Homework #8 - April 15, 2009

Due: April 22, 2009 at recitation

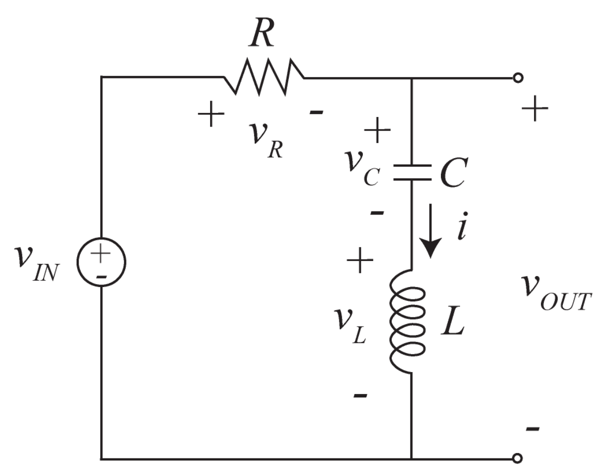
No late homework accepted

1. *[16 points]* Exercise 13.6 of Agarwal and Lang [note difference between Exercise and Problem!]

2. *[20 points]* Problem 13.4 of Agarwal and Lang.

3. *[24 points]* Problem 14.16 of Agarwal and Lang.

4. *[40 points]* This problem studies the step response of the RLC network shown below, both experimentally and theoretically. In particular, you will measure the step response of this network, extract basic parameters that charaterize it, and from there, estimate the values of the circuit elements. The experiments will be performed using the ELVIS iLab.



a) First, experimentally measure the step response of the network. To do so, log into iLab, and select the ELVIS Lab Client. When you launch the lab client, you will see the network on the canvas. Select the signal generator (FGEN) and set its parameters to WaveForm = SQUARE, Frequency = 1 kHz, Amplitude = 1 V, and Offset = 0 V. Next, select the output measurement unit (SCOPE) and determine a suitable sampling rate that will allow you to see at least one full cycle of the output waveform with adequate resolution. Remember that the system will only allow you to take a maximum of 201 data samples at the output. Third, run the experiment. To view the results, select v_{IN} for the Y1 axis and v_{OUT} for the Y2 axis, and use linear scales for both. You should obtain a sharp oscillatory transient in response to the sharp edges of the square wave input. When you are confident that you have obtain the right result, capture a screen shot of the step response.

b) The output v_{OUT} should look like a decaying sinusoid superimposed on a square wave that matches the input square wave v_{IN}. Focus now on the sinusoid. From the experimental results obtained in part (a), extract its frequency of oscillation and the time period over which it decays by a factor of $1/e$. In addition, extract its amplitude extrapolated back to the step in v_{IN}. Do this for the transients excited by both the rising and falling edges of v_{IN}. You may find it helpful to use the capabilities of the "Tracking" feature that can be switched on using the button at the bottom lower corner of the canvas.

c) Now examine the step response from a theoretical viewpoint. Derive a second-order differential equation for $i(t)$ driven by dv_{IN}/dt. One way to do this is to apply KVL around the network, and substitute the constitutive laws for the inductor, resistor and capacitor to form an equation that relates $i(t)$ to $v_{IN}(t)$. Then differentiate the equation once to reduce the integral that comes from the constitutive law for the capacitor. Alternatively, derive the differential equation by any method you wish.

d) Assume that the network is initially at rest ($d/dt = 0$), but with v_{IN} set to V_{IN1}. Further assume that the input $v_{IN}(t)$ steps to V_{IN2} at $t = 0$. For this input, determine v_C, v_L, v_R, v_{OUT}, i and di/dt just before the step at $t = 0^-$, and just after the step at $t = 0^+$. This information provides the initial conditions required to solve the differential equation found in part (c).

e) Rather than solve the second-order differential equation for $i(t)$ directly, argue that $i(\infty) = 0$ so that $i(t)$ has no constant component. Further argue that $i(t)$ takes the form $i(t) = I\sin(\omega t + \phi)e^{-\alpha t}$ for $t \geq 0$. Determine I, ω, ϕ and α in terms of R, L, C, V_{IN1} and V_{IN2}. Hint: first find ω and α from the differential equation, and then find I and ϕ from the initial conditions. Alternatively, solve for $i(t)$ by any method you wish.

f) Recognize that $v_{IN} = v_R + v_{OUT}$, and derive an expression for v_{OUT} for $t \geq 0$ in response to the step input. Do so for the transients beginning with both the rising and falling edges of v_{IN} under the assumption that each transient goes to completion before the next one begins.

g) Based on your understanding of the step response of the network and the parameters that you extracted in part b), estimate the values of the resistor, capacitor and inductor in the network.