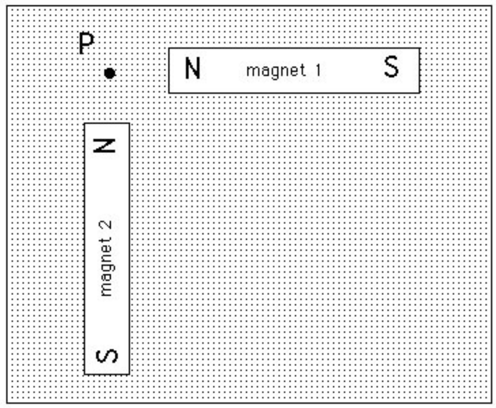
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Experiment 3 *Solutions*: B Fields of a Bar Magnet and Helmholtz Coil

Pre-Lab Questions & Solutions

1. Superposition

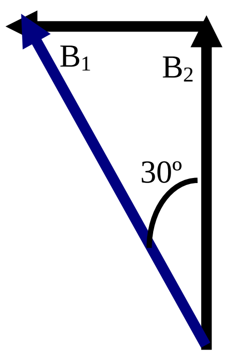


Consider two bar magnets placed at right angles to each other, as pictured at left.

(a) If a small compass is placed at point P, what direction does the painted end of the compass needle point?

It points away from each magnetic North, which means toward the upper left corner (45 degrees if they are the same magnitude).

(b) If the compass needle instead pointed 15 degrees clockwise of where you predicted in (a), what could you qualitatively conclude about the relative strengths of the two magnets?



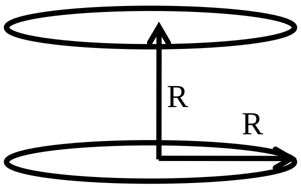
In order for it to point 15 degrees clockwise the second magnet must be stronger than the first. Since the total field is just a vector sum of the two we can see how much stronger. $\tan 30^\circ = \frac{B_1}{B_2} = \frac{1}{\sqrt{3}} \Rightarrow \boxed{B_2 = \sqrt{3} B_1}$

2. Helmholtz Coil

In class you calculated the magnetic field along the axis of a coil to be given by:

$$B_{axial} = \frac{N \mu_0 I R^2}{2} \frac{1}{(z^2 + R^2)^{3/2}}$$

where z is measured from the center of the coil.



As pictured at left, a Helmholtz coil is created by placing two such coils (each of radius *R* and *N* turns) a distance *R* apart.

(a) If the current in the two coils is parallel (Helmholtz configuration), what is the axial field strength at the center of the apparatus (midway between the two coils)? How does this compare to the field strength at the center of the single coil configuration (e.g. what is the ratio)?

We just use superposition to determine the total field. We are the same distance $z = R/2$ from each coil and since the currents are parallel they both create a field in the same direction (for example, if both currents are counter-clockwise they both create an upward magnetic field at the midpoint). The magnitude then is just twice what one would give:

$$B = 2\times\frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{\left(\left(R/2\right)^2+R^2\right)^{3/2}} = \frac{N\,\mu_0\,I}{R}\frac{1}{\left(\left(1/2\right)^2+1\right)^{3/2}} = \frac{8N\,\mu_0\,I}{5^{3/2}\,R}$$

Comparing this to the field strength at the center of a single coil:

$$B_{\text{sgl coil}} = \frac{N\,\mu_0\,I}{2R}$$

We find that the field of a Helmholtz coil is slightly larger:

$$\frac{B_{\text{Helmholtz}}}{B_{\text{Sgl Coil}}} = \left(\frac{8N\,\mu_0\,I}{5^{3/2}\,R}\right)\Big/\left(\frac{N\,\mu_0\,I}{2R}\right) = \frac{16}{5^{3/2}} \approx 1.4$$

(b) In the anti-Helmholtz configuration the current in the two coils is anti-parallel. What is field strength at the center of the apparatus in this situation?

In this case the fields from the two coils are in opposite directions so they cancel each other out. That is, $B = 0$.

(c) Our coils have a radius $R = 7$ cm and $N = 168$ turns, and we will typically run with $I = 0.3$ A in them. What, approximately, are the largest fields we should expect in the three configurations (single coil, Helmholtz & anti-Helmholtz)?

For a single coil the maximum is at the center of the coil, for a Helmholtz at the center:

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$$B_{\text{sgl coil}}^{\max} = \frac{(168)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.3\ \text{A}\right)}{2\left(7\times10^{-2}\ \text{m}\right)} = 4.5\times10^{-4}\ \text{T} = 4.5\ \text{Gauss;}$$

$$B_{\text{Helmholtz}}^{\max} = \frac{16}{5^{3/2}}\cdot 4.5\ \text{Gauss} = 6.5\ \text{Gauss}$$

For an Anti-Helmholtz coil the field between the two is given by:

$$B_{\text{Anti-Helmholtz}} = \frac{N\,\mu_0\,I\,R^2}{2}\left[\frac{1}{(z^2+R^2)^{3/2}} - \frac{1}{\left((R-z)^2+R^2\right)^{3/2}}\right]$$

where I have taken the bottom coil to be at $z = 0$ and to have the counterclockwise current, making a positive field. I then had to subtract the field from the top coil, which is now a distance (R-z) away from our observation point at z.

To maximize we could take the derivative and set it equal to zero, but this is a pretty nasty and since we are only asked to approximate, realizing that the max is at about at the centers of each of the coils makes our lives easier:

$$B^{\max}_{\text{Anti-Helmholtz}} \approx \frac{N\,\mu_0\,I}{2R}\left[\frac{1}{(u^2+1)^{3/2}} - \frac{1}{\left((1-u)^2+1\right)^{3/2}}\right]$$

$$\approx \frac{(168)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(0.3\ \text{A})}{2\left(7\times10^{-2}\ \text{m}\right)}\left[1-\frac{1}{2^{3/2}}\right] = 2.9\ \text{Gauss}$$

If you were to do the calculation (numerically is the only non-painful way) you find the max is closer to 3.2 Gauss and the maxima are just outside the two coils.