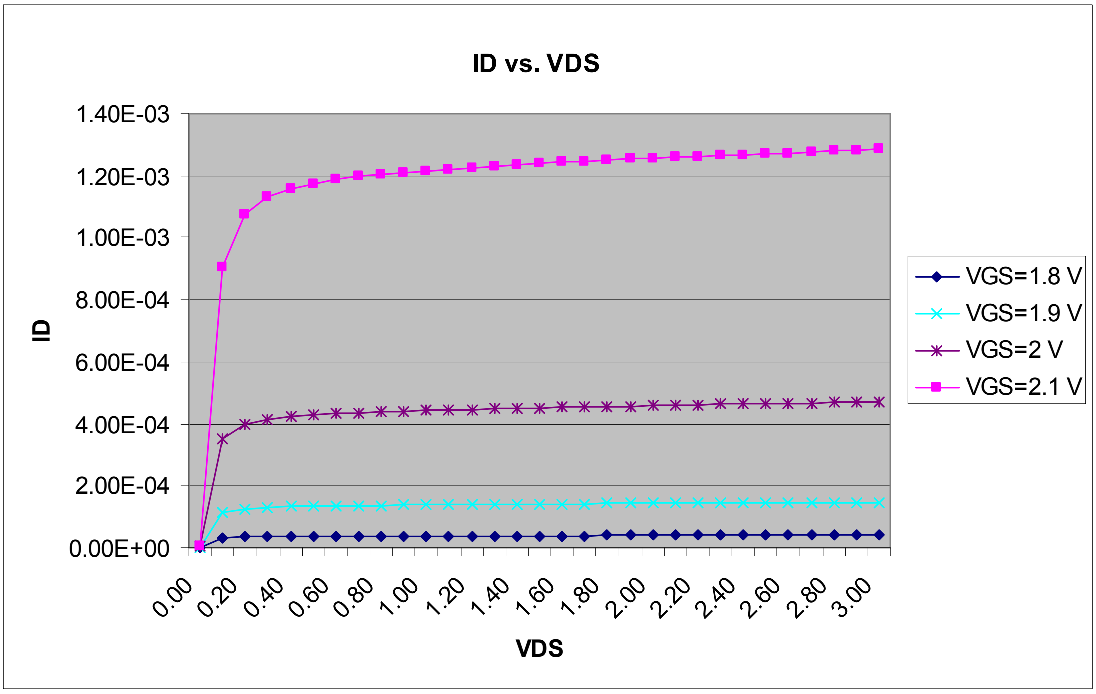
Note: The specified current range of 50µA – 1mA was essential to be taken into account when representing the graphs.

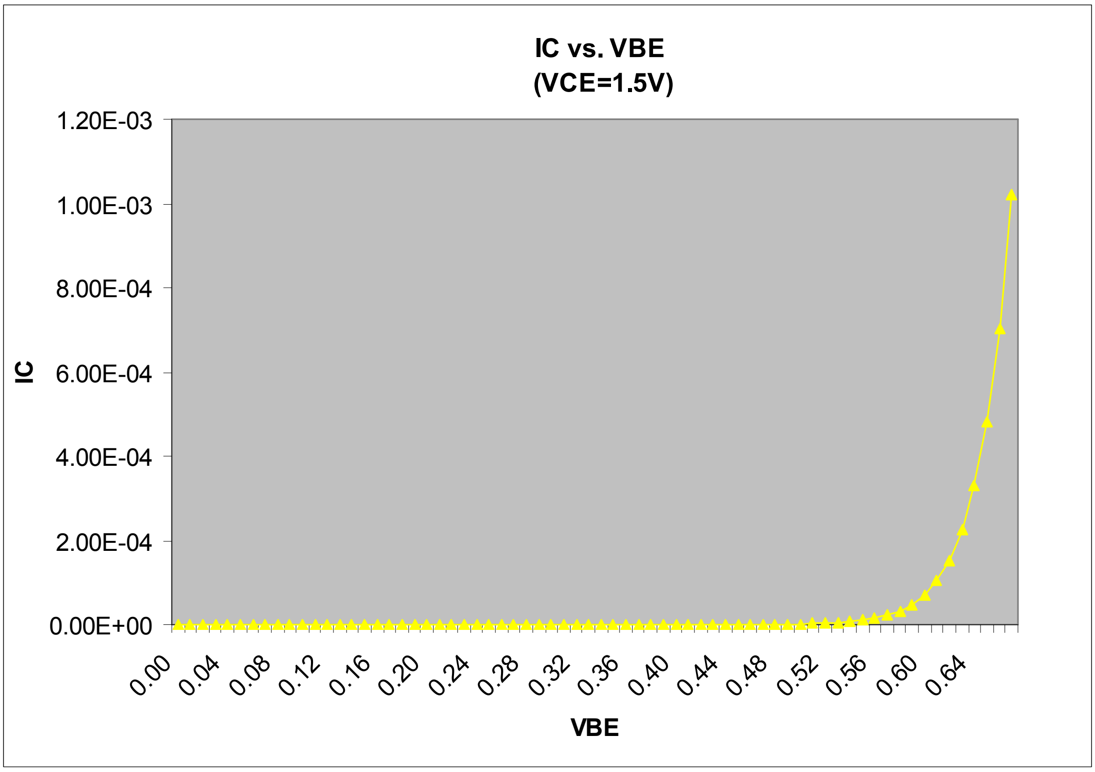
Graph 1)
I_C vs. V_{CE} for different values of I_B:



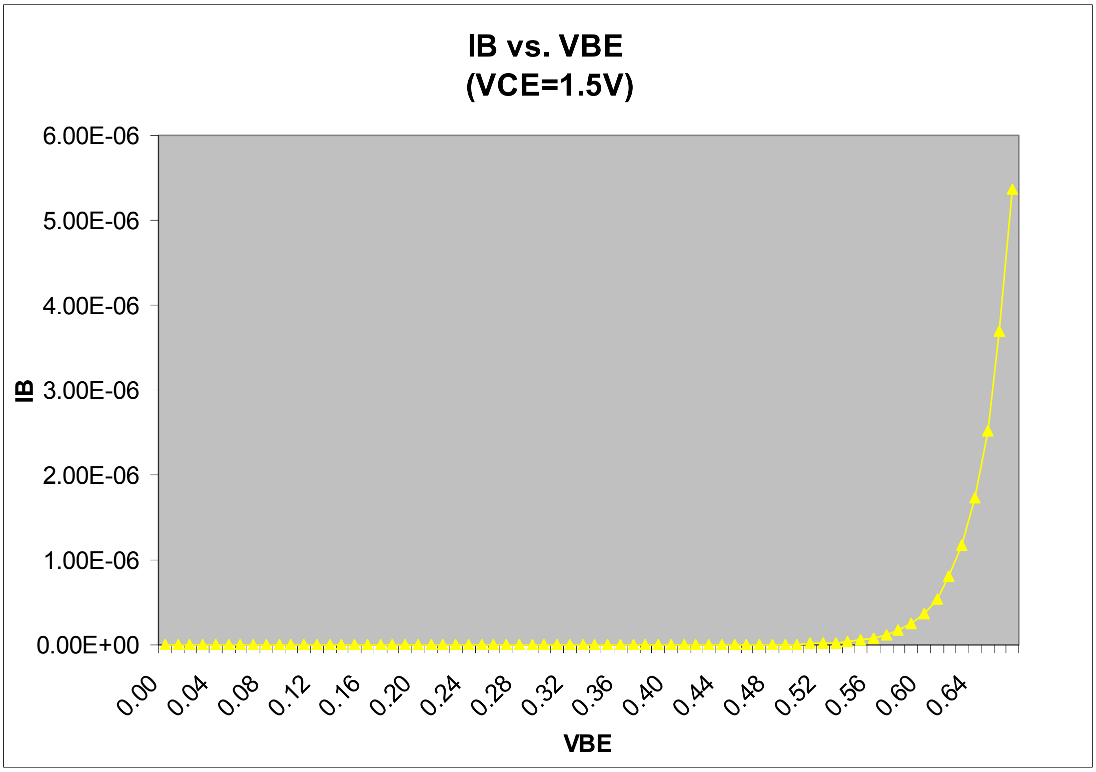
Graph 2)
I_D vs. V_{DS} for different values of V_{GS}:



Graph 3)
I_C vs. V_{BE} for V_{CE}=1.5 V:

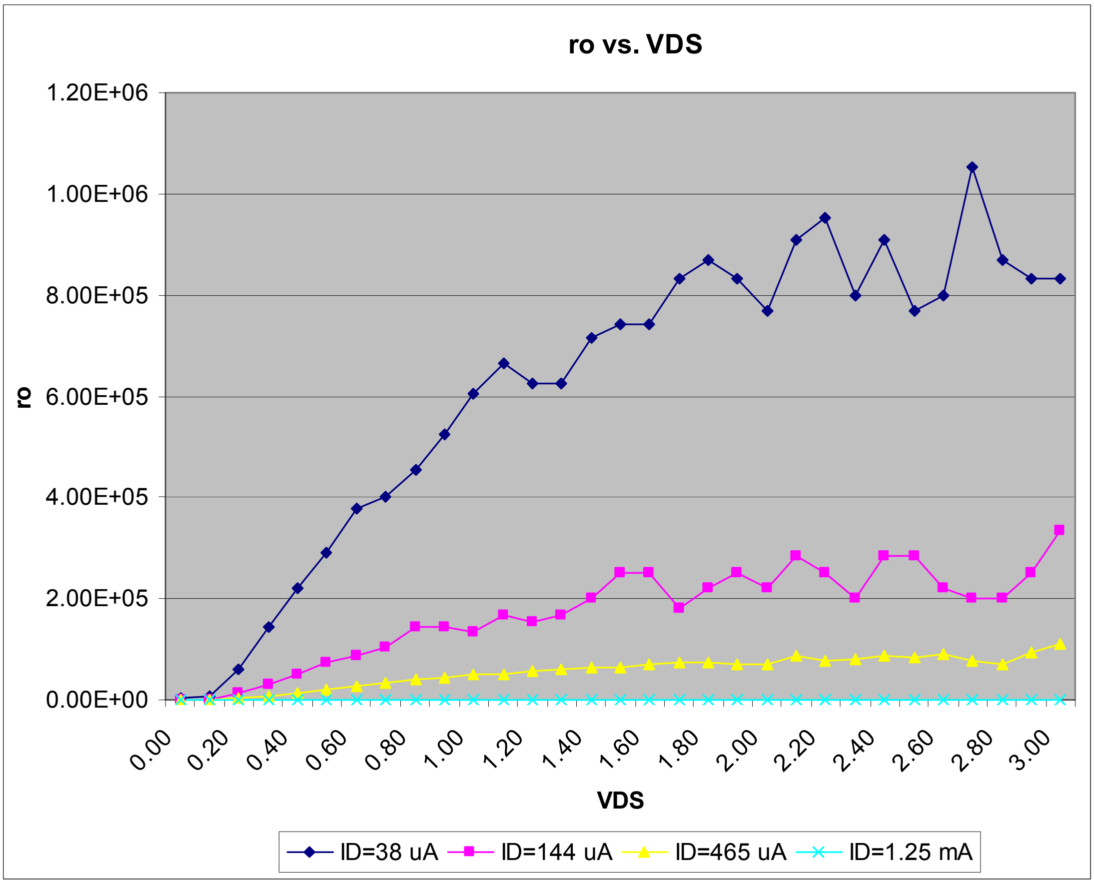


Graph 4)
I_B vs. V_{BE} for V_{CE}=1.5 V:



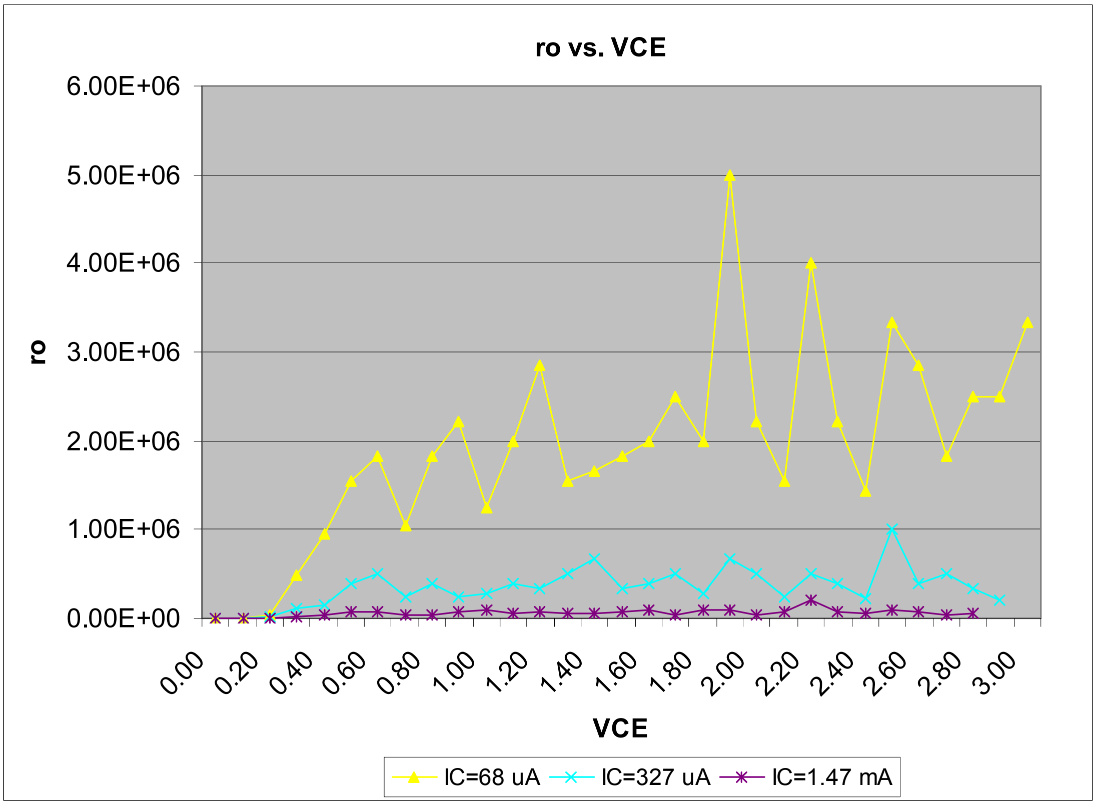
Graph 5)

ro $_{(NMOS)}$ as a function of V_{DS} for different values of I_D (or V_{GS}):

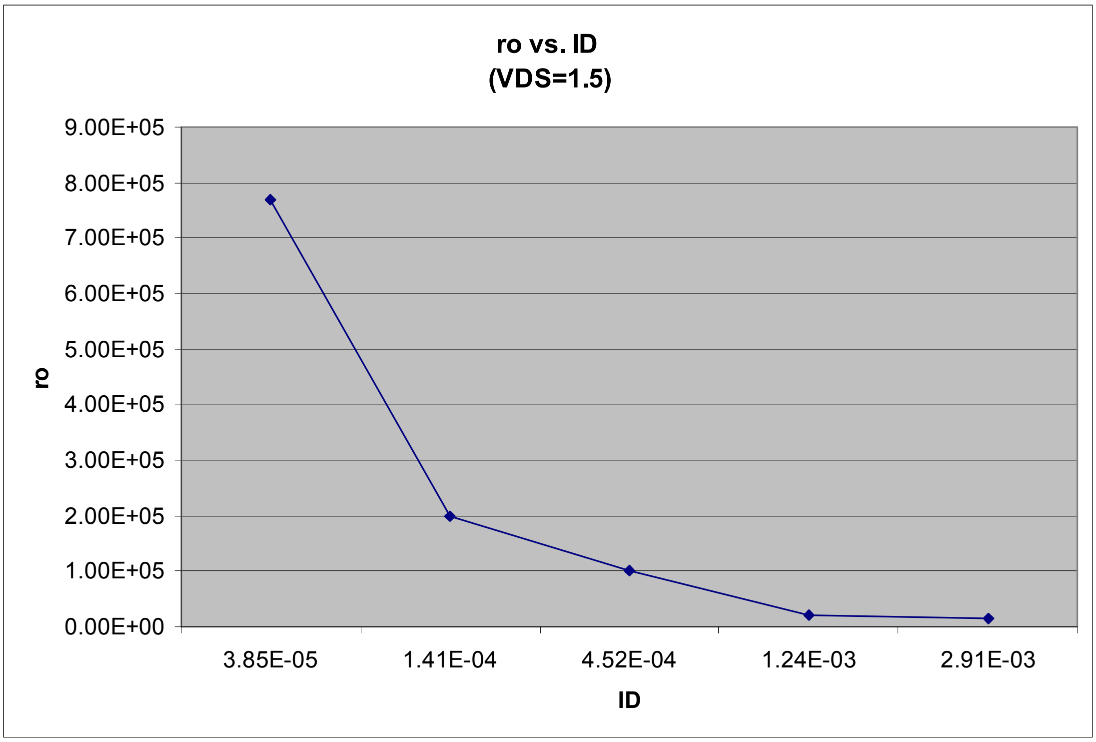


Graph 6)

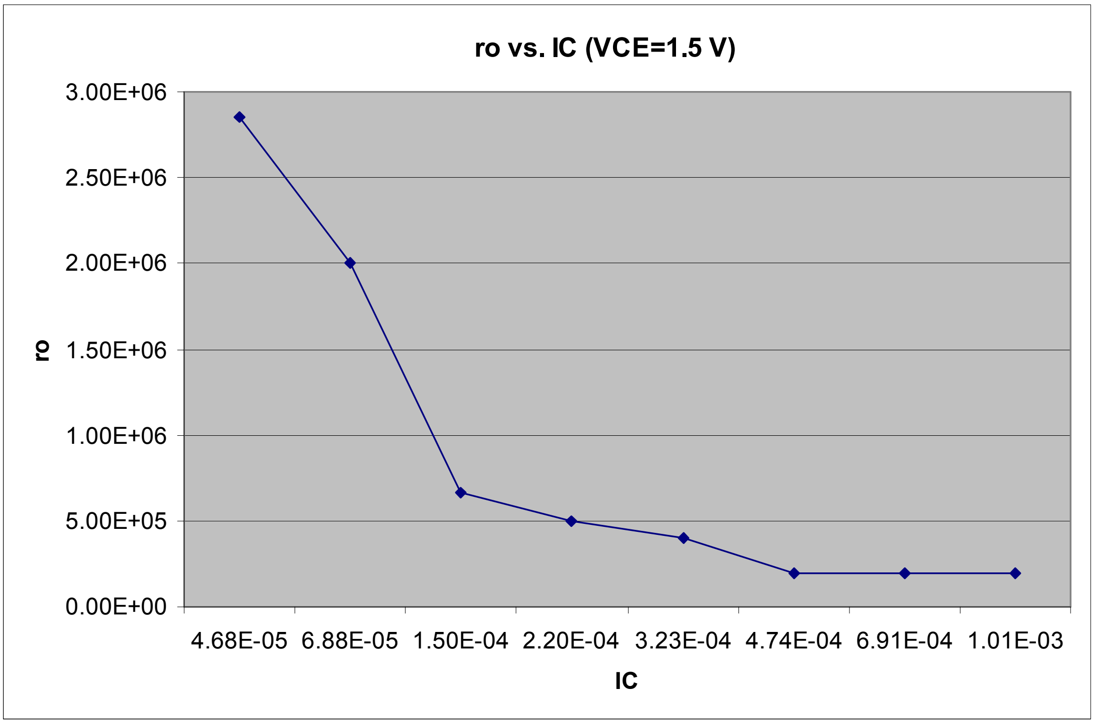
ro $_{(BJT)}$ as a function of V_{CE} for different values of I_C (or V_{BE}):



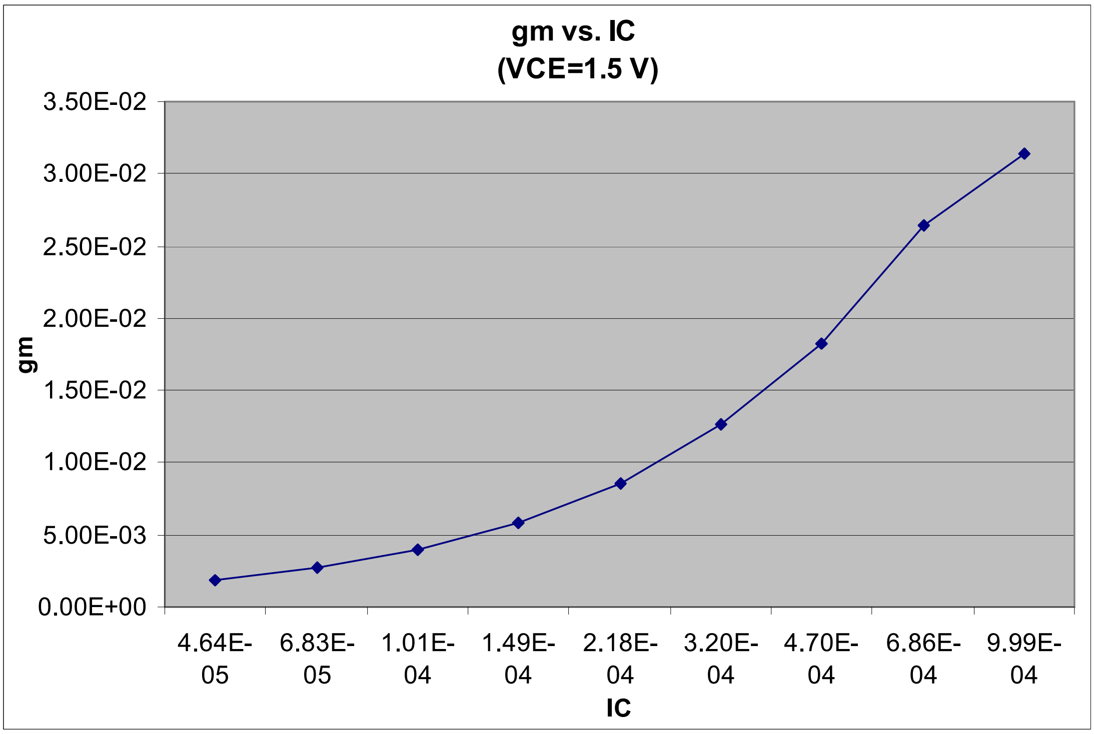
Graph 7)
ro (NMOS) as a function of I_D for V_{DS}=1.5V:



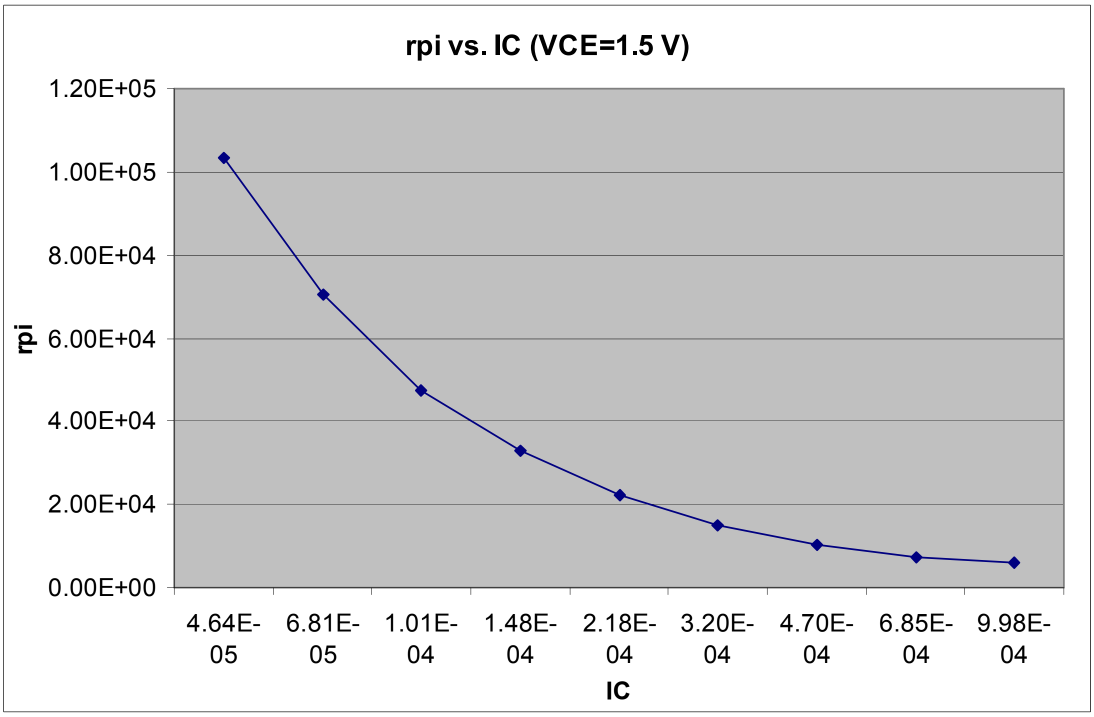
Graph 8)
ro (BJT) as a function of I_C for V_{CE}=1.5V:



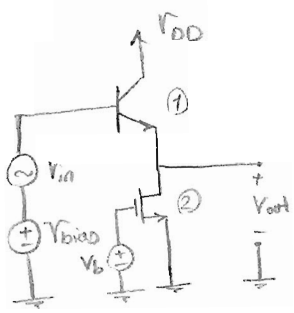
Graph 9)
gm $_{(BJT)}$ as a function of I_C for V_{CE}=1.5V:



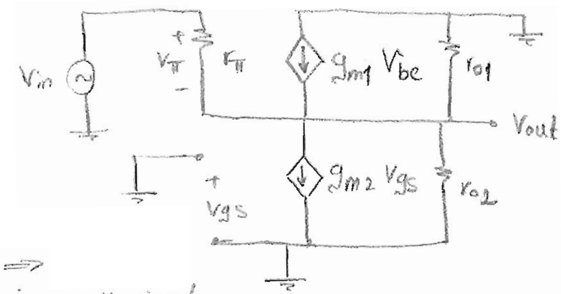
Graph 10)
$r\pi$ as a function of I_C for V_{CE}=1.5V:



6.012 — WebLab II solution (part 2, design) — Spring 2009

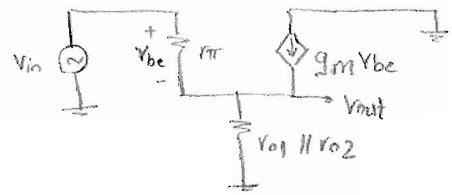


Small Signal circuit: (from scratch!)



$V_S, V_g = 0 \Rightarrow V_{gs} = 0 \Rightarrow$ Only $r_{o\,nMOS}$ remains in small signal. $(g_m = g_{m1})$

$\Rightarrow$



$V_{be} = V_{in} - V_{out}$

KCL @ Vout node:

$$-g_m \underbrace{V_{be}}_{(-V_{out}+V_{in})} + V_{out}(g_{o1}+g_{o2}) + (V_{out}-V_{in})g_\pi = 0$$

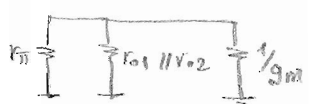
$$V_{out}(g_m + g_{o1} + g_{o2} + g_\pi) = V_{in}(g_\pi + g_m)$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{g_m + g_\pi}{g_m + g_\pi + g_{o1} + g_{o2}}$$

$$I_{in} = (V_{in} - V_{out})g_\pi = g_\pi V_{in}\left[1 - \frac{g_m + g_\pi}{g_m + g_\pi + g_{o1} + g_{o2}}\right] = \frac{g_\pi(g_{o1}+g_{o2})}{g_m + g_\pi + g_{o1} + g_{o2}} V_{in}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{g_m + g_\pi + g_{o1} + g_{o2}}{g_\pi(g_{o1}+g_{o2})}$$

$$V_{in} = 0 \Rightarrow R_{out} = r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_m} \,//\, r_\pi = \frac{1}{g_m + g_\pi + g_{o1} + g_{o2}}$$



Since usually $\{g_m \gg g_\pi \gg g_o\}$, simplifications can be made to A_v, R_{in} & R_{out} specifications:

$$A_v = \frac{g_m + g_\pi}{g_m + g_\pi + g_{o1} + g_{o2}} \underset{g_m \gg g_\pi}{\simeq} \frac{g_m}{g_m + g_{o1} + g_{o2}} = \frac{I_C/V_{th}}{\frac{I_C}{V_{th}} + \frac{I_C}{V_{A1}} + \frac{I_D}{V_{A2}}} = \frac{1}{1 + V_{th}\left(\frac{1}{V_{A1}} + \frac{1}{V_{A2}}\right)}$$

$$R_{in} = \frac{g_m + g_\pi + g_{o1} + g_{o2}}{g_\pi(g_{o1}+g_{o2})} \simeq \frac{g_m + g_\pi}{g_\pi(g_{o1}+g_{o2})} = \beta\,(r_{o1} \,//\, r_{o2})$$

$$R_{out} = \frac{1}{g_m + g_\pi + g_{o1} + g_{o2}} \simeq \frac{1}{g_m}$$

Specs: $R_{in} > 5\,M\Omega$, $R_{out} < 100\,\Omega$, $A_v > 0.98$, Swing $\pm 1\,V$

- $R_{out} < 100\ \Omega \Rightarrow g_m > 10\ \frac{mA}{V} \xrightarrow{\text{Theoretically}} \frac{I_C}{V_{th}} > 0.01 \Rightarrow I_C > 250\ \mu A \Rightarrow I_{min} = 250\ \mu A$

  $\hookrightarrow$ measurement: $I_C = 149\ \mu A,\ g_m = 6\ \frac{mA}{V}$; $I_C = 320\ \mu A,\ g_m = 12.6\ \frac{mA}{V} \Rightarrow$ ✓

- $A_v > 0.98 \Rightarrow \dfrac{1}{1 + V_{th}\left(\frac{1}{V_{A1}} + \frac{1}{V_{A2}}\right)} > 0.98 \Rightarrow 0.05 > 0.98\ \frac{V_{th}}{V_{A\,eff}} \Rightarrow \frac{V_{A\,eff}}{V_{th}} > 19.6$

  (where $\frac{1}{V_{A1}} + \frac{1}{V_{A2}} = 1/V_{A\,eff}$)

  $\Rightarrow V_{A\,eff} > 0.49\ V$! $\Rightarrow$ This would happen most probably.

  $\Rightarrow$ Voltage gain doesn't impose a hard to meet factor on specs.

- $R_{in} = \beta\,(r_{o1} \parallel r_{o2})$; plotting $\frac{I_C}{I_B}$ ratio, shows $\beta = 190$ in the current range of interest.

  $= 190\,(r_{o1} \parallel r_{o2}) > 5\ M\Omega$

  $\Rightarrow r_{o1} \parallel r_{o2} > 26.3\ k$

r_o BJT	$I_{C\,max}$
200 k	1 mA
67 k	1.5 mA

r_o MOS	$I_{D\,max}$
100 k	450 μA
20 k	1.24 mA
14 k	3 mA

$\Rightarrow r_{o\,nMOS}$ is the limiting factor.

(By creating a trendline on the available data or making finer measurements $\Rightarrow V_{A\,MOS} \simeq 40$

$\Rightarrow r_{o2} > 30\ k \Rightarrow I_D < 1.3\ mA$ ($r_o = \frac{V_A}{I_D}$)

$\Rightarrow I_{max} = 1.3\ mA \nless 1\ mA$

$\Rightarrow I_{max} = 1\ mA$ (This can be different for you, based on your measurments.)

$\Rightarrow \boxed{250\,\mu A < I_{sup} < 1\ mA}$

- Design: Choose $I_{sup} = 320\ \mu A \Rightarrow r_{o1} = 400\ k\Omega$; $r_{o2} \simeq 125\ k\Omega$

  $g_m = 12.6\ mA/V$

  $r_\pi = 15\ k\Omega$

  $V_{BE} = 0.64 \Rightarrow V_{bias} = 1.5 + 0.64 = 2.14\ V$

  $V_{GS} \cong 1.93 \Rightarrow V_b \cong 1.93$

$$\Rightarrow \begin{cases} A_v \overset{\text{exact}}{=} \dfrac{g_m + g_\pi}{g_m + g_\pi + g_{o1} + g_{o2}} = 0.999 > 0.98\ ✓ \\ R_{in} \simeq \beta\,(r_{o1} \parallel r_{o2}) = 190\,(95\,k) = 18.1\ M\Omega > 5\,M\ ✓ \\ R_{out} \simeq \dfrac{1}{g_m} = 79.4\ \Omega < 100\ \Omega\ ✓ \end{cases}$$

(As we expected A_v is not a tight spec.)

All specs are met @ $I_{sup} = 320\ \mu A$

- Voltage swing: @ $320\ \mu A = I_{sup}$

  We need $\pm 1\,V$ swing around $V_{out\,DC} = 1.5\,V$.

  $\Rightarrow$ Should check if specs are met @ $V_{out} = 0.5\ \&\ 2.5\,V$

  Corresponding to $V_{DS} = 0.5$, $V_{CE} = 2.5$ and $V_{DS} = 2.5$, $V_{CE} = 0.5\,V$ respectively.

  - Lower swing: $V_{out} = 0.5$, $V_{DS} = 0.5$, $V_{CE} = 2.5 \Rightarrow r_{o1} = 1\,M\Omega$, $r_{o2} \cong 38\,k\Omega$; $r_{o1} \parallel r_{o2} = 38\,k > 26.3\,k\Omega$

    ✓ R_{out} (exact) $= 78.7$; $A_v = 0.9977 > 0.98$ ✓ ; $7.2^M = R_{in} > 5\,M\Omega$

  - Upper swing: $V_{out} = 2.5$; $V_{DS} = 2.5$, $V_{CE} = 0.5 \Rightarrow r_{o1} = 400\,k$; $r_{o2} = 162\,k \Rightarrow R_{in} = 22^M > 5^M$ ✓

    $A_v = 0.999$ ✓ ; $R_{out} = 79\ \Omega$ ✓