System Dynamics Tools IV

Dynamics of Simple Structures: Positive & Negative Feedback Loops

December 2010 – January 2011

Paulo Gonçalves

Università della Svizzera italiana

MITSloan

Paper Folding

42 Folds:

- Paper Thickness:

  440,000 km (Earth to moon = 385,000 km)

  (.1 mm)*2^{42} ≈ .1 mm*4.4 x 10^{12}

100 Folds:

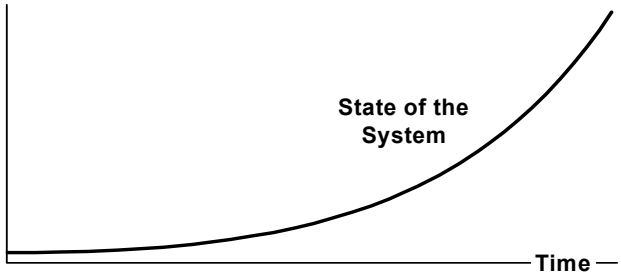
- Paper Thickness:

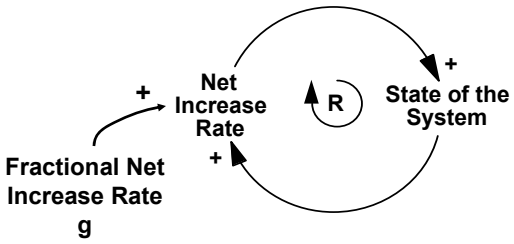
  1 billion light-years

  (*852 trillion* times the distance from the earth to the sun)

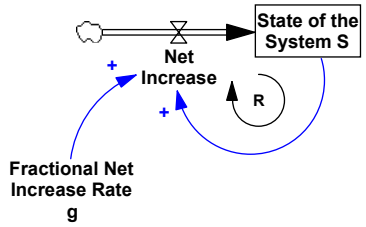
Exponential Growth

Linear First-Order Positive Feedback System

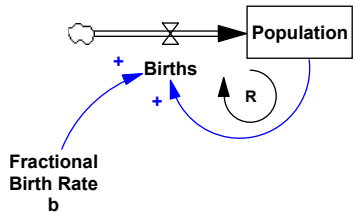




First-Order Linear Positive Feedback System



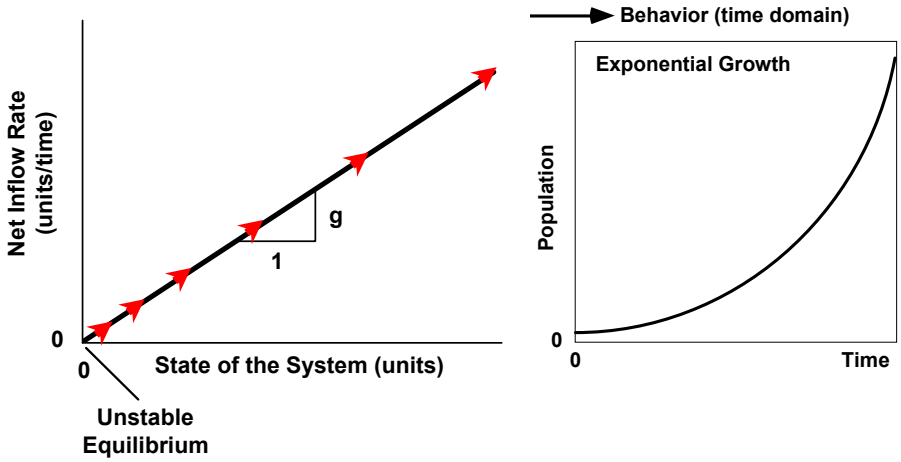
$$\frac{dS}{dt} = \text{Net Increase Rate} = gS$$



$$\frac{dP}{dt} = \text{Births} = bP$$

Phase Plot: Linear 1st Order Positive Feeback

dS/dt = Net Inflow Rate = gS



Doubling Time (t_d)

Structure: $\frac{dS}{dt} = gS$

Behavior: $S_t = S_0 \exp(gt)$

Doubling Time:

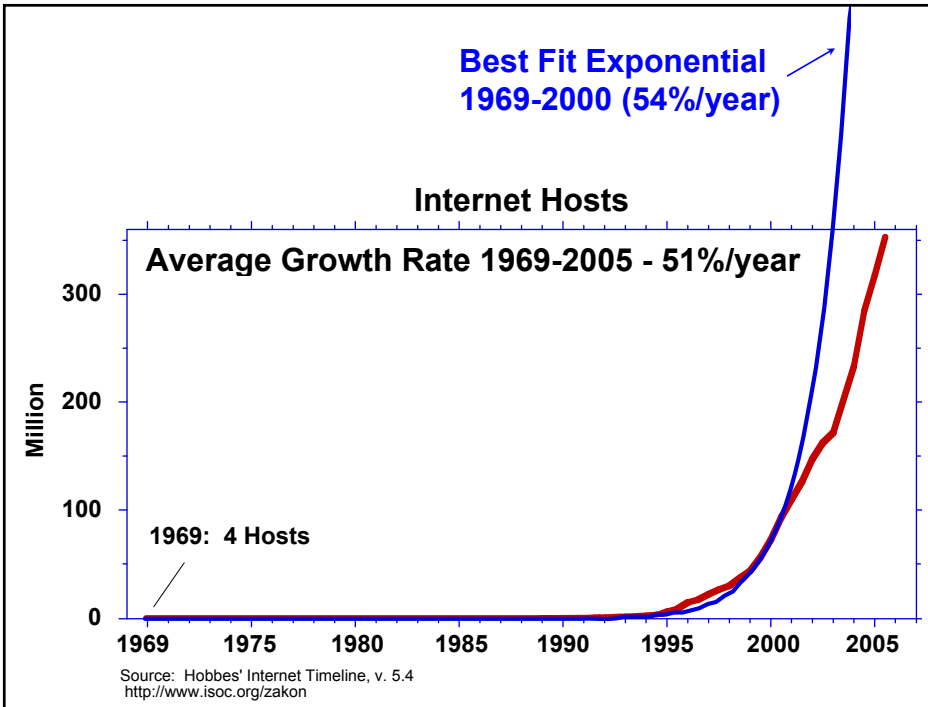
$2S_0 = S_0 \exp(gt_d)$

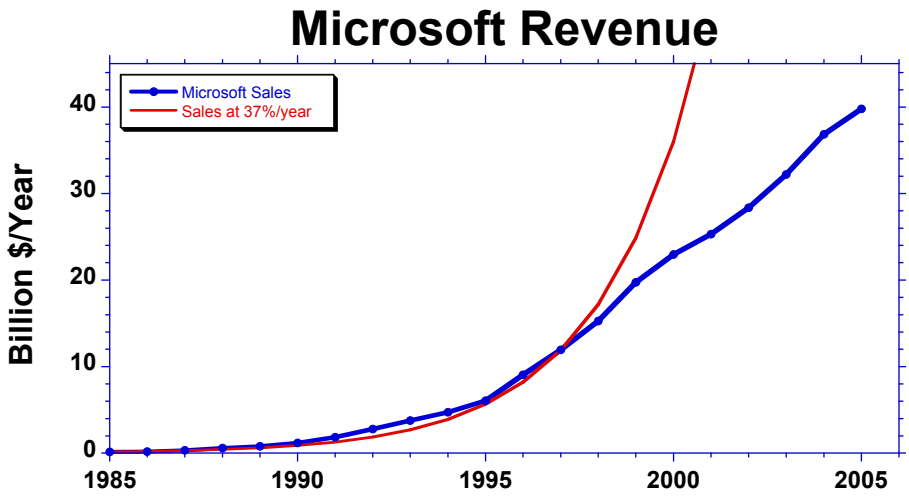
$2 = \exp(gt_d)$

$\ln 2 = gt_d$

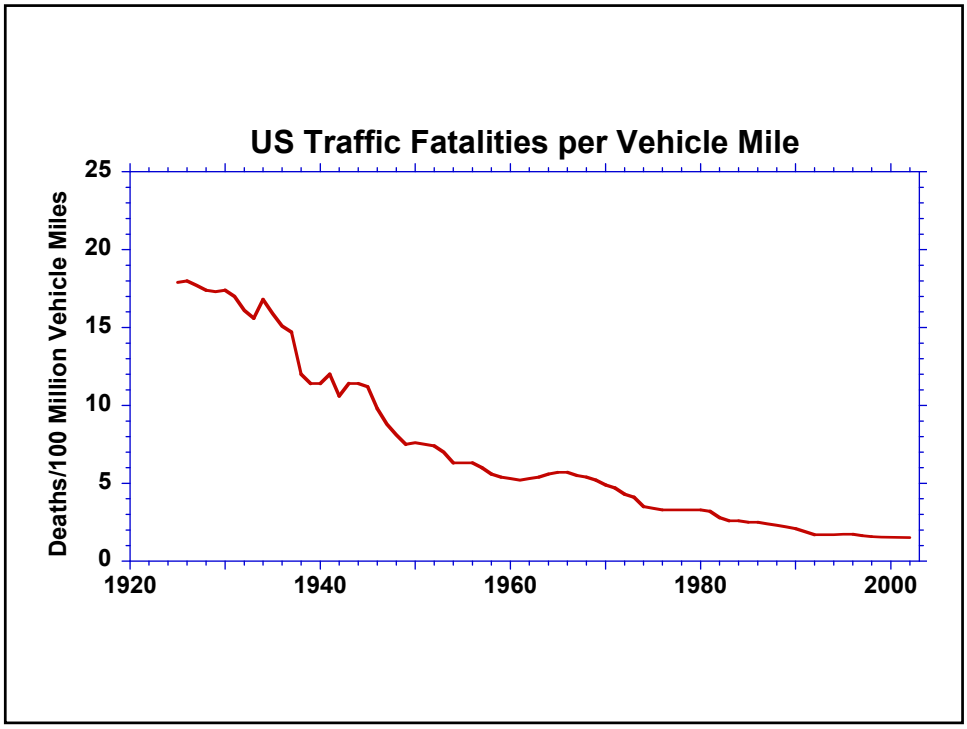
$t_d = \ln 2/g \approx 0.693/g$

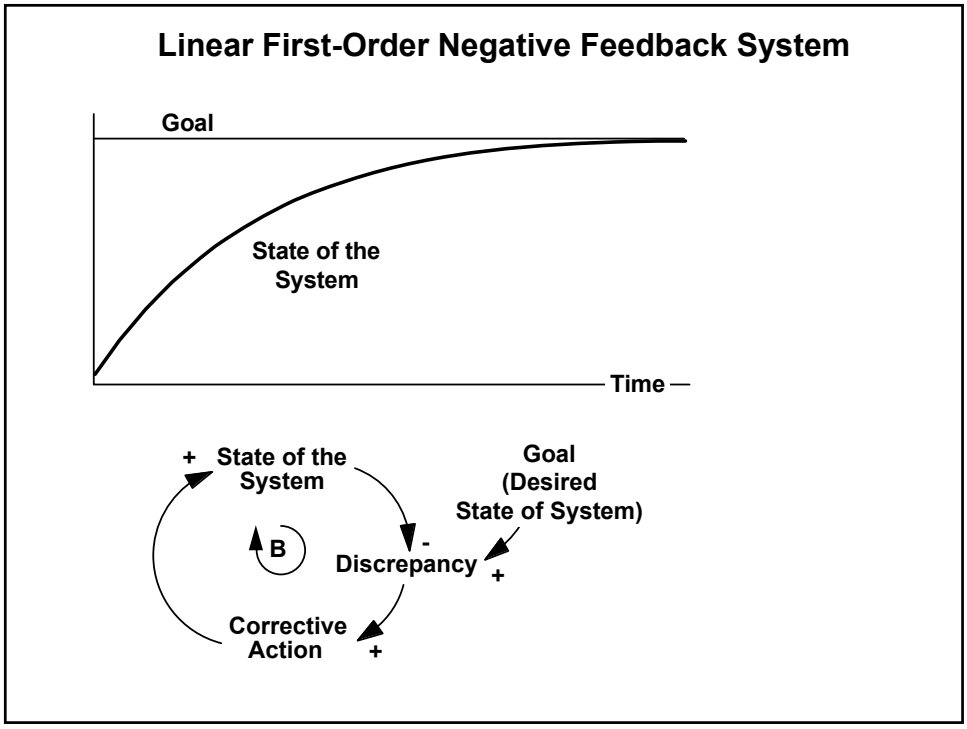
Rule of 70: $t_d \approx 70/(100g\%)$



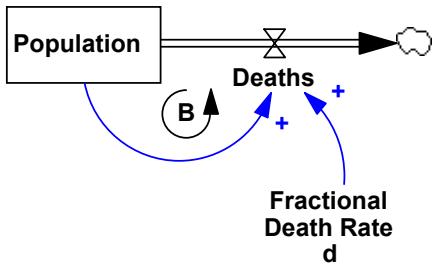


Exponential Decay


US Traffic Fatalities per Vehicle Mile
Deaths/100 Million Vehicle Miles
25
20
15
10
5
0
1920
1940
1960
1980
2000

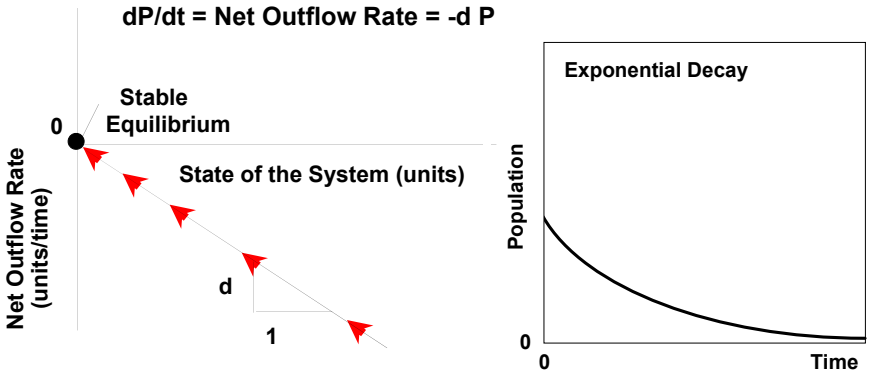

Linear First-Order Negative Feedback System
Goal
State of the System
Time
+
State of the System
Goal (Desired State of System)
B
-
Discrepancy
+
Corrective Action
+

Linear First-Order Negative Feedback System



$$\frac{dP}{dt} = -dP$$

Phase Plot: Linear 1st Order Positive Feeback



Half Life (t_h)

Structure: $\frac{dP}{dt} = -dP$

Behavior: $P_t = P_0 \exp(-dt)$

Half Life:

$(1/2)P_0 = P_0 \exp(-dt_h)$

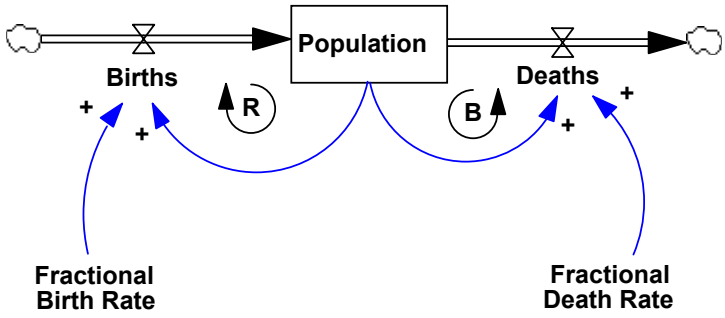
$1/2 = \exp(-dt_h)$

$\ln(1/2) = -dt_h$

$t_h = -\ln(1/2)/d \approx 0.693/d$

Rule of 70: $t_h \approx 70/(100d\%)$

Multiple Loops



Births = bP

Deaths = dP

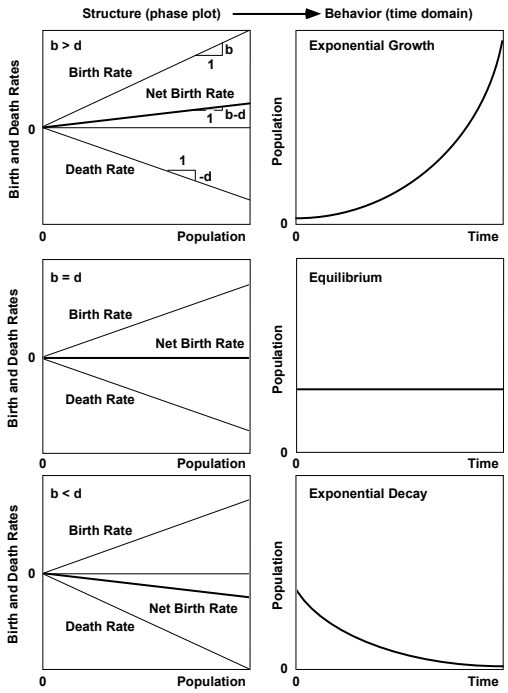
Net Births = bP – dP = (b – d)P

Structure:

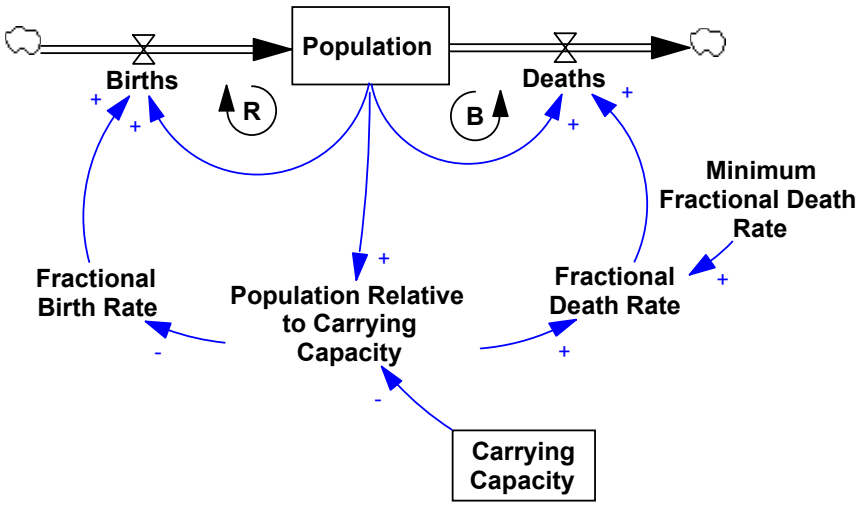
$$\frac{dP}{dt} = bP - dP$$

Behavior:

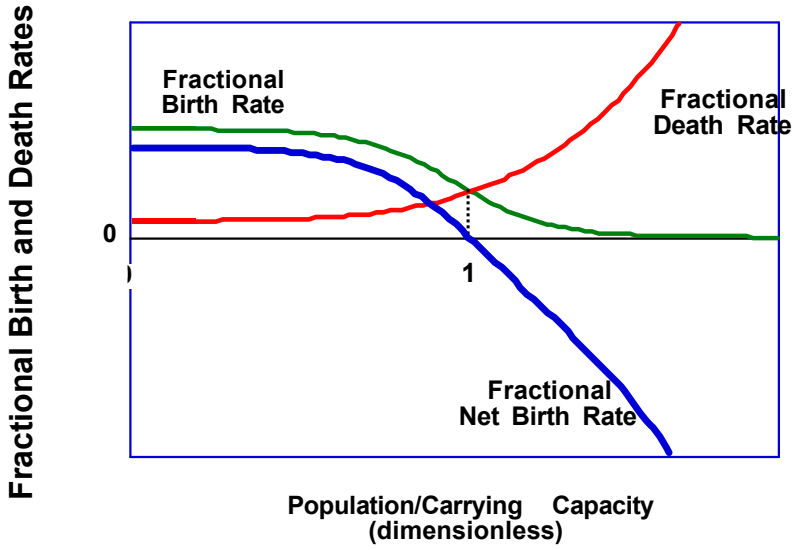
$$P_t = P_0 \exp[(b-d)t]$$



Nonlinear First Order System



Fractional Rates Depend on Population Relative to Carrying Capacity



Phase Plot