HOMEWORK 7 SOLUTIONS.

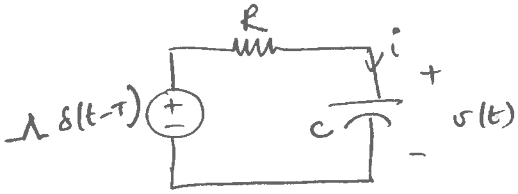
① (a)

Given

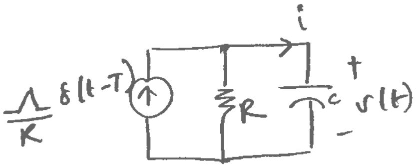
$v(0^-) = V$ $\qquad v(T^-) = V e^{-T/RC}$

$v(T^+) = ?$

$v(\infty) = ?$



⇓ Norton equivalent



The impulse current @ $t=T$ results into a charge dump on the capacitor which cause the voltage accross the capacitor to change instantaneously:

$$\therefore\ v(T^+) = v(T^-) + \frac{\Lambda}{RC} = \boxed{V e^{-\frac{T}{RC}} + \frac{\Lambda}{RC} = v(T^+)}$$

$v(0^+)$ can also be found as follows.

KCL:

$$-\frac{\Lambda}{R}\delta(t-T) + \frac{v}{R} + C\frac{dv}{dt} = 0$$

Integrate both sides from $t=T^-$ to $t=T^+$

$$\int_{T^-}^{T^+} -\frac{\Lambda}{R}\delta(t-T)\,dt + \cancel{\int_{T^-}^{T^+}\frac{v}{R}\,dt}^{\,0} + C\int_{T^-}^{T^+}\frac{dv}{dt} = 0$$

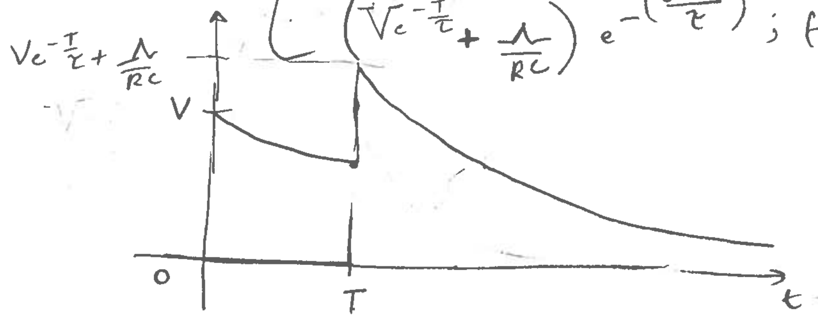
or $-\frac{\Lambda}{R} + C\left[v(T^+) - v(T^-)\right] = 0$

or, $v(T^+) = C\,v(T^-) + \frac{\Lambda}{R}$

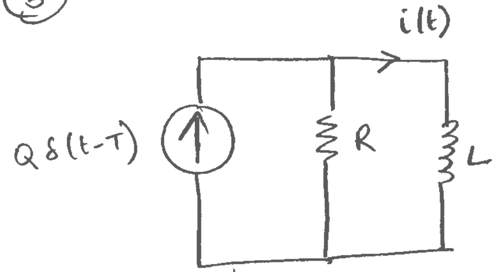
$$\boxed{v(T^+) = V e^{-\frac{T}{RC}} + \frac{\Lambda}{RC}}$$

$\boxed{v(\infty) = 0}$ capacitor discharges through the resistor.

$\therefore v(t) = \begin{cases} V e^{-t/\tau} & ; \text{ for } 0 \le t < T \\ \left(V e^{-\frac{T}{\tau}} + \frac{\Lambda}{RC}\right) e^{-\left(\frac{t-T}{\tau}\right)} & ; \text{ for } t \geq T \end{cases}$

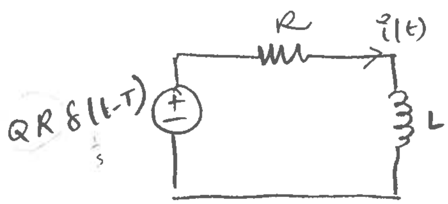


(b)



Given
$i(0) = I$ $\quad i(T^-) = I e^{-\frac{T}{\tau}}$; $\tau = \frac{L}{R}$
$i(T^+) = ?$
$i(\infty) = ?$

↓ Thevenin equivalent



Similar to that in part (a), the impulse voltage source results in a flux dump in the inductor causing the current to rise instantaneously

$$i(T^+) = i(T^-) + \frac{QR}{L}$$

$i(T^+)$ can also be found as follows.

KVL: $\quad QR\,\delta(t-T) - iR - L\frac{di}{dt} = 0$

Integrate both sides from $t = T^-$ to $t = T^+$

$$QR\int_{T^-}^{T^+} \delta(t-T) - R\cancel{\int_{T^-}^{T^+} i\,dt}^{\,0} - L\int_{T^-}^{T^+} \frac{di}{dt}\,dt = 0$$

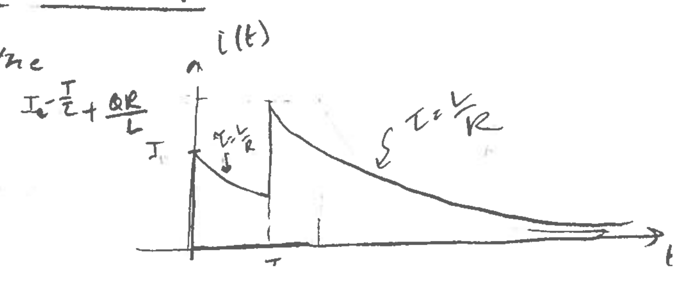
or, $\quad QR - L\left[i(T^+) - i(T^-)\right] = 0$

or, $\quad i(T^+) = i(T^-) + \frac{QR}{L} \qquad \boxed{i(T^+) = Ie^{-\frac{T}{\tau}} + \frac{QR}{L}}$

$i(\infty) = 0$ (Inductor discharges through the resistor)

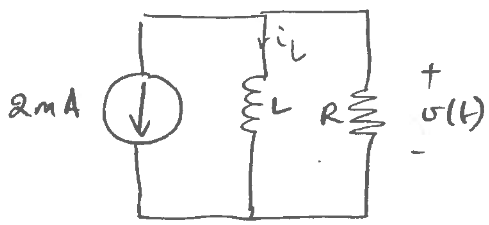
$\therefore i(t) = Ie^{-\frac{t}{\tau}} \quad 0 \le t < T$

$= \left(Ie^{-\frac{T}{L/R}} + \frac{QR}{L}\right) e^{-\frac{(t-T)}{\tau}} \qquad \tau = \frac{L}{R}$



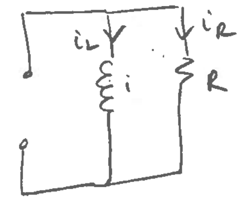
Note the duality between the circuits in part (a) & (b)

(2)



Network had been assembled for a long time. At $t=0$ the current source turns off.

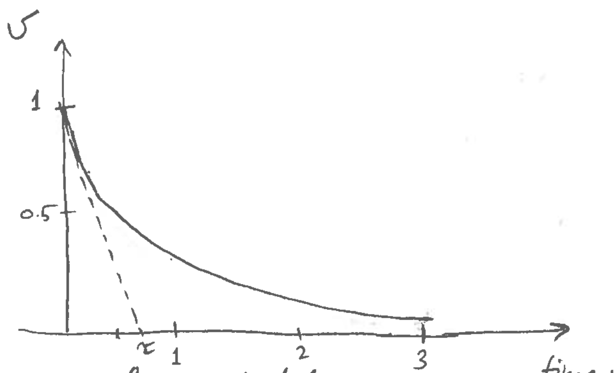
At $t=0^+$



$i_L(0^-) = -2mA$

$i_L(0^+) = -2mA$

The measured $v(t)$ is shown below.



Rough sketch.

$v(0^+) = i_R(0^+) R$

$i_R(0^+) = -i_L(0^+)$

$= 2mA$

From the graph,

$v(0^+) = 1V = 2mA * R$

$\therefore R = \frac{1}{2} k\Omega = \boxed{0.5 k\Omega = R}$

The time constant for the LC circuit is $\frac{L}{R}$. From the graph, the slope @ time $t=0^+$ is $\frac{1}{\tau} \approx \frac{1}{0.75}$.

$\boxed{\therefore \tau = 0.75 \text{ sec}}$

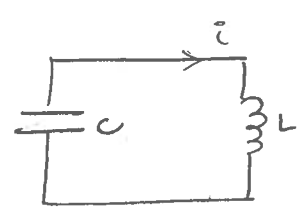
$\therefore \frac{L}{R} = 0.75$ sec

or, $L = 0.75 * 0.5$ kΩ·s

$= 0.375$ kH

$\boxed{L = 375 H}$

(3) The natural frequency of the LC circuit is 10^7 rad/s.



$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = 10^7$ rad/s. $\Rightarrow$ $C = \frac{1}{L(10^7)^2}$

The peak value of $v_c = 100mV = V^c_{peak}$

peak value of $i_L = 20mA = I^L_{peak}$

The voltage across the capacitor reaches its peak value when $i=0$. and similarly i_L reaches peak when no energy is stored in the capacitor.

∴ By energy conservation,

$$\cancel{\tfrac{1}{2}} L I_{peak}^{L\,2} = \cancel{\tfrac{1}{2}} C V_{peak}^{c\,2}$$

or $$L\,(20mA)^2 = \frac{1}{L\,(10^7)^2}(100mV)^2$$

or, $$L^2 = \frac{(100mV)^2}{(10^7)^2 \frac{1}{s^2}\,(20mA)^2} = 2.5\times 10^{-13}\ \frac{V^2 s^2}{A^2}$$

2×10^{-6}

$$\therefore\ L = 5\times 10^{-7}\ H$$

$$L = 0.5\mu H.$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$C = \frac{1}{L\omega_0^2}$

$$\therefore\ \boxed{C = \frac{1}{(10^7)^2 (0.5\mu H)} = 0.02\mu F}$$

(B) Total energy stored in the network $= \frac{1}{2} C V_{peak}^{c\,2}$

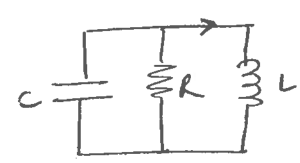
$$= \frac{1}{2}(1.86\,mF)\,(100mV)^2$$

$$= \boxed{0.1nJ}$$

[Check: $\frac{1}{2} L I_{peal}^{c\,2}$ should also yield the same answer]

$$= \frac{1}{2}(46.7mH)(20mA)^2 = 0.1nJ$$

(C) Now a resistor is placed in parallel with the inductor and capacitor.



The energy is observed to decay by the factor of $\frac{1}{e}$ in $\underline{20\,\mu s}$. $\therefore\ \tau = 20\mu s$

$e^{-t/\tau}$

For a parallel RLC circuit,

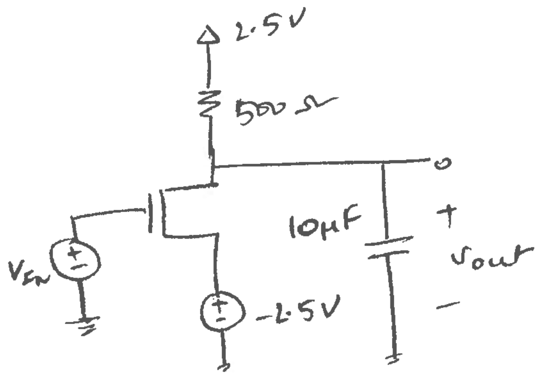
$$\frac{\partial^2 v_c}{\partial t} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{1}{LC} v_c = 0.$$

$$2\alpha = \frac{1}{RC} \qquad \omega_0^2 = \frac{1}{LC}$$

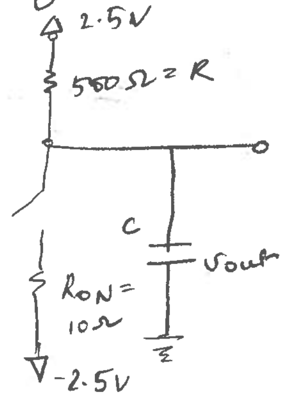
We know, $\tau = \frac{1}{\alpha} = 2RC$

$$\therefore\ R = \frac{\tau}{2C} = \frac{20\mu s}{2\times 0.02\mu F} = \boxed{500\,\Omega = R}$$

(c) Large signal model for MOSFET



For falling input



$\therefore \tau = CR$

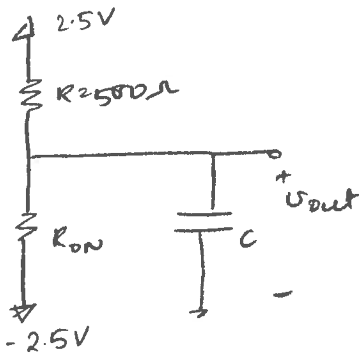
$= 10\mu F \times 500\Omega$

$= 5 \times 10^{-3}$ sec

$\boxed{\tau = 5\text{ ms}}$

This matches the time constant acquired experimentally.

For rising input



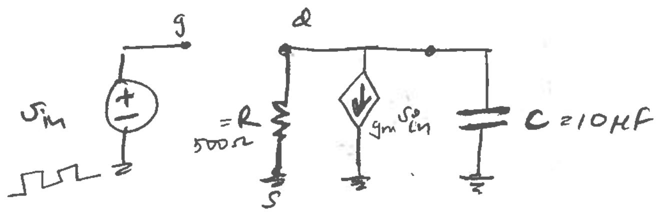
$\therefore \tau = (R_{ON} \parallel R)\, C$

$= \dfrac{R_{ON}\, R}{R_{ON} + R}\, 10\mu F$

$\boxed{\tau \approx 0.1 \text{ ms}}$

From the experimental data, it was hard to estimate τ since the fall was really sharp. However it was clear that $\tau \ll 1$ms which agrees with the value calculated above.

(D) Small signal model



$R_{TH} = R = 500\Omega$

$C = 10\mu F$

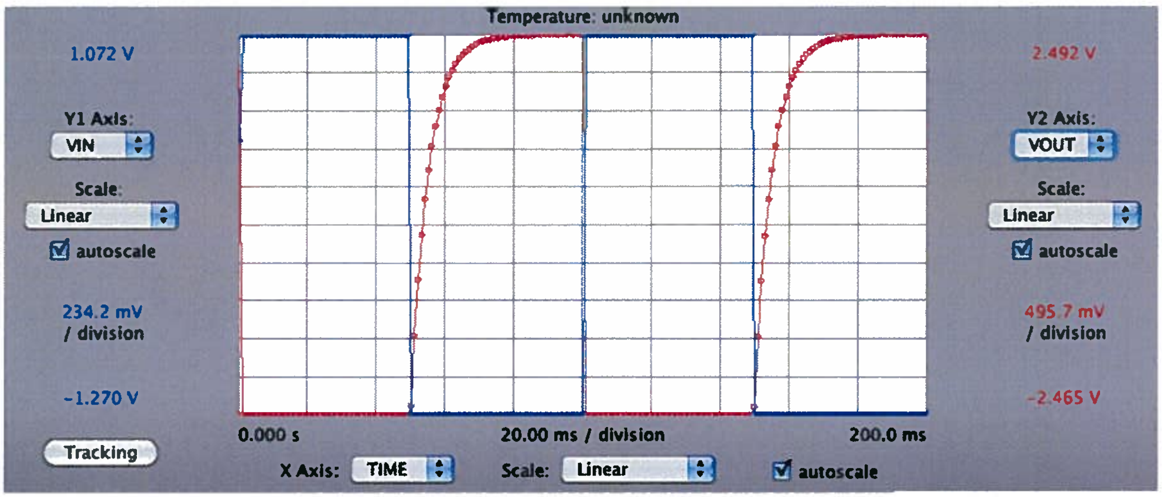
$\therefore \tau = 5$ msec.

The experimental data matches w/ the above value.

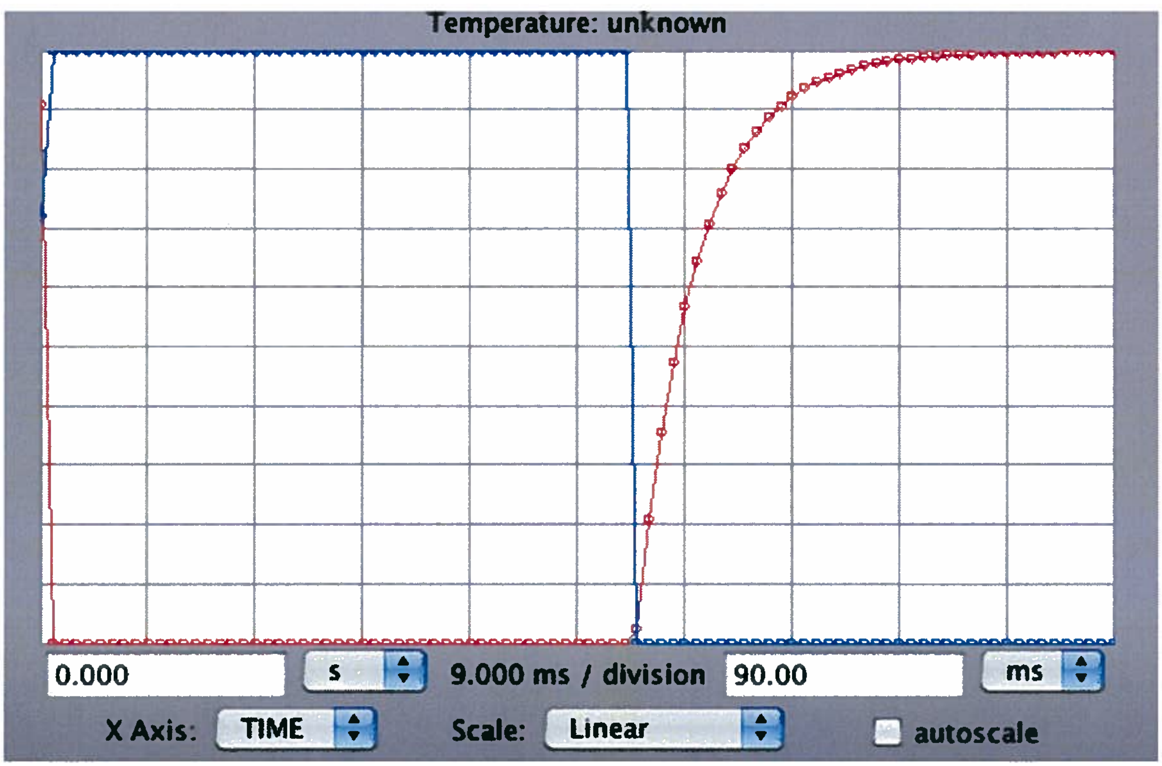
Problem 4

Blue signal – Input
Red signal - Output

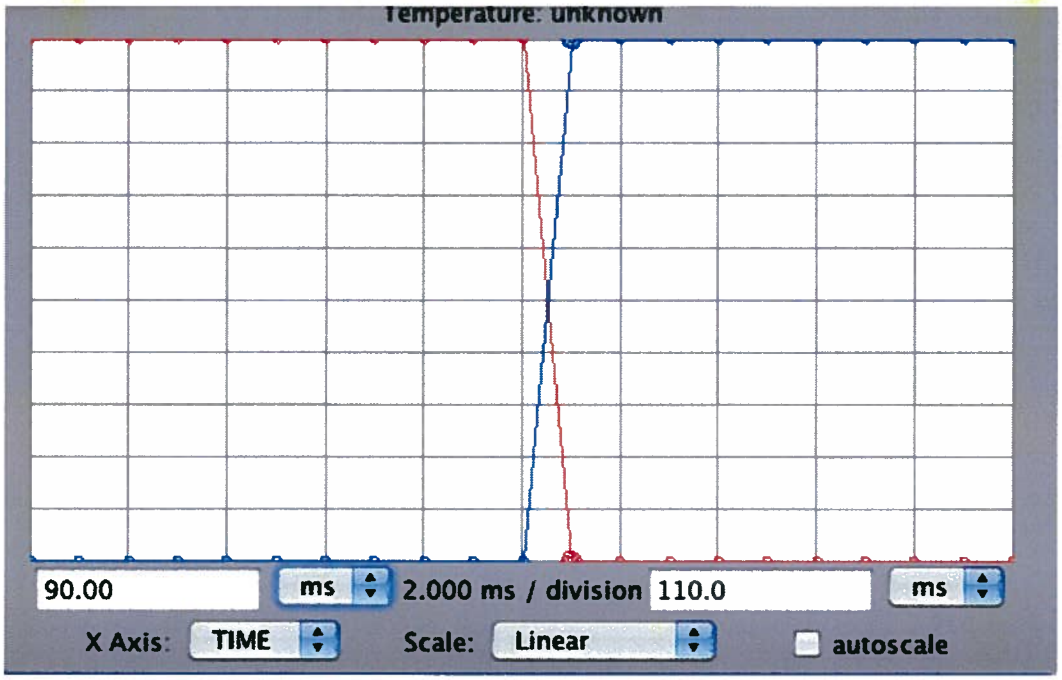
A. For large signals



The rising and falling time constants can be found by finding the slope at the time when the output is rising and falling respectively.

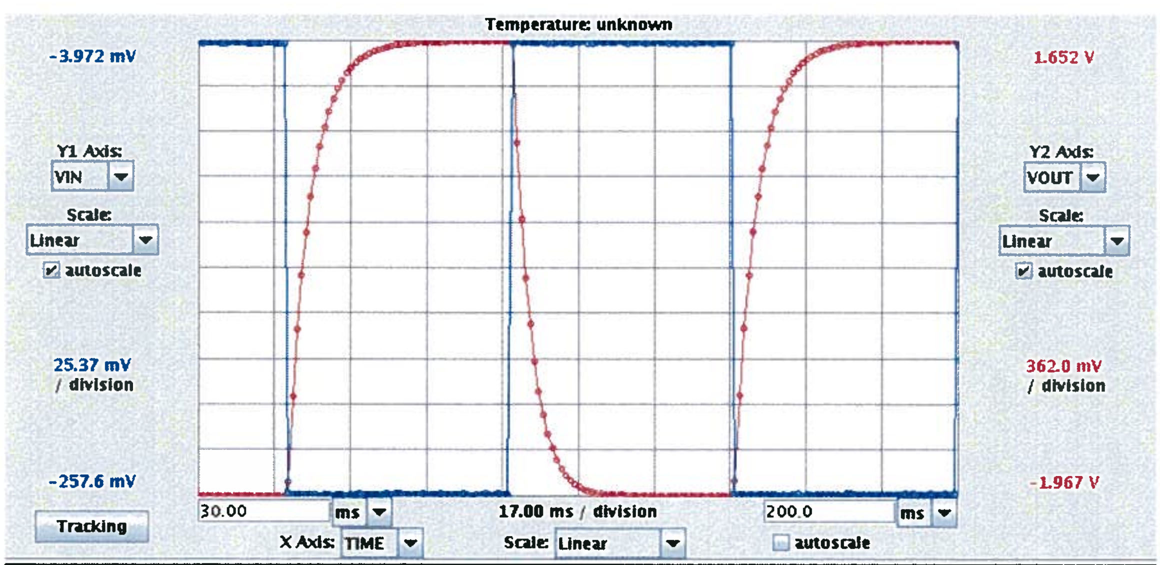


The time constant when the output is rising is ~5 ms.



At falling edge of the output the time constant is really short as seen as a sharp fall in the output. From the above figure, it is seen that the time constant is much less than 1ms since the output falls from high to low in 1ms.

B . For small signals



The time constant for both rising and falling output is ~5 ms.