

Atomic Quantum Simulation of Dynamical Gauge Fields using Quantum Links

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Outline

Ultra-cold Atoms in Optical Lattices and Trapped Ions

Wilson's Lattice Gauge Theory

Quantum Link Models

“Rishon Abacus” as a Quantum Simulator

Atomic Quantum Simulator for $(1 + 1)$ -d $U(1)$ Gauge Theory
Coupled to Fermionic Matter

Quantum Simulator for Non-Abelian Gauge Theories

Conclusions

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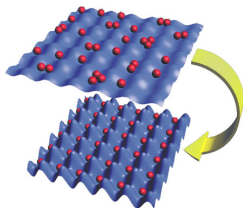
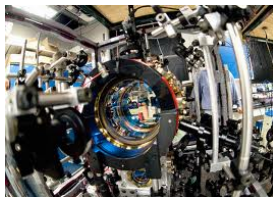
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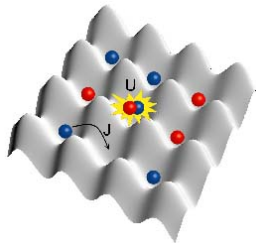
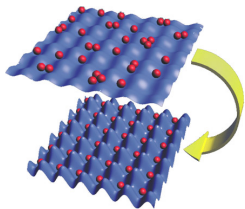
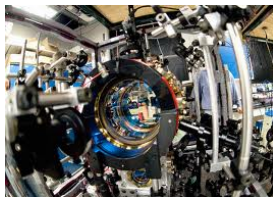
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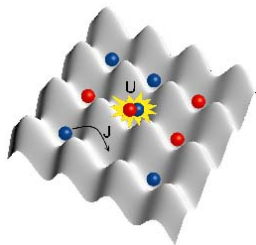
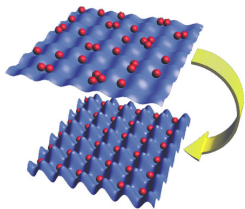
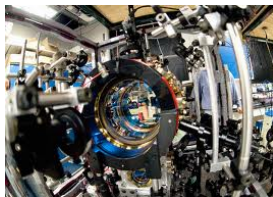
Ultra-cold atoms in optical lattices



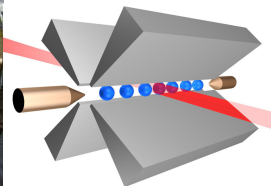
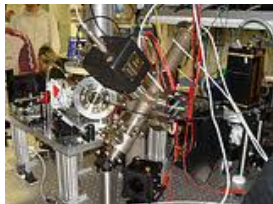
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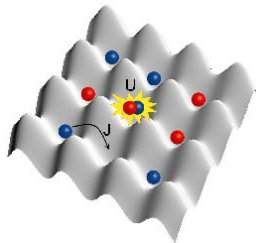
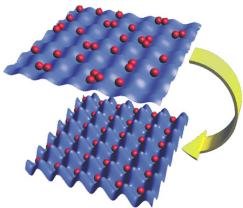
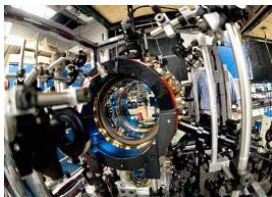
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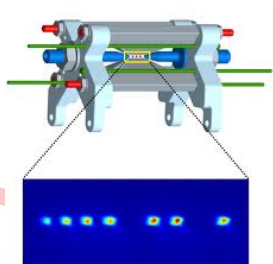
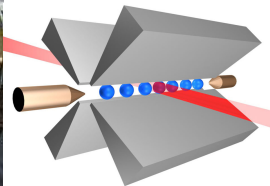
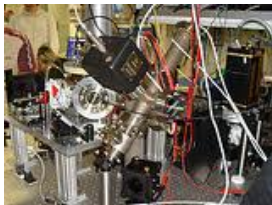
Trapped ions



Ultra-cold atoms in optical lattices



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Wilson's classical link variables

$$U_{x,\mu} = \exp(G_\mu(x + \frac{\hat{\mu}a}{2})) \in SU(N), SO(N), Sp(N)$$

and their behavior under gauge transformations

$$\begin{array}{ccc} \Omega_x & & \Omega_{x+\hat{\mu}} \\ \bullet & \text{---} & \bullet \\ x & U_{x,\mu} & x + \hat{\mu} \end{array}$$

$$\Omega U_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\hat{\mu}}^\dagger$$

Resulting anti-Hermitian non-Abelian vector potential

$$G_\mu(x) = ig G_\mu^a(x) \frac{\lambda^a}{2}, \quad a \in \{1, 2, \dots, d_G\}$$

and its behavior under gauge transformations

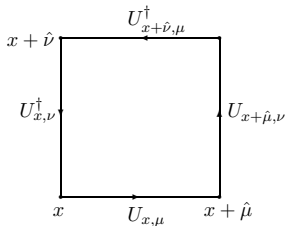
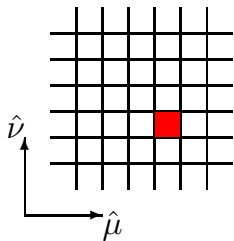
$$\Omega G_\mu(x) = \Omega(x) (G_\mu(x) + \partial_\mu) \Omega(x)^\dagger$$

Non-Abelian field strength

$$G_{\mu\nu}(x) = \partial_\mu G_\nu(x) - \partial_\nu G_\mu(x) + [G_\mu(x), G_\nu(x)]$$

and its behavior under gauge transformations

$$\Omega G_{\mu\nu}(x) = \Omega(x) G_{\mu\nu}(x) \Omega(x)^\dagger$$



Gauge invariant plaquette action

$$S[U] = -\frac{1}{g^2} \sum_{x, \mu < \nu} \text{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger + \text{h.c.})$$

Classical continuum limit

$$S[U] \rightarrow -\frac{1}{2g^2} \int d^4x \text{Tr}(G_{\mu\nu} G_{\mu\nu})$$

Functional integral using Haar measure

$$Z = \prod_{x,\mu} \int_G dU_{x,\mu} \exp(-S[U])$$

defines a quantum field theory using continuous classical field variables as fundamental degrees of freedom. [Wilson \(1974\)](#)

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$U(1)$ quantum link model

$$\begin{array}{c} T_{x,\mu} \\ \bullet \text{-----} \bullet \\ x \quad U_{x,\mu} \quad x + \hat{\mu} \end{array}$$

$$U = S_1 + iS_2 = S_+, \quad U^\dagger = S_1 - iS_2 = S_-$$

Generator E of $U(1)$ gauge transformations

$$[E, U] = U, \quad [E, U^\dagger] = -U^\dagger, \quad E = S^3, \quad [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_{\mu} (E_{x-\hat{\mu},\mu} - E_{x,\mu})$$

$U(1)$ -invariant Hamiltonian “action” operator

$$H = -J \sum_{x,\mu < \nu} (U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger + \text{h.c.}), \quad [H, G_x] = 0$$

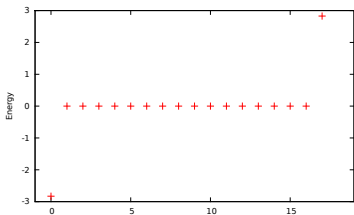
Functional integral of a quantum link model

$$Z = \text{Tr} \exp(-\beta H)$$

defines a gauge theory using discrete quantum variables

Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Spectrum of the $U(1)$ quantum link model on a 2×2 lattice



$$H \begin{array}{c} \leftarrow \\ \downarrow \quad \uparrow \\ \rightarrow \end{array} = J \begin{array}{c} \rightarrow \\ \uparrow \quad \downarrow \\ \leftarrow \end{array}$$

$$H \begin{array}{c} \rightarrow \\ \downarrow \quad \downarrow \\ \rightarrow \end{array} = 0$$

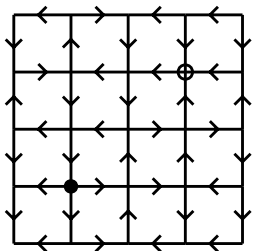
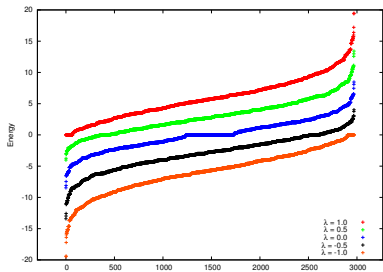
Ground state of the quantum link model on a 2×2 lattice

$$\frac{1}{2} \begin{array}{c} \leftarrow \rightarrow \\ \downarrow \quad \uparrow \\ \rightarrow \leftarrow \end{array} + \sqrt{1/8} \begin{array}{c} \uparrow \rightarrow \\ \leftarrow \quad \rightarrow \\ \downarrow \rightarrow \end{array} + \sqrt{1/8} \begin{array}{c} \leftarrow \rightarrow \\ \uparrow \quad \leftarrow \\ \downarrow \rightarrow \end{array} + \sqrt{1/8} \begin{array}{c} \leftarrow \leftarrow \\ \downarrow \quad \downarrow \\ \rightarrow \rightarrow \end{array} + \sqrt{1/8} \begin{array}{c} \leftarrow \leftarrow \\ \uparrow \quad \uparrow \\ \downarrow \leftarrow \end{array} + \frac{1}{2} \begin{array}{c} \leftarrow \leftarrow \\ \downarrow \quad \downarrow \\ \rightarrow \leftarrow \end{array}$$

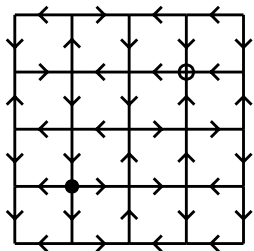
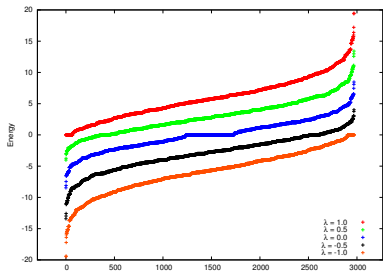
Excited state without flippable plaquettes



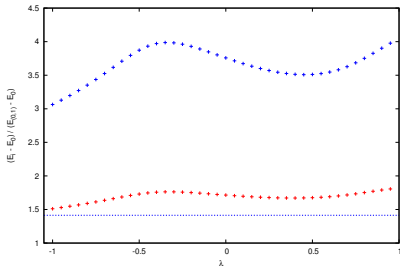
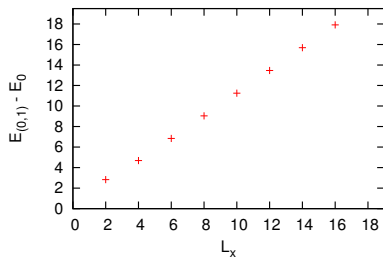
Spectra for $H = -J \sum_{\square} (U_{\square} + U_{\square}^{\dagger}) + \lambda \sum_{\square} (U_{\square} + U_{\square}^{\dagger})^2$



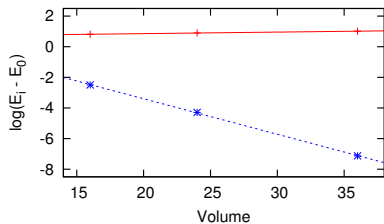
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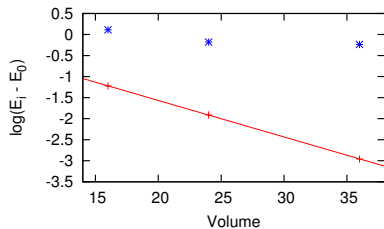
String tension and energy of fluxes $(2, 0)/(1, 0)$, $(1, 1)/(1, 0)$



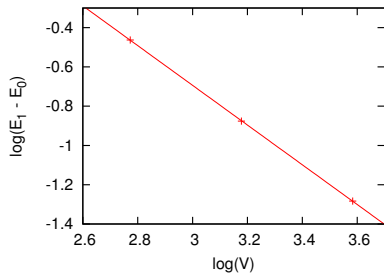
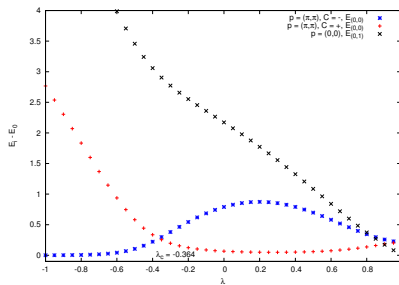
Ground state splitting as a function of the lattice volume



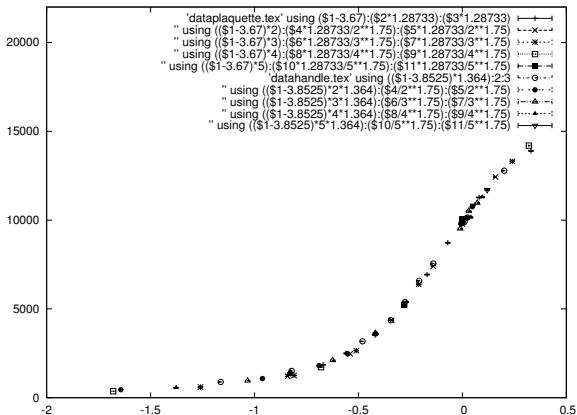
$\rho = (\pi, \pi), C = -, E_{(0,0)}$ *
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Numerical simulation of a $(2 + 1)$ -d $U(1)$ quantum link model with the first efficient cluster algorithm for a gauge theory



The system has a **second order finite-temperature phase transition** in the universality class of the 2-d Ising model, at which **charge conjugation is spontaneously broken**. Classical simulations can be used to validate a corresponding quantum simulator.

Developments related to quantum link models

- 1982: Construction of $U(1)$ and $SU(2)$ discrete gauge models by Horn

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- 2009: Design of a $U(1)$ quantum link simulator with Rydberg ions by Weimer, Müller, Lesanovsky, Zoller, and Büchler

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \quad U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \quad i, j \in \{1, 2, \dots, N\}, \quad [U^{ij}, (U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c, \quad [R^a, R^b] = if_{abc}R^c, \quad a, b, c \in \{1, 2, \dots, N^2 - 1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \quad [R^a, U] = U\lambda^a, \quad [E, U] = U$$

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$$U^{ij} = S_1^{ij} + iS_2^{ij}, \quad U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \quad i, j \in \{1, 2, \dots, N\}, \quad [U^{ij}, (U^\dagger)^{kl}] \neq 0$$

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Algebraic structures of different quantum link models

$U(N)$: $U^{ij}, L^a, R^a, E, 2N^2 + 2(N^2 - 1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij}, L^a, R^a, N^2 + 2 \frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

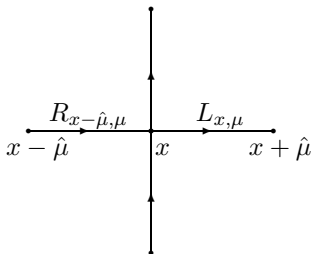
$Sp(N)$: $U^{ij}, L^a, R^a, 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

Brower, Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Consequences for non-Abelian quantum link simulators

- $U(1)$ embedded in $SU(2)$: minimal representation $\{2\}$
- $U(2)$ embedded in $SU(4)$: minimal representation $\{4\}$
- $U(3)$ embedded in $SU(6)$: minimal representation $\{6\}$
- $SU(2)$ embedded in $SU(4)$: minimal representation $\{6\}$
- $SU(3)$ embedded in $SU(6)$: minimal representation $\{20\}$
- $Sp(1) = SU(2)$ embedded in $Sp(2) = SO(5)$: minimal rep. $\{4\}$
- $SO(3) = SU(2)$ embedded in $SO(6) = SU(4)$: minimal rep. $\{4\}$

A **non-Abelian** gauge theory quantum simulator requires at least 4 states per link.



Generator of $SU(N)$ gauge transformations

$$G_x^a = \sum_{\mu} (R_{x-\hat{\mu},\mu}^a + L_{x,\mu}^a)$$

$U(N)$ -invariant Hamiltonian “action” operator

$$H = -J \sum_{x,\mu < \nu} \text{Tr}(U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^\dagger U_{x,\nu}^\dagger + \text{h.c.}), [H, G_x^a] = 0$$

Functional integral of a quantum link model

$$Z = \text{Tr} \exp(-\beta H)$$

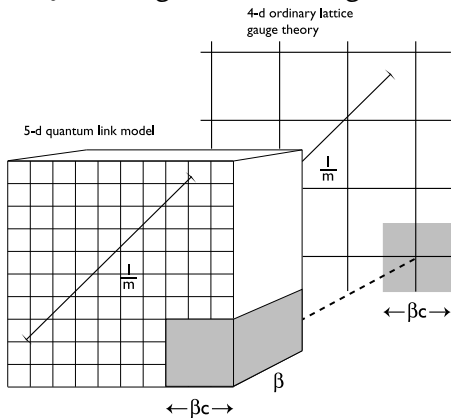
defines a quantum field theory using discrete variables

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \frac{1}{2e^2} \left(\text{Tr} G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \text{Tr} \partial_5 G_\mu \partial_5 G_\mu \right), \quad G_5 = 0$$

undergoes dimensional reduction from $4 + 1$ to 4 dimensions

$$S[G_\mu] \rightarrow \int d^4x \frac{1}{2g^2} \text{Tr} G_{\mu\nu} G_{\mu\nu}, \quad \frac{1}{g^2} = \frac{\beta}{e^2}, \quad \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



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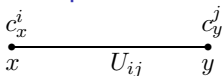
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Fermionic rishons at the two ends of a link

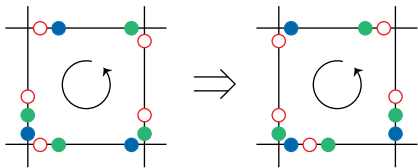
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \quad \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

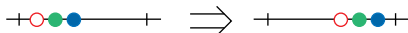


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \quad L_{xy}^a = c_x^{i\dagger} \lambda_{ij}^a c_x^j, \quad R_{xy}^a = c_y^{i\dagger} \lambda_{ij}^a c_y^j, \quad E_{xy} = \frac{1}{2}(c_y^{i\dagger} c_y^i - c_x^{i\dagger} c_x^i)$$

Can a “rishon abacus” implemented in ultra-cold atoms or trapped ions be used as a quantum simulator?



Tr Up



det $U_{x,\mu}$

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Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

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Bosonic rishon representation of the quantum links

$$U_{x,x+1} = b_x b_{x+1}^\dagger, \quad E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

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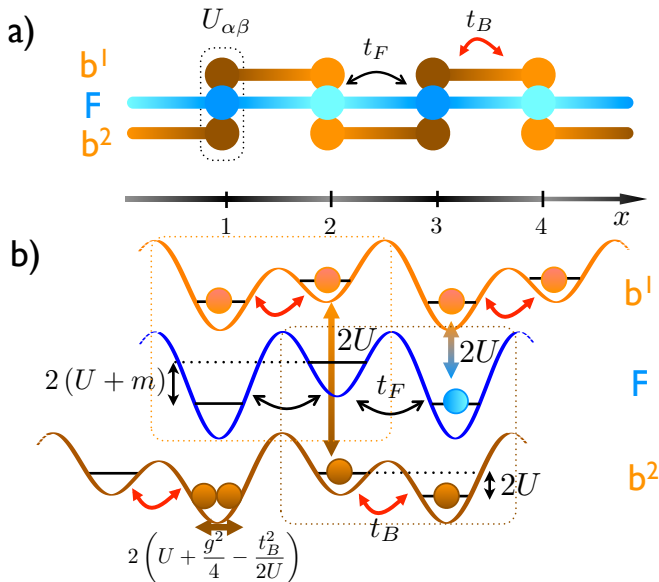
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Microscopic Hubbard model Hamiltonian

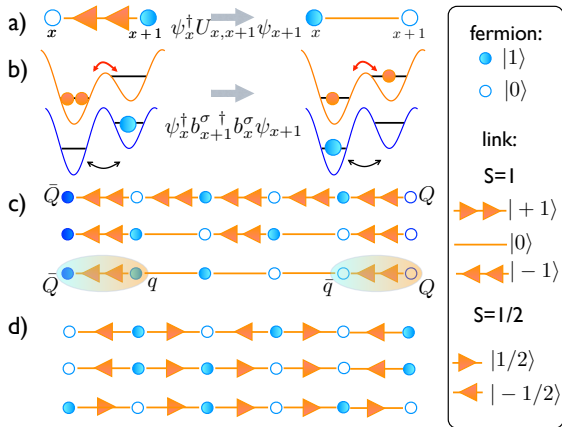
$$\begin{aligned} \tilde{H} &= \sum_x h_{x,x+1}^B + \sum_x h_{x,x+1}^F + m \sum_x (-1)^x n_x^F + U \sum_x \tilde{G}_x^2 \\ &= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.} \\ &+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha \end{aligned}$$

Optical lattice with Bose-Fermi mixture of ultra-cold atoms

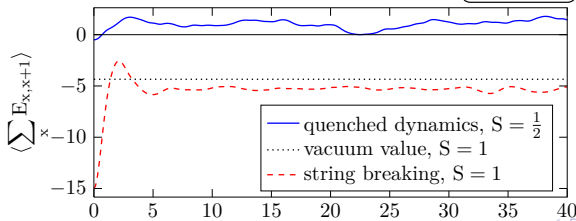
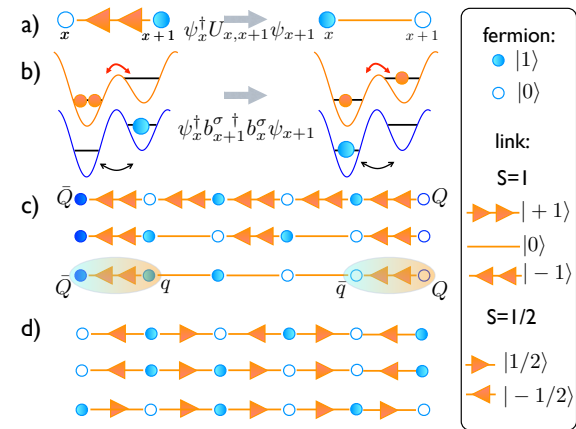


Banerjee, Dalmonte, Müller, Rico Ortega, Stebler, UJW, Zoller

Quantum simulation of string breaking and quenched dynamics



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d -dimensional $SU(N)$ gauge theory with staggered fermions

$$\begin{aligned} H = & -t \sum_{\langle xy \rangle} \left(s_{xy} \psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j + \text{h.c.} \right) + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i \\ & + \frac{g^2}{2} \sum_{\langle xy \rangle} (L_{xy}^a L_{xy}^a + R_{xy}^a R_{xy}^a) + \frac{g'^2}{2} \sum_{\langle xy \rangle} E_{xy}^2 \\ & - \frac{1}{4g^2} \sum_{\langle wxyz \rangle} (U_{wx} U_{xy} U_{yz} U_{zw} + \text{h.c.}) - \gamma \sum_{\langle xy \rangle} (\det U_{xy} + \text{h.c.}) \end{aligned}$$

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Meson, constituent quark, and glueball operators

$$M_x = \psi_x^{i\dagger} \psi_x^i, \quad Q_{x,\pm k} = c_{x,\pm k}^{i\dagger} \psi_x^i, \quad \Phi_{x,\pm k,\pm l} = c_{x,\pm k}^{i\dagger} c_{x,\pm l}^i$$

form a local $U(2d+1)$ algebra at each site x , thus providing a formulation in terms of manifestly $SU(N)$ gauge invariant objects. However, the conserved rishon number gives rise to a $U(1)$ gauge symmetry on the links

$$N_{xy} = c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i, \quad [N_{xy}, H] = 0$$

1-d $SO(3)$ gauge theory with adjoint staggered fermions

$$\begin{aligned} H &= -t \sum_{\langle xy \rangle} \left(\psi_x^{i\dagger} O_{xy}^{ij} \psi_y^j + \text{h.c.} \right) + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i \\ &= -t \sum_{\langle xy \rangle} \left(B_{x,+}^\dagger B_{y,-} + \text{h.c.} \right) + m \sum_x (-1)^x M_x \end{aligned}$$

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Majorana rishons in the $\{4\}$ representation of $SO(6)$

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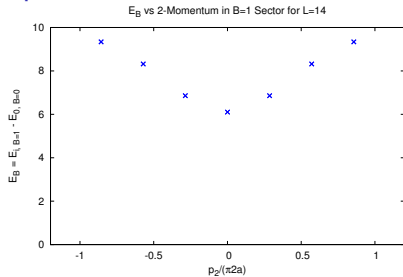
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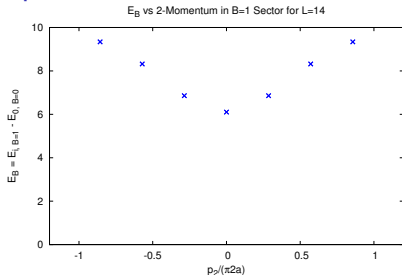
form a local $SO(4)$ algebra at each site x , thus providing a formulation in terms of manifestly $SO(3)$ gauge invariant objects, in this case without any additional gauge symmetry on the links.

Spectrum in the $B = 1$ sector (including 4-fermion coupling)



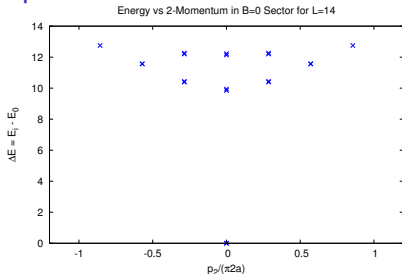
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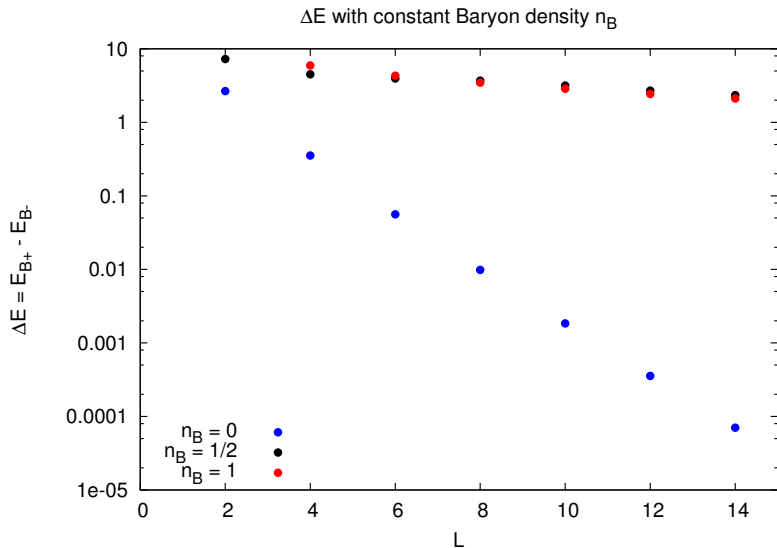
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Spectrum in the $B = 0$ sector



$$M_{B\bar{B}} \approx 10t < 2M_B$$

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



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