From SVC to CVC4

15 Years of Decision Procedures SAT/SMT Summer School

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Outline

From SVC to CVC4

- SVC
- CVC
- CVC Lite
- CVC3
- CVC4

2 Verification of Low-Level Code

- Satisfiability Modulo Theories
- Processing Packets
- Memory Models
- Example



Motivation for a Validity Checker

- Processor Verification via Symbolic Simulation
- Prove that Abstract Specification Machine matches Implementation
- Burch-Dill Commuting Diagram
- Check equality of two big formulas

Burch-Dill Commuting Diagram



SVC

SVC

- Stanford Validity Checker [Barrett, Dill, & Levitt '96]
- Authors: Clark Barrett, Jeremy Levitt, Aaron Stump, Robert Jones, David Dill
- First source release: 1998

Innovations

- Theory reasoning based loosely on Shostak's method [Shostak '84, Levitt '98]
- Powerful rewriter/simplifier
- Helpful built-in support for backtracking data structures
- Novel decision procedures (e.g. bit-vectors, arrays, records)
- Modular theory solver design

SVC

Applications

- Processor Verification [Levitt & Olukotun '97]
- Specification Checking [Park et al. '98]
- Theorem prover assistance [Heilmann '99]

Headaches

- Shostak's method too complicated and restrictive
- Equational solvers required to respect restrictive total order
- Boolean reasoning too primitive
- Software architecture too entangled

CVC

- Cooperating Validity Checker [Stump, Barrett, & Dill '02]
- Authors: Aaron Stump, Clark Barrett, David Dill, Sergey Berezin, Vijay Ganesh
- First release: 2002

Innovations

- Use of SAT solver (Chaff) for Boolean reasoning [Barrett, Dill, & Stump '02]
- Theory combination framework based on Nelson-Oppen with features of Shostak [Barrett '03]
- Proof production [Stump, Barrett, & Dill '02]

Applications

- Predicate Abstraction [Das & Dill '02]
- Software Verification (BLAST tool) [Henzinger et al. '03]
- Compiler Validation [Barrett, Goldberg, & Zuck '03]

Headaches

• Software architecture - too entangled

CVC Lite

CVC Lite

- CVC Lite [Barrett & Berezin '04]
- Authors: Clark Barrett, Sergey Berezin, David Dill, Vijay Ganesh
- Additional Contributors: Cristian Cadar, Jake Donham, Yeting Ge, Deepak Goyal, Ying Hu, Sean McLaughlin, Mehul Trivedi, Michael Veksler, Daniel Wichs, Mark Zavislak, Jim Zhuang
- First release: 2004

Innovations

- Theorem-based computation
- Handling of partial functions via TCC's [Berezin et al. '04]
- Mixed integer-real arithmetic (plus some non-linear reasoning)
- Quantifiers
- Predicate sub-typing

CVC Lite

Applications

- Translation validation for compilers [Goldberg, Zuck, & Barrett '04]
- Trusted theorem prover assistance [McLaughlin, Barrett, & Ge '05]
- Hardware equivalence checking at Calypto Systems

Headaches

- Performance
- Software architecture too entangled

CVC3

• CVC3 [Barrett & Tinelli '07]

- Authors: Clark Barrett, Cesare Tinelli, Chris Conway, Morgan Deters, Alexander Fuchs, Yeting Ge, George Hagen, Mina Jeong, Dejan Jovanović, Tim King
- First release: 2007

Innovations

- Enhanced MiniSat Boolean core with proof capability
- New decision procedures (bit-vectors, data types, quantifiers)
- Improved support for non-linear arithmetic
- Extensive support for SMT-LIB and format translation

Applications

- Deductive program verification with Why [Filliâtre & Marché '07]
- Symbolic analysis of software at IBM [Chandra, Fink, & Sridharan '09]
- Static analysis of C programs [Conway & Barrett '10]
- Many more...

Headaches

- Performance
- Incompleteness due to non-stably-infinite theories
- Software architecture too entangled

CVC4

• CVC4 [Barrett et al. '11]

- Designers and Authors: Kshitij Bansal, Clark Barrett, Christopher Conway, Morgan Deters, Liana Hadarean, Tim King, Dejan Jovanović, Andrew Reynolds, Cesare Tinelli
- First release: 2011

Innovations

- New efficient expression package
- Decentralized and more powerful theory combination techniques (polite theories, care functions) [Jovanović & Barrett '10]
- New state-of-the-art theory implementations (uninterpreted functions, real arithmetic, arrays, bit-vectors)
- Performance-neutral proof production
- Designed to be easily parallelizable



Applications

- BMC of Hybrid Systems [King & Barrett '11]
- More to come...

Headaches

• Trying to keep the software architecture from becoming too entangled



Applications

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Headaches

Trying to keep the software architecture from becoming too entangled

A Sneak Peek at CVC4

- CVC4 vs CVC3 (time and memory)
- CVC4 vs other solvers (time and memory)

CVC4 vs CVC3 (time)



CVC4 vs CVC3 (memory)



Cumulative Time Cactus Plot



Cumulative Memory Cactus Plot



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Satisfiability Modulo Theories

For a theory T, the *T*-satisfiability problem consists of deciding whether there exists a model A and variable assignment α such that $(A, \alpha) \models T \cup \varphi$ for a given formula φ .

Theories of Inductive Data Types

An *inductive data type* (IDT) defines one or more *constructors*, and possibly also *selectors* and *testers*.

Example: list of int

- Constructors: $cons:(int, list) \rightarrow list, null: list$
- Selectors: car: $list \rightarrow int$, cdr: $list \rightarrow list$
- Testers: *is_cons*, *is_null*

The *first order theory* of a inductive data type associates a function symbol with each constructor and selector and a predicate symbol with each tester.

Example: $\forall x : list. (x = null \lor \exists y : int, z : list. x = cons(y, z))$

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For IDTs with a single constructor, a conjunction of literals is decidable in polynomial time [Oppen '80].

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For more general IDTs, the problem is NP complete, but reasonbly efficient algorithms exist in practice [Barrett et al. '07].



Network packets are highly structured



Network packets are highly structured but usually processed with low-level bit-twiddling code

One solution: packet-processing DSLs (e.g., binpac, Melange, Morpheus, Prolac)

```
type List =
    cons {
        tag:1 = 0b1,
        count: 7,
        data: u_char[count],
        cdr: List
    }
| nil {
        tag:8 = 0x00
}
```

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High level

• Type safe

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- High level
- Type safe
- Slower than C
- Need to rewrite existing code

Packet Types as Specification

Instead of synthesizing a performant implementation, let's use packet types as the basis of a specification

```
while( (n = *p++) & 0x80 ) {
    assert( isCons(prev(p)) );
    p += n & 0x7f;
    assert( p == cdr(prev(p)) );
}
```

We can use bit-precise reasoning to prove that the code satisfies the assertions using CASCADE.

Cascade Verification Framework





- High-precision verification of program paths
- Intended for use in a *multi-stage analysis*
- Path is defined and assertions are injected using an XML control file

Cascade/C



Cvc3 Encoding

- Encode verification conditions as SMT instances
- Use Cvc3 SMT solver to decide validity
- CVC3 includes theories for:
 - Arrays
 - Uninterpreted functions
 - Bit vectors
 - Inductive datatypes
- Connect the high-level assertions and the low-level code by generating:
 - An inductive datatype
 - Functions mapping datatype values to arrays of bytes
 - Encode program semantics using bit vectors

CVC3 Encoding

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```

CVC3 Encoding

```
ptrType : BITVECTOR(N);
byteType : BITVECTOR(8);
memType : ARRAY ptrType OF byteType;
DATATYPE
List =
  cons( tag: BITVECTOR(1),
        len: BITVECTOR(7),
        data: memType,
        cdr: List )
| nil( tag: BITVECTOR(8) )
undefined;
END:
toList : (memType, ptrType) -> List;
\forall m:memType, i:ptrType.
  isNil(toList(m,i)) \iff m[i] = 0;
\forall m: memType, i: ptrType.
  isCons(toList(m,i)) \iff m[i][7] = 1;
\forall m: memType, i: ptrType.
  isCons(toList(m, i)) \implies
      cdr(toList(m,i)) = toList(m,i+len(toList(m,i))+1);
etc...
```

Verification Condition Generation

```
n = *p++;
assume( (n & 0x80) != 0 );
assert( isCons(prev(p)) );
```

becomes

$$egin{aligned} m_1 &= m_0 [\& n \mapsto m_0 [m_0 [\& p]]] \ m_2 &= m_1 [\& p \mapsto m_1 [\& p] + 1] \ m_2 [\& n] \& 0 {
m x80}
eq 0 {
m x00} \ is Cons(toList(m_2,m_0 [\& p])) \end{aligned}$$

- "Flat" memory model
 - Memory is one big array:

$$egin{aligned} m_1 &= m_0 [\& \mathbf{n} \mapsto m_0 [m_0 [\& \mathbf{p}]]] \ m_2 &= m_1 [\& \mathbf{p} \mapsto m_1 [\& \mathbf{p}] + 1] \end{aligned}$$

- No "frame rule" is implied.
 - E.g., the following isn't necessarily valid:

- We can't rule out &i being reachable if toList is unrolled enough times.
- Detailed non-aliasing assumptions have to be added by hand

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- No "frame rule" is implied.
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- We can't rule out &i being reachable if toList is unrolled enough times.
- Detailed non-aliasing assumptions have to be added by hand
- And they don't help much

- Burstall model [Burstall '72, Bornat '00]
 - A separate memory array for each static type:

$$m'_{char} = m_{char} [\&n \mapsto m_{char} [m_{char*} [\&p]]]$$

 $m'_{char*} = m_{char*} [\&p \mapsto m_{char*} [\&p] + 1]$

- Can't handle safe dynamic casts
- Can't handle promiscuous pointer manipulation

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 $m'_{char*} = m_{char*} [\&p \mapsto m_{char*} [\&p] + 1]$

- Can't handle safe dynamic casts
- Can't handle promiscuous pointer manipulation
- Which is exactly what packet processing is

An "in between" model, based on separation analysis [Hubert & Marché '07, Rakamaric & Hu '09]

- Memory is partitioned into disjoint regions.
- Every pointer expression is associated with a region



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• Each region can be represented by a separate "memory"

Flat:

$$egin{aligned} m_1 &= m_0 [\& {
m n} \mapsto m_0 [m_0 [\& {
m p}]]] \ m_2 &= m_1 [\& {
m p} \mapsto m_1 [\& {
m p}] + 1] \ m_2 [\& {
m n}] \& 0 {
m x80}
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m p}))) \end{aligned}$$

Partitioned:

$$egin{aligned} &m_{ extsf{n}}' = m_{ extsf{n}} [\& extsf{n} \mapsto m_{ extsf{p}} [m_{ extsf{p}} [\& extsf{p}]]] \ &m_{ extsf{p}}' = m_{ extsf{p}} [\& extsf{p} \mapsto m_{ extsf{p}} [\& extsf{p}] + 1] \ &rac{m_{ extsf{n}}' [\& extsf{n}] \ \& \ 0 extsf{x} \otimes 0 \ extsf{x}$$

Partitioned:

$$\begin{split} & \textbf{m}_{n}' = \textbf{m}_{n}[\&n \mapsto \textbf{m}_{\star p}[\textbf{m}_{p}[\&p]]] \\ & \textbf{m}_{p}' = \textbf{m}_{p}[\&p \mapsto \textbf{m}_{p}[\&p] + 1] \\ & \textbf{m}_{n}'[\&n] \& 0 \times 80 \neq 0 \times 00 \\ & \textbf{isCons(toList(\textbf{m}_{\star p}, \textbf{m}_{p}[\&p]))} \end{split}$$

- Separation creates a "frame" around datatype values
- Makes hard problems easy and easy problems trivial
- The verification condition is sound if the partition is sound

"Real World" Example: Encoded Domain Name

```
type Dn =
  label {
    tag:2 = 0b00,
    len:6 != 0b000000,
    name:u_char[len],
    rest:Dn
  }
  indirect {
    tag:2 = 0b11,
    offset:14
  }
 nullt {
    tag:8 = 0x00
  }
```

```
#define NS_CMPRSFLAGS (0xc0)
int ns_name_skip(const u_char **ptrptr, const u_char *eom) {
  { allocated(*ptrptr, eom) }
  const u_char *cp; u_int n;
  cp = *ptrptr;
  { @invariant: cp <= eom =>
                cp + sizeOfDn(cp) = init(cp) + sizeOfDn(init(cp)) }
  while (cp < eom \&\& (n = *cp++) != 0) {
    switch (n & NS_CMPRSFLGS) {
      case 0: /* normal case, n == len */
        { isLabel(prev(cp)) }
        cp += n:
        { rest(prev(cp)) = toDn(cp) }
        continue:
      case NS CMPRSFLGS: /* indirection */
        { isIndirect(prev(cp)) }
        cp++; break;
      default: /* illegal type */
        __set_errno (EMSGSIZE); return (-1);
    }
    break:
  }
  if (cp > eom) { __set_errno (EMSGSIZE); return (-1); }
  { cp = eom _ cp = init(cp) + sizeOfDn(init(cp)) }
  *ptrptr = cp;
  return (0);
}
```

Experimental results

- Verification times for ns_name_skip.
- 30 LOC, 4 assertions + a loop invariant

		Time (seconds)	
Name	Lines	Flat	Part.
Init	5–12	0.34	0.03
Case 0 (1)	12-16	13.94	0.05
Case 0 (2)	12-28	33.42	0.06
Case 0 (3)	12-19	*	0.12
CASE $0xc0$ (1)	12-14, 20-21	6.14	0.04
CASE $0xc0$ (2)	12-14, 20-23, 30, 34	*	0.07
Term (1)	12, 30, 34	0.63	0.06
Term (2)	12, 30, 34	*	0.05

Final Thoughts

15 years of checking formulas

- SMT has come a long way in last 15 years
- Dramatic advances in theory and practice
- Explosion of application areas

Lessons

- Balancing high-performance and software flexibility is a challenge
- Modularity and solid theoretical foundations can help
- But in a rapidly advancing area, may have to reimplement every few years anyway

CVC4 is coming

- Goals: open source, high-performance, full-featured SMT solver
- Contributions and collaborations welcome after first release

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