# Nucleon structure near the physical pion mass

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# Outline

- Introduction: isovector nucleon observables
- Lattice methodology
  - Challenges
  - BMW action; ensembles
  - Systematic errors
- Results
  - Vector form factors; radii and magnetic moment
  - Momentum fraction
  - Axial charge
  - Tensor and scalar charges
- Conclusions

# Nucleon observables

Electromagnetic form factors:

$$\langle p(P',s')|\bar{q}\gamma^\mu q|p(P,s)\rangle = \bar{u}(p',s')\left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_p}F_2^q(Q^2)\right)u(p,s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.

Quark momentum fraction:

$$\langle p(P,s')|\bar{q}\gamma_{\{\mu}D_{\nu\}}q|p(P,s')\rangle = \langle x\rangle_q \bar{u}(P,s')\gamma_{\{\mu}P_{\nu\}}u(P,s)$$

Axial, tensor, and scalar charges:

$$\langle p(P,s')|\bar{u}\gamma^\mu\gamma_5 d|n(P,s)\rangle = g_A\bar{u}_p(P,s')\gamma^\mu\gamma_5 u_n(P,s)$$
$$\langle p(P,s')|\bar{u}d|n(P,s)\rangle = g_S\bar{u}_p(P,s')u_n(P,s)$$
$$\langle p(P,s')|\bar{u}\sigma^{\mu\nu}d|n(P,s)\rangle = g_T\bar{u}_p(P,s')\sigma^{\mu\nu}u_n(P,s)$$

# Vector form factors

Dirac and Pauli form factors:

$$\langle p(P',s')|\bar{q}\gamma^\mu q|p(P,s)\rangle = \bar{u}(p',s')\left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_p}F_2^q(Q^2)\right)u(p,s),$$

where $\Delta = P' - P$, $Q^2 = -\Delta^2$.

- Isovector combination:

$$F_{1,2}^v = F_{1,2}^u - F_{1,2}^d = F_{1,2}^p - F_{1,2}^n,$$

  where $F_{1,2}^{p,n}$ are form factors of the electromagnetic current in a proton and in a neutron.
- Dirac and Pauli radii defined via slope at $Q^2 = 0$:

$$F_{1,2}(Q^2) = F_{1,2}(0)(1 - \tfrac{1}{6}r_{1,2}^2 Q^2 + O(Q^4));$$

  $F_2(0) = \kappa$, the anomalous magnetic moment.
- Proton charge radius, $(r_E^2)^p = (r_1^2)^p + \frac{3\kappa^p}{2m_p^2}$, has $7\sigma$ discrepancy between measurements from $e$–$p$ scattering and from Lamb shift in muonic hydrogen.

# Quark momentum fraction

Measure forward matrix element of quark energy-momentum operator:

$$\langle p(P,s')|\bar{q}\gamma_{\{\mu}D_{\nu\}}q|p(P,s)\rangle = \langle x\rangle_q \bar{u}(P,s')\gamma_{\{\mu}P_{\nu\}}u(P,s).$$

$\langle x\rangle_q$ is the average momentum fraction carried by quarks $q$ and $\bar{q}$; focus on isovector combination $\langle x\rangle_{u-d}$.

# Axial charge

"Benchmark" observable:

$$\langle p(P,s')|\bar{u}\gamma^\mu\gamma_5 d|n(P,s)\rangle = g_A \bar{u}_p(P,s')\gamma^\mu\gamma_5 u_n(P,s).$$

- Naturally isovector observable; PDG: $g_A/g_V = 1.2701(25)$ from $\beta$ decay of polarized neutrons.
- Is also the difference between contributions from the spin of $u$ and $d$ quarks to the total proton spin.

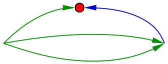
# Scalar and tensor charges

$$\langle p(P,s')|\bar{u}d|n(P,s)\rangle = g_S \bar{u}_p(P,s')u_n(P,s)$$
$$\langle p(P,s')|\bar{u}\sigma^{\mu\nu}d|n(P,s)\rangle = g_T \bar{u}_p(P,s')\sigma^{\mu\nu}u_n(P,s)$$

- Not measured experimentally.
- Recent interest because they are needed to know leading contributions to neutron $\beta$ decay from BSM physics.
- There are plans to measure the tensor charge at JLab using the upcoming 12 GeV upgrade.

# Lattice calculations

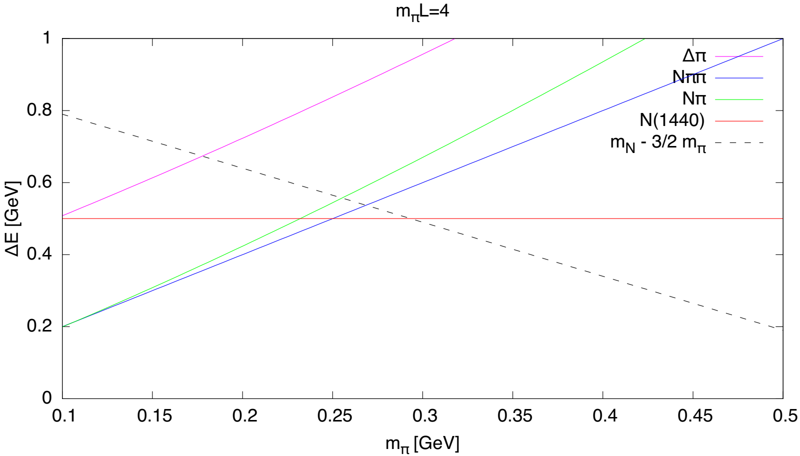
Use sequential propagator through the sink: propagators from fixed source and sink allow for measurement at all intermediate times.



Cost grows linearly with number of source-sink separations $T$; past calculations typically used just one.

- Excited-state contamination $\sim \exp(-\Delta E T)$.
- Signal-to-noise $\sim \exp(-(m_N - \frac{3}{2}m_\pi)T)$.

# Challenges at low pion mass



# BMW action

- $N_f = 2 + 1$ tree-level clover-improved Wilson fermions coupled to double-HEX-smeared gauge fields.
- Pion mass ranging from 149 MeV to 356 MeV.
- Nine coarse lattices with $a = 0.116$ fm;
  one fine lattice with $a = 0.09$ fm.
- No disconnected diagrams, so we focus on isovector observables.
- Three source-sink separations for controlling excited-state contributions: $T \in \{0.9, 1.2, 1.4\}$ fm.

# HEX smearing

Like HYP smearing, constructed using links from the unit hypercubes that touch $U_\mu(x)$.



Fat links in $SU(3)$ are obtained analytically from staples like in stout smearing.

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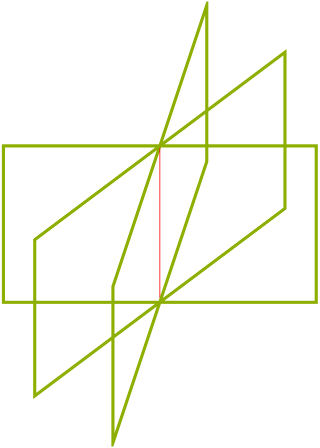

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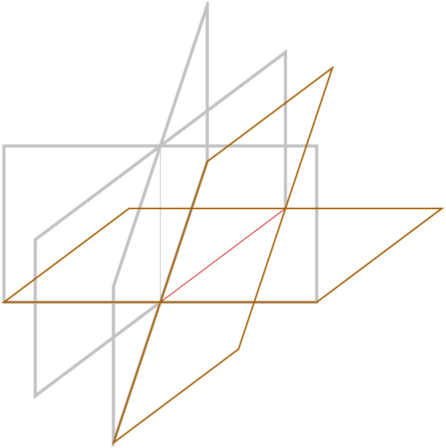

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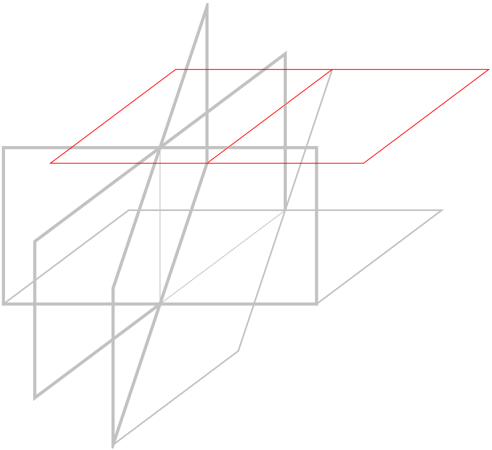
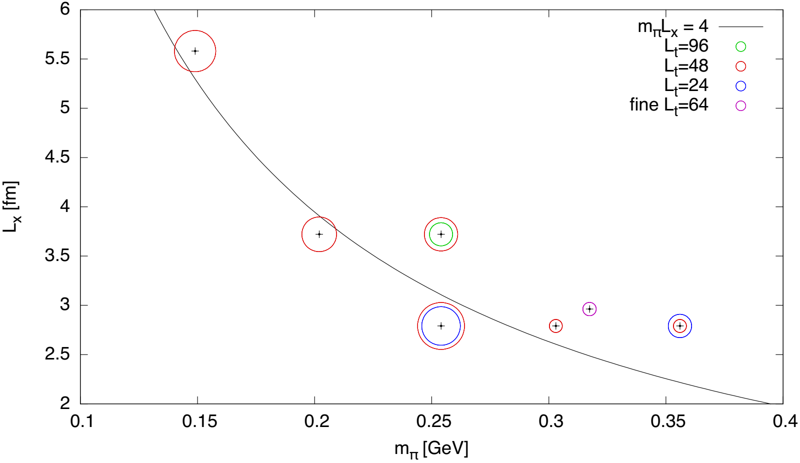


Fat links in $SU(3)$ are obtained analytically from staples like in stout smearing.

# Ensembles




Areas of circles scale with number of measurements: largest is 10,000.

# Contributions to systematic error

From best-controlled to worst-controlled:

- Quark masses: smallest pion mass is 149 MeV, so $m_\pi \to 135$ MeV chiral extrapolation is under control.
- Excited states: use of multiple source-sink separations allows for clear identification of observables where excited states are a problem; the summation method seems to work well for reducing excited-state contamination.
- Finite volume: effects expected to be small with $m_\pi L \approx 4$, and two volumes at $m_\pi = 254$ MeV allow for fully-controlled test of finite-size effects.
- Finite temperature: at small $m_\pi$, $L_t$ is smaller than the typically used $L_t \approx 2L_x$, but three different time extents at $m_\pi \approx 250$ MeV are useful for identifying possible problems.
- Discretization: one finer lattice at $m_\pi = 317$ MeV for consistency check, but no $a \to 0$ extrapolation.

# Systematic error: excited states

Usual approach for extracting matrix elements (forward case):

$$C_{2\text{pt}}(t) = \langle N(t)\bar{N}(0)\rangle$$
$$C_{3\text{pt}}(T,\tau) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle$$

Take ratio:

$$\begin{aligned} R(T,\tau) &= C_{3\text{pt}}(T,\tau)/C_{2\text{pt}}(T) \\ &= c_{00} + c_{10}e^{-\Delta E\tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \dots, \end{aligned}$$

where $c_{00}$ is the desired ground-state matrix element. Averaging a fixed number of points around $\tau = T/2$ yields asymptotic errors that fall off as $e^{-\Delta ET/2}$. Alternatively, use summation method: compute

$$S(T) = \sum_\tau R(T,\tau) = b + c_{00}T + dTe^{-\Delta ET} + \dots,$$

and then find its slope, which gives $c_{00}$ with errors that fall off as $Te^{-\Delta ET}$.

# Results

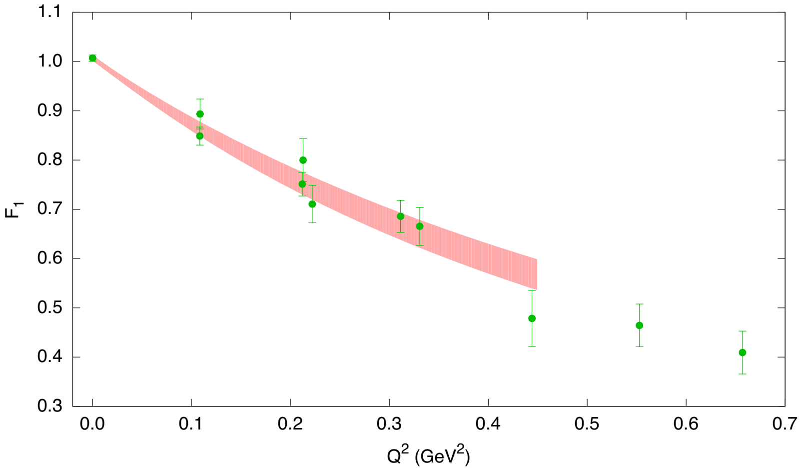
JRG, M. Engelhardt, S. Krieg, J. W. Negele, A. V. Pochinsky, S. N. Syritsyn, *Nucleon Structure from Lattice QCD Using a Nearly Physical Pion Mass*, arXiv:1209.1687

- $F_1^v(Q^2)$
  - $(r_1^2)^v$
- $F_2^v(Q^2)$
  - $\kappa^v$
  - $(r_2^2)^v$
- $\langle x \rangle_{u-d}$
- $g_A$

JRG, J. W. Negele, A. V. Pochinsky, S. N. Syritsyn, M. Engelhardt, S. Krieg, *Nucleon Scalar and Tensor Charges from Lattice QCD with Light Wilson Quarks*, arXiv:1206.4527

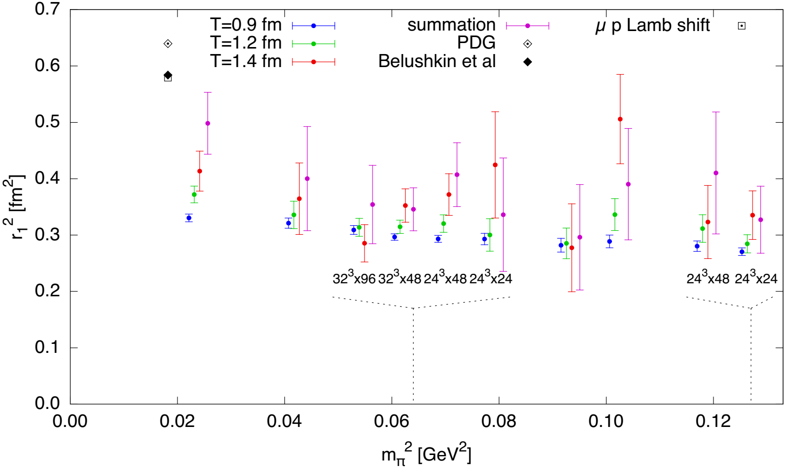
- $g_S$
- $g_T$

# $F_1^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation

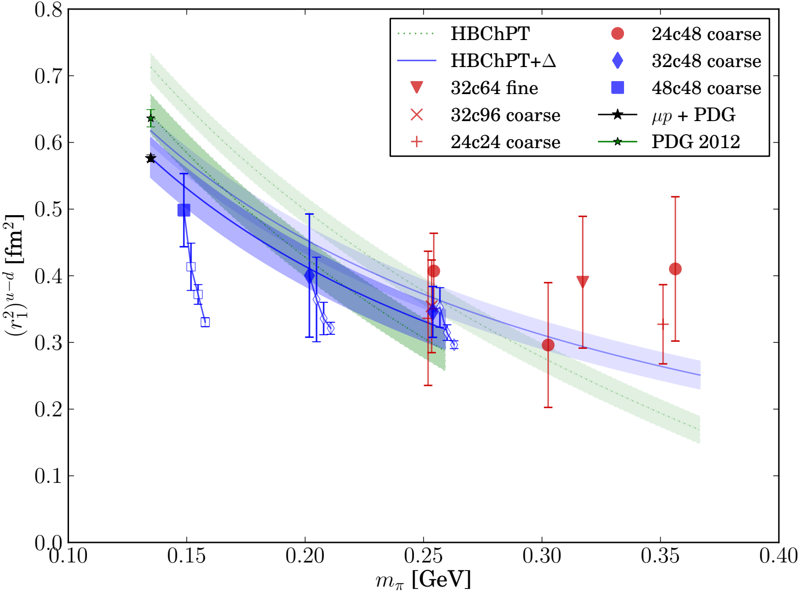



Dipole fit $F_1(Q^2) \sim \frac{F_1(0)}{(1+Q^2/M_D^2)^2}$ to range $0 \leq Q^2 \leq 0.5$ GeV$^2$.

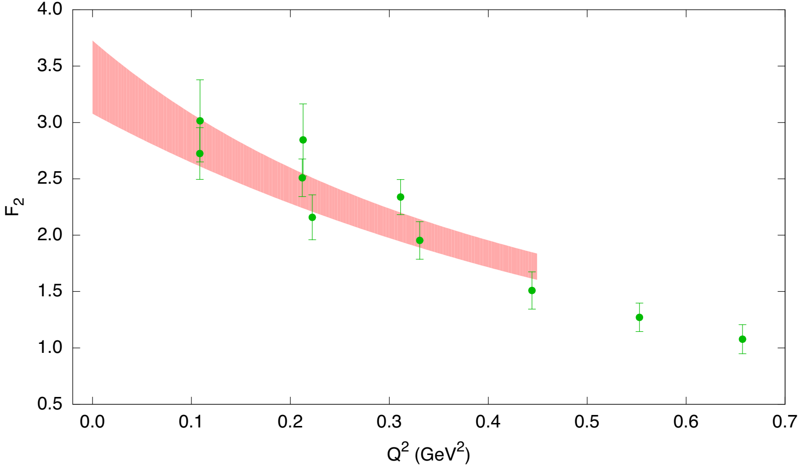
# Isovector Dirac radius $(r_1^2)^v$



# $(r_1^2)^v$: chiral extrapolation of summation points

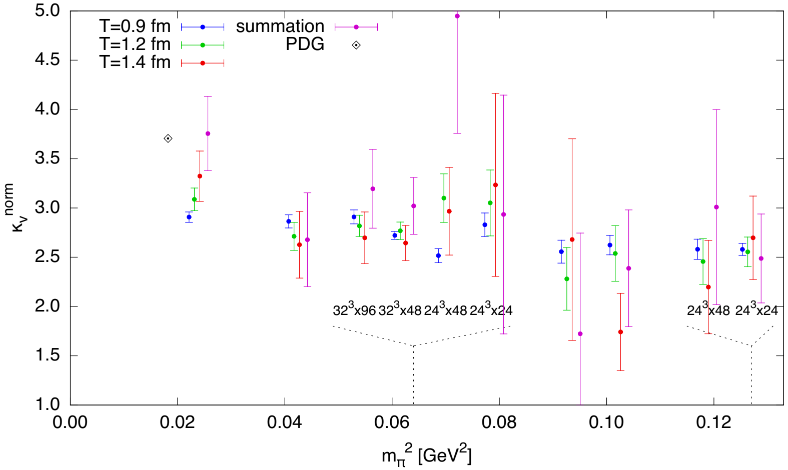


# $F_2^v(Q^2)$: $m_\pi = 254$ MeV, $32^3 \times 48$, summation




Dipole fit $F_2(Q^2) \sim \frac{F_2(0)}{(1+Q^2/M_D^2)^2}$ to range $0 < Q^2 \leq 0.5\ \text{GeV}^2$.

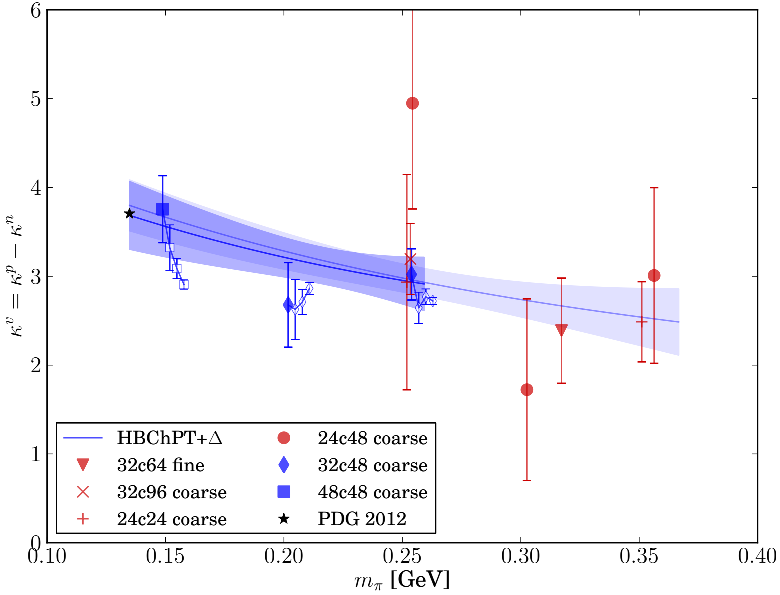
# Isovector anomalous magnetic moment $\kappa^V$



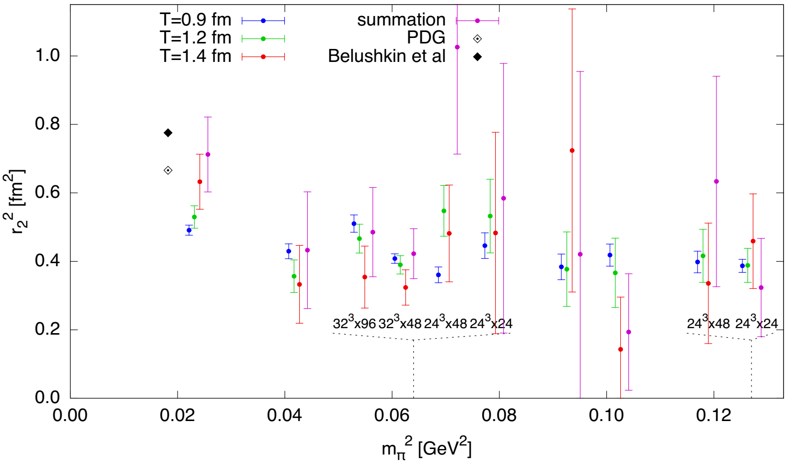

Normalized relative to the physical magneton:

$$\kappa^{\text{norm}} = \frac{m_N^{\text{phys}}}{m_N^{\text{lat}}} F_2^{\text{lat}}(0).$$

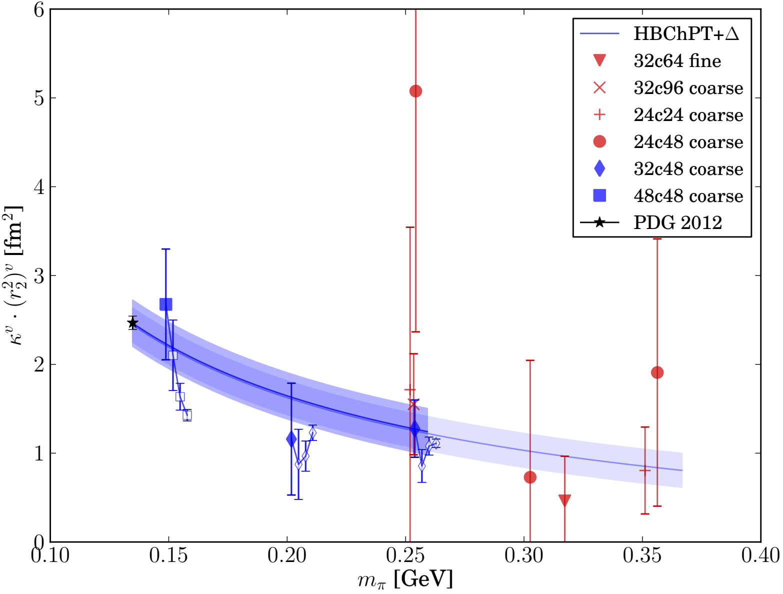
# $\kappa^V$: chiral extrapolation of summation points



# Isovector Pauli radius $(r_2^2)^v$



# $\kappa^v (r_2^2)^v$: chiral extrapolation of summation points

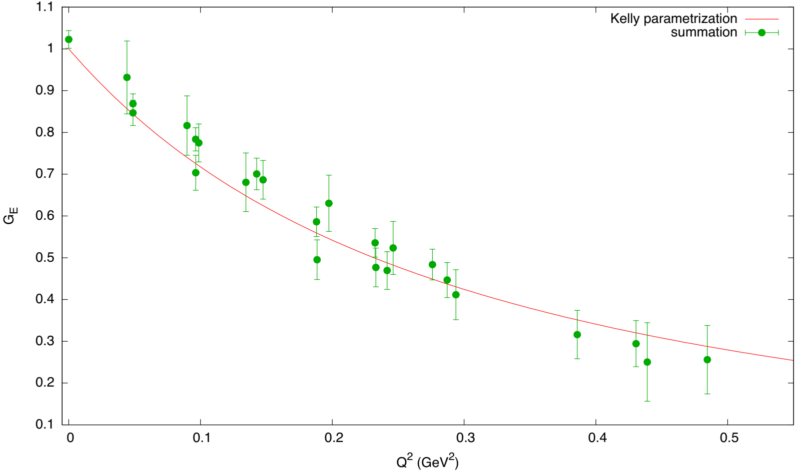


# Sachs form factors

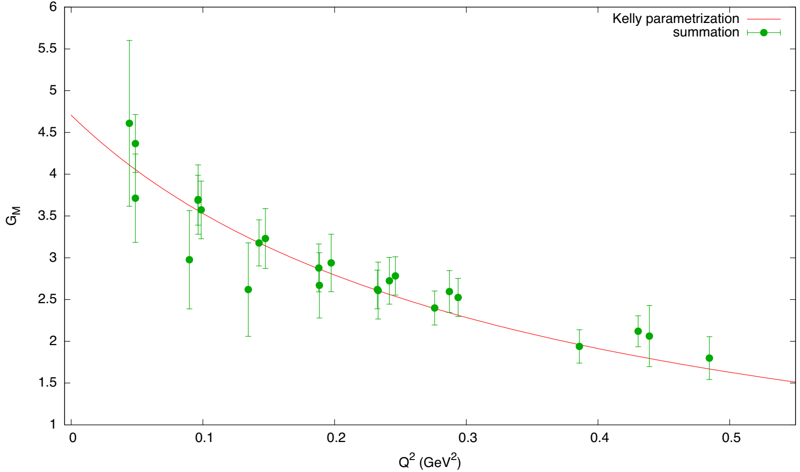
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{2m_N} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- Slopes at $Q^2 = 0$ give rms charge and magnetic radii.
- Compare:
  1. Isovector $m_\pi = 149$ MeV, summation data from lattice calculation.
  2. Parameterization of experimental data:
     J. J. Kelly, "Simple parametrization of nucleon form factors," Phys. Rev. C **70**, 068202 (2004).
     4 parameters for each of $G_{Ep}$, $G_{Mp}$, $G_{Mn}$; 2 parameters for $G_{En}$: determined from fit to experiment.

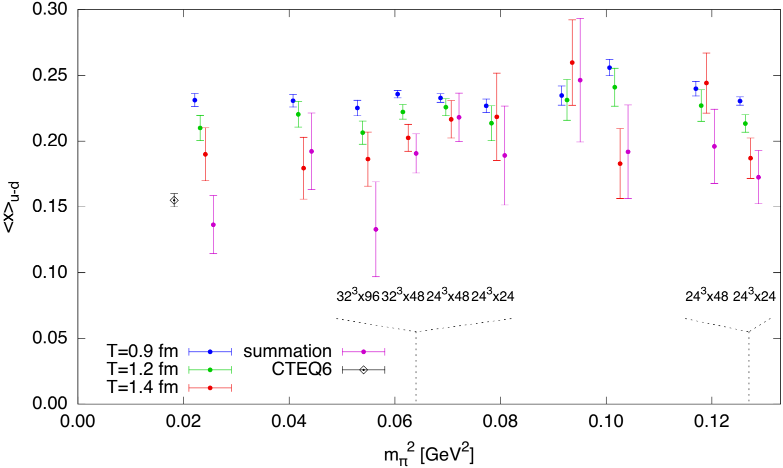
# $G_E^v(Q^2)$: lowest pion mass



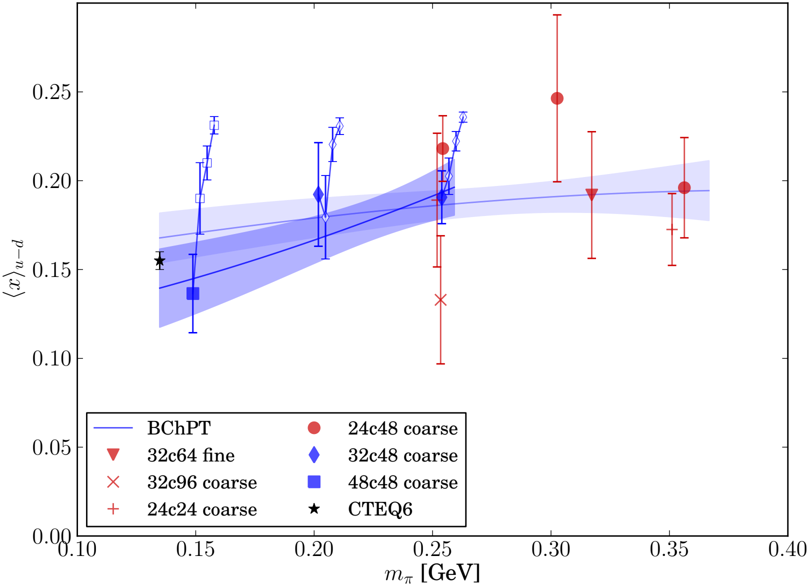
# $G_M^v(Q^2)$: lowest pion mass



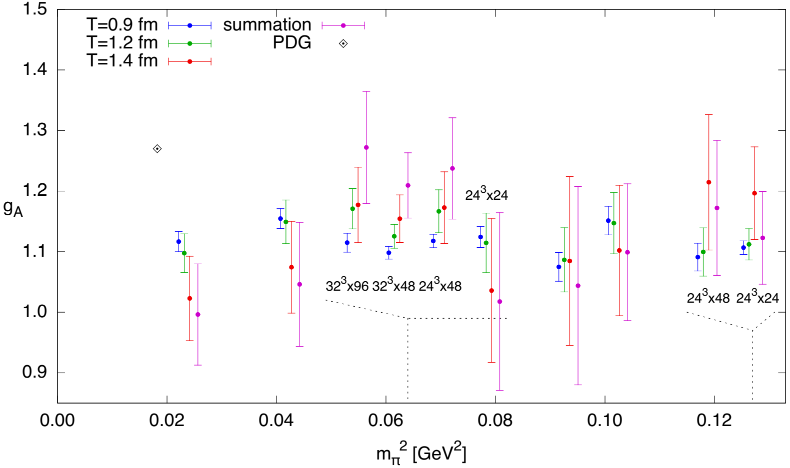
# Isovector quark momentum fraction $\langle x \rangle_{u-d}$



# $\langle x \rangle_{u-d}$: chiral extrapolation of summation points



# Axial charge $g_A$



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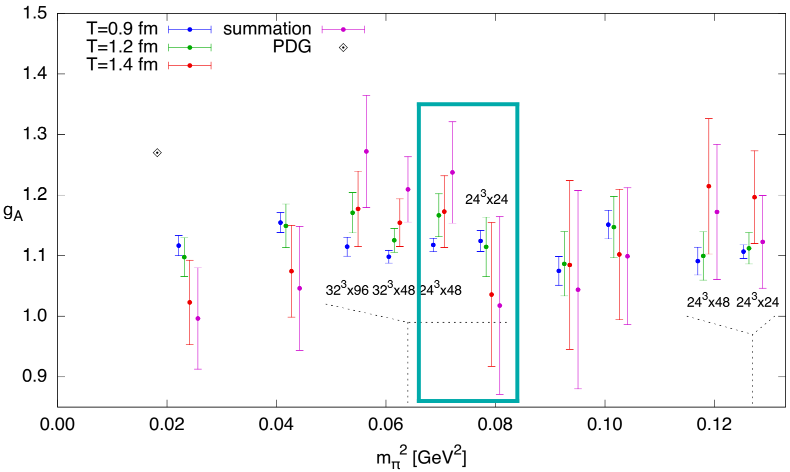
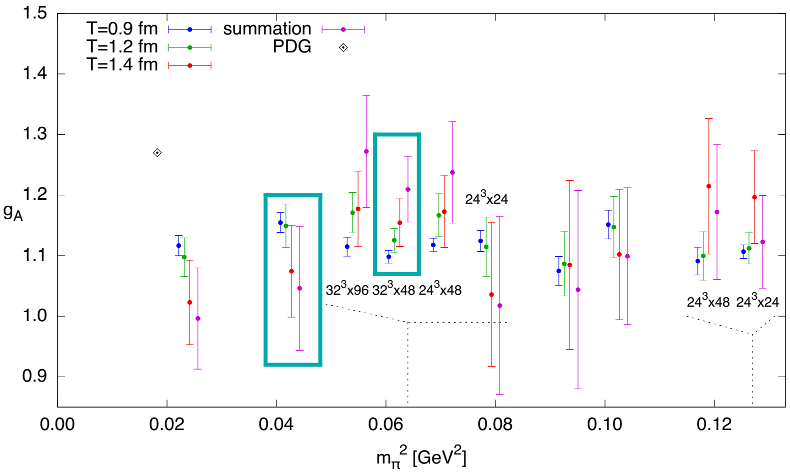


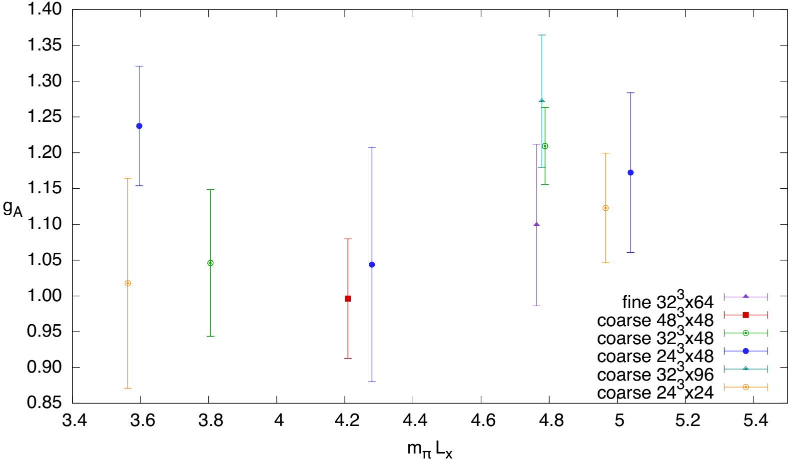

$m_\pi \approx 250$ MeV, $L_x = 24$; $L_t = 48$ versus $L_t = 24$.

# Axial charge $g_A$

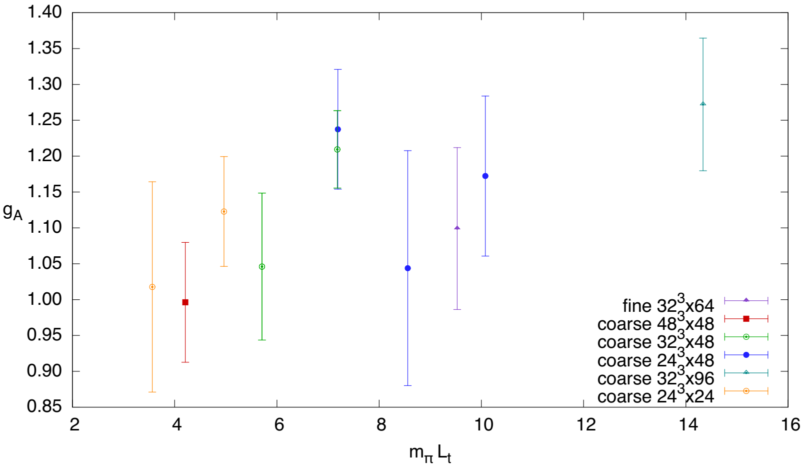



$32^3 \times 48$; $m_\pi = 200$ MeV versus $m_\pi = 250$ MeV.

# $g_A$ vs. $m_\pi L_x$: summation data

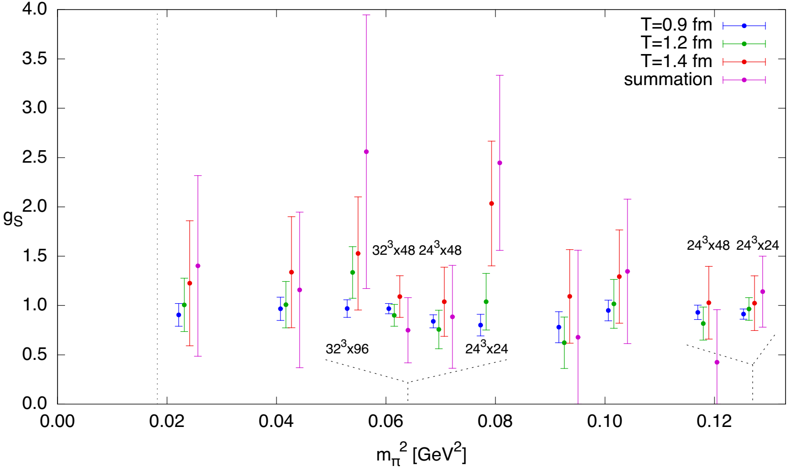


# $g_A$ vs. $m_\pi L_t$: summation data

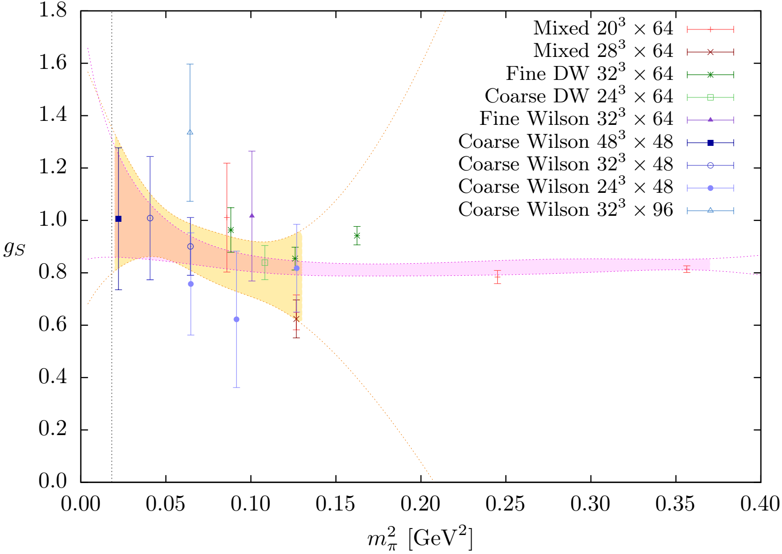



Thermal effects?

# Scalar charge $g_S$

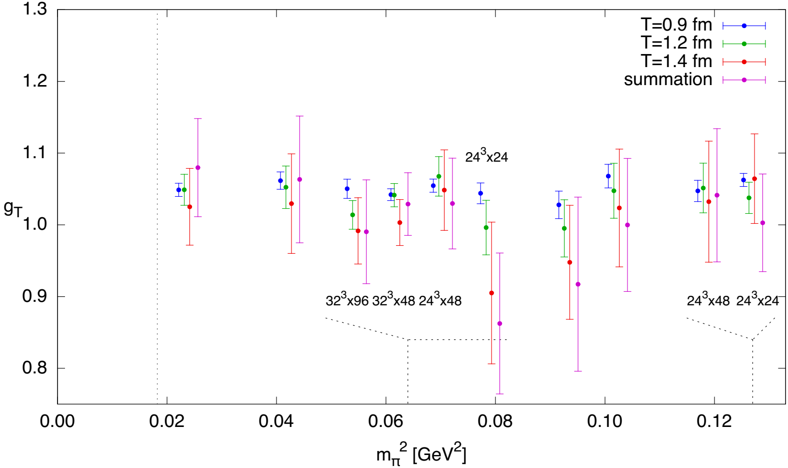


# $g_S$: chiral extrapolation

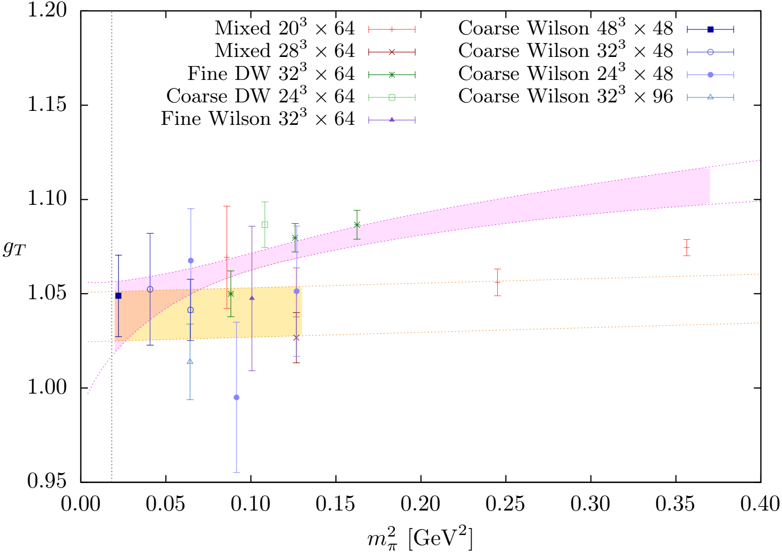



Using middle source-sink separation; also analyzed data from earlier studies.

# Tensor charge $g_T$



# $g_T$: chiral extrapolation




Using middle source-sink separation; also analyzed data from earlier studies.

# Conclusions

- Key new ingredient for calculations of nucleon structure observables: combination of both
  1. $m_\pi$ close to physical
  2. serious effort toward control over excited-state contamination
- Calculated isovector Dirac and Pauli radii, anomalous magnetic moment, and momentum fraction are consistent with experiment.
- "Benchmark" axial charge still inconsistent with experiment, but the influence of thermal states has been identified as a serious candidate for being the culprit.
  - We are planning to do a controlled study of finite volume and time effects using $32^3 \times 48$, $24^3 \times 48$, $32^3 \times 24$, and $24^3 \times 24$ ensembles.
- Agreement with experiment will increase confidence in predictions such as $g_S$ and $g_T$.