Homework # 8 Solution

Due: Never

1. *[16 Points]*: Exercise 13.6

Answer:

(a) *[4 Points]* Plot **(2)**

It looks capacitive at low frequency, and resistive at high frequency.

(b) *[4 Points]* Plot **(4)**

$H = [V_1(1/j\omega C)/(R+1/j\omega C) - V_1R/(R+1/j\omega C)]/V_1$

$= (1-j\omega CR)/ (1+j\omega CR)$

=> $|H| = 1$

(c) *[4 Points]* Plot **(8)**

At low (or high) frequency, the capacitor (or inductor) looks like an open circuit. Therefore,

$Z(\omega=0) = Z(\omega=\infty) = R$

At resonance, the LC in series looks like a short circuit. That is,

$Z(\omega=\omega_0) = 0$

(d) *[4 Points]* Plot **(5)**

At low frequency, the inductor looks like a short circuit. Therefore,

$Z(\omega=0) = R_1$

At high frequency, the inductor looks like an open circuit. Therefore,

$Z(\omega=\infty) = R_1+R_2$

In between, it's a first order system, so the slope of Z(jω) is 1.

2. *[20 Points]*: Problem 13.4

Answer:

(a) *[5 Points]*:

$$H = \frac{R_1 + j\varpi L_1}{R_1 + R + j\varpi (L_1 + L)}$$

(b) *[20 Points]*:

$H(\omega=0) = R_1/(R_1+R) = 1/10$

R = 9kΩ

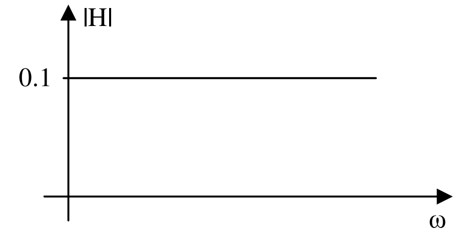
(c) *[5 Points]*:

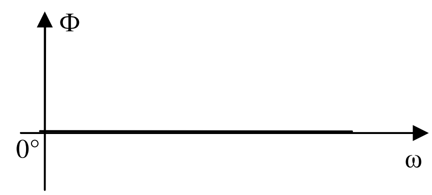
$H(\omega=\infty) = L_1/(L_1+L) = 1/10$

L = 90mH

(d) *[5 Points]*:

H(jω) = 1/10





3. *[24 Points]*: Problem 13.4

Answer:

(a) *[5 Points]*:

$i(t) = I\cos(\omega t)$

i $= Ie^{j\omega t}$

$$v = Zi = \frac{i}{\frac{1}{j\varpi L} + j\varpi C + \frac{1}{R}}$$

$$= \frac{i}{j\left(\varpi C - \frac{1}{\varpi L}\right) + \frac{1}{R}}$$

Therefore,

$v(t) = V\cos(\omega t+\Phi)$

$$V = \frac{I}{\sqrt{\frac{1}{R^2} + \left(\varpi C - \frac{1}{\varpi L}\right)^2}}$$

$$\Phi = \tan^{-1}\left(R\left(\frac{1}{\varpi L} - \varpi C\right)\right)$$

(b) *[4 Points]*:

At $\varpi = \frac{1}{\sqrt{LC}}$,

$$V = V_{MAX} = IR$$

(c) *[5 Points]*:

$$\varpi = \frac{1}{\sqrt{LC}},$$

$$C = \frac{1}{L\varpi^2}$$

$C_1 = 1/L\omega_1^2$
$= 1/(365u*(2*3.14*540k)^2$
$= 2.38*10^{-10}$ C
$= 238$ pF

$C_2 = 1/L\omega_2^2$
$= 1/(365u*(2*3.14*1600k)^2$
$= 2.71*10^{-11}$ C
$= 27.1$ pF

Therefore, C must be able to vary from **27.1pF to 238pF**.

(d) *[5 Points]*:

$$V = \frac{IR}{\sqrt{1+\left(\varpi C - \frac{1}{\varpi L}\right)^2 R^2}} = 0.25IR$$

$$1+\left(\varpi C - \frac{1}{\varpi L}\right)^2 R^2 = 16$$

$$R = \sqrt{15}\left(\varpi C - \frac{1}{\varpi L}\right)^{-1}$$

$$= \sqrt{15}\left((\varpi_0 + \Delta\varpi)C - \frac{1}{(\varpi_0+\Delta\varpi)L}\right)^{-1}$$

$$= \sqrt{15}\left(\Delta\varpi C + \frac{\Delta\varpi}{\varpi_0^2 L}\right)^{-1}$$

Notice $C = \frac{1}{L\varpi_0^2}$,

$$R = \sqrt{15}\left(\frac{2\Delta\varpi}{\varpi_0^2 L}\right)^{-1} = \frac{\sqrt{15}\varpi_0^2 L}{2\Delta\varpi}$$

$$= \frac{\sqrt{15}\times(6.28\times 1M)^2\times 365\mu}{2\times 6.28\times 5k}$$

$$= 888k\Omega$$

(e) *[5 Points]*:

$$\frac{V}{V_{MAX}} = \frac{1}{\sqrt{1+\left(\varpi C - \frac{1}{\varpi L}\right)^2 R^2}}$$

$$\approx \frac{1}{\left(\varpi C - \frac{1}{\varpi L}\right)R} \approx \frac{\omega_0^2 L}{2\Delta\omega R}$$

$$= \frac{(6.28\times 1M)^2\times 365\mu}{2\times 6.28\times 10k\times 888k}$$

$$= 0.13$$

$$Q = \frac{R}{\omega_0 L} = \frac{888k}{6.28\times 1M\times 365u} = 389$$

4. *[40 Points]*:

Answer:

(a) *[5 Points]*:
See the figure on the next page.

(b) *[6 Points]*:
T = 150us
Frequency:
f = 1/T = **6.67 kHz**
Time constant:
τ = 1/α = **300 us**
Amplitude at t=0+:
V = **330 mV**

These values are similar at the rising edge and falling edge of the square wave.

(c) *[5 Points]*:

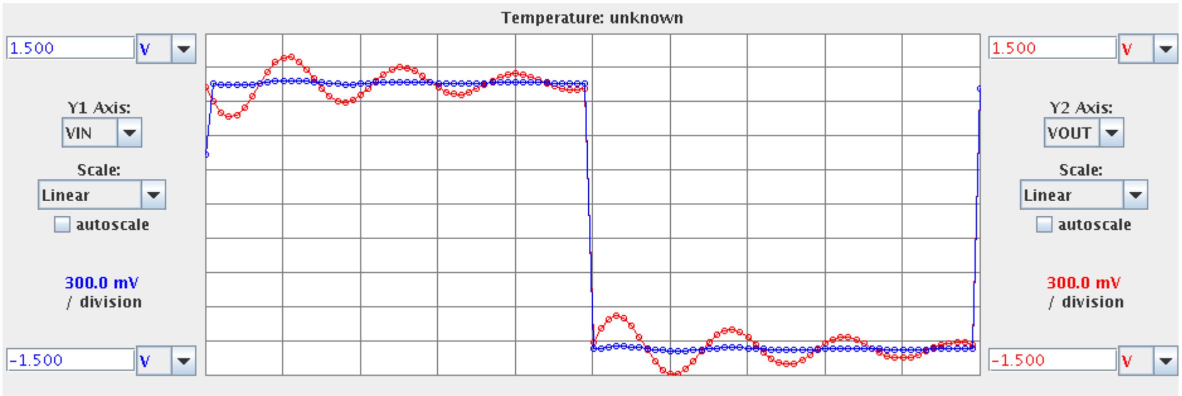
$$v_{IN} = v_R + v_C + v_L$$

Differentiate,

$$\frac{dv_{IN}}{dt} = R\frac{di}{dt} + \frac{1}{C}i + L\frac{d^2 i}{dt^2}$$

(d) *[5 Points]*:

	v_C	v_L	v_R	v_{OUT}	i	di/dt
t=0-	V_{IN1}	0	0	V_{IN1}	0	0
T=0+	V_{IN1}	V_{IN2} - V_{IN1}	0	V_{IN2}	0	(V_{IN2} - V_{IN1})/L



(e) *[8 Points]*:
In steady state, v_C = constant, therefore,

$i(\infty) = 0$,

because there is no DC current through the capacitor.

The general solution to a second order differential equation is a linear combination of two exponential functions, and if we know it's oscillating, then it must have the following form,

$i(t) = I\sin(\omega t+\Phi)e^{-\alpha t}$

$$\frac{d^2 i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i+=\frac{1}{L}\frac{dv_{IN}}{dt}$$

1) $i(t=0+) = 0$, therefore,

$\Phi = 0$

2) $$\omega=\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

3) **$\alpha = R/2L$**

4) $$\left.\frac{di}{dt}\right|_{t=0+}=\omega I=\frac{V_{IN2}-V_{IN1}}{L}$$

$$I=\frac{V_{IN2}-V_{IN1}}{\sqrt{\frac{L}{C}-\frac{R^2}{4}}}$$

(f) *[6 Points]*:

$v_{OUT} = v_{IN} - v_R$

$= \mathbf{v_{IN2} - RI\sin(\omega t+\Phi)e^{-\alpha t}}$

For the rising edge,

V_{IN1} = -1V, V_{IN2} = 1V (nominal)

For the falling edge,

V_{IN1} = 1V, V_{IN2} = -1V (nominal)

I, ω, Φ, and α are calculated in part (e).
Remember, I is also a function of V_{IN1} and V_{IN2}.

(g) *[5 Points]*:
From part (b),

$$RI=\frac{R(V_{IN2}-V_{IN1})}{\omega L}=0.33V$$

From the graph above,

$V_{IN2} - V_{IN1}$ = 2.3 V

Therefore,

$\omega L/R = 2.3/0.33 = 6.9$

$L/R = 1.6 \times 10^{-4}$ sec

which agrees with the measurement

$1/\alpha = 2L/R = 300$ us

$L/R = 1.5 \times 10^{-4}$ sec

Substitute this into the equation of ω,

$1/LC = \omega^2+(R/2L)^2 = 2.9 \times 10^8$ Hz^2

$LC = 2.5 \times 10^{-9}$ sec^2

However, we are not able to figure out the resistor, capacitor, and inductor values from the measurements of voltage transfer function (VTF). As long as L/R and LC have the values specified above, we will get the same VTF no matter what absolute values of R, L, and C we have chosen.