SU(3) gauge theory with 12 flavours in a twisted box

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Collaborators

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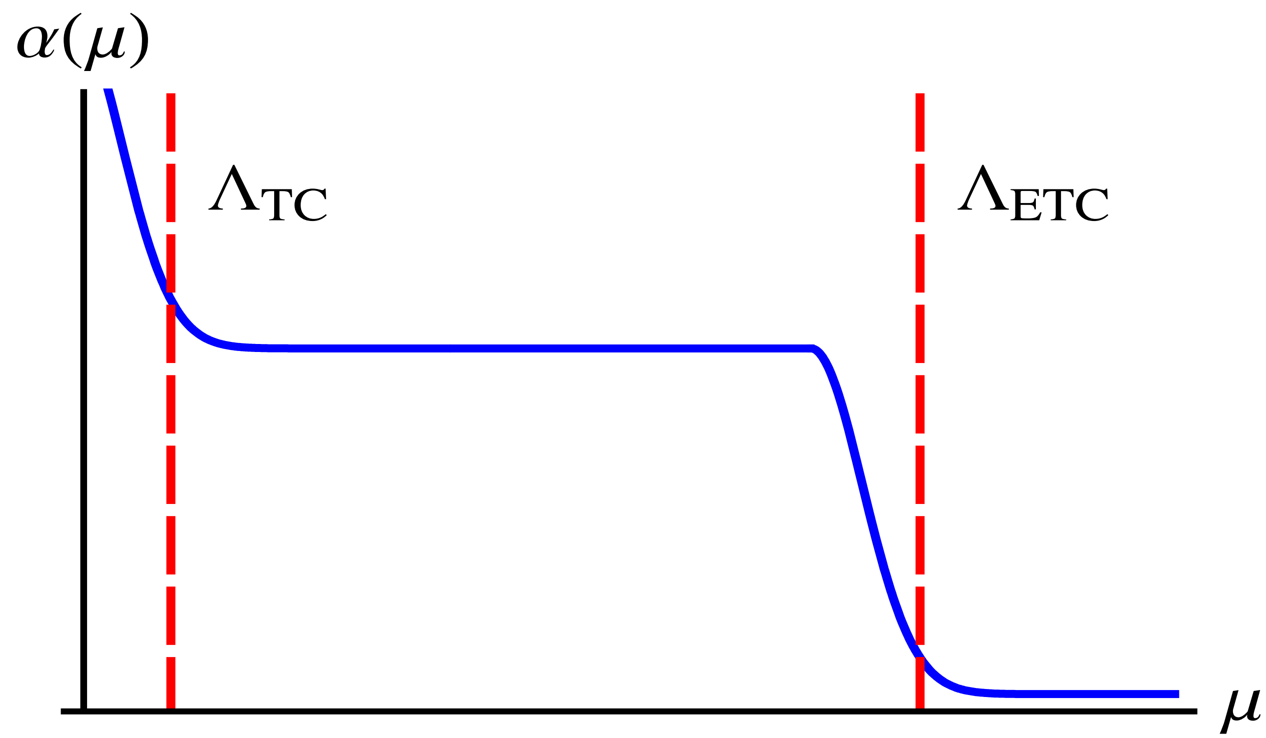
Outline

- Motivation.
- Step scaling.
- Two schemes on the twisted box:
  - ☆ Twisted Polyakov Loop (TPL) scheme.
  - ☆ Wilson flow (WF) scheme.
- Numerical (preliminary) results.
- Outlook.

Why SU(3) with many flavours

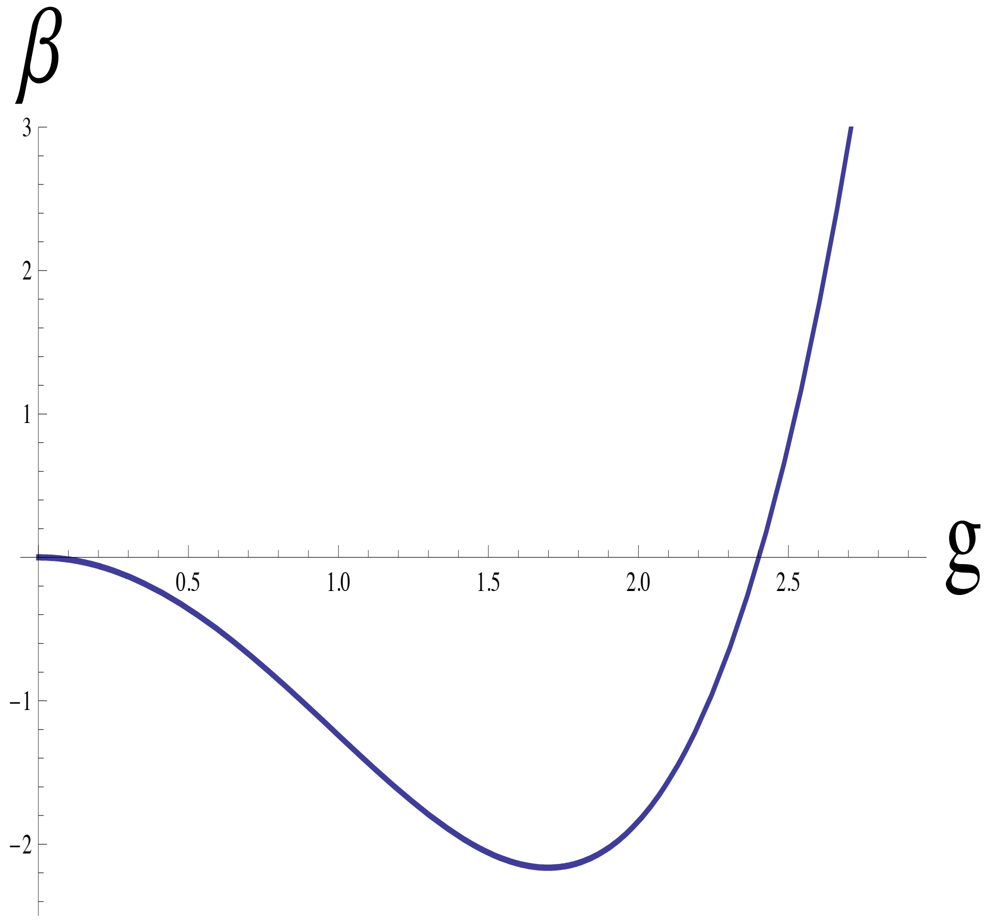
- Infrared conformality as an interesting field-theoretic problem.
- Walking technicolour model building.
- Understanding of the relation/distinction between confinement and chiral symmetry-breaking scales in QCD.

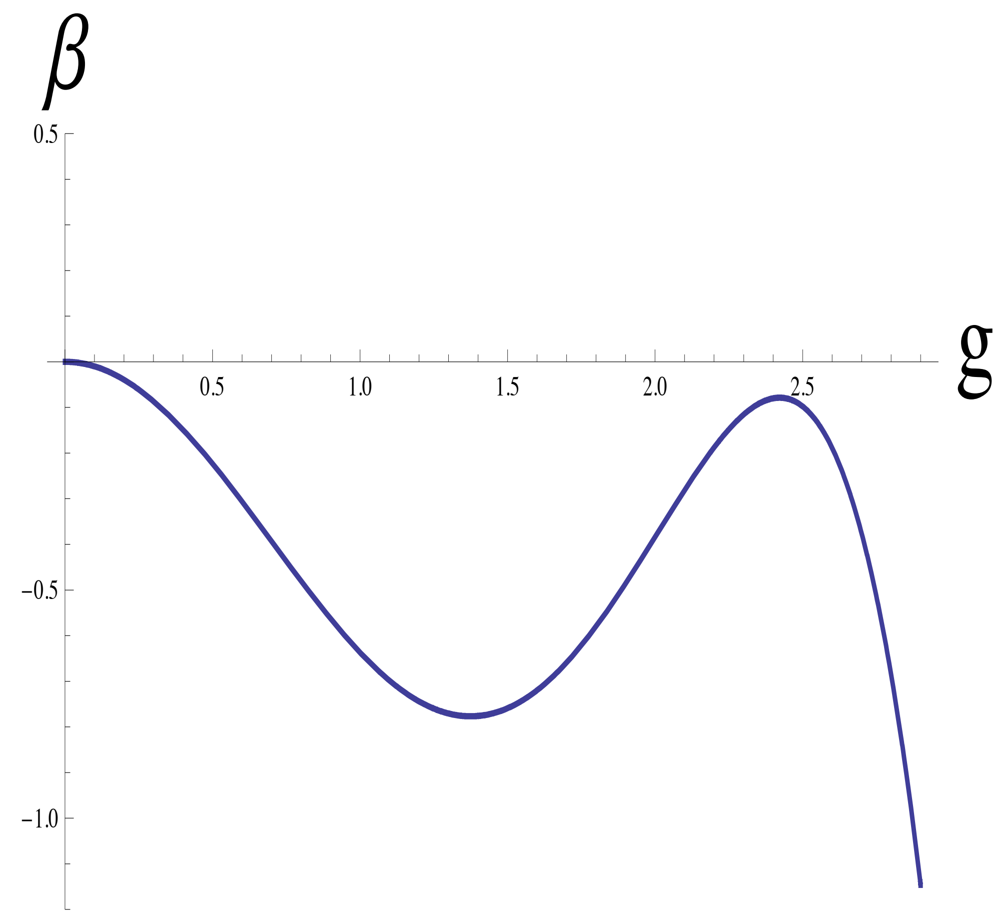
Walking technicolour



- With large mass anomalous dimension
  ➡ Solve the FCNC and S-param problems.
- $\Lambda_{\text{ETC}}/\Lambda_{\text{TC}} \sim 10^2 \sim 10^3$
  ➡ Compare to typical $L/a \sim 30$.

The beta function



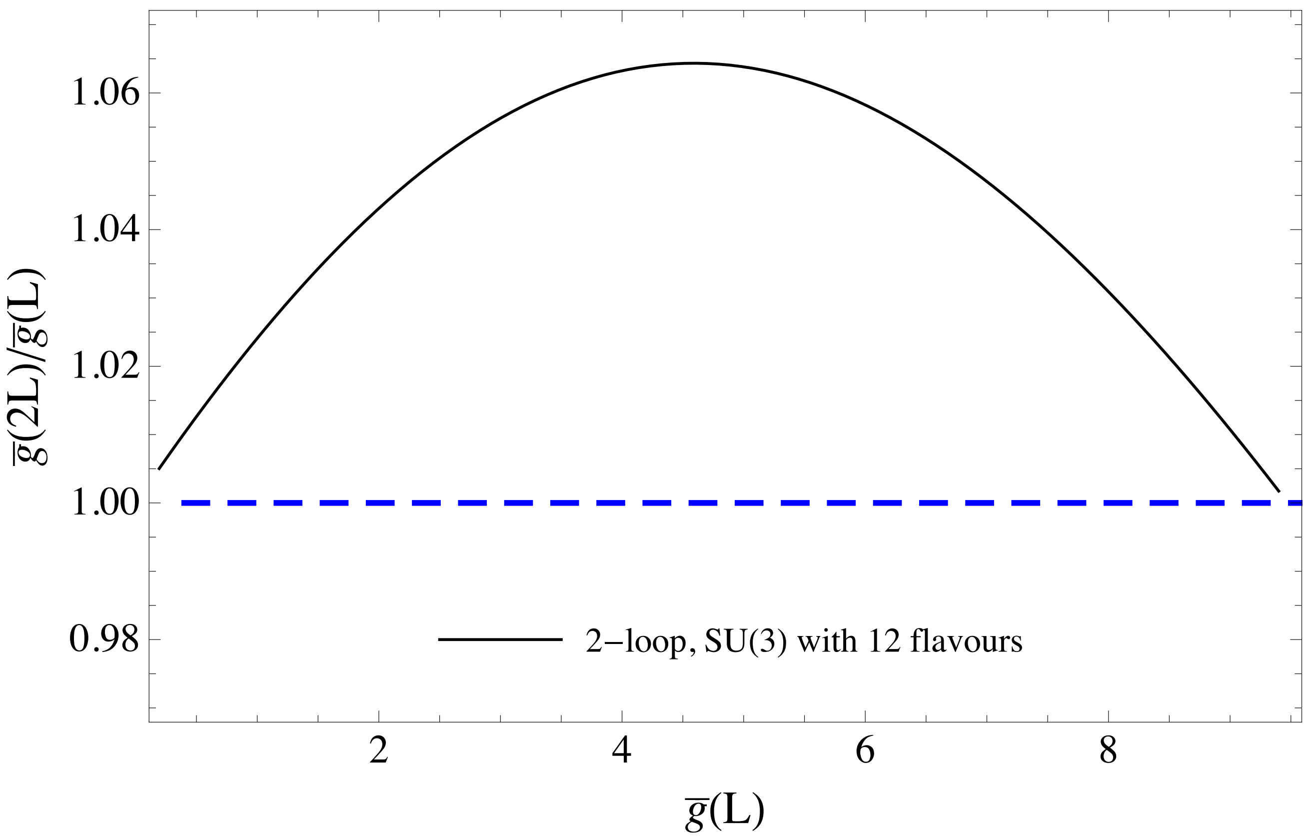


- Large-N_f gauge theories with asymptotic freedom.
- Need a scale to generate a gap

⇒ Just below the conformal window $N_f^* < N_f < N_f^{\mathrm{af}}$

Lattice strategy for the search of IRFP

- Spectrum: Large finite-volume effects?
- Finite-size scaling *a'la* M. Fisher : universal curves?
- Running coupling: (slow) running within error?



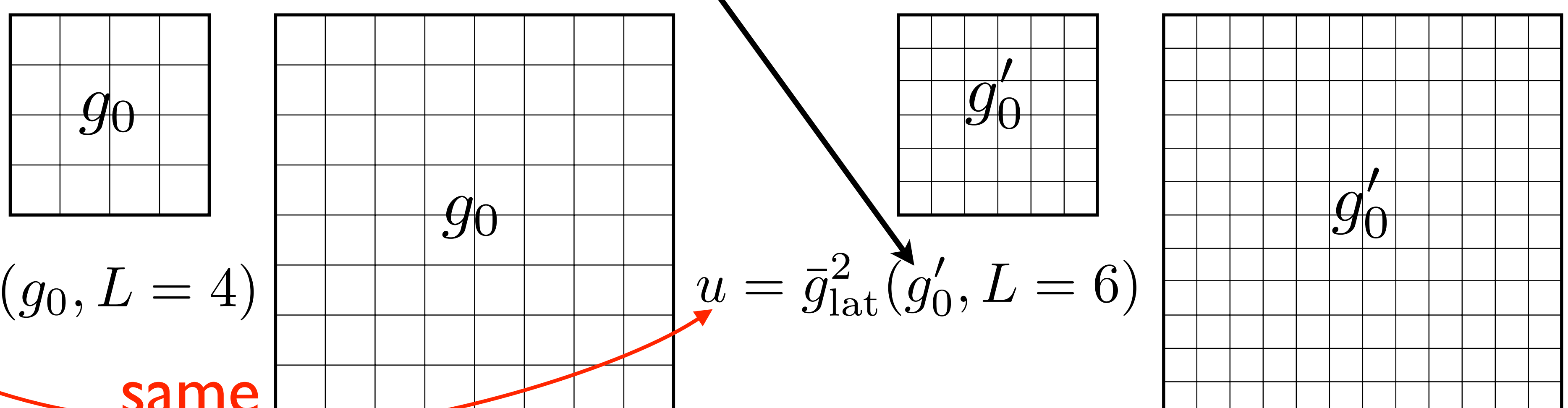
Need high-precision calculations.

The step-scaling method
The idea

M. Luscher, P. Weisz, U. Wolff, 1991.

Tune g'$_0$ to give the same value of u.

$L = 4$ $\qquad\qquad\qquad\qquad$ $L = 6$



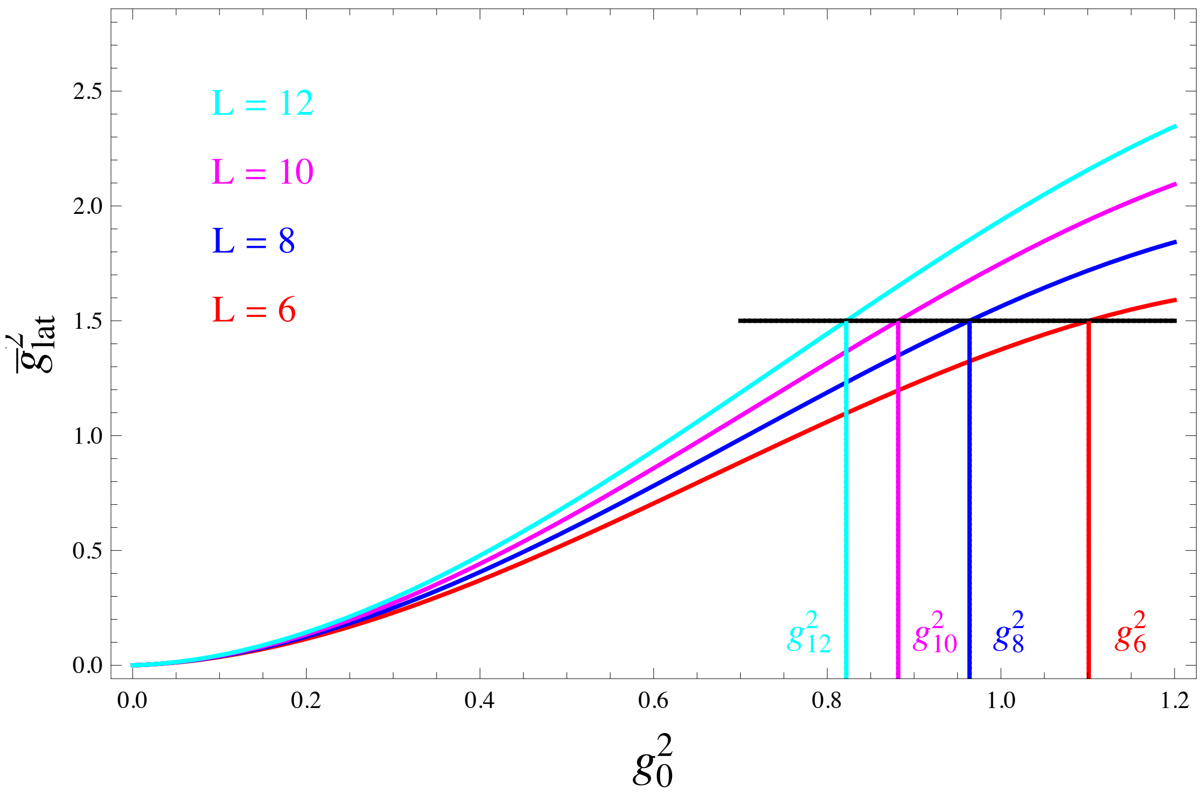
$u = \bar{g}^2_{\rm lat}(g_0, L = 4)$ $\qquad$ $u = \bar{g}^2_{\rm lat}(g'_0, L = 6)$

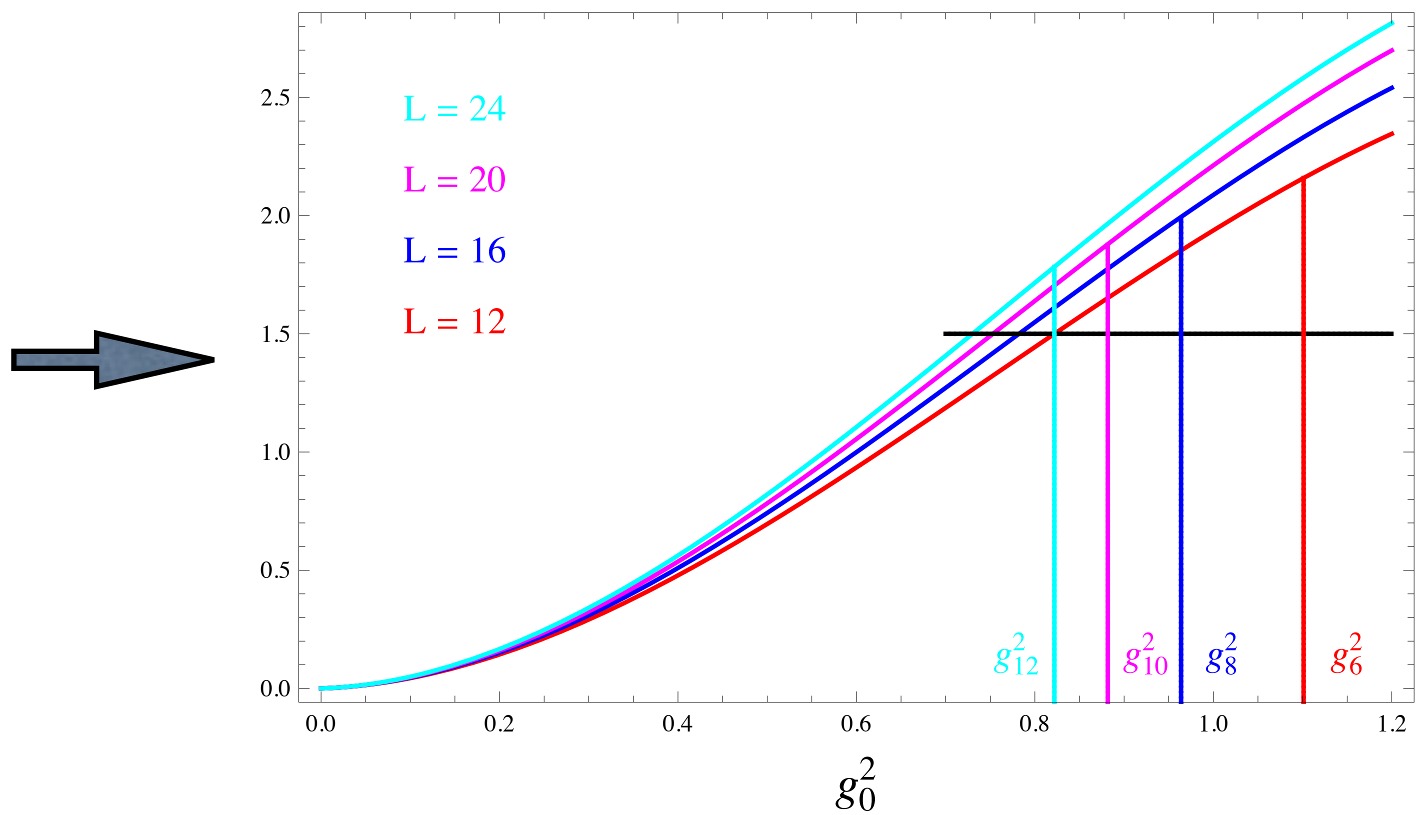
same

$$\Sigma(u, L = 4) = \bar{g}^2_{\rm lat}(g_0, L = 8) \qquad \Sigma(u, L = 6) = \bar{g}^2_{\rm lat}(g'_0, L = 12)$$

$$\sigma(u) = \lim_{a \to 0} \Sigma, \quad r_\sigma = \frac{\sigma(u)}{u} \xrightarrow{\text{fixed points}} 1.$$

The step-scaling method
The practice





- Massless unimproved staggered fermions with Wilson's plaquette gauge action.
- Compute $\bar{g}^2_{\rm lat}$ at many g_0^2 for each volume, and then interpolate volume by volume.
- Very challenging to pin down percentage-level effects in $r_\sigma = \dfrac{\sigma(u)}{u}$.

Bare-coupling interpolation

- Impose the non-decreasing constraint,

$$u_{\text{latt}} = f(u_0) = \int du_0 \left(\sum_{m=0}^{N_{\text{deg}}} c_m u_0^m \right)^2 = \sum_{n=0}^{N_h} h_n u_0^n, \; u_0 \equiv \frac{1}{\beta} = \frac{g_0^2}{6}$$

in order to avoid the Runge phenomenon.

- Impose the perturbation-theory constraint,

$$h_0 = 0, \; h_1 = 6 \; (\text{then } c_0 = \sqrt{6}).$$

Continuum extrapolation

- Using various polynomials in $\left(\frac{a}{L}\right)^2$.
- Central issue in controlling the systematic error.
- Can we go IR enough before hitting any bulk phase transition?

Twisted box
removing the zero modes

- **Gauge field:** G.'t Hooft, 1979

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger, \;\; \nu = 1,2,$$

where the twist matrices Ω_ν satisfy

$$\Omega_1\Omega_2 = \mathrm{e}^{2i\pi/3}\Omega_2\Omega_1, \;\; \Omega_\mu\Omega_\mu^\dagger = 1, \;\; (\Omega_\mu)^3 = 1, \;\; \mathrm{Tr}(\Omega_\mu) = 0.$$

- **Fermion:** If $\psi(x+\hat{\nu}L) = \Omega_\nu\psi(x)$

$$\Rightarrow \psi(x+\hat{\nu}L+\hat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$$

- **The fermion "smell" dof:** $N_s = N_c$ G. Parisi, 1983

$$\psi^a_\alpha(x+\hat{\nu}L) = \mathrm{e}^{i\pi/3}\Omega_\nu^{ab}\psi^b_\beta(x)(\Omega_\nu)^\dagger_{\beta\alpha}.$$

TPL scheme

- Polyakov loops in the twisted directions:

$$P_1(y,z,t) = \mathrm{Tr}\langle \Pi_j U_1(j,y,z,t)\Omega_1 \mathrm{e}^{2iy\pi/3L}\rangle$$

with gauge and translation invariance.

- The renormalised coupling constant:

$$g^2_{\mathrm{TP}}(L) = \frac{1}{k}\frac{\langle \sum y,z P_1(y,z,L/2)P_1^*(0,0,0)\rangle}{\langle \sum x,y P_3(x,y,L/2)P_3^*(0,0,0)\rangle},$$

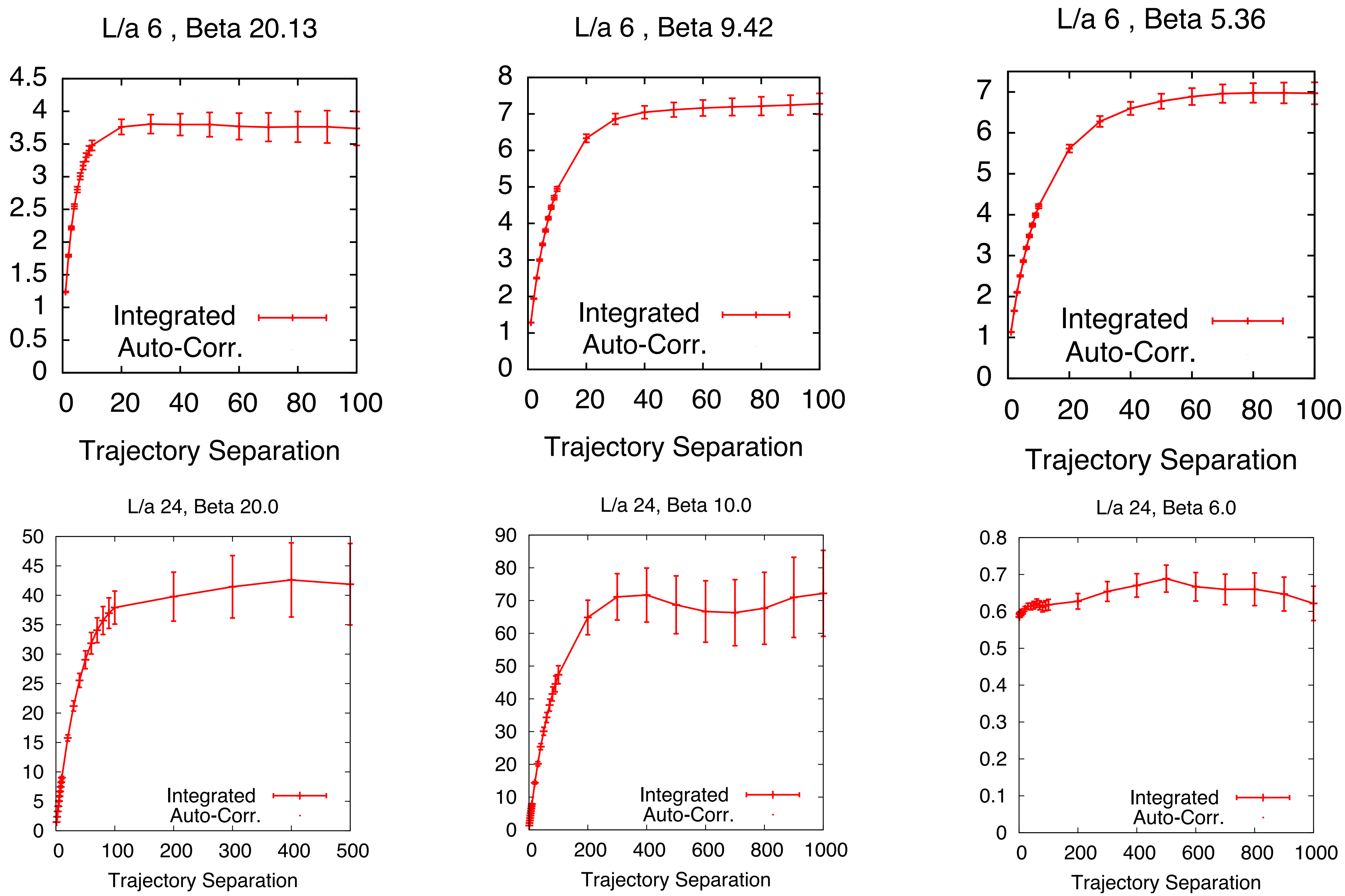
where $k = \frac{1}{24\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+(1/3)^2} \sim 0.031847$

- Special feature:

At $L \to \infty$, $g^2_{\mathrm{TP}} \to \frac{1}{k} \sim 32$ if there is no IRFP.

Challenge in using the TPL scheme

Autocorrelation of the coupling

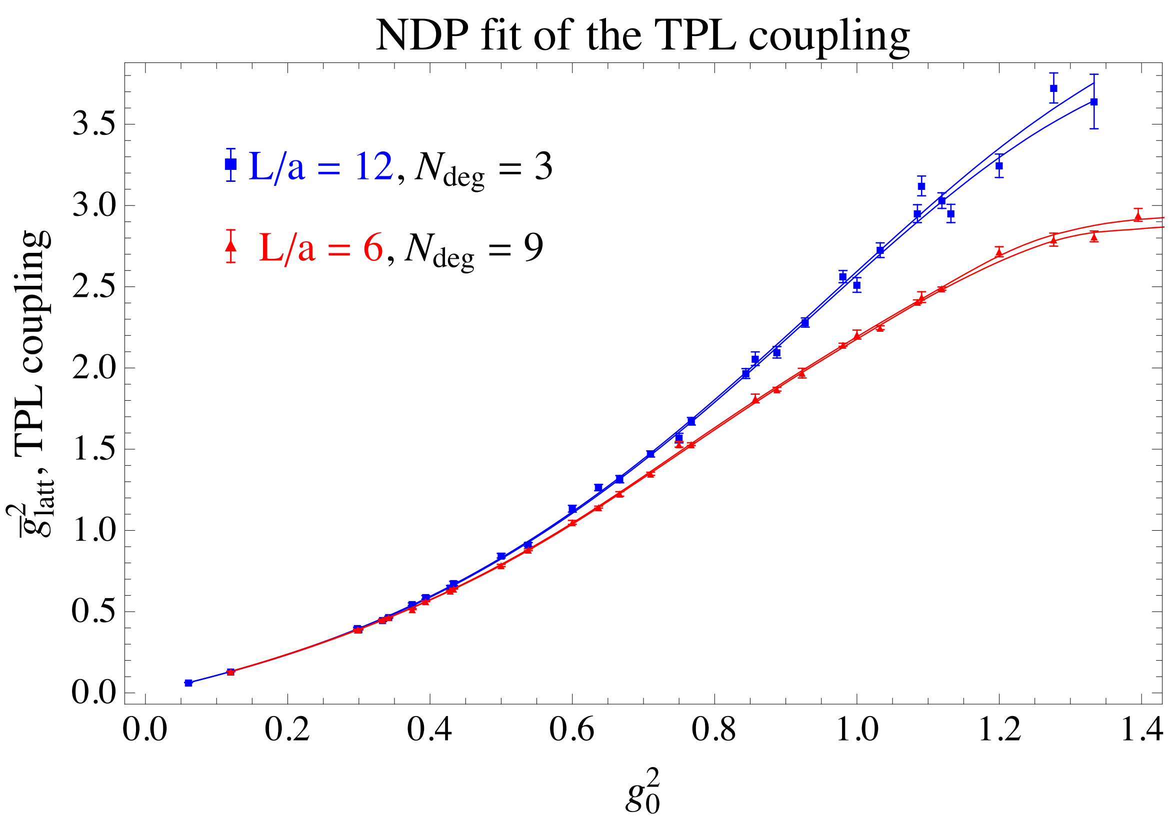


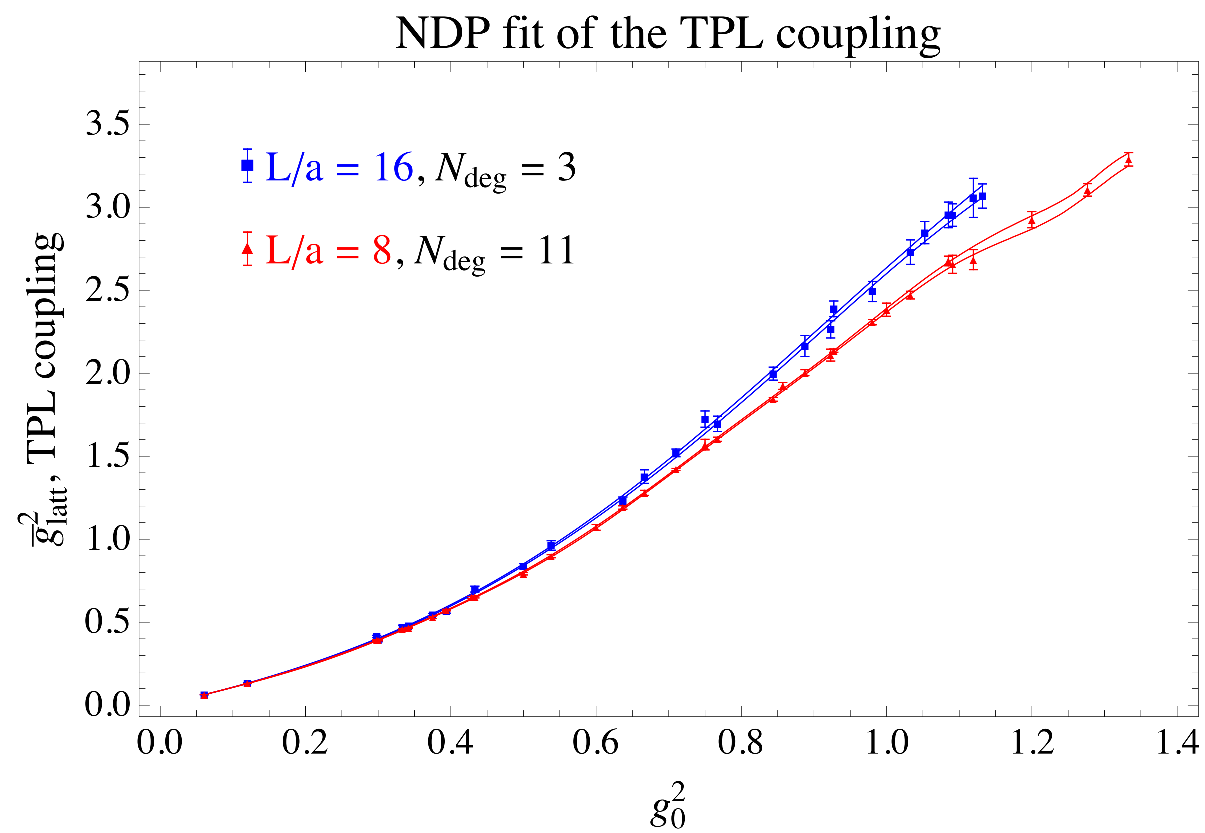
- Autocorrelation time grows with physical volume.

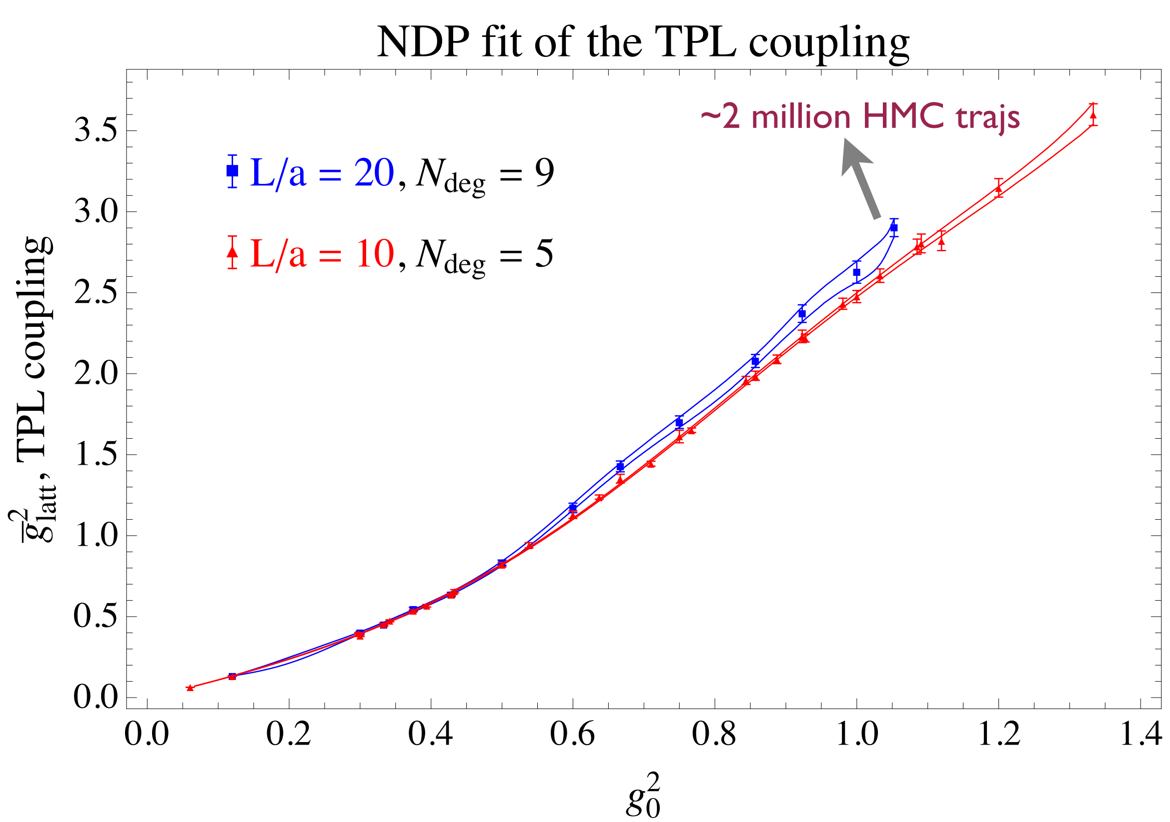
  ➡ Very challenging to have good statistics for large volumes at low beta.

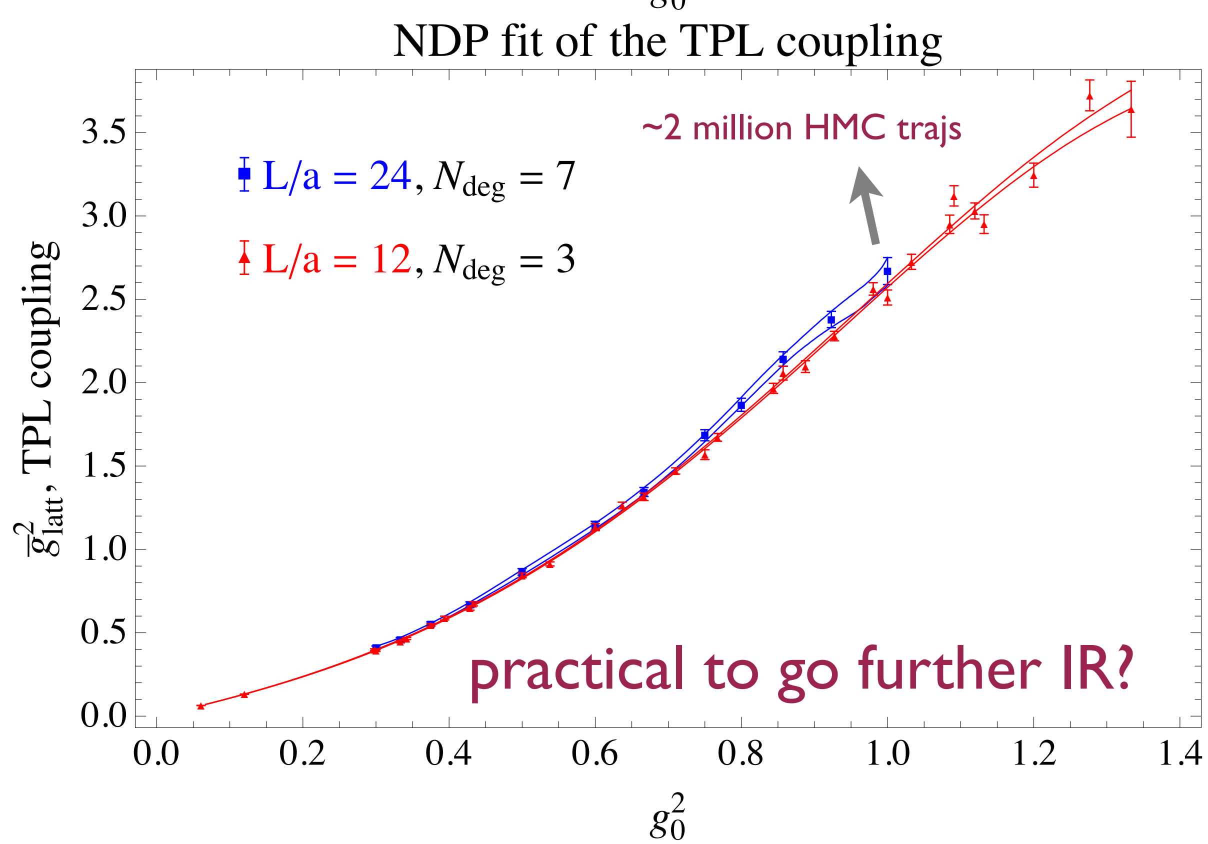
Bare-coupling interpolation

TPL scheme

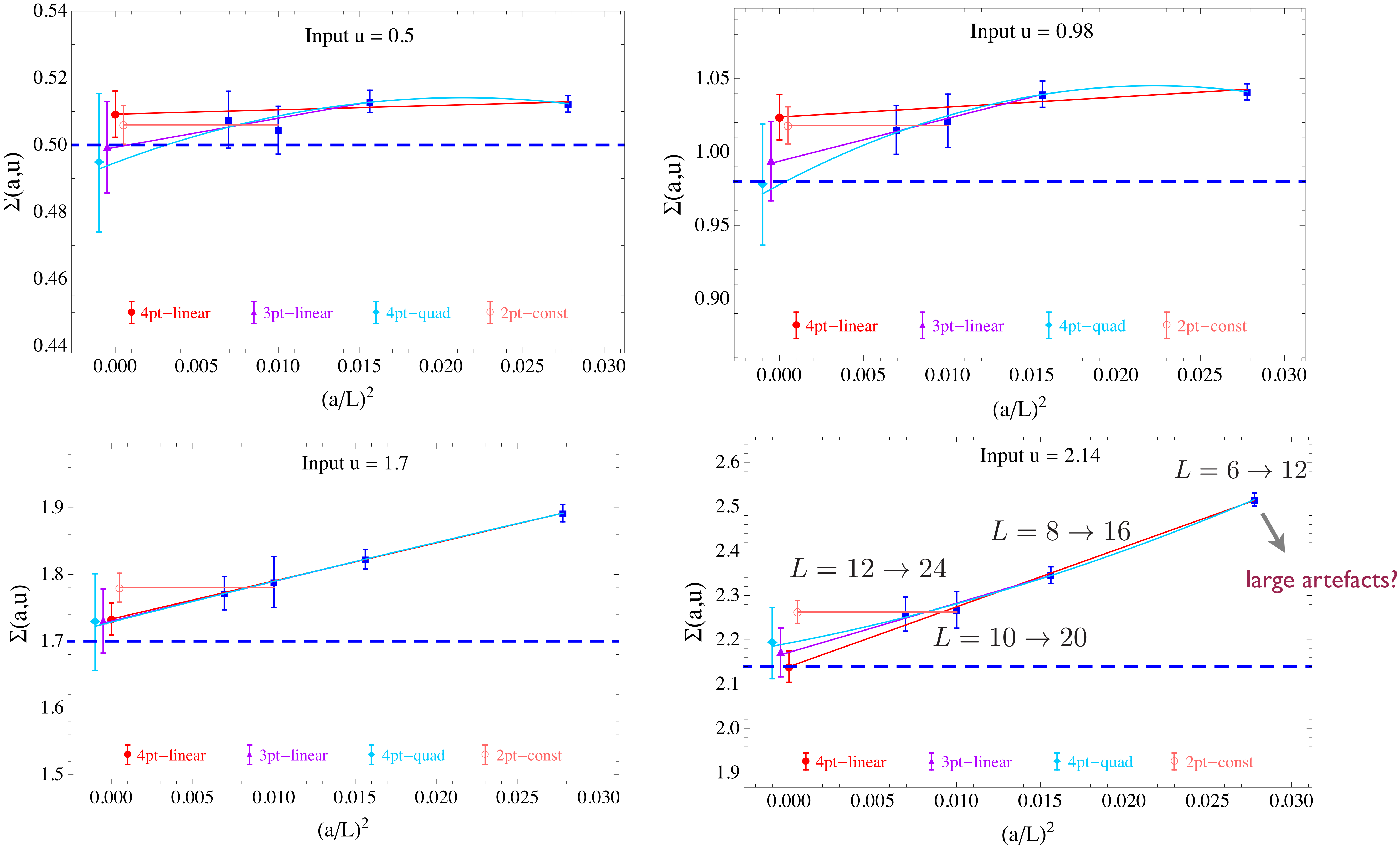








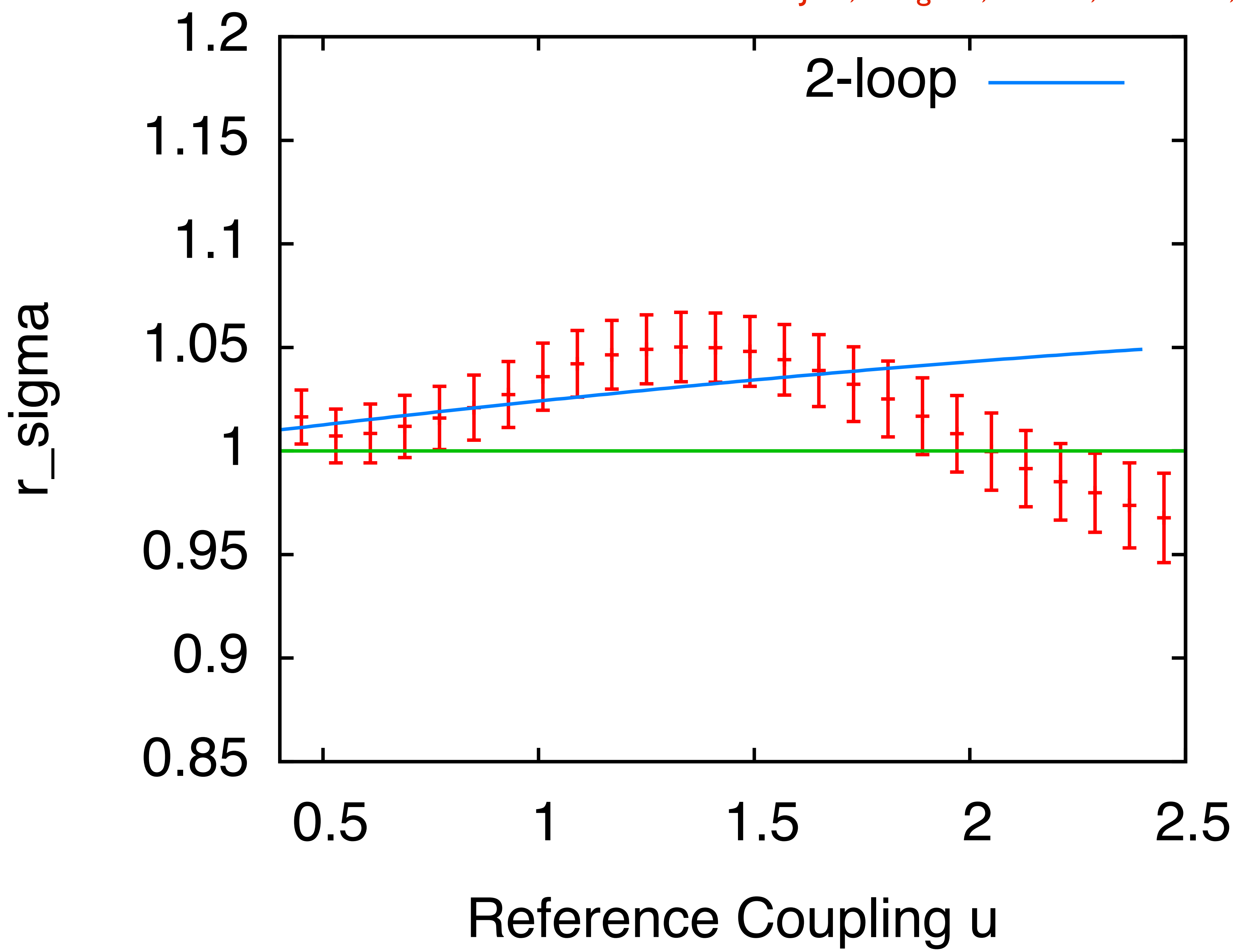
Continuum extrapolation
TPL scheme



Result without the L/a=24 lattices

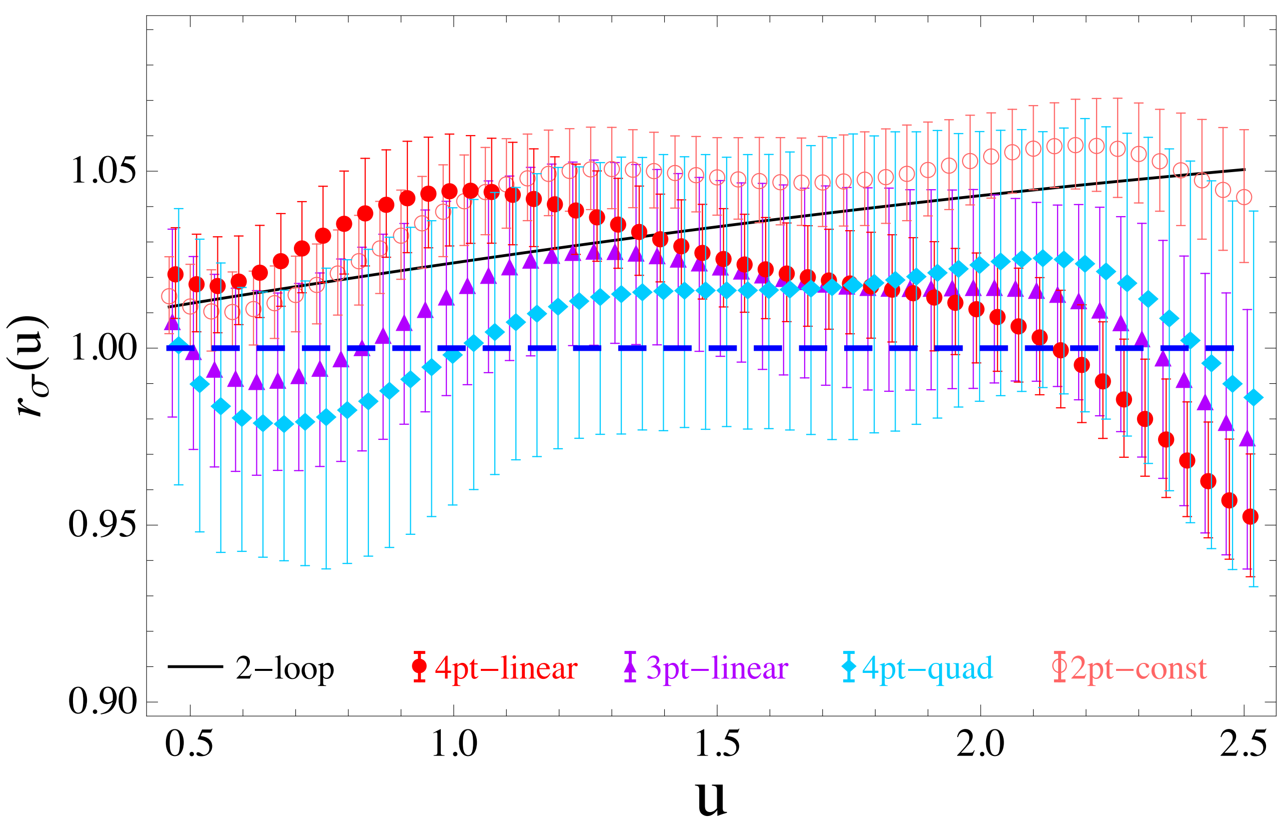
TPL scheme

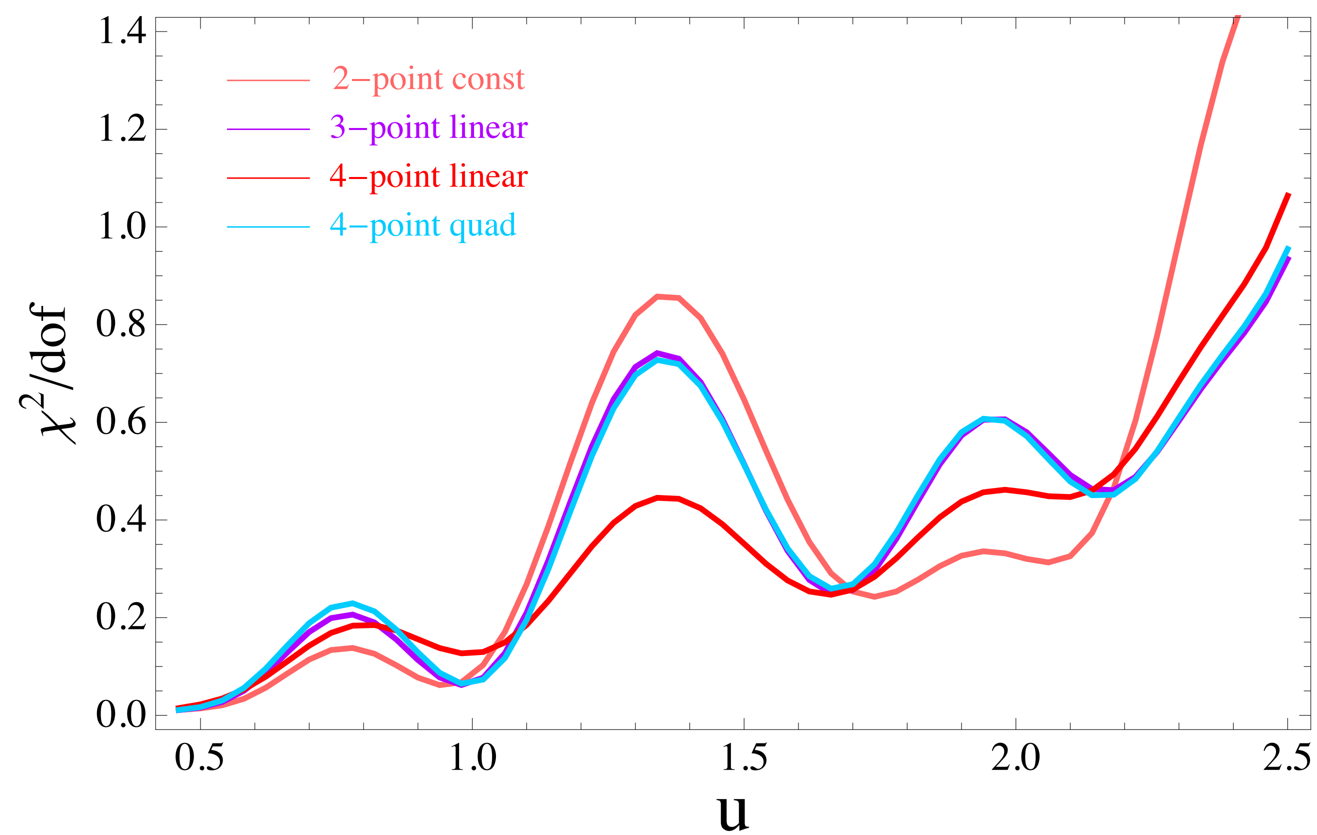
C-JDL, K.Ogawa, H.Ohki, E.Shintani, JHEP 1208 (2012) 096



Result with the L/a=24 lattices

TPL scheme





Systematic error was severely underestimated without the L/a=24 data.

The Wilson flow

- Diffusion of the gauge fields:

$$\dot{V}_t(x,\mu) = -g_0^2 \{\partial_{x,\mu} S_{\mathrm{w}}(V_t)\} V_t(x,\mu),\ V_t(x,\mu)|_{t=0} = U(x,\mu).$$

- The radius of diffusion is $\sqrt{8t}$.

$$c_\tau = \frac{\sqrt{8t}}{L}$$

- Local operators are also diffused.

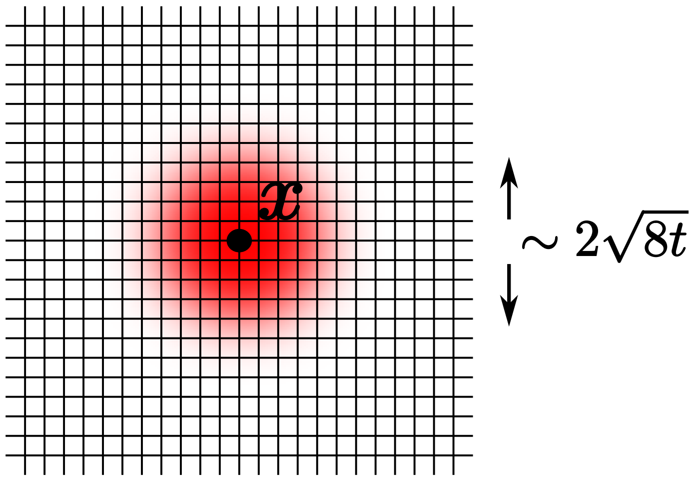


Figure taken from M.Luscher, Lattice 2013

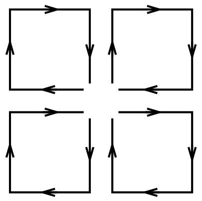
The Wilson flow scheme

- The quantity, $\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.

- In a colour-twisted box, can define,

$$\overline{g}^2_{\rm GF}(L) = \mathcal{N}^{-1} t^2 \langle E(t)\rangle = \overline{g}^2_{\rm MS} + \mathcal{O}(\overline{g}^4_{\rm MS}),$$

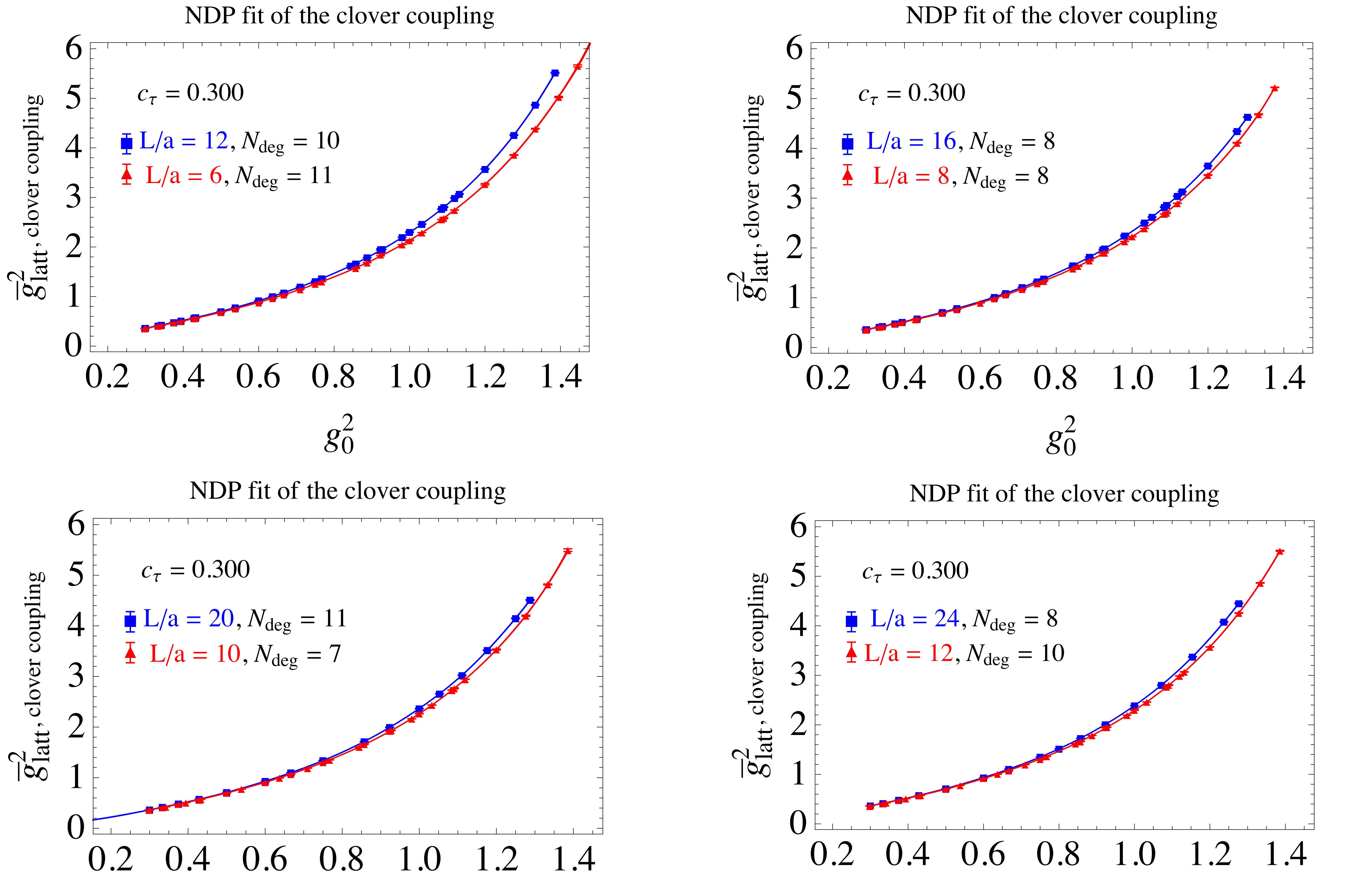
  where $\mathcal{N}$ can be computed in perturbation theory.

- Use the clover operator,  , to extract $\langle E(t)\rangle$.

- Autocorrelation time ~25 HMC trajectories for all simulations.

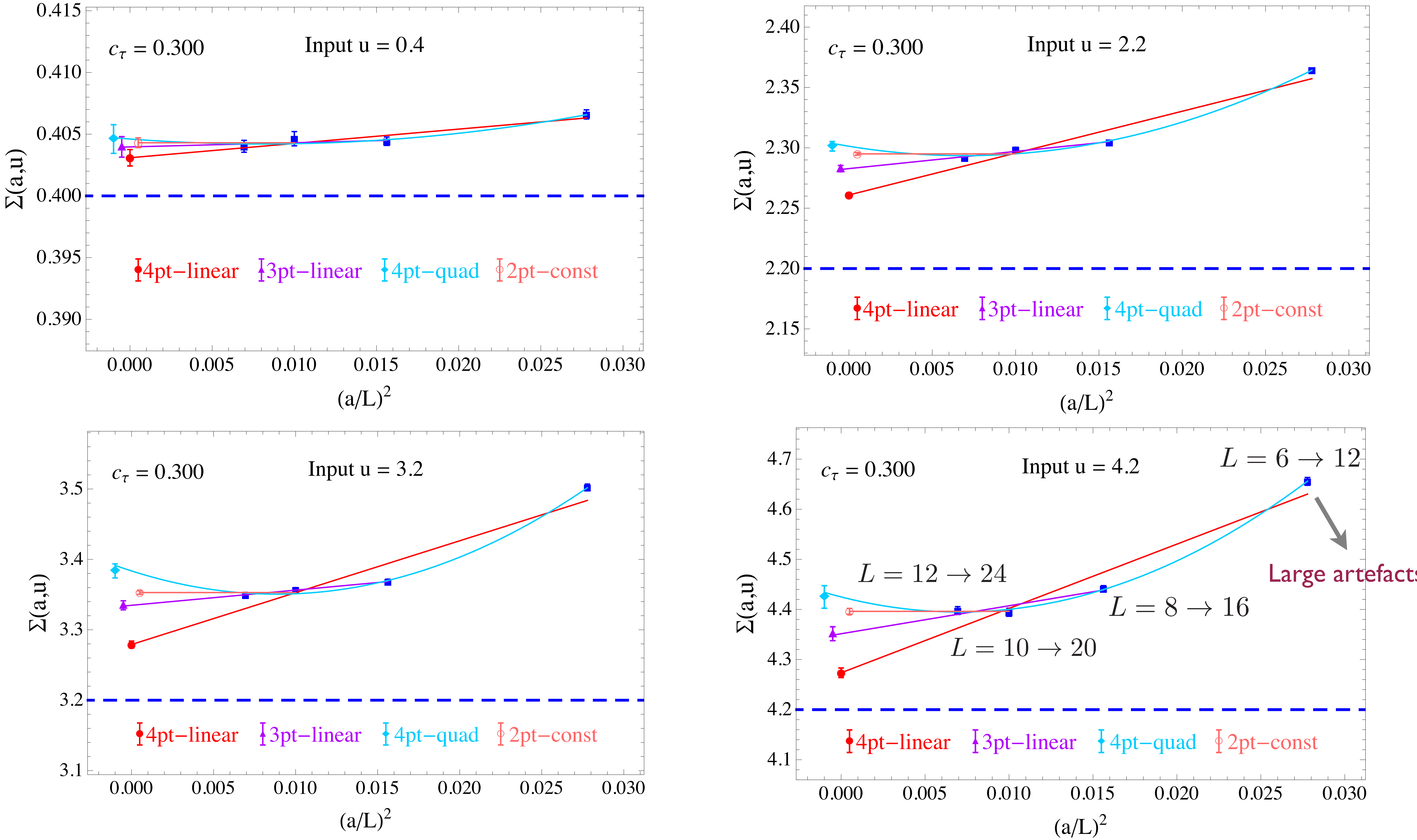
Bare-coupling interpolation

Wilson flow scheme



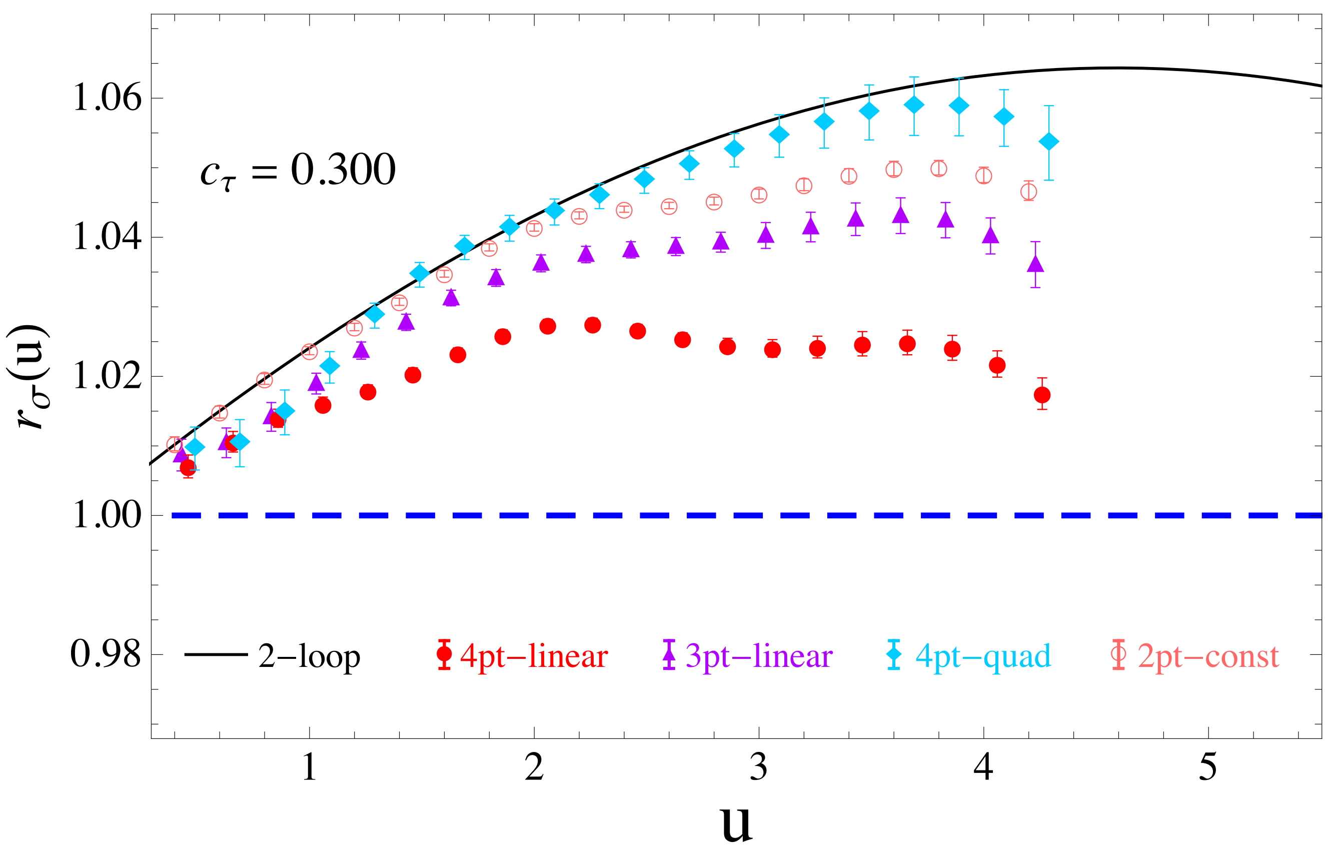
Continuum extrapolation

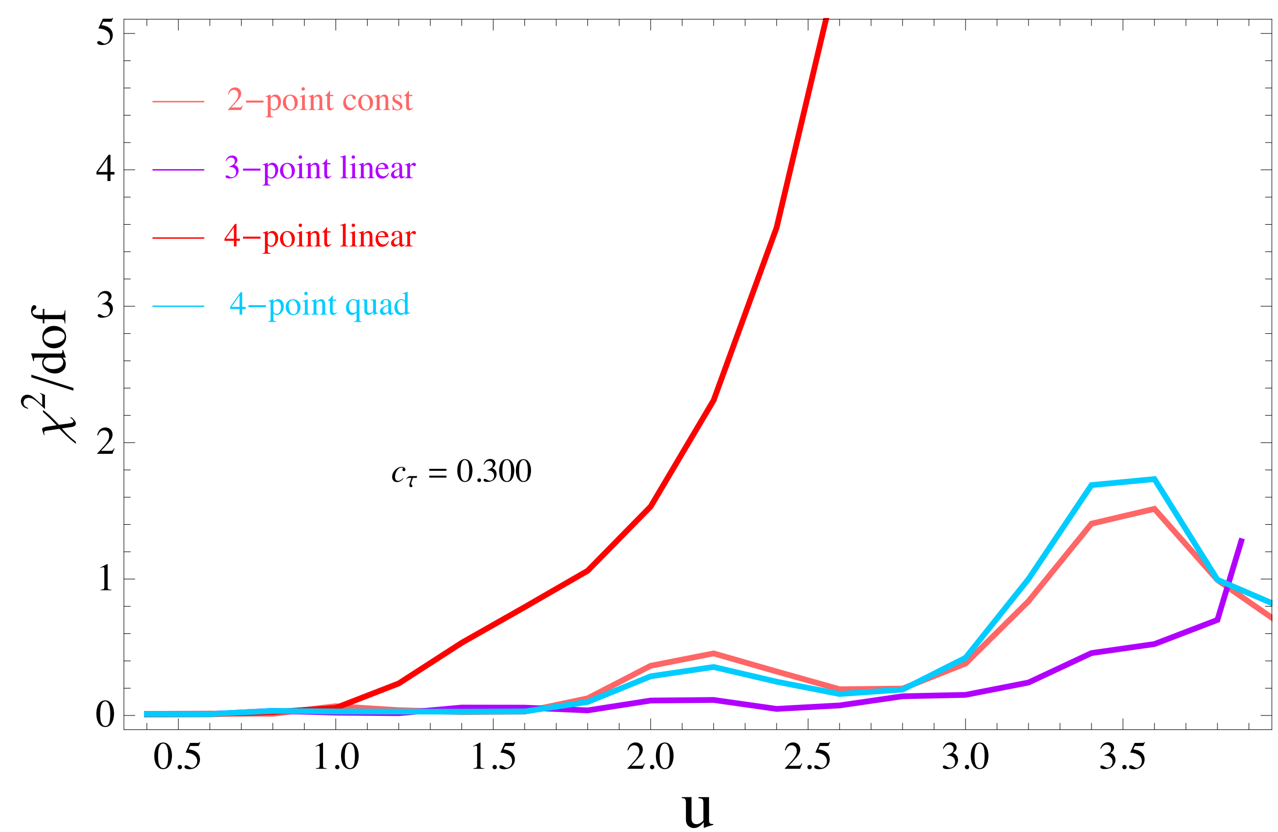
Wilson flow scheme



Preliminary result

Wilson flow scheme





Continuum extrapolation

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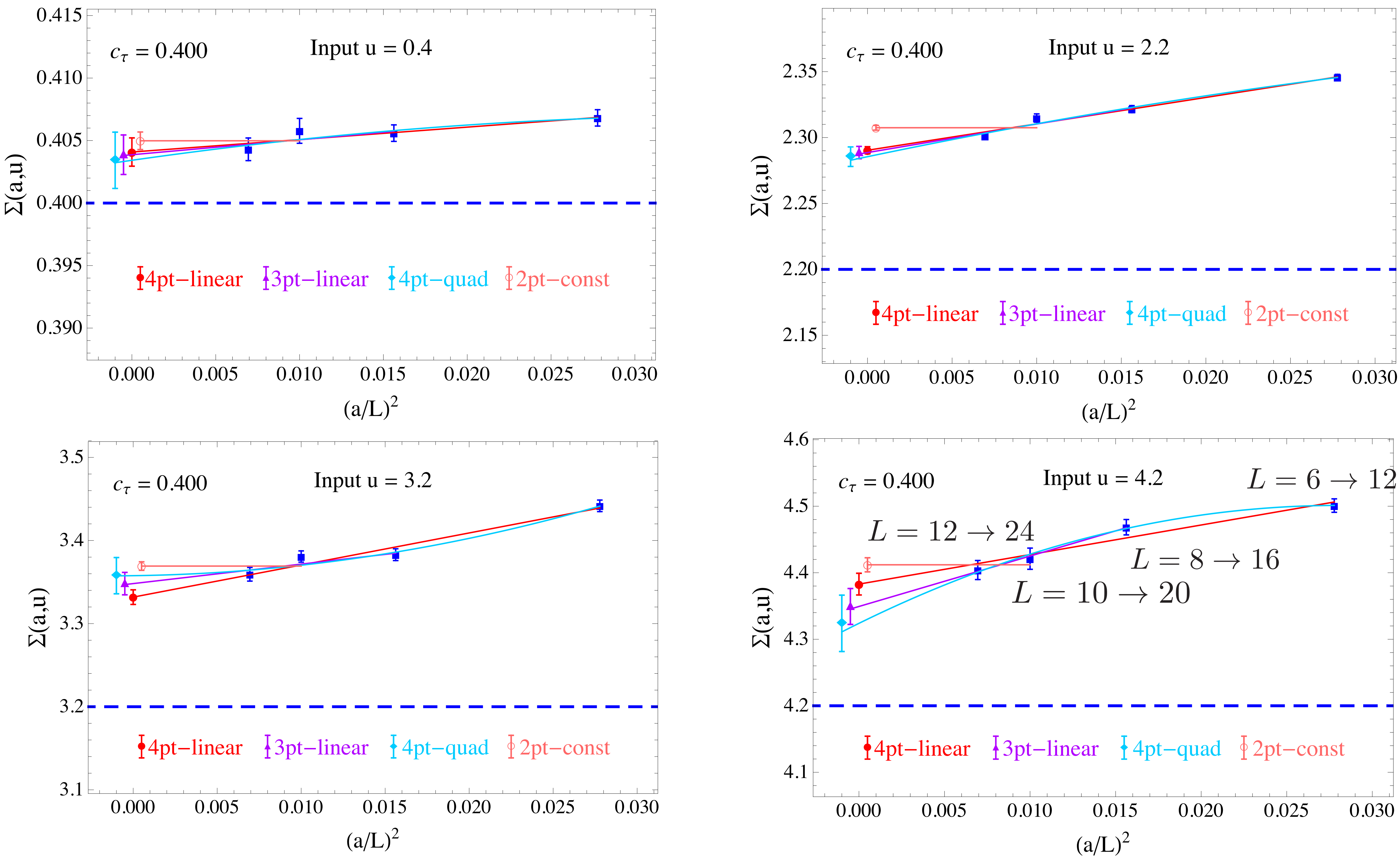

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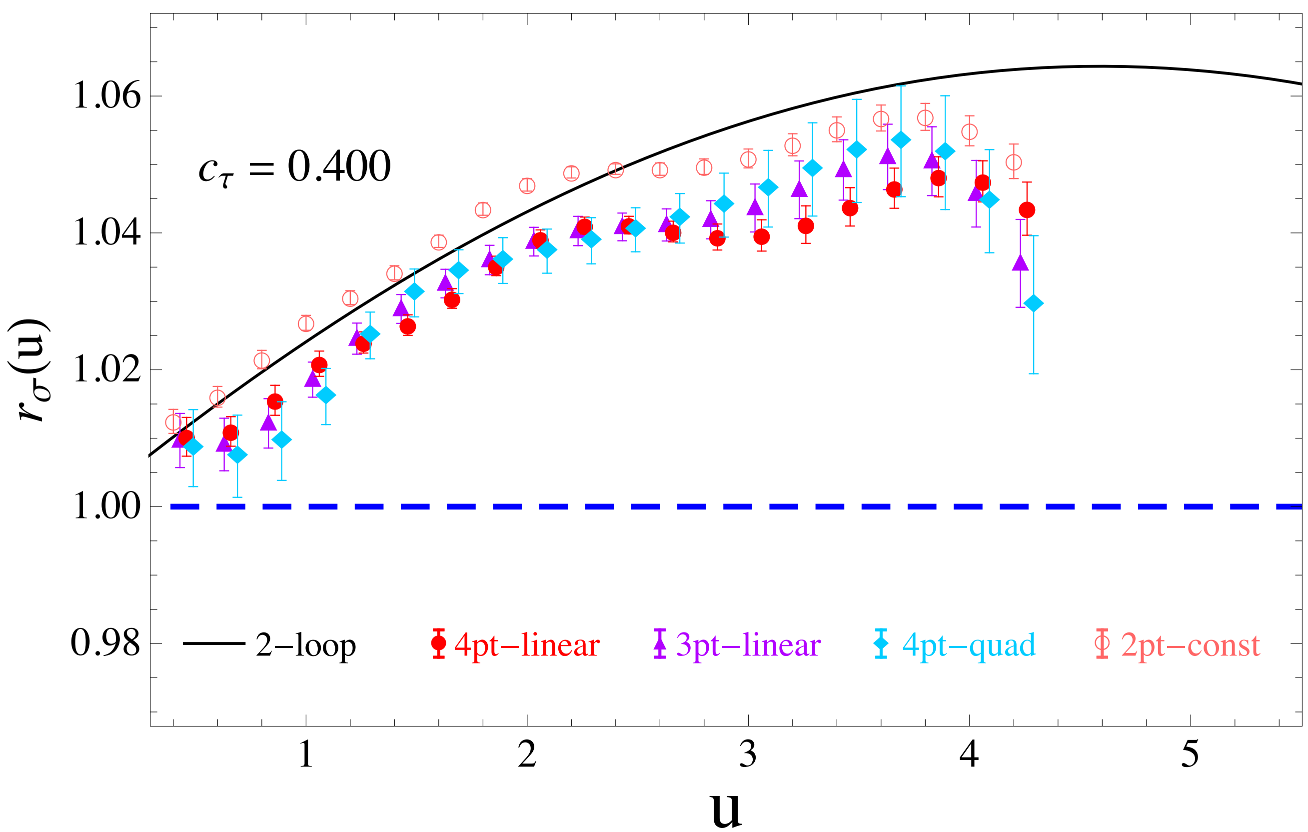

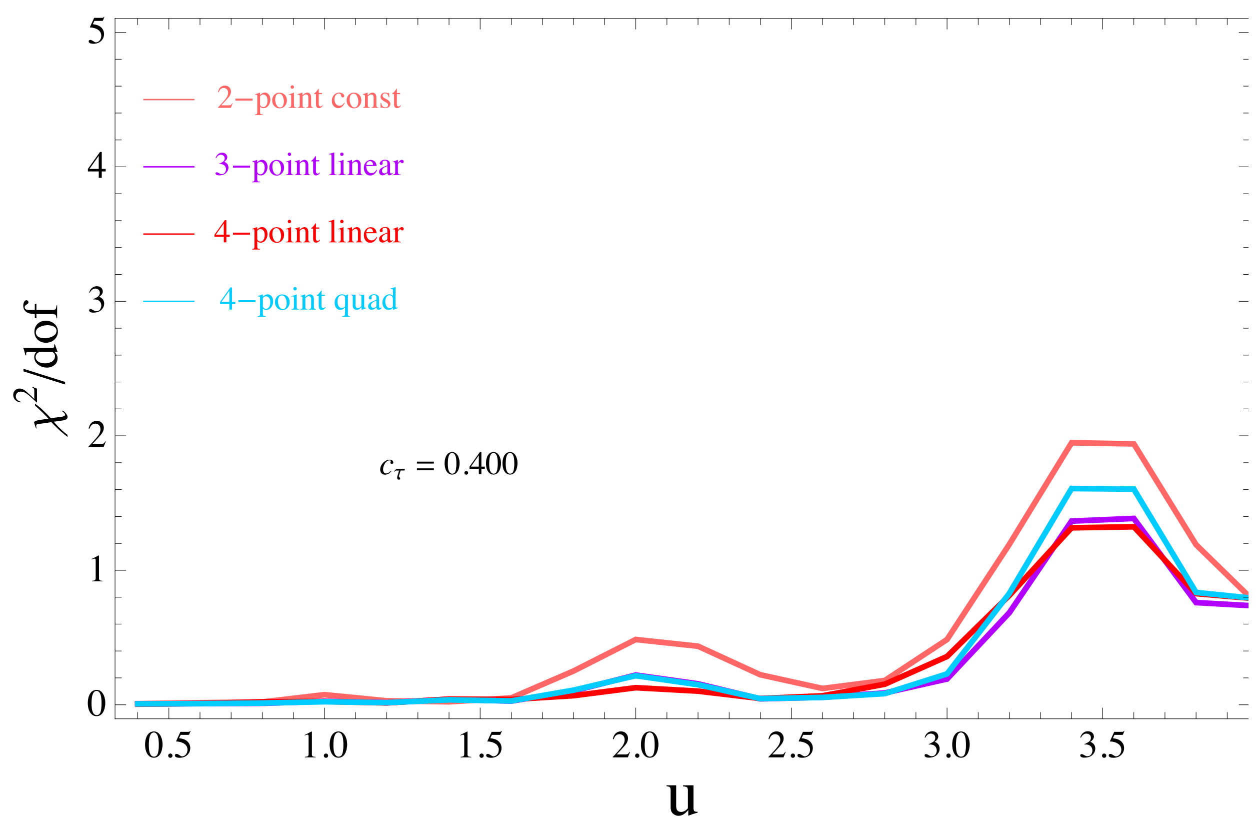

Remarks and outlook

- Calculation in the TPL scheme shows no definite conclusion for IR comformality in SU(3) gauge theory with 12 flavours hitherto.
- It is very challenging to use the TPL scheme to study the evolution of the coupling in the IR.
- On the other hand, the Wilson flow scheme offers a very nice/promising tool.
- We are currently generating data to go further IR

Backup slides

Vacuum structure