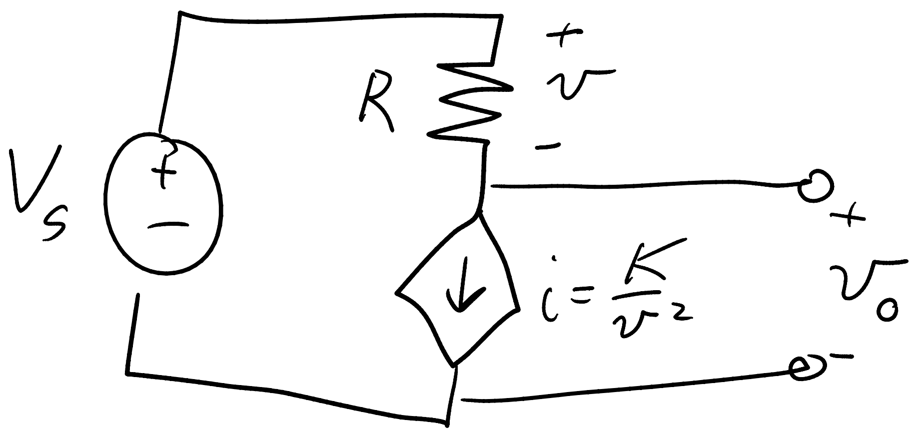
Exercise 1):



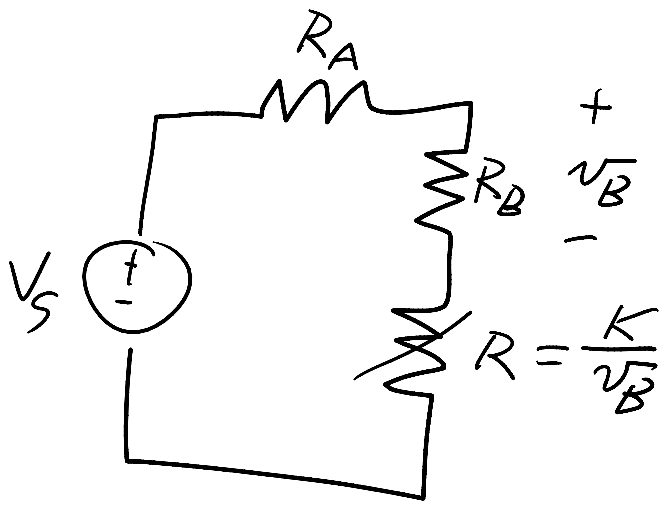
$$i(v) = \frac{K}{v^2}$$

$$V_s - v - v_o = 0$$

$$v = iR = \frac{KR}{v^2} \rightarrow v = \sqrt[3]{KR}$$

$$\boxed{v_o = V_s - \sqrt[3]{KR}}$$

Exercise 2:

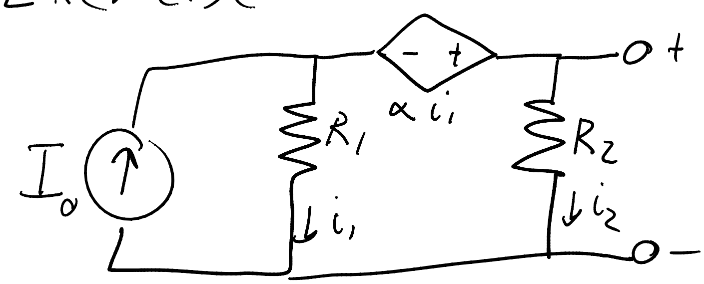


$$i = \frac{V_S}{R_A + R_B + R}$$

$$v_B = i R_B = \frac{V_S R_B}{R_A + R_B + \frac{K}{v_B}}$$

$$\boxed{v_B = \frac{V_S R_B - K}{R_A + R_B}}$$

Exercise 3:



Open Circuit:

$$v_{oc} - \alpha i_1 - R_1 i_1 = 0$$

$$v_{oc} = i_2 R_2 = (I_0 - i_1) R_2$$

$$I_0 R_2 - R_2 i_1 - \alpha i_1 - R_1 i_1 = 0 \rightarrow i_1 = \frac{I_0 R_2}{R_1 + R_2 + \alpha}$$

$$\boxed{V_{th} = \frac{(\alpha + R_1) R_2 I_0}{R_1 + R_2 + \alpha}}$$

R_{th}: Remove INDEPENDENT sources, then apply test source.

$$R_{th} = \frac{v_{test}}{i_{test}}$$

$$i_1 R_1 + \alpha i_1 - v_{test} = 0 \rightarrow i_1 = \frac{v_{test}}{R_1 + \alpha}$$

$$v_{test} = i_2 R_2 \rightarrow i_2 = \frac{v_{test}}{R_2}$$

$$i_{test} = i_1 + i_2 = \left(\frac{1}{R_1 + \alpha} + \frac{1}{R_2}\right) v_{test}$$

$$\boxed{R_{th} = \frac{R_2 (R_1 + \alpha)}{R_1 + R_2 + \alpha}}$$

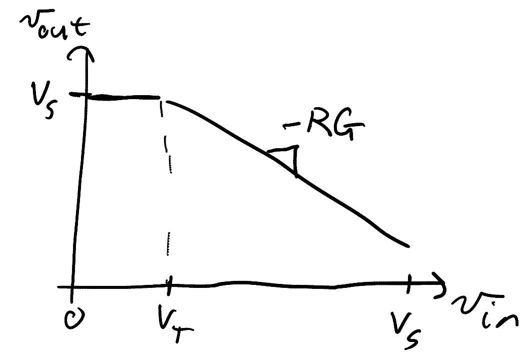
Problem 1:

a) $$i_d = \begin{cases} 0 & v_a < V_T \\ G(v_a - V_T) & v_a > V_T \end{cases}$$

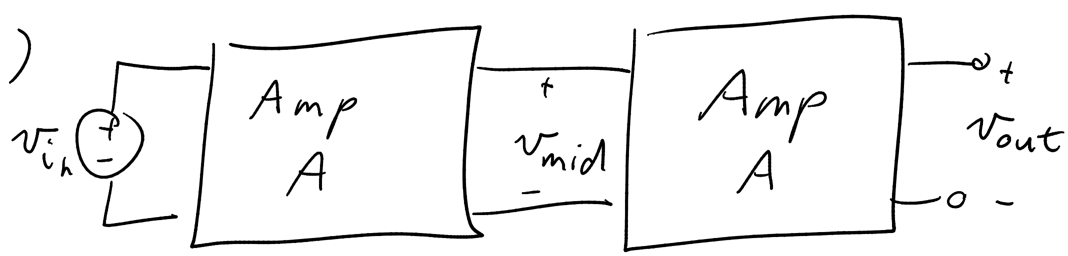
$$v_{out} = V_s - i_d R$$

$$v_{out} = \begin{cases} V_s & v_{in} < V_T \\ V_s - RG(v_{in} - V_T) & v_{in} > V_T \end{cases}$$

b)



c)



$$v_{mid} = \begin{cases} V_s & v_{in} < V_T \\ V_s - RG(v_{in} - V_T) & v_{in} > V_T \end{cases}$$

$$v_{out} = \begin{cases} V_s & v_{mid} < V_T \\ V_s - RG(v_{mid} - V_T) & v_{in} > V_T \end{cases}$$

Problem 1 (cont):

c) $v_{mid} < V_T$: $\quad v_{out} = V_s$

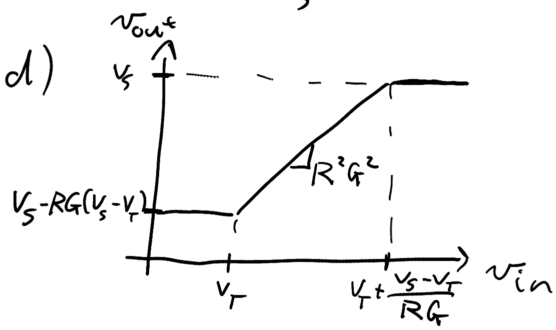
$$V_T = V_s - RG(v_{in} - V_T)$$

$$v_{in} = V_T + \frac{V_s - V_T}{RG}$$

$$v_{out} = \begin{cases} V_s - RG(V_s - V_T) & v_{in} < V_T \\ V_s - RG\left(V_s - RG(v_{in} - V_T) - V_T\right) & V_T < v_{in} < V_T + \frac{V_s - V_T}{RG} \\ 0 & V_T + \frac{V_s - V_T}{RG} < v_{in} \end{cases}$$

$$v_{out} = \begin{cases} V_s - RG(V_s - V_T) & v_{in} < V_T \\ (1 - RG)(V_s + RG\,V_T) + (RG)^2 v_{in} & V_T < v_{in} < V_T + \frac{V_s - V_T}{RG} \\ V_s & V_T + \frac{V_s - V_T}{RG} < v_{in} \end{cases}$$

d)



e) $P_{diss} = v_{out} \cdot i_d$

i_d is always in same direction, so $P_{diss} > 0$ when $v_{out} > 0$ (sinking power) and $P_{diss} < 0$ when $v_{out} < 0$ (sourcing power

Problem 1 (cont):

f)

single amplifier:

$v_{out} \geq 0$

$v_{out} = 0 = V_s - RG(v_{in} - V_T) \rightarrow v_{in} = V_T + \frac{V_s}{RG}$

$$v_{out} = \begin{cases} V_s & v_{in} < V_T \\ V_s - RG(v_{in} - V_T) & V_T < v_{in} < V_T + \frac{V_s}{RG} \\ 0 & V_T + \frac{V_s}{RG} < v_{in} \end{cases}$$

for $G \rightarrow \infty$: $V_T = V_T + \frac{V_s}{RG}$

$$v_{out} = \begin{cases} V_s & v_{in} < V_T \\ 0 & v_{in} > V_T \end{cases}$$

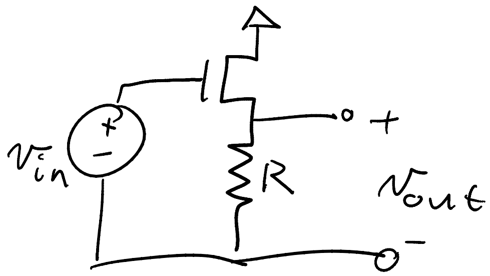
* looks like inverter!

chained amplifiers:

$$v_{out} = \begin{cases} 0 & v_{in} < V_T \\ V_s & v_{in} > V_T \end{cases}$$

* looks like logic buffer

Problem 2:



a) $v_{GS} = v_{in} - v_{out}$

$$v_{out} = i_d \cdot R = \frac{RK}{2}(v_{in} - V_T - v_{out})^2$$

$$(v_{in} - V_T)^2 - 2v_{out}\left[(v_{in} - V_T) + \frac{2}{RK}\right] + v_{out}^2 = 0$$

$$v_{out} = (v_{in} - V_T) + \frac{2}{RK} \pm \sqrt{\left[(v_{in} - V_T) + \frac{2}{RK}\right]^2 - (v_{in} - V_T)^2}$$

$$= (v_{in} - V_T) + \frac{2}{RK} - \sqrt{\frac{4}{RK}(v_{in} - V_T) + \frac{4}{RK^2}}$$

$$v_{out} = \left[\frac{\sqrt{\frac{2}{RK} + (v_{in} - V_T)} - \sqrt{\frac{2}{RK}}}{2}\right]^2$$

b) $v_{DS} = V_s - v_{out}$

$v_{DS} \geq v_{GS} - V_T$

$V_s - v_{out} \geq v_{in} - v_{out} - V_T$

$v_{in} \leq V_s + V_T$

$v_{GS} \geq V_T$

$v_{in} - v_{out} \geq V_T$

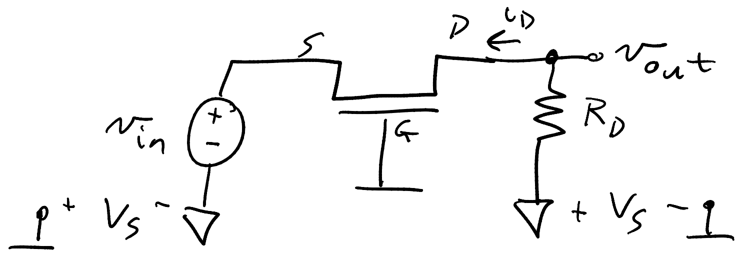
$v_{in} = V_T, \; i_d = 0 \rightarrow v_{out} = 0$

$v_{in} \geq V_T$

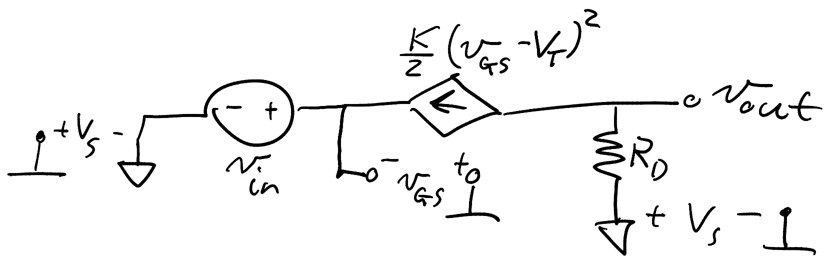
$$V_T \leq v_{in} < V_s + V_T$$

$$0 \leq v_{out} \leq V_s + \frac{2}{RK} - \sqrt{\frac{4}{RK}\left(V_s + \frac{1}{RK}\right)}$$

Problem 3:



a)



b) $v_{GS} = 0 - (-V_S + v_{in}) = V_S - v_{in}$

$i_D = \frac{K}{2}(V_S - V_T - v_{in})^2$

$v_o = V_S - R_D i_D = V_S - \frac{R_D K}{2}(V_S - V_T - v_{in})^2$

c) $v_{GS} > V_T$:

$V_S - v_{in} > V_T \rightarrow v_{in} < V_S - V_T$

$v_{DS} > v_{GS} - V_T$:

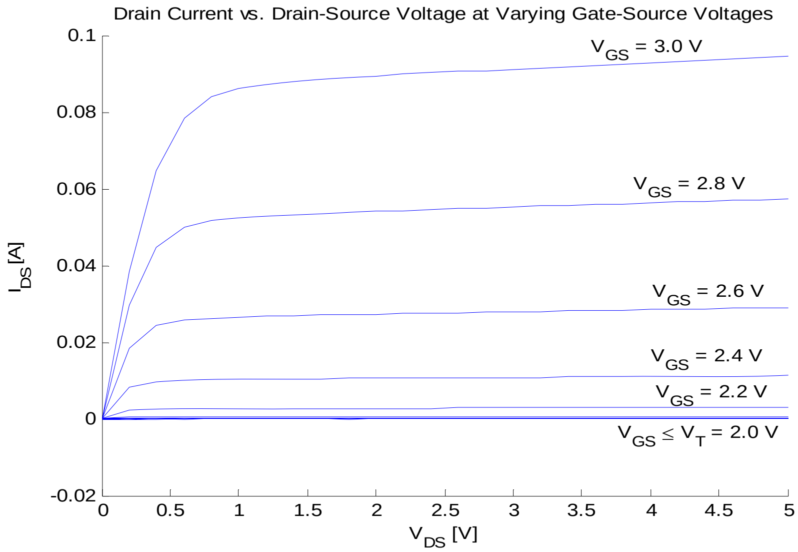
$v_{DS} = v_{out} - (-V_S + v_{in}) = V_S + v_{out} - v_{in}$

$2V_S - \frac{R_D K}{2}(V_S - V_T - v_{in})^2 - \cancel{v_{in}} > V_S - \cancel{v_{in}} - V_T$

$(V_S - V_T - v_{in})^2 < \frac{2(V_S + V_T)}{R_D K} \rightarrow v_{in} > V_S - V_T - \sqrt{\frac{2(V_S+V_T)}{R_D K}}$

Problem 4:

a)



b) $V_T \approx 1.8V$ (note when $I_{DS} \approx 0$ for $V_{DS} > 0$)

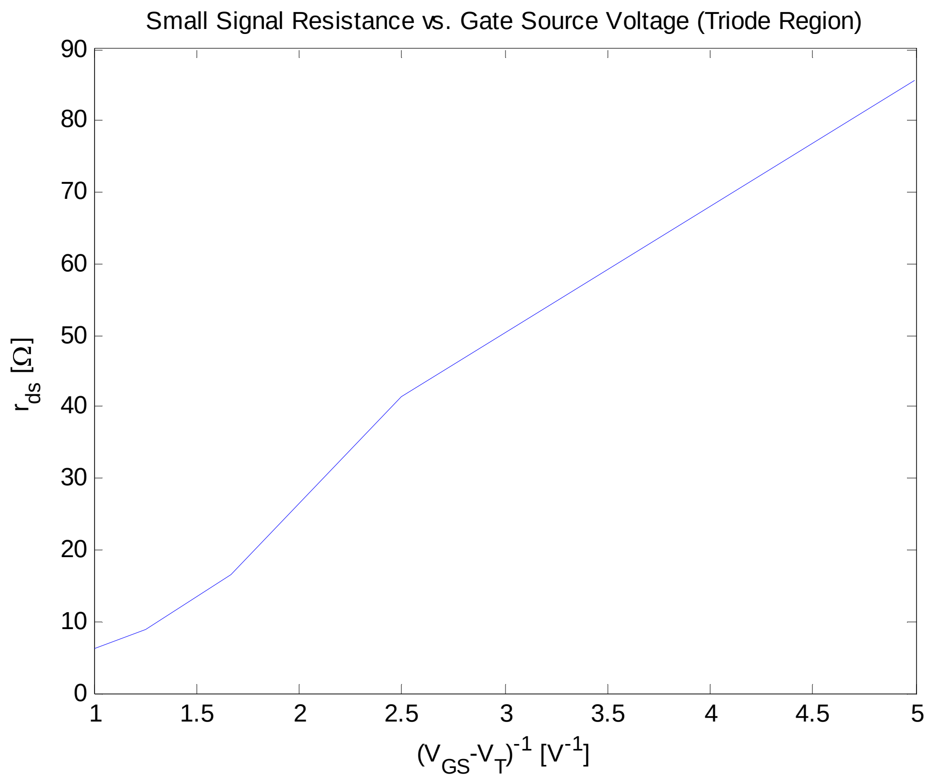
K: $I_D = \frac{K}{2}(V_{GS} - V_T)^2$

@ $V_{GS} = 2.8V$, $I_D \approx 55mA$

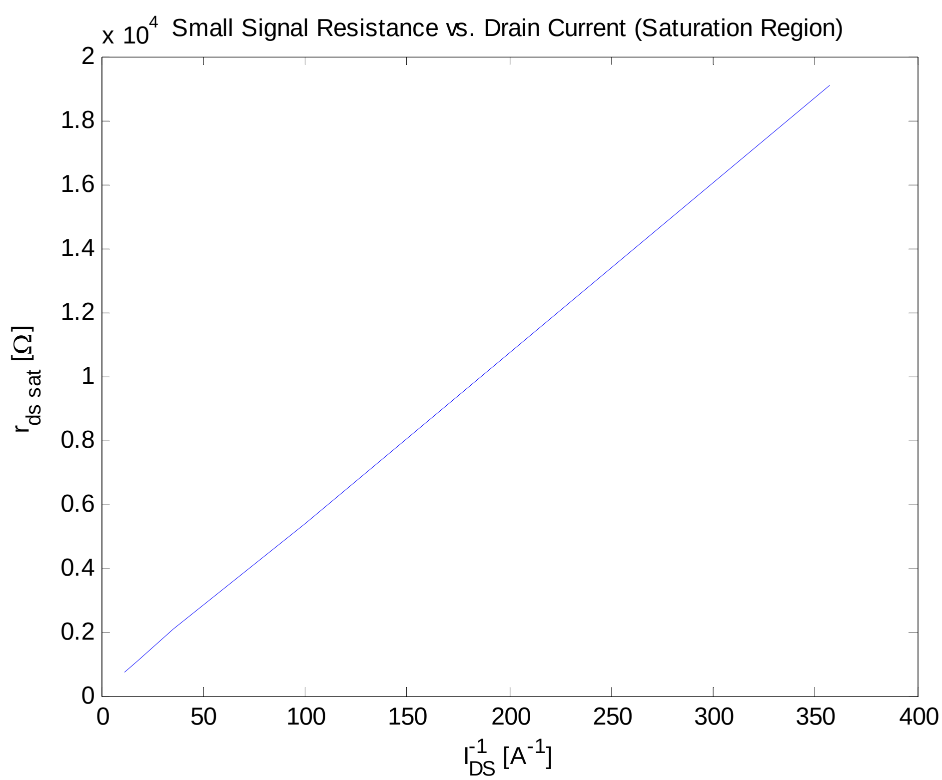
$K = 110 \, mA/V^2$

Problem 4:

c)


Small Signal Resistance vs. Gate Source Voltage (Triode Region)
90
80
70
60
50
40
30
20
10
0
r_ds [Ω]
1
1.5
2
2.5
3
3.5
4
4.5
5
(V_GS-V_T)^-1 [V^-1]

d)


x 10^4
Small Signal Resistance vs. Drain Current (Saturation Region)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
r_ds sat [Ω]
0
50
100
150
200
250
300
350
400
I_DS^-1 [A^-1]