Reverse Supply Chain Forecasting and Decision Modeling for Improved Inventory Management

by

Brian J Petersen

B.S., Webb Institute, 2007

Submitted to the MIT Sloan School of Management and Engineering Systems Division in partial fulfillment of the requirements for the degrees of

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and

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Abstract

This thesis details research performed during a six-month engagement with Verizon Wireless (VzW) in the latter half of 2012. The key outcomes are a forecasting model and decision-support framework to improve management of VzW's reverse supply chain inventory. The forecasting model relies on a reliability engineering formulation and incorporates a learning component to allow incremental forecast improvement throughout the device lifecycle. The decision-support model relies on Monte Carlo simulations to quantify the uncertainty and risk associated with different inventory management policies.

These tools provide VzW stakeholders with a full-lifecycle perspective so that inventory planners can avoid costly end-of-life underages and overages. Prior to this effort, inventory planners at VzW relied on a three month returns forecast despite the fact that customers can return devices more than three years after launch. The decision-support model replaces existing heuristics to improve inventory management.

Model efficacy is demonstrated through case studies. For a variety of representative SKUs, the returns forecast model is found to predict cumulative lifecycle returns within 10% using data available six months from launch. Had inventory been managed according to the policies recommended by the decision support model instead of policies from existing heuristics, VzW could have avoided an end-of-life stockout of more than 20,000 devices for a particular SKU.

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Chapter 1

Introduction

The research presented in this thesis derives from a six-month engagement with Verizon Wireless (VzW) in the latter half of 2012. The key outcome is a forecasting model and decision-support framework to improve management of VzW's reverse supply chain inventory. These tools provide VzW stakeholders with a full-lifecycle perspective so that inventory planners can avoid costly end-of-life underages and overages. Model efficacy is demonstrated through representative case studies. Additionally, this thesis details several insights derived from the core research that improved understanding among VzW stakeholders of the company's reverse supply chain operations.

All data contained in this thesis, whether in figures, tables, or text, are masked or otherwise obfuscated to protect VzW's trade secrets. Nonetheless, to the extent possible, the outcomes and conclusions presented herein are consistent with those derived from VzW proprietary data during the internship.

1.1 Verizon Wireless

Cellco Partnership, doing business as Verizon Wireless, is the largest wireless phone operator in the United States by sales and subscribers. Founded in 1995 as a joint-venture of Verizon Communications (55%) and UK-based Vodafone (45%), VzW is headquartered in Basking Ridge, NJ and employs more than 80,000 people. [17]

In 2011, the company generated \$70.1B in net sales to nearly 110 million consumer, business, and government customers. It offers subscriptions for pre- and post-paid mobile telephony and data services, including text messaging, multimedia content, and internet access. In addition to providing mobile telecommunications services, VzW also sells mobile devices from several manufacturers including Apple, Samsung, Motorola, and LG. VzW's offering of mobile devices includes basic or feature phones, smart phones, tablets, and mobile internet hotspots. [17]

A key driver of VzW's industry-leading sales is the company's impressive history of maintaining high customer satisfaction. As a testament to this fact, VzW boasts a post-paid customer churn rate of less than 1%, best among the nation's major wireless providers. As further evidence, in 2012, the company was awarded J.D. Power and Associates' "Highest Ranked Customer Service Performance among Full Service Wireless Providers."

1.2 VzW Reverse Supply Chain

An important contributor to VzW's high customer satisfaction and retention is the company's generous customer warranty programs. VzW supports its warranty programs through its reverse supply chain operations. This section details the operation of the reverse supply chain, how it is used to service customer warranty claims, and how it has been transformed over the last several years to provide context for the remainder of this thesis.

1.2.1 Customer Warranty Programs

VzW warranties the devices it sells to customers during the first year of ownership for any electrical or mechanical malfunction due to manufacturing defect. The company provides a certified like new replacement (CLNR), a refurbished device of a like or comparable SKU, to customers making warranty claims.

For an additional monthly fee and deductible, VzW offers enhanced warranty options. The Extended Warranty (EW) program allows customers to make warranty claims for devices with manufacturing defects after the basic warranty expires. In a partnership with Asurion, a third-party licensed insurance provider, VzW offers the Wireless Phone Protection (WPP) program. WPP covers not only manufacturing defects but also insures against the device being lost, stolen, or accidentally damaged by the customer. The Total Equipment Coverage (TEC) program combines the benefits of both the WPP and EW programs. Each of these programs relies on an inventory of like or comparable refurbished devices to provide to customers making warranty claims.

1.2.2 Reverse Supply Chain Operation

Figure 1-1 illustrates the concept of operation for VzW's warranty programs as well as highlights major additions to and subtractions from CLNR inventory pool.

An overview of the direct fulfillment returns process follows:

- 1. a customer makes a warranty claim in store or over the phone
- 2. the customer receives a CLNR from the Centralized Returns Warehouse (CRW) in the mail
- 3. the customer mails his or her device to the CRW and it enters work-in-process inventory
- 4. the customer's previous device is triaged, tested, and sent to a third party for remanufacture
- 5. the remanufactured device is returned to the CRW, enters finished goods CLNR inventory, and is used to satisfy future warranty claims

1.2.3 Centralized Returns Warehouse

CLNR inventory is housed at the Centralized Returns Warehouse (CRW), which is located in Fort Worth, Texas and is operated by VzW's third-party logistics provider



Figure 1-1: VzW's CLNR program provides refurbished devices to customers making warranty claims.

(3PL) New Breed. The CRW is an ISO 9001:2000 certified facility, which requires that New Breed maintain an ongoing process of quality management and measurement. In addition, New Breed also complies with ANSI Z1.4-2003 to ensure all functional and cosmetic tests are performed to rigorous quality standards.

The CRW is the nexus of VzW's reverse supply chain, which serves VzW's various warranty programs and maintains an adequate supply of refurbished devices for CLNR inventory.

1.2.4 Historical Context

VzW's reverse supply chain has changed significantly over the past several years, and it is in the context of this change that the effort detailed in this thesis was completed.

The current Supply Chain Management (SCM) organization, which is responsible for the operation of VzW's reverse supply chain and warranty programs, was created in 2009 to drive transformation across the enterprise. The timing coincides with the arrival of Viju Menon, the SCM organization Vice President. Prior to 2009, the business functions performed today by the SCM organization were dispersed throughout the enterprise, mostly under the aegis of the Marketing organization.

The CRW was relocated to its current location from another returns facility in

the Fort Worth area in 2011. Prior to 2003, VzW maintained reverse supply chain inventory in stores. Defective devices returned by customers were refurbished in the forward supply chain distribution centers. The decision to open the CRW was made primarily to reduce inventory carrying costs, standardize the handling of warranty claims and the customer experience, and simplify operations in the forward DCs.

Under the leadership of Menon, the reverse supply chain has undergone significant transformation. For the purposes of this thesis, the most important change is the introduction of programs to sell CLNR inventory into secondary markets as part of a concerted effort to reduce CLNR inventory levels. These efforts have been very successful, reducing excess inventory at CRW by more than a factor of five since 2009. However, this reduction in CLNR inventory exposes VzW to the risk of inventory constraints in the CLNR loop and necessitates more advanced forecasting tools and inventory management models.

1.3 CLNR Loop

To first approximation, the CLNR program is a closed-loop inventory system. Under ideal circumstances, a CLNR exchange has net-zero impact on the CLNR inventory levels held at CRW: a CLNR device leaves the inventory and is provided to a customer, and that customer's defective device is remanufactured and enters into the CLNR inventory pool. In reality, however, there are many deviations from this closed-loop ideal that can result in additions to or subtractions from the CLNR inventory pool over time.

The most important contributor to the CLNR inventory pool is the Customer Guarantee (CG) program. The CG program permits customers to return a recently-purchased device (optionally replacing it with a device of the same or different SKU) within the first 14 days of ownership for any reason. The used device enters the CLNR inventory pool after cosmetic refurbishment. Another addition to the CLNR inventory pool is OEM-provided seed stock. Most of the OEMs with which VzW collaborates are contractually-obligated to prime the CLNR inventory pool; the quantity of this

OEM-provided seed stock is a fraction of the quantity sold in the primary channel.

There are several ways the level of CLNR inventory held at CRW can be depleted. Some defective devices sent for remanufacture are found to be unrepairable (UR) or beyond-economic-repair (BER). In other cases, customers who receive a CLNR fail to return their defective device to CRW. In each of these situations, collectively referred to for the purposes of this thesis as yield loss, there is a net reduction in CLNR inventory. VzW has little control over the level of yield loss for a particular SKU.

More importantly for the purposes of this thesis, however, is when VzW *chooses* to deplete the CLNR inventory for a particular SKU. There are two primary situations where VzW will purposefully deplete CLNR inventory: device substitutions and sales to secondary channels.

1.3.1 CLNR Substitutions

VzW will allocate CLNR inventory of one SKU for use as a substitution for another SKU that is constrained. In this way, VzW is able to risk pool inventory across the reverse supply chain. Although offering flexibility, substitutions are not a panacea for inventory planners.

First, excessive use of CLNR substitutions can drive unwanted customer behavior. To ensure customer satisfaction, customers must generally perceive the substitute SKU as an "upgrade" over the SKU for which the substitute is made. Substitutions are often noted on online industry-watch forums, and as a result persistent long-lived substitutions can drive customers to make unnecessary warranty-claims in an attempt to obtain a "free" upgrade.

Additionally, there are many situations where substitutions are not a viable option for covering a CLNR shortfall. For example, during the transition of 3G to 4G, VzW was not able to substitute 4G devices for a 3G device because the technology was not yet widely available and the customer experience would have suffered as a result of the substitution. Furthermore, as discussed in the inventory management chapter of this thesis, there are a set of conditions for which substitutions are not a sustainable business practice. Indeed, the use of substitutions is viewed somewhat skeptically

among VzW stakeholders, who frequently invoke the analogy of "robbing Peter to pay Paul."

Furthermore, there is an opportunity cost associated with CLNR substitutions. As discussed in more detail in the following subsection, VzW operates a Certified Pre-Owned program that uses CLNR devices. Unlike allocating CLNR for substitutions, using it in the Certified Pre-Owned program results in a revenue stream and drives not only customer retention but also new customer acquisition.

Finally, from a process improvement perspective, reliance on substitutions masks operational problems. For example, the impetus for improved forecasting capabilities is mitigated by relying on substitutes, and as a result, improvements that can benefit reverse supply chain operations aren't as highly prioritized because the true cost of neglecting process improvement is not borne if substitutions are utilized to cover inventory shortfalls.

1.3.2 OCPO and Sideways Sales

VzW also purposefully depletes reverse supply chain inventory when allocating CLNR devices for sales to secondary channels. This is primarily done to avoid holding excess reverse supply chain inventory. The Online Certified Pre-Owned (OCPO) program makes CLNR available to customers at discounted prices. VzW also offers returned devices in bulk quantities through a Sideways Sales program to resellers and repair organizations.

The high-clockspeed nature of the mobile telecommunications industry requires that VzW make decisions about allocating CLNR inventory to secondary channels early in the device lifecycle to avoid rapid price deterioration. Historically, this decision has been made before there was a thorough understanding of the devices cumulative lifetime inventory needs, which prior to this internship were assessed via a three month returns forecast despite the fact that customers can return devices more than three years after launch. Put another way, inventory was allocated to secondary channels before it was determined that the inventory won't likely be needed to satisfy end-of-lifecycle warranty claims. It is expensive to satisfy these late lifecycle claims

without CLNR inventory, but there are also significant opportunity costs associated with having excess CLNR inventory at the end of the device lifecycle.

1.4 Thesis Objective

The objective of this internship is to implement and deploy computational models and tools to help VzW stakeholders best manage the CLNR inventory pool. Specifically, these models address the following two questions:

- 1. how many returns should VzW expect to process monthly for a given SKU for the three years following launch?
- 2. how much CLNR inventory is needed to satisfy full-lifecycle returns (while accounting for yield loss and avoiding unnecessary substitutions) and how much instead can be allocated for sale to secondary markets?

Chapter 2

Forecasting CLNR Lifecycle

Returns

This chapter details the formulation and implementation of a strategic returns fore-casting model for devices warrantied by VzW. This model supplements an existing 13-week Logility forecast model owned by New Breed, VzW's third party logistics provider. Prior to this effort, inventory planners at VzW relied exclusively on this short-horizon returns forecast for decision-making despite the fact that VzW warranties customers for as many as three years after a device is launched.

The forecast model presented offers a full lifecycle view of projected monthly return volumes, is flexible enough to model VzW's entire portfolio of devices, incorporates a learning component such that the forecast improves over time, and is general enough to be robust despite rapid technological changes in the mobile telecommunications industry.

The model is intended to be used strategically: it provides insight into the overall lifecycle trends and accurately predicts cumulative returns, which as discussed in decision-framework chapter of this thesis, is the most crucial component of forecast efficacy for improving decision-making with respect to CLNR allocations. The forecasting tool detailed here is not appropriate, however, for planning on a weekly or monthly timescale because it ignores exogenous factors that must be incorporated for an accurate tactical forecasting tool. Thus, it should be viewed as complementary to the existing Logility model and not as a substitute for that forecast.

2.1 Strategic Forecasting Requirements

A set of requirements for the strategic forecasting model is presented that reflects and synthesizes the opinions of five VzW stakeholders. These requirements informed the forecast model formulation and implementation. Specifically, the strategic returns forecasting model shall:

- provide rough monthly estimates of returns volume for a particular SKU over a planning horizon of at least three years to reflect the fact that customers can make warranty claims years after launch
- provide an accurate estimate of cumulative lifecycle returns to support CLNR inventory decision-making, which is the most important metric for assessing lifecycle CLNR inventory needs
- be generally-applicable across VzW's diverse product portfolio
- be responsive to technology changes across VzW's device portfolio, which is particularly important in light of the recent resurgence of BlackBerry (formerly RIM) and Nokia
- incorporate returns data to improve the forecast over time, which is especially useful in the first three months after launch because CLNR inventory is typically not sold into secondary markets to avoid channel conflict

2.2 Lifecycle Trends

In order to ensure the strategic forecasting model is generally applicable, its formulation reflects and is motivated by lifecycle returns data from devices across VzW's device portfolio. Figure 2-1 shows monthly lifecycle returns volume for five representative SKUs. These SKUs represent a variety of product types (smart phones,

feature phones, and tablets), OEMs, operating systems, and features. These are also high-returns volume drivers, with each being among the top 30% of SKUs in terms of cumulative life-to-date returns since 2009.

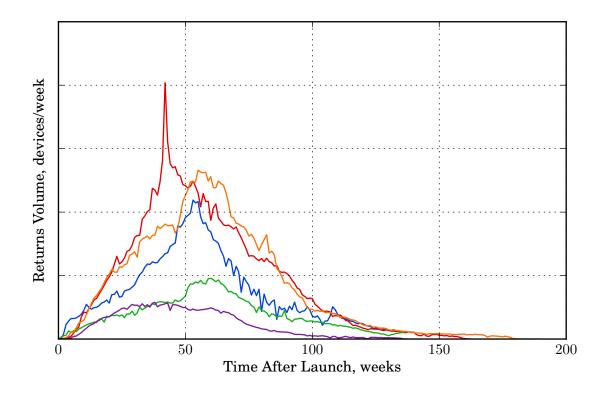


Figure 2-1: Weekly returns volume for five representative high returns volume SKUs.

To characterize the similarities among these return lifecycles, the data are processed with a third-order, lowpass Butterworth zero-phase digital filter [21] with a cutoff frequency corresponding to two months. The filtered data are then normalized to allow easy comparison among the SKUs. Figure 2-2 shows the filtered lifecycles, as well as a comparison between the filtered result and the original data for one of the SKUs.

There are two important things to note about Figure 2-2:

1. the lifecycle trends are similar: the returns volume for each of the SKUs experiences a near-linear rise after launch, peaks between 40 and 60 weeks after launch, and has an exponential tail that extends to more than three years after launch

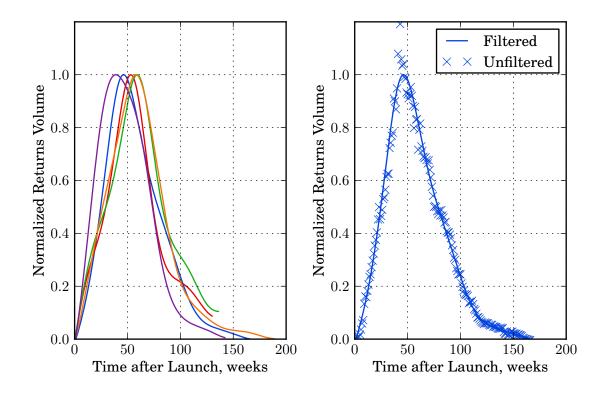


Figure 2-2: When the lifecycles are filtered, similarities among the different SKUs are evident.

2. the smoothed returns data accurately reflect the true lifecycle: the error between the smoothed returns data and the actual returns data are small as demonstrated qualitatively by the right-hand-side of Figure 2-2

Expanding this analysis to all of the SKUs for which VzW has supported device returns since 2009 (more than 200 SKUs) reveals that these conclusions are generally-applicable. They inform the formulation described in the next section.

2.3 Model Formulation

Denote the number of devices sold during time period j (eg, day j) by S[j]. Observe the total number of returns for a given SKU during time period i as R[i]. The returns can be expressed as:

$$R[i] = \sum_{j \le i} R[i, j]$$

$$(2.1)$$

Here, R[i,j] is the number of devices sold in period j and returned in period i.

Assume that the rate of the returns depends on the number of days since purchase; symbolically:

$$R[i,j] = S[j] \lambda[i-j] + \epsilon_{i,j}$$
(2.2)

In Equation 2.2, λ denotes a failure (or return) rate and $\epsilon_{i,j}$ is a zero-mean random variable. That is, $\lambda [i-j]$ denotes the percent of devices that are returned i-j days from purchase.

To examine this conjecture and to determine these return rates, consider Figure 2-3. This figure shows for a single device the number of returns as a function of the days from purchase. More specifically, $\rho[k]$ is plotted, where $\rho[k]$ is defined as:

$$\rho[k] = \sum_{j} R[j+k, j] \tag{2.3}$$

A few comments concerning the trends shown in Figure 2-3:

- the trends demonstrated in the data are common across the high returns volume SKUs supported by VzW
- the number of returns during the first two weeks of ownership spikes due to CG returns
- from two weeks after purchase to approximately one year from purchase, the returns are approximately constant
- there is a distinct discontinuity, with the number of returns reduced significantly, at one year after purchase owing to the fact that VzW's standard device warranty only covers the first year after launch; returns processed after one year of ownership are due to customers participating in the EW and TEC programs

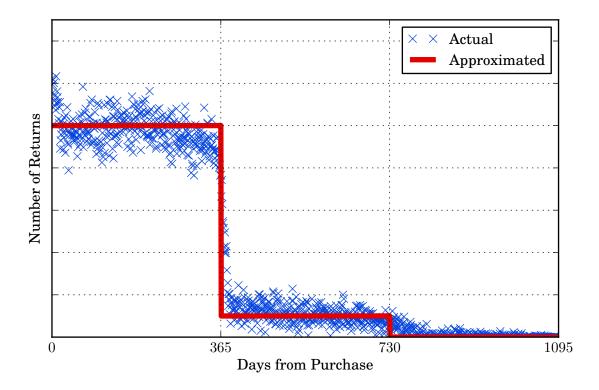


Figure 2-3: The number of devices returned as a function of days after purchase for a representative high returns volume SKU. Given the consistency of the trend over the lifecycle, it can be assumed that the trend is stationary with respect to time after launch.

- the number of returns during the second year is approximately constant
- after about two years from purchase, the number of returns reduces to the point where it is negligible

The consistency among the data $\rho[k]$ in Figure 2-3 suggests that the return rate distribution λ is largely independent of the time after purchase, or stationary with respect to the time since purchase. Appling this conjecture to Equation 2.3 and dropping the noise term:

$$\rho[k] = \sum_{j} R[j+k,j] = \sum_{j} S[j] \lambda[k]$$

$$\rho[k] = \lambda[k] \sum_{j} S[j]$$
(2.4)

Since $\sum_{j} S[j]$, equal to the total number of purchases, is the same for all choices of k, this implies that λ has the same characteristic shape as ρ , which can be approximated as shown in the red line of Figure 2-3 as:

$$\lambda [i - j] \approx \begin{cases} \lambda_0 & 0 < i - j \le 12 \\ \lambda_1 & 12 < i - j \le 24 \end{cases}$$
 (2.5)

If the returns rate is assumed stationary, the returns during time period i simplifies to:

$$R[i] = \sum_{j,i-j\geq 0}^{\infty} \lambda[i-j] \cdot S[j]$$
(2.6)

Note that this is simply the discrete convolution of λ and S.

$$R = \lambda * S \tag{2.7}$$

2.3.1 Parameterizing the Model

This convolution-based approach for forecasting lifecycle returns is demonstrated in Figure 2-4. Note the returns volume history is approximated as the discrete convolution of the sales and the stylized defect rate distribution λ . Note also that the sales history has also been stylized, approximated as follows:

$$S[j] \approx \begin{cases} s_0 & 0 < j \le 6 \\ s_0 \left(2 - \frac{j}{6}\right) & 6 < j \le 12 \end{cases}$$
 (2.8)

In general, the majority of sales occur during the first six months after launch and decrease to negligible quantities after 12 months from launch. Exceptions result from the timing of sales promotions and peak selling seasons relative to launch.

Given life-to-date monthly sales and returns and the five month sales forecast currently produced by VzW supply chain analysts, the required values λ_0 , λ_1 and s_0 can be estimated. The sales forecast is augmented to sales-to-date, and in cases

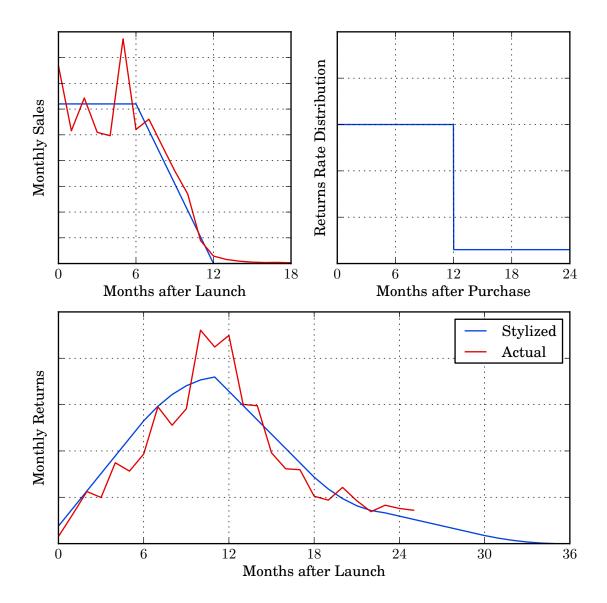


Figure 2-4: A simple stylized model of sales (top left) and stationary return rate distribution (top right) is able to replicate the returns actual volume (bottom) quite well.

where it is fewer than 7 months from launch, Equation 2.8 is fit to the data to project the remainder of the sales history over the first year after launch.

The parameterization of λ is accomplished by solving an overdetermined system of linear equations defined by a Toeplitz matrix that encodes the convolution operation [12]. The least-squares solution is found numerically by forming the Moore-Penrose pseudo-inverse through application of a singular-value decomposition [12]. In other words, the values λ_0 and λ_1 are estimated using a least-squares fit of Equation 2.7 to

life-to-date returns and life-to-date and forecasted sales.

This system of equations is degenerate for both λ_0 and λ_1 pre-launch and for λ_1 prior to 12 months from launch. The degeneracy results from the fact that, prior to launch, there isn't life-to-date returns data to regress against. In these cases, the underdetermined parameters are estimated using reference classes. Specifically, if the parameter cannot be estimated from the returns history, it is instead estimated from the average value of that parameter across historical precedents with the same OEM and device type as the SKU in question.

It is interesting to consider the source of the deviations between forecasted and actual returns shown in Figure 2-4. Assume that the actual returns can be decomposed into the number of returns predicted by the model presented in this chapter, $\lambda * S$, and some error ϵ as follows:

$$R[t] = \lambda * S + \epsilon[t] \tag{2.9}$$

There are two potential sources of deviations ϵ from the modeled returns: exogenous events and sampling error. The sampling error explanation posits that the deviations are due to having a finite population from which the failures are occurring. For example, consider a hypothetical situation where N devices are sold and experience random failures according to a Bernoulli process with failure rate r. The number of failures expected in a Bernoulli trial is given by a Binomial distribution.

The coefficient of variation of the number of failures is found from the Binomial distribution to be $\sqrt{\frac{1-r}{rN}}$. For a high-volume SKU such as the one shown in Figure 2-4, during the first six months, $s_0 = 200,000$ is a reasonable average monthly sales volume; further, a reasonable returns rate is r = 0.8%. For this hypothetical scenario, the coefficient of variation is less than 0.25%, significantly less than the deviations seen in Figure 2-4.

The other explanation for the deviation is more plausible. Many exogenous factors affect the number of returns processed by VzW in a given month and result in perturbations in the returns rate. This explanation is consistent with the experience of

VzW analysts, who are able to identify fluctuations in returns volume due to discrete events such as an over-the-air software update. It is very difficult to predict these exogenous events far in advance; fortunately, by regressing the forecasting model presented in this chapter to historical precedents, the deviations between the forecasted and actual monthly returns are approximately zero-mean.

Nonetheless, this does introduce a limitation of the forecasting model presented here. Specifically, the deviations ϵ due to exogenous events, while zero mean, tend to be correlated temporally. Intuitively, this makes sense. However, early in the lifecycle without substantial life-to-date data to regress against, these deviations can impact the quality of the forecast. Fortunately, a way of quantifying the uncertainty associated with estimating s_0 , λ_0 , and λ_1 is presented in the decision framework chapter of this thesis.

2.4 Forecast Model Efficacy

Figure 2-5 and 2-6 show forecasts made for a representative high returns volume SKU. Figure 2-5 shows the forecast made pre-launch using the five month sales forecast extrapolated to end-of-life using Equation 2.8. The returns rate parameters λ_0 and λ_1 are estimated from a reference class with 35 historical precedents. The forecast error of the cumulative returns is approximately 20%.

Figure 2-6 is a forecast made with data available four months after launch. The forecast improves due both to the forecast horizon being four months shorter as well as the fact that returns and sales data are available to update the model. The forecast error of the cumulative returns is slightly greater than 5%.

The highlighted area in Figure 2-6 shows the time horizon for the existing returns forecast. Note that without a full lifecycle forecasting model, inventory planners are unable determine the magnitude of peak returns or the character of the returns volume tail.

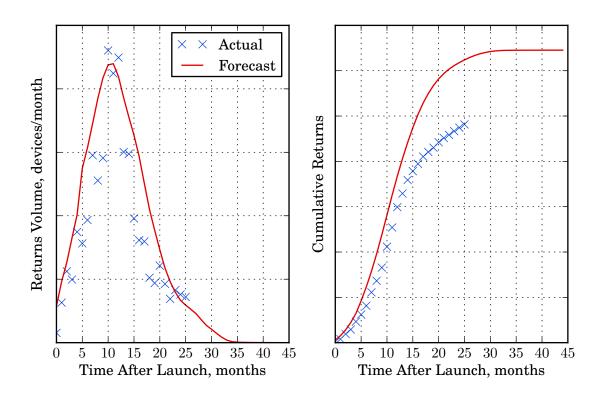


Figure 2-5: The pre-launch forecast using the formulation presented in this chapter is able to predict cumulative returns with about 20%.

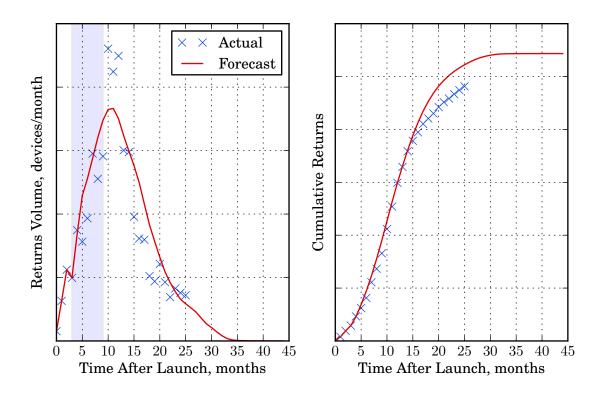


Figure 2-6: The forecast made using data available four months from launch shows a drastic reduction in forecast error for cumulative returns. Note that the model presented in this chapter gives a full lifecycle assessment of the returns compared to three month tactical forecast available before this effort.

Chapter 3

Explaining CLNR Return Seasonality

The data mining completed in support of developing of a full-lifecycle returns forecast model yielded several compelling ancillary insights into the operation of VZW's reverse supply chain. Most notable is an explanation for the substantial summerover-winter seasonality in CLNR exchange volume.

Figure 3-1 demonstrates the magnitude of this seasonality. The graphic shows the number of weekly CLNR exchanges summed over all devices processed by VzW since 2009. There are two things to note about this graphic:

- 1. the magnitude of returns processed by VzW has declined at an average rate of about 12.5% year-over-year during the past four years; this reduction was accomplished through a variety of initiatives to better train and incent VzW customer service representatives to avoid unnecessary returns as well as through an evolving product portfolio with a larger percentage of sales comprised of higher quality devices
- 2. during the summer months, highlighted in Figure 3-1 as the period between May 15 and September 15, the aggregate returns volume increases 25% over the volume during the rest of the year for the period of January 2009 to present

This chapter details explanations for the seasonal fluctuation in returns volume

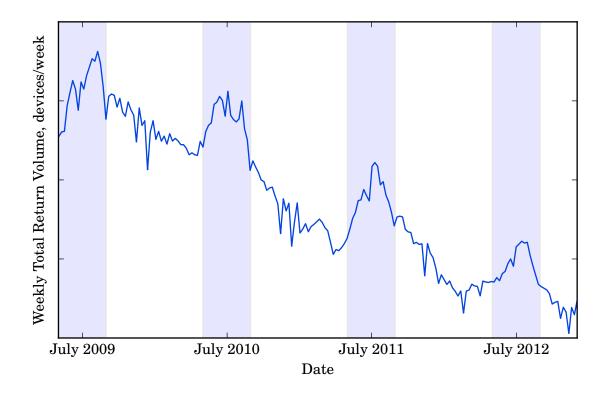


Figure 3-1: The total returns volume processed by VzW during the summer has exceeded the returns volume processed in the winter by more than 25% over the last four years.

postulated by VzW stakeholders and offers a data-driven conclusion: the seasonality is caused by a combination of predominant lifecycle trends in device returns and VzW's device launch schedule.

3.1 Postulated Explanations

Prior to the author's internship, there were several competing explanations for the returns volume seasonality. These included:

- Climate: high temperatures and humidity can damage electronic equipment, including the mobile devices warrantied by VzW. On average, extreme high temperatures are most likely to occur in the summer months in the US. Climate is discussed in more detail in the next section.
- Customer Damage: during the summer, customers are more likely to engage

in activities that can result in damaged phones. For example, the US Census estimates that more than 25% of all Americans attend ocean beaches in a given year [6], and saltwater and sand both can damage electronic equipment and interfaces.

- Customer Usage: customers tend to participate in more outdoor activities during the summer than during cooler months. The US Census reports that more than 100m Americans participated in outdoor activities typically associated with summer, such as fishing, hiking, swimming, and boating, compared to about 7m Americans participating in activities typically associated with winter, such as snowboarding and skiing [6]. Not only could this increase in summer outdoor activity lead to more customer-induced damage, as described above, but it may also result in more usage that could reveal latent manufacturing defects and drive warranty exchanges.
- Customer Convenience: customers may be more apt to return devices during the summer because it is more convenient to do so. For example, the US Census reports that there were 7.2m primary teachers employed in 2009 [7], and the vast majority of these teachers enjoy summer vacations. Therefore, the seasonality could reflect customers making warranty claims for minor defects they otherwise might overlook were the summer not a convenient period to make the warranty claim.

It is important to note that these explanations can reinforce certain mental models that might affect decision-making. For example, if devices aren't robust to higher temperatures despite being advertised as such, VzW could seek remediation from OEMs. Alternatively, if customer-induced damage is driving the seasonality, this might suggest that VzW should invest more in preventing such returns from occurring for customers without WPP or TEC protection.

3.2 Climate Deep Dive

One of the more plausible explanations particularly popular among VzW stakeholders was that the devices are not designed to withstand the high temperatures typically encountered during the summer in the US. Apple, Motorola, and Samsung advise that customers should not expose cell phones to temperatures below -20 degrees Celsius (-4 degrees Fahrenheit) or above 45 degrees Celsius (113 degrees Fahrenheit) for extended periods [2]. Outside this range, device components such as the battery are at high risk of failing.

This is a reasonable explanation for the summer seasonality. Consider Figure 3-2, which indicates the spatial distribution of extreme temperatures throughout the 48 contiguous states [1]. Specifically, the figure shows the 95th percentile of daily high temperatures for each state since 2009. For example, the figure suggests that the highest daily temperature in Pennsylvania over the last four years exceeds approximately 95 degrees Fahrenheit only 5% of the time.

It should be noted from Figure 3-2 that all of the top 5% daily high temperatures since 2009 occurred between May 15 and September 15, or during the period when VzW's return volume is highest. Thus, there is a strong temporal correlation between extreme temperature and high return volumes.

However, compare Figure 3-2 to Figure 3-3, which shows the fraction of all returns that were made between May 15 and September 15 during the last four years. If the seasonality in CLNR exchange volume were caused by extreme temperature, it would be expected that the percentage of returns occurring during the summer in regions such as the southwest would be greater than in regions where there are fewer extreme temperature events. However, note that while there is a clear bias in extreme temperature in the south and southwest relative to the north and northeast, Figure 3-3 reflects no such geographic patterns, and in fact, the percentage of all returns to those that are made in the summer is approximately 32% independent of geography. This evidence undermines the conclusion that the CLNR exchange seasonality results from the devices not being robust to the summer's high temperatures.

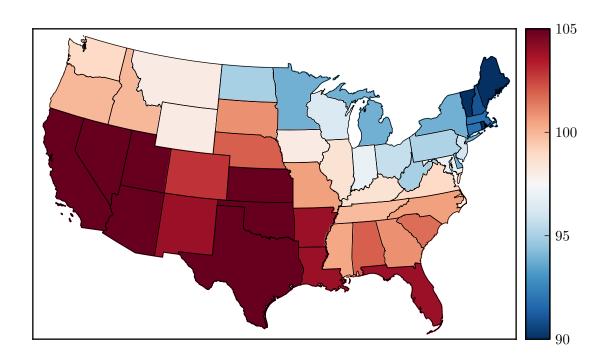


Figure 3-2: The 95th percentile of daily high temperatures, in degrees Fahrenheit, for each state since 2009 is a good proxy for the geographic distribution of extreme temperatures in the US.

3.3 Seasonality Hypothesis

It is hypothesized that the seasonality in CLNR exchanges is caused by a combination of two factors:

- 1. more than 45% of all SKUs in VzW's product portfolio are launched in Q4 to support the holiday selling season
- the returns lifecycle for a given SKU tends to increase monotonically from launch until approximately 10.5 months after launch at which point they begin to decrease monotonically

These factors are discussed in more depth throughout the remainder of this chapter.

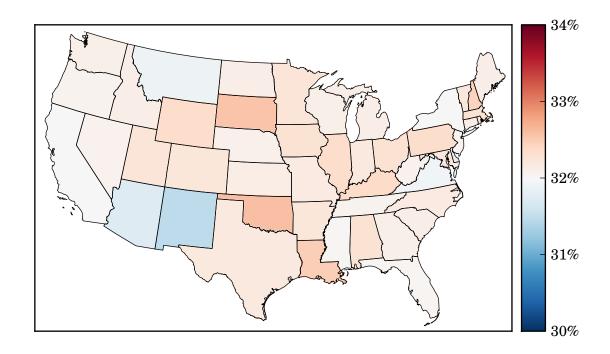


Figure 3-3: The percentage of returns made in the summer over the last three years is roughly constant across geography despite significantly higher extreme summer temperatures in the south and southwest.

3.3.1 Device Lifecycle Homogeneity

As discussed in the forecasting chapter of this thesis, there are similarities among the time series data for device returns that are independent of device characteristics such as OEM and device type.

Figure 3-4 shows the monthly returns as a function of time for approximately 75 SKUs that account for more than 85% of VzW returns volume since 2009. As in the forecast model chapter, these time series data were low-pass filtered and normalized to allow easy comparison among the lifecycles. The lifecycles for the high volume SKUs are colored according to whether or not they exhibited peak returns during the summer months. Qualitatively, there is no discernible difference among the lifecycles.

That the lifecycle trend is independent of peak returns month is better quantified by the histograms in Figure 3-5. The left hand side of the figure is a histogram of the duration from launch to peak for the high volume SKUs since 2009. All SKUs in this

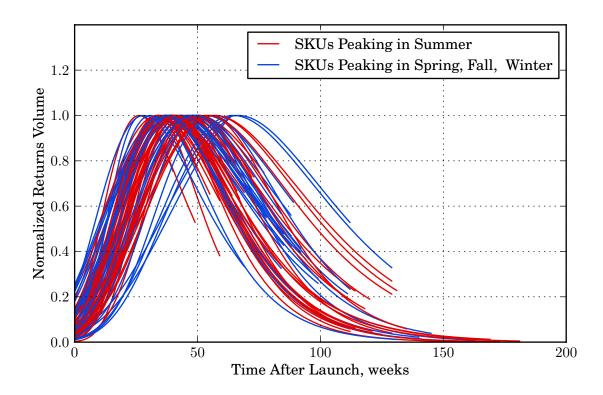


Figure 3-4: The lifecycle trends of monthly returns volume is independent of whether the returns peak during the summer or not.

time period peaked between 5 and 15 months after launch; the mean launch-to-peak duration is nearly 11 months, and the mode is 10 months.

If some exogenous factor were responsible for the seasonality, such as extreme temperatures causing device hardware failure, it would be expected that lifecycles would vary significantly as a function of time after launch. For example, a SKU launched in March should peak during summer or at least be bimodal to represent peaks from the user base building over time and the exogenous summer event. However, the data are not supporting that conclusion. Indeed, with few exceptions, the lifecycles are unimodal and exhibit peak returns between 5 and 15 months after launch, independent of launch date.

The right hand side of Figure 3-5 shows the corresponding histograms for those high volume SKUs peaking in the summer and those SKUs peaking in other months. The ordinate axis limits are consistent between the left and right portions of the figure. There are 32 SKUs that peak in the summer and 44 SKUs that peak in the

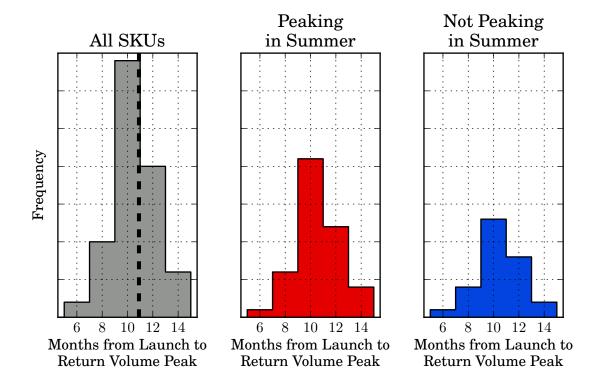


Figure 3-5: The period of peak returns for a given SKU occurs on average between 10 and 11 months after launch, independent of whether the SKU peaks in the summer or during the spring, fall, and winter.

other seasons.

A two-sample Kolmogorov-Smirnov (KS) test is applied to the unbinned data to test whether the observed launch-to-peak durations for SKUs peaking in the summer are drawn from the same statistical distribution as the observed launch-to-peak durations for SKUs peaking in other seasons. The KS test is a general non-parametric method for comparing two samples and assumes as a null hypothesis that the two sets of data are samples from the same continuous distribution. In this case, the test statistic (the supremum distance between the empirical distribution functions for the two sample sets) is 0.179, and the p-value is 0.55 implying that we cannot reject the null hypothesis with significance $\alpha = 0.05$.

3.3.2 Device Launch Cycles

Figure 3-6 reiterates the uniformity of returns lifecycle trends for high volume SKUs since 2009. The uniformity is demonstrated by the linearity of the SKU peak dates as a function of SKU launch date. A linear regression to these four years of data reveals a intercept of almost 11 months (consistent with the observations in the previous subsection), a slope of approximately 1 year; the linear goodness-of-fit is $r^2 = 0.94$.

Also shown in Figure 3-6 is the clustering of SKU launches during Q4 (October-December). In fact, despite the fact that this period represents just 25% of the year, more than 45% of all SKUs are launched in anticipation of the holiday selling season.

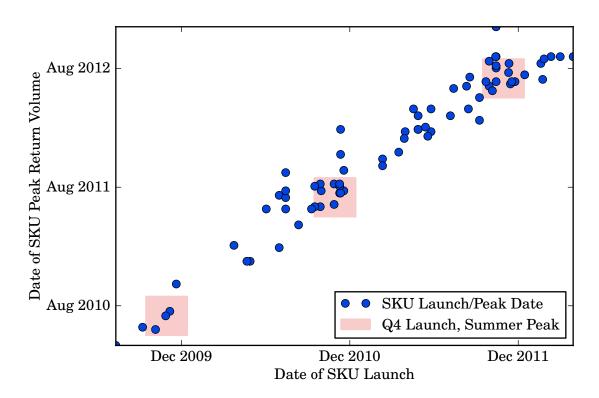


Figure 3-6: SKUs are most likely to be launched in Q4 and peak in the following summer as demonstrated by the clusters in these regimes. This fact drives the summer seasonality in CLNR exchanges.

Combining the fact that SKU lifecycle are independent of launch date with the fact that VzW launches and sells many phones in Q4 explains the summer seasonality observed in VzW's returns data. Specifically, if a phone is launched in October, on average it will experience peak returns volume about 10.5 months later, or during

the middle of July. The same is true of phones launched during other periods: a phone launched in April is most likely to peak in returns volume in the middle of January of the following year, on average. The seasonality is due to the fact that VzW launches many more devices during the late fall and early winter than it does during the spring.

Figure 3-7 provides further evidence for this conclusion using the summer of 2012 as an example. The total monthly returns volume processed by VzW was partitioned into SKUs launched between October and December 2011 and all other SKUs. Subtracting the return volume from the Q4 launches from the total returns volume demonstrates that the peak volume is almost exclusively resulting from the returns of SKUs launched the previous holiday selling season. This is consistent with the findings of this chapter.

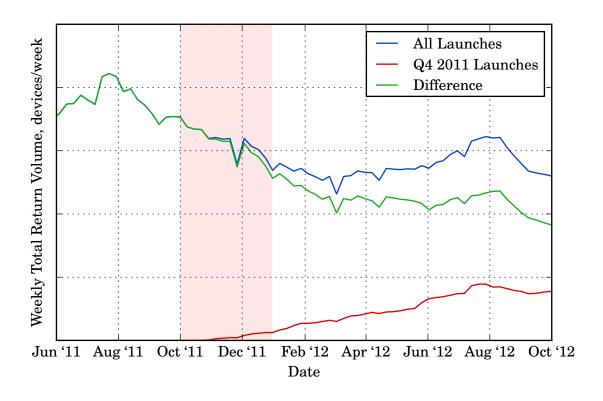


Figure 3-7: Partitioning monthly returns volume in 2012 between those SKUs launched in Q4 demonstrates that the summer seasonality is caused by a combination of predominant lifecycle trends in device returns and VzW's device launch schedule.

Chapter 4

Decision Support Framework

One of the major outcomes of this research is a decision-support framework to help VzW stakeholders manage CLNR inventory well. Specifically, the framework addresses one of the two questions raised in the objective of this research effort: how much CLNR inventory is needed to satisfy full-lifecycle returns (while accounting for yield loss and avoiding unnecessary substitutions), and how much instead can be allocated for sale to secondary markets? The decision support framework builds on the returns forecast models and returns lifecycle insights detailed in the earlier chapters of this thesis.

4.1 CLNR Inventory Dynamics

This section details a model of CLNR inventory dynamics that, when combined with estimates of key system parameters such as the returns rate, can be used to forecast the evolution of CLNR inventory levels over the entire SKU lifecycle; the model can also be used to assess how sensitive end-of-life CLNR inventory levels are to the different factors influencing the system and evaluate the efficacy of different inventory management policies.

4.1.1 Modeling Inventory Dynamics

Assume that no CLNR devices are allocated to secondary markets and that no substitutions are made (the "do-nothing" policy). Under these assumptions, the CLNR virtual inventory (which includes the physical inventory held at CRW and the WIP devices being remanufactured), also known as the CLNR inventory position, during time period t, $I_{clnr}[t]$, evolves according to the following equation:

$$I_{clnr}[-1] \equiv 0$$

$$I_{clnr}[t] = I_{clnr}[t-1] + CG[t] + SS[t] - Y[t]$$
(4.1)

In this equation, CG [t] and SS [t] are the customer guarantee returns and OEM-provided seed-stock, respectively, and Y[t] is the yield loss.

The OEM-provided seed stock is contractually-obligated as a percentage of sales for nearly every OEM with which VzW partners. As in the forecasting chapter of this thesis, assume the returns can be expressed as a discrete convolution between monthly sales and a stationary returns rate distribution. Furthermore, assume that the customer guarantee returns and yield loss can be assumed proportional to sales and returns, with the proportionality constants denoted cg and y:

$$I_{clnr}[t] = I_{clnr}[t-1] + (cg + ss) S[t] - y R[t]$$

 $I_{clnr}[t] = I_{clnr}[t-1] + (cg + ss) S[t] - y (\lambda * S)$ (4.2)

In reality, the customer guarantee returns are nearly proportional to sales: the coefficient of variation of monthly CG returns for a given SKU, averaged across VzW's product portfolio since 2009, is 15%, indicating low month-to-month variability of the ratio of monthly CG returns to monthly sales.

The approximation of yield loss being proportional to returns over the entire lifecycle is less realistic. In general, the yield loss as a percent of returns tends to increase later in the lifecycle. For the purposes of this stylized model, yield loss is modeled as proportional to sales via an average lifecycle constant-of-proportionality; for the decision models presented in this chapter, this assumption is not a constraint as the most important metric is entire lifecycle yield loss, which can be reliably estimated using an average yield loss fraction.

Equation 4.2 demonstrates that the inventory evolution can be approximated as only a function of monthly sales and five parameters (λ_0 and λ_1 are implicit within the variable λ) that can be assumed constant over the lifecycle.

Application of this model using plausible inputs reveals the strong motivation for selling CLNR inventory to secondary markets. Consider Figure 4-1, which shows the evolution of CLNR inventory according to Equation 4.2 assuming $\lambda_0 = 0.04$, $\lambda_1 = 0.004$, ss = 0.01, cg = 0.03, and y = 0.05. Sales are assumed to evolve according to Equation 2.8. Also shown in Figure 4-1 are the monthly additions and subtractions to the closed loop. All curves are normalized by total lifecycle sales.

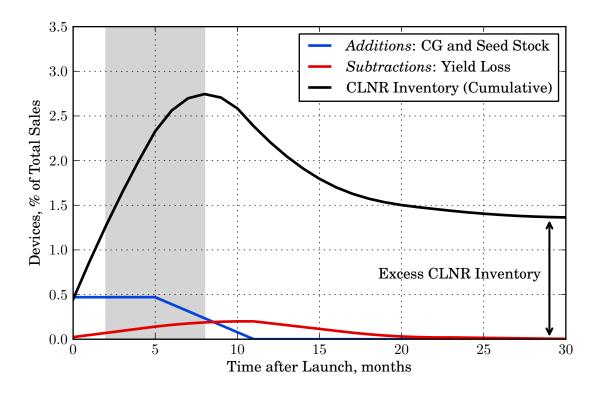


Figure 4-1: Using plausible inputs to the model reveals that under a "do-nothing" policy, there is significant CLNR inventory remaining at the end-of-life.

There are three important things to note about Figure 4-1. The first is the timing and magnitude of additions and subtractions to the CLNR inventory pool. Additions to the CLNR inventory pool, modeled proportional to sales, generally occur exclusively within the first 12 months after launch and are largest in magnitude during the first six. Depletions from the CLNR inventory pool from yield loss occur throughout the entire lifecycle and generally peak around 11 months after launch. Thus, inventory is built up early in the lifecycle and depleted throughout.

The second important thing to note is that there is significant excess CLNR inventory available at the end-of-life under a "do-nothing" policy given plausible model inputs. The dynamics shown in Figure 4-1 are responsible for the fact that CLNR inventory levels were historically much higher before VzW implemented programs to sell CLNR to secondary markets.

This excess end-of-life inventory has high associated opportunity cost because the devices have much lower salvage value at the end of the lifecycle than at the beginning of the lifecycle. This is due to rapid price-deterioration in an industry where customers replace their mobile devices on average between one and two years after purchase and most SKUs are only sold on the primary channel for a year or less after launch.

The remedy to excess end-of-life CLNR inventory is selling devices from the CLNR pool into secondary markets. The Online Certified Pre-Owned (OCPO) program performs precisely that function, offering CLNR to new and existing VzW customers at a discounted price. This reduces the level of CLNR inventory and also satisfies latent demand from customers interested in obtaining used devices at a discount.

The third notable thing about Figure 4-1 is that the opportunity to sell into the OCPO program begins two months after a SKU is launched (to avoid conflict with the primary channel) and can extend for several months thereafter depending on demand. A hypthetical OCPO selling window is highlighted in grey in the figure. The timing of the OCPO selling window is problematic from an inventory planning perspective because the decision to allocate CLNR to the OCPO program is made early in the lifecycle when there is still uncertainty associated with full lifecycle inventory needs

to support the CLNR loop.

Although presented as a deterministic process in the context above, in reality, the inventory evolution defined by Equation 4.2 is stochastic. There is uncertainty in each of the parameters defining the model, and during the OCPO selling window, there is risk in over- or under-allocating CLNR to the OCPO program. Decisions made early in the lifecycle can have consequences more than 30 months later. In a high-clockspeed environment such as mobile telecommunications industry, such a long duration feedback loop can make it difficult for stakeholders to identify causal relationships, and as a result organizational learning suffers and problems remain unaddressed.

Prior to this effort, the inherent uncertainty associated with inventory dynamics was compounded by the fact that inventory planners could only predict yield loss over a three month horizon because the returns forecast made available by New Breed is a tactical forecast. In order to compensate for lack of sufficiently long-horizon forecasts, inventory planners used heuristics for decision-making. Two heuristics were popular:

- the weeks-on-hand inventory management policy requires that no CLNR allocation should result in fewer than ten weeks of inventory on hand relative to recent "usage"
- 2. the **percentage of sales** inventory management policy requires that no CLNR allocation should cause cumulative allocations to exceed a percentage of cumulative sales-to-date

Using the stylized model of Equation 4.2, it is possible to evaluate these inventory management policies under the assumption of plausible $\operatorname{cg}[t]$, $\operatorname{y}[t]$, λ_0 , and λ_1 . Figure 4-2 demonstrates that the simple heuristics that guided VzW decision-makers prior to this effort can result in an end-of-life inventory underage in typical circumstances. VzW satisfies customer warranty claims even if there is no CLNR inventory available of the same SKU. In the case of a shortage, VzW will allocate CLNR inventory of another SKU for use as a substitute for the constrained SKU. As discussed in the introductory chapter, there are several reasons to avoid substitutions and the

CLNR inventory shortages that necessitate them. From an accounting perspective, the worst case scenario for an end-of-life CLNR shortage is that VzW cannot identify an appropriate substitute and has to purchase new devices or take packaged inventory from forward distribution centers for use as CLNR. In this case, VzW absorbs the entire cost of the device (including the subsidy normally offered to the customer) without generating a new revenue stream through the transaction.

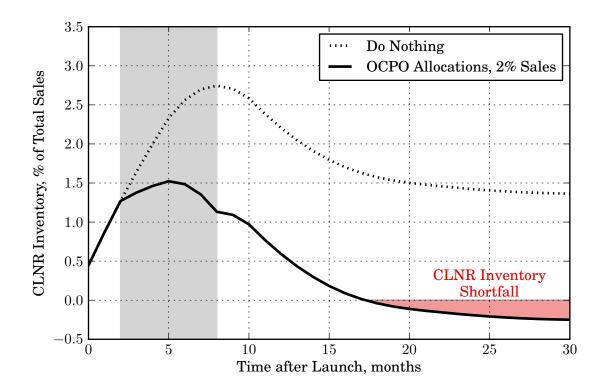


Figure 4-2: The 2% life-to-date sales CLNR allocation policy results in an end-of-life CLNR inventory shortfall.

In summary, end-of-life CLNR inventory overages and underages are expensive, and given the complexity of CLNR inventory dynamics and the inherent uncertainty in forecasting the evolution of CLNR inventory levels, it is imperative to have a model that can be used by stakeholders to develop good inventory management policies that best balance the overage and underage risks and costs. The following subsection derives a deterministic expression that addresses this need.

4.1.2 Strategic Inventory Planning

Using the stylized model defined in Equation 4.2, it is possible to express the end-of-life CLNR inventory level as a simple function of a few key inputs. For the purposes of this analysis, the end-of-life is assumed to occur in period T=36 months, after which some end-of-life policy is assumed to go into effect. In practice, returns are negligible more than three years after a SKU is launched because customers are generally eligible for new contracts and can receive subsidized upgrades prior to that date; only those customers with EW and TEC are able to make returns more than three years after a phone is launched.

Denoting Q_{vzw} as the quantity of inventory Verizon Wireless chooses to remove from the CLNR pool (for example, for use in the OCPO program or as a substitution for a constrained SKU), the end-of-life inventory defined by Equation 4.2 can be simplified by evaluating the convolution product and collecting terms as follows:

$$I_{clnr}[T = 36] = s_0 (8.5 (cg + ss) - 102 y (\lambda_0 + \lambda_1)) - Q_{vzw}$$
 (4.3)

Without loss of generality, the stylized functional form of the sales history defined in Equation 2.8 has been assumed. While this was a useful construct for exploring CLNR inventory dynamics, in reality, the true sales data can differ from this assumed form. Fortunately, the inventory evolution scales proportionally to cumulative sales independent of the assumed temporal distribution of the sales. Normalizing Equation 4.3 by the total lifecycle sales, $\sum_t S[t] = 8.5 s_0$, and defining q_{vzw} as the quantity of CLNR inventory VzW removes from the reverse supply chain as a percentage of total lifecycle sales, Equation 4.3 becomes:

$$\frac{I_{clnr}[T = 36]}{\sum_{t} S[t]} = cg + ss - 12 y (\lambda_0 + \lambda_1) - q_{vzw}$$
(4.4)

As an aside, it should be noted that the subexpression 12 $(\lambda_0 + \lambda_1)$ is equivalent to the cumulative end-of-life exchanges normalized by cumulative end-of-life sales. Note that the ratio of total lifecycle exchanges to total lifecycle sales is also the lifecycle average return rate, and therefore $\overline{\lambda} = 12 (\lambda_0 + \lambda_1)$.

Given deterministic knowledge of the key system parameters, the condition to guarantee no end-of-life stockouts is simply:

$$q_{vzw} \le cg + ss - 12 y (\lambda_0 + \lambda_1) \tag{4.5}$$

It is not possible to estimate λ_1 directly from early-lifecycle returns data. However, a plausible value to use is $\lambda_1 \approx 0.1 \lambda_0$, which reduces Equation 4.5 to:

$$q_{vzw} \le cg + ss - 13.2 \text{ y } \lambda_0 \tag{4.6}$$

For example, assuming deterministic knowledge and that customer guarantee returns for a particular SKU are 5% of sales, that OEM-provided seed stock is 1% of sales, yield loss is 5%, and the return rate during the first year of ownership is 3%, VzW should allocate up to 4% of sales to secondary markets.

Equation 4.6 is a simple deterministic heuristic that can be used by inventory planners to estimate the efficacy of potential inventory management policies. As will be demonstrated in the remainder of this chapter, this heuristic is more accurate than those previously used by planners, such as limiting weeks-on-hand or a constant percentage of cumulative sales, while only being marginally more sophisticated.

In the next section, the inventory evolution model presented here is extended to incorporate the inherent uncertainty associated with estimating the inputs to Equation 4.4 from life-to-date data for a given SKU.

4.1.3 Tactical Inventory Planning

Although this effort is primarily focused on strategic forecasting, it is important to note that the formulation described here offers insight for tactical decision-making and planning as well. Given the high-clockspeed nature of the mobile telecommunications industry, it is not surprising that inventory planners are preeminently occupied with near-term events. According to one stakeholder, "85% of what we do [at CRW] is tactical."

Tactical inventory planning in the VzW reverse supply chain involves avoiding

and managing CLNR inventory shortages that occur mid-lifecycle. Such shortages occur when finished-goods CLNR inventory is insufficient to cover CLNR usage. For the purposes of this thesis, a stockout is referred to as "strategic" when sales to the primary channel end (implying no further additions to the CLNR loop inventory via customer guarantee returns and OEM seed stock) and virtual inventory (finished-goods plus work-in-process) is not sufficient to cover warranty claims. In other words, whereas a tactical stockout is not persistent and inventory levels can rebound after the stockout without significant intervention by VzW, a strategic stockout is persistent and VzW must intervene continually throughout the remainder of the lifecycle to satisfy warranty claims.

It is possible to estimate the work-in-process (WIP) inventory, I_{wip} , via application of Little's Law. In this case, r is the rate at which VzW receives defective devices from customers and T is the typical duration required to restore a defective device to a CLNR state. Assume T=2 weeks = 0.5 months, which is plausible, and note that the rate at which VzW receives defective devices is simply the returns volume, R[t]. In this case, the WIP can be expressed as follows:

$$I_{wip} = r \cdot T$$
 (4.7)
 $I_{wip}[t] = R[t] \cdot 0.5 \text{ months}$

Note that the finished goods CLNR inventory in every time period, $I_{clnr}[t] - I_{wip}[t]$, must exceed the CLNR provided to customers making warranty claims during that time period in order to avoid an inventory shortage. As a first approximation, assume that returns volume is evenly distributed throughout the month; in this case, the CLNR provided is simply the monthly returns, R[t].

Simplifying the tactical no-stockout condition using these assumptions:

$$I_{clnr}[t] - I_{wip}[t] \geq R[t]$$

$$I_{clnr}[t] - 0.5 \cdot R[t] \geq R[t]$$

$$I_{clnr}[t] - 1.5 \cdot R[t] \geq 0$$

$$(4.8)$$

Unfortunately, no simple closed-form inequality that bounds the entire lifecycle exists to guarantee no tactical stockout. Each month the inequality must be evaluated. Figure 4-3 shows two scenarios. In the left subplot, which assumes a yield loss ratio of 5% of returns, there is enough finished-goods inventory to support the CLNR loop. Assuming a yield loss ratio of 7%, however, results in a stockout beginning during the month of peak returns volume. This is a typical tactical stockout scenario.

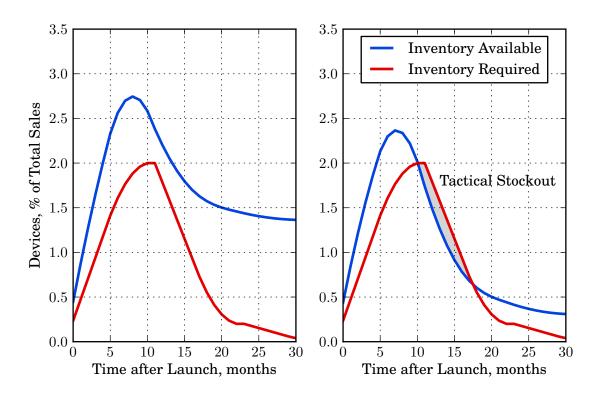


Figure 4-3: To avoid tactical stockouts, the available CLNR inventory must exceed the number of warranty claims made by customers. The left subplot demonstrates no tactical stockout with y = 0.05. There is a tactical stockout, however, when y = 0.07, as shown in the left subplot.

Although tactical inventory management can be improved through the models presented in this thesis, the short-horizon returns forecasts currently provided by New Breed are to be preferred for tactical decision-making over the strategic models presented in this thesis.

4.2 Decision Framework Formulation

Per Equation 4.4, given estimates of cg, y, λ_0 , λ_1 , it is possible to estimate how much CLNR inventory will be available at the end of a SKU's lifecycle assuming a policy by which VzW depletes Q_{vzw} devices from the CLNR pool. Thus far, these inputs have been assumed deterministic. If the parameters of Equation 4.4 are instead assumed to be random variables, the expression for end-of-life inventory also becomes a random variable and this formulation can be used to evaluate different inventory management policies probabilistically. Doing so incorporates into the model the risk and uncertainty inherent in inventory planning.

The uncertainty in the estimates of the various inputs to Equation 4.4 can be characterized by combining a life-to-date value of the input and historical data. For example, to estimate a full lifecycle average value of some input for a particular SKU, $\bar{\mathbf{x}}$, first calculate the life-to-date value from the most current data available at time t, $\mathbf{x}[t]$. Define $\epsilon[t]$ as the random variable defining the percent difference between the true end-of-life value $\bar{\mathbf{x}}$ and the life-to-date value $\mathbf{x}[t]$. Then the random variable characterizing the distribution of $\bar{\mathbf{x}}$ is expressed in Equation 4.10 as:

$$\epsilon [t] \equiv \frac{\overline{\mathbf{x}} - \mathbf{x}[t]}{\overline{\mathbf{x}}} \tag{4.9}$$

$$\overline{\mathbf{x}} = \frac{\mathbf{x}[t]}{1 - \epsilon[t]} \tag{4.10}$$

Then, identify appropriate historical precedents with characteristics similar to the SKU in question. Note that if these historical precedents are at or have exceeded the end-of-life month T, both $\overline{\mathbf{x}}$ and $\mathbf{x}[t]$ are known and it is possible to use Equation

4.9 to sample from the distribution characterizing random variable $\epsilon[t]$. Then, use Equation 4.10 to derive a distribution over the end-of-life value $\bar{\mathbf{x}}$.

This technique is referred to as reference class forecasting. Here, the reference class is comprised of set of values $\epsilon[t]$ calculated using historical precedent SKUs. This formulation assumes that what previously happened to the reference class is a good proxy for what will happen to the SKU in question. This assumption can be violated, for example, if there are exogenous factors relevant to the forecast error of the SKUs in the reference class that aren't relevant to the SKU in question, or vice versa.

The historical samples of $\epsilon[t]$ were found to be largely independent of key SKU characteristics. This was determined by comparing the samples via Kolmogorov-Smirnov (KS) over all pair-wise combinations of OEM and device type at a significance of $\alpha=0.05$. As a result, to maximize the number of historical precedents in the reference class, all SKUs with end-of-life data available are used for estimating the monthly forecast accuracy distribution $\epsilon[t]$. For λ_1 , full lifecycle data are required, so only those SKUs launched more than 36 months ago are considered. For λ_0 and y, only those SKUs launched more than 24 months ago are considered. Because cg is proportional to sales, and because the sales period is generally limited to the first year after launch, all SKUs launched between January 2009 and January 2012 are included in the reference class.

Monte Carlo simulation is used to quantify the probability distribution defining the end-of-life CLNR inventory levels through combination of Equation 4.4 and Equation 4.10. Because the reference classes for the inputs are relatively large (N > 25), it is possible to bootstrap the $\epsilon[t]$ samples drawn from the historical precedence to perform the Monte Carlo simulation.

For more resolution in the tails of the CLNR inventory distribution or in situations where large reference classes aren't available, canonical probability distributions can be fit to the sample reference class distributions.

4.2.1 Demonstrating Efficacy: Device X Case Study

In the remainder of this chapter, two formulations are presented for determining how much CLNR inventory should be allocated to secondary markets. In order to demonstrate the efficacy of the models developed in this chapter, a canonical case study is presented. Specifically, a decision-analysis is applied retrospectively to the historical data and compared to actual decisions and outcomes.

In order to protect the trade secrets of VzW and its OEM partner, both the name of the SKU and the precise details of the historical context of the case study are obfuscated. A situation similar to the one actually faced by VzW inventory planners is presented instead.

This case study involves Device X, a high-sales volume SKU. Beginning three months after launch and ending nine months after launch, approximately 56,000 CLNR devices were allocated to the OCPO program according to the percentage of cumulative sales heuristic. These devices were sold approximately uniformly over the six month period. The relevant inputs to Equation 4.4 are from life-to-date data and compared to the end-of-life values in Table 4.1:

Table 4.1: Estimating End-of-Life Inputs from Life-to-Date Data for Device X

Variable	t=3 months	t=9 months	t=36 months
$\sum_{t} S$	2.8e6	3.0e6	3.0e6
cg	0.045	0.05	0.05
у	0.05	0.075	0.08
λ_0	0.04	0.0475	0.045

During the second year after launch, this policy resulted in a persistent stockout for Device X, which likely would have to be covered with 20,000 substitution devices of a newer model to satisfy customer warranty exchanges.

The risk of an end-of-life stockout given the CLNR allocation made by VzW is illustrated in Figure 4-4, which shows the forecasted end-of-life CLNR inventory under a "do-nothing" policy made three months and nine months from launch. There are three interesting things to note about Figure 4-4:

- 1. As expected, the nine month inventory forecast has lower coefficient of variation ($\approx 25\%$) than the three month forecast ($\approx 45\%$). This is due to the fact that the reference class forecast accuracies, $\epsilon[t]$, for the inputs to Equation 4.4 reduce significantly with longer durations from launch.
- 2. It is clear from both the three month and the nine month inventory forecast that an allocation of 56,000 devices exposes VzW to significant risk of a stockout. In fact, the three month forecast suggests an 55% chance of a stockout if 56,000 CLNR devices are removed from the reverse loop. In the more than 10 million bootstrapped simulations performed at nine months, less than 1% projected inventory levels would exceed 56,000 devices at end-of-life under a "do nothing" policy.
- 3. That said, it is also not prudent for VzW to follow a "do nothing" policy. Under such a policy, the expected end-of-life inventory levels forecasted at three months and nine months are 53,000 and 34,000 devices, respectively, at T = 36. Given that approximately 10,000 devices were sold each month between three and nine months from launch, it is reasonable that VzW could have used the inventory forecasts presented here to decide whether to allocate devices each month, updating the forecasts as new data arrived.

The next two subsections detail ways to balance the risk associated with overage with the reward of selling to secondary markets in principled manner to maximize benefit to VzW.

4.2.2 Single-Period Formulation

It is possible to maximize the expected profit to the reverse supply chain using a simple single-period model. While derived as a single period model, as noted above, it can be applied sequentially because CLNR allocation decisions are rarely irrevocable: the the number of CLNR devices to be allocated over the entire lifecycle is substantially less than the number of devices that can be sold to the secondary markets in a given month.

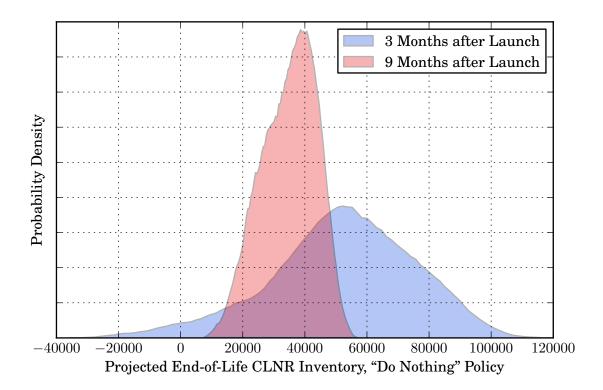


Figure 4-4: Using the model presented in this chapter, CLNR inventory levels were forecasted for Device X.

Defining c as the expected cost of underages and p as the revenue generated from an OCPO sale, the expected full-lifecycle profit π generated from a CLNR allocation of size Q_{vzw} is given by:

$$\pi \left(Q_{vzw} \right) = p \ Q_{vzw} - c \ \mathbb{E} \left[I_{clnr} - Q_{vzw} \ | \ I_{clnr} - Q_{vzw} \le 0 \right]$$

$$\tag{4.11}$$

If f is the probability density function defining the quantity of end-of-life CLNR inventory remaining under a "do-nothing" policy, then the expected value in the above equation can be expressed as an integral:

$$\pi \left(Q_{vzw} \right) = p \ Q_{vzw} - c \ \int_{-\infty}^{Q_{vzw}} \left(Q_{vzw} - q \right) f \left(q \right) dq \tag{4.12}$$

Differentiating the profit with respect to Q_{vzw} to find the CLNR allocation that maximizes VzW profit:

$$\frac{\mathrm{d}\,\pi\left(Q_{vzw}\right)}{\mathrm{d}\,Q_{vzw}} = p - c \int_{-\infty}^{Q_{vzw}} f\left(q\right) \,\mathrm{d}q \tag{4.13}$$

$$\frac{\mathrm{d}\,\pi\left(Q_{vzw}\right)}{\mathrm{d}\,Q_{vzw}} = p - c\,F\left(Q_{vzw}\right) \equiv 0 \tag{4.14}$$

Here, the cumulative distribution function for forecasted end-of-life CLNR inventory F(q) has been substituted. Isolating Q_{vzw}^* , the optimal CLNR quantity to sell to secondary markets takes the familiar form:

$$F\left(Q_{vzw}^*\right) = \frac{p}{c} \tag{4.15}$$

Note that this differs from the traditional Newsvendor solution in one key aspect: if the revenue generated from selling the CLNR to secondary markets exceeds the cost of a stockout, the profit-maximizing solution is to always sell the device because there is essentially no risk in doing so and it is the dominant strategy under any realization of the end-of-life CLNR inventory random variable.

4.2.3 Service Level Formulation

When presented with the single-period result, VzW stakeholders disagreed about the appropriate values for c and p. Suggestions for the underage cost, c, included:

- the CLNR device book value. This is the accounting cost the Supply Chain Management organization absorbs if the underage is covered using new devices from the forward DC.
- 2. the **out-of-warranty repair cost** for a defective device. In the past, if VzW had sufficient CLNR inventory, it would not send all the defective devices it received from customers making warranty claims for remanufacture to avoid costs associated with disposition alignment (in some cases, VzW will have to pay to have a device remanufactured even if it is still covered by the OEM warranty because the remanufacturer disagrees with the testing performed at New Breed

and claims there is nothing wrong with the device). As a result, in some past instances, VzW could cover shortages by remanufacturing devices it had chosen not to earlier in the lifecycle. In this case, the devices sometimes were no longer covered by the OEM warranty, and VzW had to cover the remanufacture cost even in cases without disposition alignment issues.

3. **no cost**. From an economic perspective, if VzW uses new kits to cover the shortfall, it already owns those devices so there is no incremental cost to the enterprise. The same is true if the shortfall is covered using substitutions. However, there are opportunity costs associated with both of these activities.

Similarly, there was disagreement among the stakeholders regarding the appropriate revenue p to consider for a CLNR device sold to secondary markets:

- 1. the **OCPO** sales price. This is the immediate incremental revenue generated through the sale.
- 2. the average revenue per user (ARPU). This is the revenue generated both by the sale and by the contract subscription over the life of the contract.

Stakeholders did, however, agree that stockouts should be avoided and believed the type-I service level for the CLNR loop should be high (between 90% and 95%). In general, the stakeholders felt more comfortable imposing a service level for reverse supply chain management than specifying overage and underage costs. Using the CLNR inventory forecasts and the probability density functions derived using the techniques described in this chapter, it is possible to express the type-I service level as follows:

$$\alpha = \Pr\left(I_{clnr} - Q_{vzw} \ge 0\right) \tag{4.16}$$

Expanding and solving for α , the complement of stockout probability:

$$\alpha = \int_{Q_{vzw}}^{\infty} f(q) \, \mathrm{d}q = 1 - F(Q_{vzw}^*)$$

$$(4.17)$$

Note that it is possible to relate the two decision-frameworks presented in this chapter as follows:

$$(1 - \alpha) = \frac{p}{c} \tag{4.18}$$

Thus, a high service-level implies a high stockout cost relative to the revenue generated by an OCPO sale. Reviewing the suggested values of p and c suggests that the stakeholders attribute a very high intangible cost to having CLNR available late in lifecycle to satisfy warranty exchanges without resorting to substitutions or use of forward DC inventory.

4.2.4 Case Study Results

Figure 4-5 shows the cumulative probability distribution associated with the three and nine month forecasts of the end-of-life CLNR inventory levels for Device X. Also shown in the figure are three different inventory allocation policies:

- 1. a 95% service level
- 2. a single-period model assuming p = \$100 and c = \$400, implying a service level of 75%
- 3. the 56,000 CLNR devices VzW stakeholders sold to the OCPO program under a percentage of cumulative sales policy

Assume the true costs and revenues associated with sale and shortage of CLNR inventory reflect the actual costs and revenues borne by VzW. With the caveat that VzW stakeholders disagree about the actual values, but assuming p = \$100 and c = \$400 are plausible, Table 4.2 compares the policies recommended by this effort to the "do-nothing" case, the percent-of-sales heuristic, the service level model, and the single period model.

It is clear that the policies recommended by the decision-framework presented in the chapter are improvements over the "do-nothing" policy and the heuristic-driven

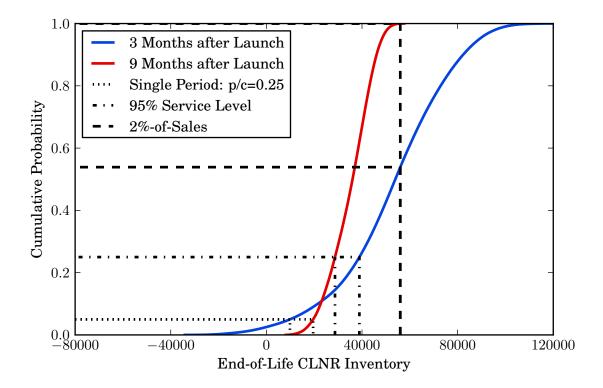


Figure 4-5: Applying the decision-frameworks developed in this chapter to the Device X case study reveals differences among the service level, single period, and actual policies.

policy. Additionally, it is of note that neither policy recommended at nine months by the models presented in this chapter would have resulted in a shortage.

4.3 Implementation

The service level formulation was implemented in Excel as an array workbook function using Visual Basic for Applications (VBA). Excel was chosen as the platform for its ubiquity across the enterprise and because much of the forward supply chain planning is performed in Excel. The tool is currently being used by VzW stakeholders for inventory planning.

Table 4.2: A Comparison of the Policies Developed in this Chapter to the "Do Nothing" Policy and Percentage of Cumulative Sales Heuristic

	CLNR	End-of-Lif	e Inventory	Actual
Policy	Allocation	Forecast	Actual	Profit, \$m
Do Nothing, Three Month	-	53,000	36,000	-
Do Nothing, Nine Month	-	34,000	36,000	-
95% Service Level 3 Month	10,000	43,000	26,000	1.0
95% Service Level, 9 Month	20,000	16,000	16,000	2.0
Single Period, 3 Month	39,000	14,000	-3,000	2.7
Single Period, 9 Month	29,000	6,000	7,000	2.9
Percentage of Sales Heuristic	56,000	=	-20,000	(2.4)

4.3.1 Communicating the Decision

The decision-framework and models presented in this chapter are one component of the decision-making process. Another key component is communication of the decision to stakeholders. This is especially important in the case of VzW's reverse supply chain, as the CRW is remote from the rest of the Supply Chain Management organization at headquarters in Basking Ridge, NJ. Furthermore, decisions made by VzW stakeholders must be communication to New Breed, which operates the CRW.

During the internship that supported this research, it became evident that VzW could improve the processes used to communicate inventory management decisions. For example, although records of historical inventory allocations could be retrieved, this was often a time-intensive and error-prone process involving several interactions between New Breed and VzW. Furthermore, although the transactional data regarding inventory allocations was persisted, the context around those decisions was not included with the data. As a result, the reverse supply chain had limited institutional memory and was unable to fully incorporate learnings from previous decisions into best practices. Finally, there was no standard protocol for communicating CLNR allocations between VzW stakeholders and New Breed personnel; often, the allocations were made verbally during meetings and were acted upon before a formal requisition was made.

To address these opportunities, an ancillary outcome of this research effort was

a tool that can be used by VzW stakeholders to make CLNR allocation requests. A screenshot of the tool is shown in Figure 4-6. The tool is written in VBA and integrates with Outlook and Excel. It provides the user a form to standardize CLNR allocations. It supports substitutions and sales to the various programs supported by VzW. The tool not only captures the date, relevant SKUs, number of devices allocated, and the person making the allocation, but it also captures the context around that decision. An email is automatically generated from the form that is sent to all relevant stakeholders, and an Excel workbook on a publicly available network resource is updated.

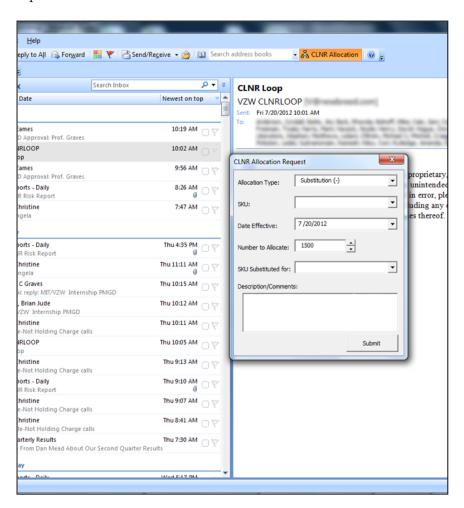


Figure 4-6: A widget was implemented to improve and standardize processes around making CLNR allocations.

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Chapter 5

Conclusions

This thesis details research completed during a six-month engagement with Verizon Wireless (VzW) in the latter half of 2012. A full-lifecycle returns forecast model and decision-support framework were successfully implemented and are currently being utilized by VzW stakeholders for planning and inventory management. These analytic tools help stakeholders avoid costly end-of-life CLNR inventory underages and overages.

5.1 Key Takeaways

There are two key takeaways from this effort:

1. Devices have very long lifecycles in the reverse supply chain. It is important to understand the inherent uncertainty that results from long planning horizons when analyzing and managing VzW's reverse supply chain. This is especially true given the fact that mobile telecommunications is a high clockspeed industry: it is important that stakeholders familiar with forward operations remember that VzW is obligated to provide CLNR long after sales are discontinued for a given SKU. Decisions made early in the reverse supply chain lifecycle can have very significant consequences years later, making posterior causal inferences difficult.

For this reason, the models developed in this thesis (a) take into account the entire lifecycle, with forecasts and planning horizons more than three years after a SKU is launched and (b) are stochastic, to incorporate the inherent uncertainty with planning over such a long horizon.

2. The patterns in device returns are consistent across nearly all relevant dimensions that VzW uses for characterization. This takeaway was leveraged in explaining returns volume seasonality. Recognizing these trends and how they affect the reverse supply chain qualitatively can give VzW stakeholders competitive advantage when managing the CLNR inventory pool both tactically and strategically. As one stakeholder noted, "85% of what is done at CRW is tactical," but the execution and decision-making when confronted with those tactical situations could benefit significantly from a longer-horizon understanding of, for example, how many months before returns will peak after launch for a particular SKU.

5.2 Opportunities for Future Work

There is a significant opportunity for future work at VzW related to the research presented in this thesis. One area that VzW stakeholders may realize significant return on investment is focusing on optimizing decision-making across the forward and reverse supply chains. For example, with the tools developed in this thesis, stakeholders can estimate how many devices will be returned over the course of a SKUs lifecycle. These returns have large associated costs to VzW, and these expected costs should be incorporated into forward logistics and supply chain planning to maximize benefit to the entire enterprise.

Bibliography

- [1] Federal climate complex global surface summary of day data. http://www1.ncdc.noaa.gov/pub/data/gsod/, 2012. [Accessed 22-July-2012].
- [2] iphone technical and environmental specifications. http://www.apple.com/iphone/specs.html, 2012. [Accessed 22-July-2012].
- [3] E. T. Anderson, K. Hansen, and D. Simester. The option value of returns. 2006.
- [4] Dimitri P. Bertsekas. *Dynamic Programming & Optimal Control, Vol. I.* Athena Scientific, 3rd edition, May 2005.
- [5] W.R. Blischke and D.N.P. Murthy. Product warranty management i: A taxonomy for warranty policies. *European Journal of Operational Research*, 62(2):127–148, October 1992.
- [6] U.S. Census Bureau. 2010 census. http://www.census.gov/compendia/statab/cats/arts_recreation_travel/recreation_and_leisure_activities.html, 2010. [Accessed 22-July-2012].
- [7] U.S. Census Bureau. 2010 census. http://www.census.gov/compendia/statab/2012/tables/12s1249.pdf, 2010. [Accessed 22-July-2012].
- [8] Grard P. Cachon. Supply chain coordination with contracts.
- [9] W. Emons. The theory of warranty contracts. *Journal of Economic Surveys*, 3(1):4357, 1989.
- [10] D. P. De Farias and B. Van Roy. The linear programming approach to approximate dynamic programming. *Operations Research*, 51(6):850–865, November 2003. ArticleType: research-article / Full publication date: Nov. Dec., 2003 / Copyright 2003 INFORMS.
- [11] Roland Geyer, Luk N Van Wassenhove, and Atalay Atasu. The economics of remanufacturing under limited component durability and finite product life cycles. *Management Science*, 53(1):88–100, January 2007.
- [12] G.H. Golub and C.F. Van Loan. *Matrix Computations*. Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, 1996.

- [13] Stephen C. Graves and Sean P. Willems. Optimizing the supply chain configuration for new products. *Management Science*, 51(8):1165–1180, 2005. Article-Type: research-article / Full publication date: Aug., 2005 / Copyright 2005 INFORMS.
- [14] V. Daniel R Guide and Luk N Van Wassenhove. OR FORUMThe evolution of closed-loop supply chain research. *Operations Research*, 57(1):10–18, January 2009.
- [15] V. Daniel R Guide and Luk N Van Wassenhove. OR FORUMThe evolution of closed-loop supply chain research. *Operations Research*, 57(1):10–18, January 2009.
- [16] Wei Huang, Vidyadhar Kulkarni, and Jayashankar M Swaminathan. Managing the inventory of an item with a replacement warranty. *Management Science*, 54(8):1441–1452, August 2008.
- [17] Chris Huston. Cellco Partnership. http://www.hoovers.com/, 2012. [Hooverś Online; accessed 19-July-2012].
- [18] M. A. Jolson, J. Weiner, and R. Rosecky. Correlates of rebate proneness. *Journal of Advertising Research*, 27(1):3343, 1987.
- [19] Peter Kelle and Edward A. Silver. Forecasting the returns of reusable containers. Journal of Operations Management, 8(1):17–35, January 1989.
- [20] John Khawam, Warren H Hausman, and Dinah W Cheng. Warranty inventory optimization for hitachi global storage technologies, inc. *Interfaces*, 37(5):455–471, September 2007.
- [21] V.K. Madisetti and D.B. Williams. *The Digital Signal Processing: Handbook*. The Electrical Engineering Handbook Series. CRC Press, 1998.
- [22] D.N.P. Murthy and W.R. Blischke. Product warranty management II: an integrated framework for study. *European Journal of Operational Research*, 62(3):261–281, November 1992.
- [23] D.N.P. Murthy and W.R. Blischke. Product warranty management III: a review of mathematical models. *European Journal of Operational Research*, 63(1):1–34, November 1992.
- [24] D.N.P Murthy and I Djamaludin. New product warranty: A literature review. *International Journal of Production Economics*, 79(3):231–260, October 2002.
- [25] D.N.P Murthy, O Solem, and T Roren. Product warranty logistics: Issues and challenges. *European Journal of Operational Research*, 156(1):110–126, July 2004.

- [26] G. L. Priest. A theory of the consumer product warranty. *Economic Analysis of the Law*, page 111121, 1981.
- [27] R. Canan Savaskan, Shantanu Bhattacharya, and Luk N. Van Wassenhove. Closed-loop supply chain models with product remanufacturing. *Management Science*, 50(2):239 –252, February 2004.
- [28] R. Canan Savaskan, Shantanu Bhattacharya, and Luk N. Van Wassenhove. Closed-loop supply chain models with product remanufacturing. *Management Science*, 50(2):239 –252, February 2004.
- [29] R. Canan Savaskan, Shantanu Bhattacharya, and Luk N. van Wassenhove. Closed-loop supply chain models with product remanufacturing. *Management Science*, 50(2):239–252, February 2004. ArticleType: research-article / Full publication date: Feb., 2004 / Copyright 2004 INFORMS.